



UNIVERSITÀ DI TRENTO

Formal Method Mod. 2 (Model Checking)

Laboratory 11

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Timed systems

Real time systems

- ▶ Correctness depends not only on the logical result but also on the time required to compute it.
- ▶ Common in safety-critical domains like: defense, transportation, health-care, space and avionics.

Timed Transition System (TTS)

transitions are either discrete

or time-elapses,

all clocks increase of the same amount in time-elapses.

Model checking for TTS is **undecidable**.

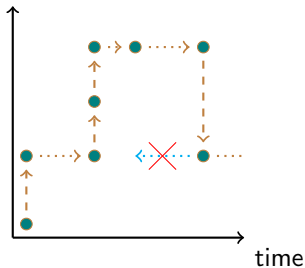
Timed Automata (TA)

decidable restriction of TTS,

finite time abstraction:

clocks compared only to constants.

discrete

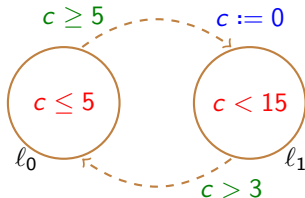


Timed systems: representation

Timed Automata (TA)

Explicit graph representation of discrete states (nodes) and transitions (edges).

Symbolic representation of temporal aspects via (convex) constraints (**location invariants**, **transition guards** and **resets**).



Symbolic TTS

Logical formulae represent sets of states: $p := \{s \mid s \models p\}$.

Transition system symbolically represented by formula $\varphi(X, X')$.

There is a discrete transition from s_0 to s_1 iff $s_0(X), s_1(X') \models \varphi(X, X')$.

$$I = l_0 \rightarrow c \leq 5 \quad \wedge$$

$$I = l_1 \rightarrow c < 15 \quad \wedge$$

$$(I = l_1 \wedge I' = l_0) \rightarrow c > 3 \quad \wedge$$

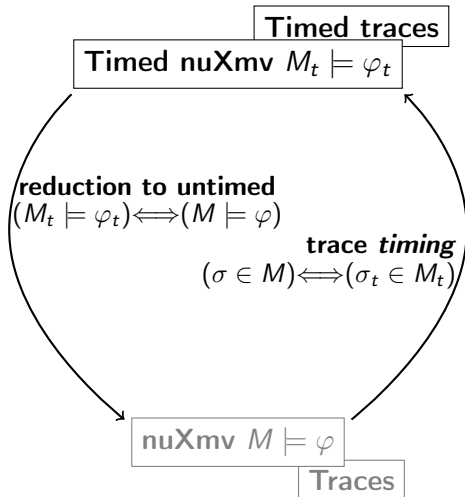
$$(I = l_0 \wedge I' = l_1) \rightarrow (c \geq 5 \wedge c' = 0)$$

Outline

1. Timed nuXmv
2. Timed and infinite traces
3. Exercises



nuXmv for timed system: architecture



1. Timed nuXmv

Timed nuXmv: input language [1/4]

Overview

- ▶ Must start with `@TIME_DOMAIN continuous;`
- ▶ Symbolic description of infinite transition system using: `INIT`, `INVAR` and `TRANS` to specify initial, invariant and transition conditions.
- ▶ Model described as a synchronous composition of `MODULE` instances.
- ▶ Clock variables,
 - ▶ `time`: built-in clock variable,
 - ▶ convex invariants over clocks,
 - ▶ `URGENT`: forbid time elapse.

Timed nuXmv: input language [2/4]

Timed nuXmv adds

- ▶ `clock` variable type, all `clocks` increase of the same amount during timed transitions;
- ▶ `time`: built-in `clock`, can be used only in comparisons with constants;
- ▶ `noncontinuous` type modifier: symbol can change its assignment during timed transitions;
- ▶ `URGENT`: freeze time: when one of the `URGENT` conditions is satisfied only discrete transitions are allowed;
- ▶ `MTL0,∞` specifications, by “extending” LTL;

Timed nuXmv: input language [3/4]

Timed nuXmv updates

- ▶ TRANS constrain the discrete behaviour only,
- ▶ INVAR: clocks allowed in invariants with shape:
`no_clock_expr -> convex_clock_expr;`
- ▶ LTL operators: X , Y , U , S ,
- ▶ Bounded LTL operators.



Timed nuXmv: input language [4/4]

Specification

- ▶ Different operators to refer to the *discrete* next and *timed* next: X , X_{\sim} ; symmetrically for the past: Y , Y_{\sim} .
- ▶ Time interval semantic to handle open intervals: a predicate p might hold in an interval $(a, b]$ for $a, b \in \mathbb{R}$.
- ▶ Operators to retrieve value of expression the next/last time an expression will hold/held: time_until , time_since , $@F_{\sim}$ and $@O_{\sim}$.

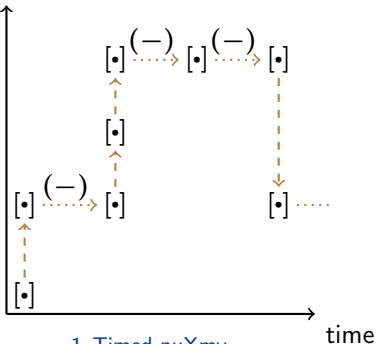


Timed nuXmv: untiming

Timed to untimed model

- ▶ clock symbols and time: variables of type real.
- ▶ δ : continuous positive variable, prescribes the amount of time elapse for every transition.
- ▶ ι : prescribes the alternation of singular $[\cdot]$ and open $(-)$ time intervals.

discrete



1. Timed nuXmv

Timed nuXmv: untiming

Properties rewriting

MTL *fragment*

$$F_{[0,5]} p$$

↓ rewrite

LTL *timed*

$$((\neg p U p) \wedge \text{time_until}(p) \leq 5) \vee \\ ((\neg p U \tilde{X} p) \wedge \text{time_until}(p) < 5)$$

↓ untime

LTL *untimed*

$$((\neg p U p) \wedge (\text{time@}\tilde{F}p - \text{time} \leq 5)) \vee \\ ((\neg p U ((\neg \iota \wedge p) \vee X(\neg \iota \wedge p))) \wedge (\text{time@}\tilde{F}p - \text{time} < 5))$$

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1. Timed nuXmv
2. Timed and infinite traces
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Timed and infinite traces

From untimed model execution to timed trace.

Issue

nuXmv traces must have shape: $\alpha\beta^\omega$,
 α and β sequences of states.

Complete for finite state systems.

TTS: time monotonically increasing,
 infinite state system, **undecidable**.

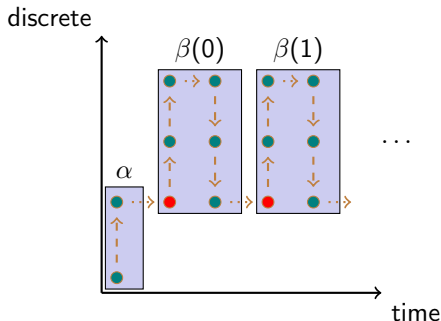
Identify traces expressible as: $\alpha\beta(i)^\omega$.

Same problem can be found in
 infinite state transition systems.

Solution

Value assigned to variables at state
 s is function of the previous
 configuration assignments.

e.g. $next(time) := time + \delta$



Timed and infinite traces: operations

Three main operations on traces: **simulation**, **execution** and **completion**.

Simulation

Build a possible execution of the model. The trace can be built automatically by the system or the user can choose each state from the list of possible ones.

Exploit SMT-solver to perform a discrete transition or time-elapse to obtain next configuration.



Timed and infinite traces: operations

Execution

Check if a trace belongs to the language of the model.

Exploit SMT-solver to prove that **for all** possible iterations all prescribed transition can be performed.

Completion

A partial trace is completed so that it belongs to the model language.

Sound and complete technique requires to check if there **exists** a possible completion so that the completed trace belongs to the model language: quantifier alternation ($\exists\forall$).

Adopt sound but incomplete approach.

How to run: model [1/3]

- ▶ `./nuXmv -time -int`: start nuXmv interactively and enable commands for timed models.
- ▶ `go_time`: process the model.
- ▶ `write_untimed_model`: dump SMV model corresponding to the input timed system.



How to run: verify [2/3]

- ▶ `timed_check_invar`: check invariants.
- ▶ `timed_check_ltlspec`: check LTL.

Mostly the same command line options of the corresponding commands for untimed models.



How to run: simulation and traces [3/3]

- ▶ `timed_pick_state`: pick initial state.
- ▶ `timed_simulate`: simulate the model starting from a given state.
- ▶ `execute_traces`: re-execute stored traces.
- ▶ `execute_partial_traces`: try to complete stored traces.



Semantics of temporal operators

Formally nuXmv uses a super-dense weakly-monotonic time model $T \subset \mathbb{N} \times \mathbb{R}_0^+$.

A time point is a pair $\langle i, r \rangle$ where $i \in \mathbb{N}$ “counts the discrete steps” and $r \in \mathbb{R}_0^+$ is the time.

We say that $\langle i, r \rangle < \langle i', r' \rangle$ iff $i < i'$ or $i = i'$ and $r < r'$.



Semantics of temporal operators

$\sigma, t \models \phi$ is defined recursively on the structure of ϕ :
usual definition for predicates, conjunction and negation.

$\sigma, t \models \phi_1 U \phi_2$ iff there exists $t' \geq t, \sigma, t' \models \phi_2$ and
for all $t'', t \leq t'' < t', \sigma, t'' \models \phi_1$

$\sigma, t \models \phi_1 S \phi_2$ iff there exists $t' \leq t, \sigma, t' \models \phi_2$ and
for all $t'', t' < t'' \leq t, \sigma, t'' \models \phi_1$

$\sigma, t \models X\phi$ iff there exists $t' > t, \sigma, t' \models \phi$ and
there exists no $t'', t < t'' < t'$

$\sigma, t \models \tilde{X}\phi$ iff for all $t' > t$, there exists $t'', t < t'' < t', \sigma, t'' \models \phi$

$\sigma, t \models Y\phi$ iff $t > 0$ and there exists $t' < t, \sigma, t' \models \phi$ and
there exists no $t'', t' < t'' < t$

$\sigma, t \models \tilde{Y}\phi$ iff $t > 0$ and for all $t' < t$,
there exists $t'', t' < t'' < t, \sigma, t'' \models \phi$

2. Timed and infinite traces

LTL- MTL properties [1/2]

Let k , k_1 and k_2 be some constant real values such that $0 \leq k \leq k_1 < k_2$ and let b a boolean symbol.

The following properties true or false?

► \tilde{Y}_T



LTL- MTL properties [1/2]

Let k , k_1 and k_2 be some constant real values such that $0 \leq k \leq k_1 < k_2$ and let b a boolean symbol.

The following properties true or false?

- ▶ \tilde{Y}_T : false in the initial state.
- ▶ $(\neg Xb) \rightarrow (X\neg b)$



LTL- MTL properties [1/2]

Let k , k_1 and k_2 be some constant real values such that $0 \leq k \leq k_1 < k_2$ and let b a boolean symbol.

The following properties true or false?

- ▶ $\tilde{Y} \top$: false in the initial state.
- ▶ $(\neg Xb) \rightarrow (X\neg b)$: false, the first one holds in every time elapse, the second one holds only in discrete steps where $\neg b$ holds in the next state.
- ▶ $(\neg \tilde{X} b) \rightarrow (\tilde{X} \neg b)$

LTL- MTL properties [1/2]

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- ▶ $(\neg \tilde{X} b) \rightarrow (\tilde{X} \neg b)$: false, as above but for time elapses.
- ▶ $(X\neg b) \rightarrow (\neg Xb)$

LTL- MTL properties [1/2]

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- ▶ $(\neg \tilde{X} b) \rightarrow (\tilde{X} \neg b)$: false, as above but for time elapses.
- ▶ $(X\neg b) \rightarrow (\neg Xb)$: true, the first one holds iff there is a discrete step and $\neg b$ holds in the next state, hence Xb is false.
- ▶ $(\tilde{X} \neg b) \rightarrow (\neg \tilde{X} b)$

LTL- MTL properties [1/2]

Let k , k_1 and k_2 be some constant real values such that $0 \leq k \leq k_1 < k_2$ and let b a boolean symbol.

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- ▶ $(\tilde{X}\neg b) \rightarrow (\neg \tilde{X}b)$: true, as above but for time elapses.
- ▶ $(G\tilde{X}\top) \rightarrow ((Gb) \vee (G\neg b))$

LTL- MTL properties [1/2]

Let k , k_1 and k_2 be some constant real values such that $0 \leq k \leq k_1 < k_2$ and let b a boolean symbol.

The following properties true or false?

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- ▶ $(\tilde{X}\neg b) \rightarrow (\neg \tilde{X}b)$: true, as above but for time elapses.
- ▶ $(G\tilde{X}\top) \rightarrow ((Gb) \vee (G\neg b))$: true, the first part implies that we never perform a discrete transition and the truth value of b can only change in discrete transitions.



LTL- MTL properties [2/2]

See files in examples.



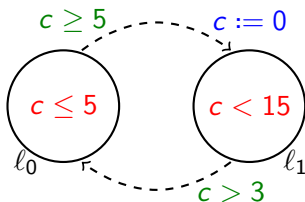
Outline

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Simple timed automaton

Write the SMV model corresponding to the timed automaton in the figure.



Properties

- ▶ from location ℓ_0 we always reach ℓ_1 within 5 time units;
- ▶ if we are in ℓ_1 then for the next 3 time units we remain in ℓ_1 ;
- ▶ if just arrived in ℓ_1 then for the next 3 time units we remain in ℓ_1 .

Timed thermostat

- ▶ a thermostat has 2 states: *on* and *off*;
 - ▶ if the temperature is below 18 degrees the thermostat switches *on*.
 - ▶ if the temperature is above 18 degrees the thermostat switches *off*.
- ▶ Every time the thermostat misure the temperature in the room, the temperature increases (if *on*) or decreases (if *off*) by dt (with respect to the previous check);
- ▶ the thermostat measures the temperature at most (\leq) every max_dt time units.
- ▶ the temperature initially is in $[18 - max_dt; 18 + max_dt]$.

Verify that the temperature is always in $[18 - 2max_dt; 18 + 2max_dt]$

Homework

Try to encode the timed automata shown in the theoretical slide about timed automata.

