



# UNIVERSITÀ DI TRENTO

## Formal Method Mod. 2 (Model Checking)

Laboratory 10

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# Outline

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1. Bounded Model Checking
2. K-Invariant
3. Exercises



# Bounded Model Checking

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## Idea

- ▶ look for a **counter-example** path of increasing length  $k$ 
  - ▶ **bug oriented**: *is there a bad behaviour?*

# Bounded Model Checking

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  - ▶ **bug oriented:** *is there a bad behaviour?*
- ▶ for each  $k$ : build a Boolean formula that is satisfiable iff there is a counter-example of length  $k$   
(can be expressed using  $k \cdot |s|$  variables)



# Bounded Model Checking

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## Idea

- ▶ look for a **counter-example** path of increasing length  $k$ 
  - ▶ **bug oriented:** *is there a bad behaviour?*
- ▶ for each  $k$ : build a Boolean formula that is satisfiable iff there is a counter-example of length  $k$   
(can be expressed using  $k \cdot |s|$  variables)
- ▶ use of a *SAT procedure* to check the satisfiability of the Boolean formula
  - ▶ can manage complex formulas on several 100K variables
  - ▶ returns satisfying assignment (i.e. a counter-example)

# Commands for Bounded Model Checking

## NuSMV/ nuXmv

- ▶ `go_bmc`: initializes the system for the BMC verification.
- ▶ `bmc_pick_state`, `bmc_simulate [-k]`: simulate the system
- ▶ `check_ltlspec_bmc` checks LTL specifications
- ▶ `check_invar_bmc` checks INVAR specifications

## nuXmv only

- ▶ `go_msat`: initializes the system so as to use the **MathSAT 5** *SMT Solver* as back-end
- ▶ `msat_pick_state`, `msat_simulate [-k]`: simulate the system
- ▶ `msat_check_ltlspec_bmc`: checks LTL specifications
- ▶ `msat_check_invar_bmc`: checks INVAR specifications

# Example: BMC simulation

## modulo 8 counter

```

MODULE main
VAR
  b0   : boolean;
  b1   : boolean;
  b2   : boolean;
ASSIGN
  init(b0) := FALSE;
  init(b1) := FALSE;
  init(b2) := FALSE;
  next(b0) := !b0;
  next(b1) := (!b0 & b1)
  | (b0 & !b1);
  next(b2) := ((b0 & b1) & !b2)
  | (!(b0 & b1) & b2);
DEFINE
  out := toint(b0)
  + 2*toint(b1)
  + 4*toint(b2);

```

```

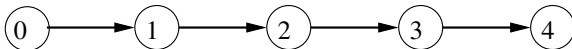
NuSMV > read_model -i counter8.smv
NuSMV > bmc_go;
NuSMV > bmc_pick_state;
NuSMV > bmc_simulate -k 3 -p
-> State: 1.1 <-
b0 = FALSE
b1 = FALSE
b2 = FALSE
out = 0
-> State: 1.2 <-
b0 = TRUE
out = 1
-> State: 1.3 <-
b0 = FALSE
b1 = TRUE
out = 2
-> State: 1.4 <-
b0 = TRUE
out = 3

```

# Checking LTL specifications

The following specification is **false**:

LTLSPEC  $G (out = 3 \rightarrow X out = 5)$



- ▶ It is an example of **safety** property: *nothing bad ever happens*.
  - the counterexample is a **finite** trace (of length 4)
  - **important**: there are no counterexamples of length up to 3



# Checking LTL specifications: output

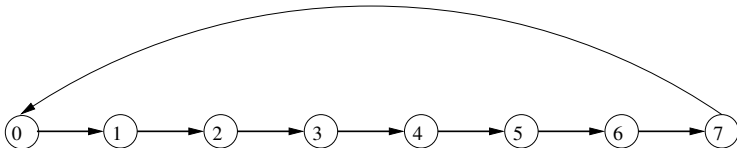
```
NuSMV > check_ltlspec_bmc -p "G (out = 3 -> X out = 5)"
-- no counterexample found with bound 0 for specification ...
-- no counterexample found with bound 1 for specification ...
-- no counterexample found with bound 2 for specification ...
-- no counterexample found with bound 3 for specification ...
-- specification G (out = 3 -> X out = 5) is false
-- as demonstrated by the following execution sequence
-> State 1.1 <-
...
out = 0
-> State 1.2 <-
...
out = 1
-> State 1.3 <-
...
out = 2
-> State 1.4 <-
...
out = 3
-> State 1.5 <-
...
out = 4
```

# Checking LTL specifications

The following specification is **false**:

LTLSPEC ! G ( F (out = 2) );

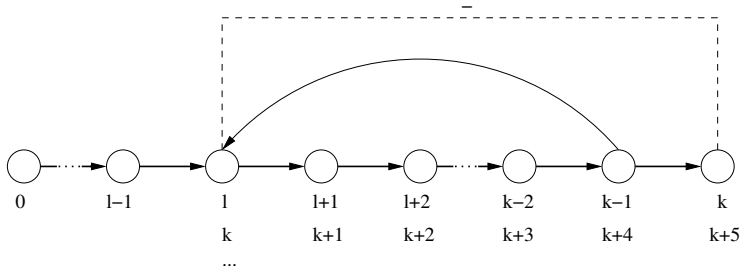
LTLSPEC F ( G ! (out = 2) );



- ▶ It is an example of **liveness** property: *something desirable will eventually happen*
  - the counterexample is an **infinite** trace (loop of length 8)
  - since the state where out = 2 is entered infinitely often, the property is **false**

# Bounded Model Checking: counter-examples

## Looping counterexample



**prefix** : assignments from 0 to  $l - 1$ ,

**loop** : infinitely repeat assignments  $l$  to  $k - 1$ ,

**loop-back** :  $k^{th}$  assignment, always identical to  $l^{th}$  assignment.

# Length and loopback condition

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- ▶ `check_ltlspec_bmc` looks for counterexamples of length up to  $k$ .
- ▶ `check_ltlspec_bmc_onepb` looks for counterexamples of length  $k$ .
- ▶ To set the loopback conditions use: `-l bmc_loopback`.
  - ▶ `bmc_loopback >= 0` : loop to a precise time point
  - ▶ `bmc_loopback < 0` : loop length
  - ▶ `bmc_loopback = 'X'`: no loopback
  - ▶ `bmc_loopback = '*'`: all possible loopbacks
- ▶ To set the bounded length use: `-k bmc_length`.
- ▶ Default values: `bmc_loopback = '*'`, `bmc_length = 10`
- ▶ Default values can be changed using:
  - ▶ `set bmc_length k` sets the length to  $k$
  - ▶ `set bmc_loopback l` sets the loopback to  $l$

# Checking LTL specifications

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Let us consider again the specification ! G ( F (out = 2))

```
NuSMV > check_ltlspec_bmc_onepb -k 9 -l 0 -p "! G ( F (out = 2))"
-- no counterexample found with bound 9
   and loop at 0 for specification ...
```



# Checking LTL specifications

Let us consider again the specification ! G ( F (out = 2))

```
NuSMV > check_ltlspec_bmc_onepb -k 9 -l 0 -p "! G ( F (out = 2))"
-- no counterexample found with bound 9
   and loop at 0 for specification ...
```

```
NuSMV > check_ltlspec_bmc_onepb -k 8 -l 1 -p "! G ( F (out = 2))"
-- no counterexample found with bound 8
   and loop at 1 for specification ...
```

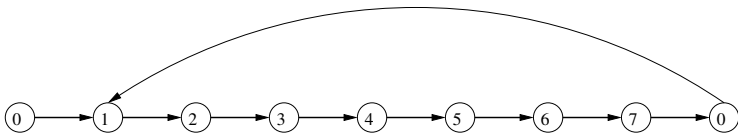
# Checking LTL specifications

Let us consider again the specification ! G ( F (out = 2))

```
NuSMV > check_ltlspec_bmc_onepb -k 9 -l 0 -p "! G ( F (out = 2))"
-- no counterexample found with bound 9
   and loop at 0 for specification ...
```

```
NuSMV > check_ltlspec_bmc_onepb -k 8 -l 1 -p "! G ( F (out = 2))"
-- no counterexample found with bound 8
   and loop at 1 for specification ...
```

```
NuSMV > check_ltlspec_bmc_onepb -k 9 -l 1 -p "! G ( F (out = 2))"
-- specification ! G F out = 2   is false
-- as demonstrated by the following execution sequence
...
```



## 1. Bounded Model Checking

# Checking LTL specifications

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Let us consider again the specification  $\neg G (F (out = 2))$

```
NuSMV > check_ltlspec_bmc_onepb -k 9 -l X -p "! G ( F (out =2))"  
-- no counterexample found with bound 9 and no loop for specification ...
```



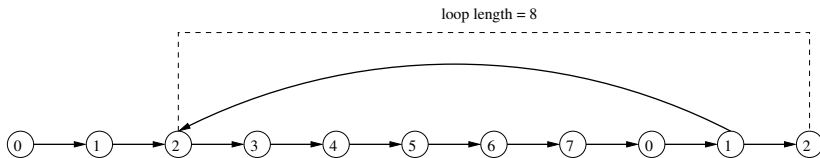


# Checking LTL specifications

Let us consider again the specification  $\neg G (F (out = 2))$

```
NuSMV > check_ltlspec_bmc_onepb -k 9 -l X -p "! G ( F (out =2))"
-- no counterexample found with bound 9 and no loop for specification ...
```

```
NuSMV > check_ltlspec_bmc_onepb -k 10 -l -8 -p "! G ( F (out =2))"
-- specification ! G F out = 2 is false
-- as demonstrated by the following execution sequence
...
```



# Outline

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1. Bounded Model Checking
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# Checking invariants

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- ▶ Bounded model checking can be used also for checking invariants
- ▶ Invariants are checked via the `check_invar_bmc` command
- ▶ Invariants are checked via an **inductive reasoning**, i.e. nuXmv tries to prove that:
  - ▶ the property **holds in** every **initial state**
  - ▶ the property **holds in** every state that is **reachable from another state in which the property holds**



# Checking invariants

Consider the following example:

```
MODULE main
```

```
VAR
```

```
out : 0..15;
```

```
ASSIGN
```

```
init(out) := 0;
```

```
TRANS
```

```
case
```

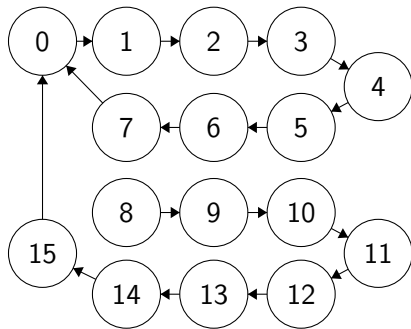
```
out = 7 : next(out) = 0;
```

```
TRUE   : next(out) = ((out + 1) mod 16);
```

```
esac
```

```
INVARSPEC out in 0..10
```

```
INVARSPEC out in 0..7
```



# Checking invariants

```
NuSMV > check_invar_bmc
-- cannot prove the invariant out in (0 .. 10) : the induction fails
-- as demonstrated by the following execution sequence
-> State 1.1 <-
out = 10
-> State 1.2 <-
out = 11
-- invariant out in (0 .. 7)   is true
```

- ▶ The invariant `out in 0..10` is **true**, but the the induction **fails** because a state in which `out=11` can be reached from a state in which `out=10`
- ▶ **Thus:** if an invariant cannot be proved by inductive reasoning, it does not necessarily mean that the formula is false
- ▶ The stronger invariant `out in 0..7` is proved true by BMC, therefore also the invariant `out in 0..10` is true

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# Exercise: Cleaning Robot [1/5]

## Exercise

Model a rechargeable cleaning **robot** which task is to move around a  $10 \times 10$  room and clean it.

The robot state is so composed:

- ▶ variables “x” and “y”, ranging from 0 to 9, keep track of the robot’s position;
- ▶ variable “state”, with values in MOVE, CHECK, CHARGE, CLEAN, OFF, keeps track of the next action taken by the robot;
- ▶ variable “budget” in  $\{ 0..100 \}$  which signals the remaining power;
- ▶ output variable “pos”, *defined* to be equal  $y \cdot 10 + x$ .

## Exercise: Cleaning Robot [2/5]

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- ▶ At the beginning, the robot is in state “CHECK” and all other *vars* are 0.
- ▶ The budget is decreased by a single unit each time the robot is in state “MOVE” or “CLEAN” (and *budget* > 0)
- ▶ The budget is restored to 100 if the robot is in “CHARGE” state.
- ▶ Otherwise, the budget doesn't change.



## Exercise: Cleaning Robot [3/5]

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The robot changes state according to this **ordered** set of rules:

- ▶ if the robot is in “pos” 0 and the budget is smaller than 100, then the next state is “CHARGE”
- ▶ if the budget is 0, then the next state is “OFF”
- ▶ if the robot is in state “CHARGE” or “MOVE”, then the next state is “CHECK”
- ▶ if the robot is in state “CHECK”, then the next state is either “CLEAN” or “MOVE”
- ▶ otherwise, the next state is “MOVE”.



# Exercise: Cleaning Robot [4/5]

---

Encode, using the **constraint-style** (**easier!**), the following constraints:

- ▶ if the state is different than “MOVE”, then the position of the robot never changes.
- ▶ if the state is equal to “MOVE”, then the robot moves by a single square in one of the cardinal directions: it increases or decreases either “x” or “y”, but not both at the same time.



# Exercise: Cleaning Robot [5/5]

---

Encode and verify the following properties:

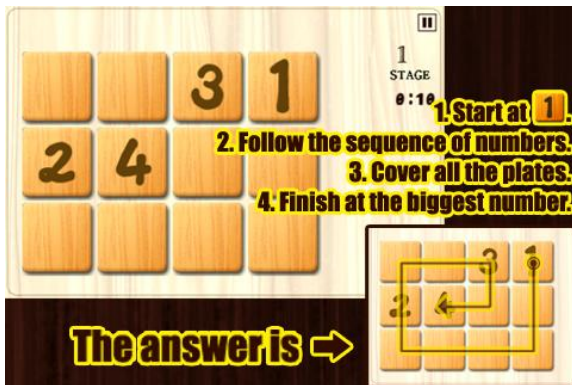
- ▶ in all possible executions, the robot changes position infinitely many times (**false**)
- ▶ it is never the case that the robot's action is either "MOVE" or "CLEAN" and the available budget is zero (**false**)
- ▶ if the robot charges infinitely often, then it changes position infinitely many times (**true**)
- ▶ the robot does not move along the diagonals (**true**)



# Exercise: Number Paranoia

## Numbers Paranoia

Encode and solve the following puzzle as a planning problem using nuXmv or NuSMV



# Homework [1/3]

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## Cannibals

Three missionaries and three cannibals want to cross a river but they have only one boat that holds two. If the cannibals ever outnumber the missionaries on either bank, the missionaries will be eaten. The boat cannot cross the river by itself with no people on board. The problem consists of finding a strategy to make them cross the river safely.

## Goals

- ▶ model the problem in SMV
- ▶ use nuXmv or NuSMV to prove that there exists a solution to the planning problem

# Homework [2/3]

## Gnome sort

Model the following code as a **module**:

```
procedure gnomeSort(arr, len):
10:   pos := 0
11:   while (pos < len):
12:     if (pos == 0 or arr[pos] >= arr[pos - 1]):
13:       pos := pos + 1
14:     else:
15:       swap(arr[pos], arr[pos - 1])
16:       pos := pos - 1
17:   return # self-loop here!
}
```

Declare, inside the **main** module, the following variables:

- ▶ arr: array initialised to { 9, 7, 5, 3, 1 }
- ▶ sorter: instance of gnomeSort(arr, 5)

# Homework [2/3]

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## Verify

- ▶ the algorithm always terminates;
- ▶ eventually in the future, the array will be sorted forever;
- ▶ eventually the array is sorted, and the algorithm is not done until the array is sorted.



# Homework [3/3]

## Leaping frogs

The puzzle involves seven rocks and six frogs. The seven rocks are laid out in a horizontal line and the six frogs are evenly divided into a green trio and a brown trio. The green frogs sit on the rocks on the right side and the brown frogs sit on the rocks on the left side. The rock in the middle is vacant. Can you move the frogs to the opposite side? Notice that you can only move one frog at a time, and they can only move forward to an empty rock or jump over one (and only one) frog, to reach an empty rock.

## Goals

- ▶ model the problem in SMV
- ▶ use nuXmv or NuSMV to prove that there exists a solution to the planning problem