RIF: 15.2 e 15.3

Legame costitutivo $\underline{T} \leftrightarrow \underline{E}$

relazione lineare

materiali isotropi (acciaio, calcestruzzo)

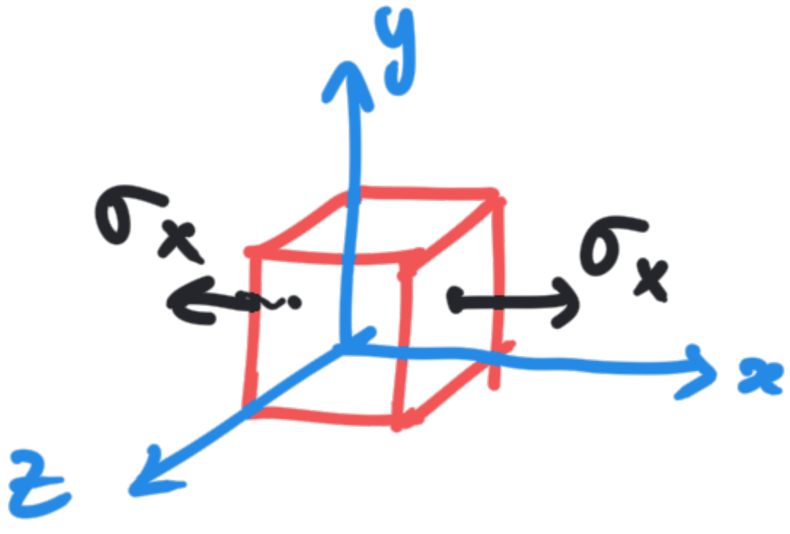
non il legno

3 costanti:

- E modulo di Young $[E] = FL^{-2}$
- ν coeff. di Poisson $[\nu] = 1$
- G modulo di scorrimento $[G] = FL^{-2}$

NB: $$G = \frac{E}{2(1+\nu)}$$

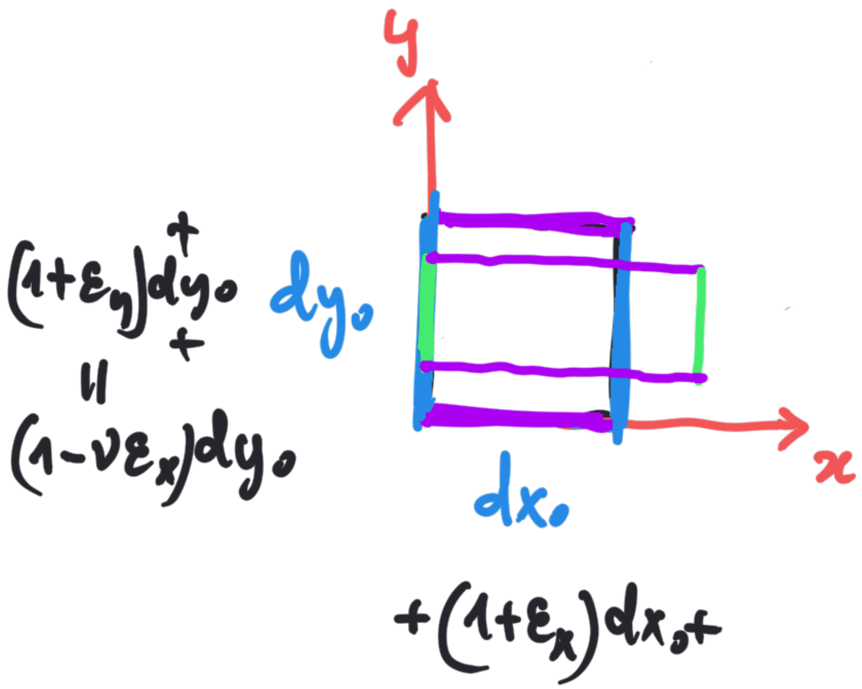
$$\underline{T} = \begin{bmatrix} \sigma_x & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$



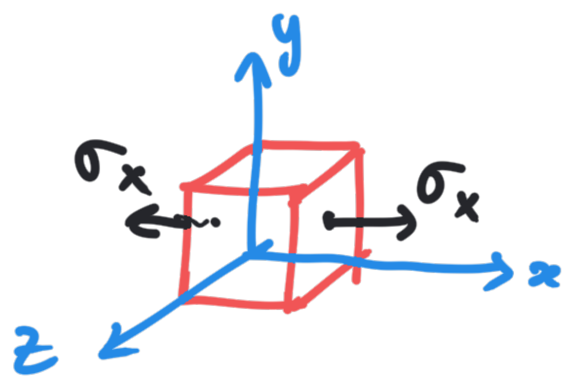
$$\underline{E} = \begin{bmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix}$$

$$\varepsilon_x = \frac{\sigma_x}{E} > 0$$

$$\varepsilon_y = \varepsilon_z = -\nu \frac{\sigma_x}{E} = -\nu \varepsilon_x < 0$$



$$\underline{T} = \begin{bmatrix} \sigma_x & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$



$$\sigma_x > 0$$

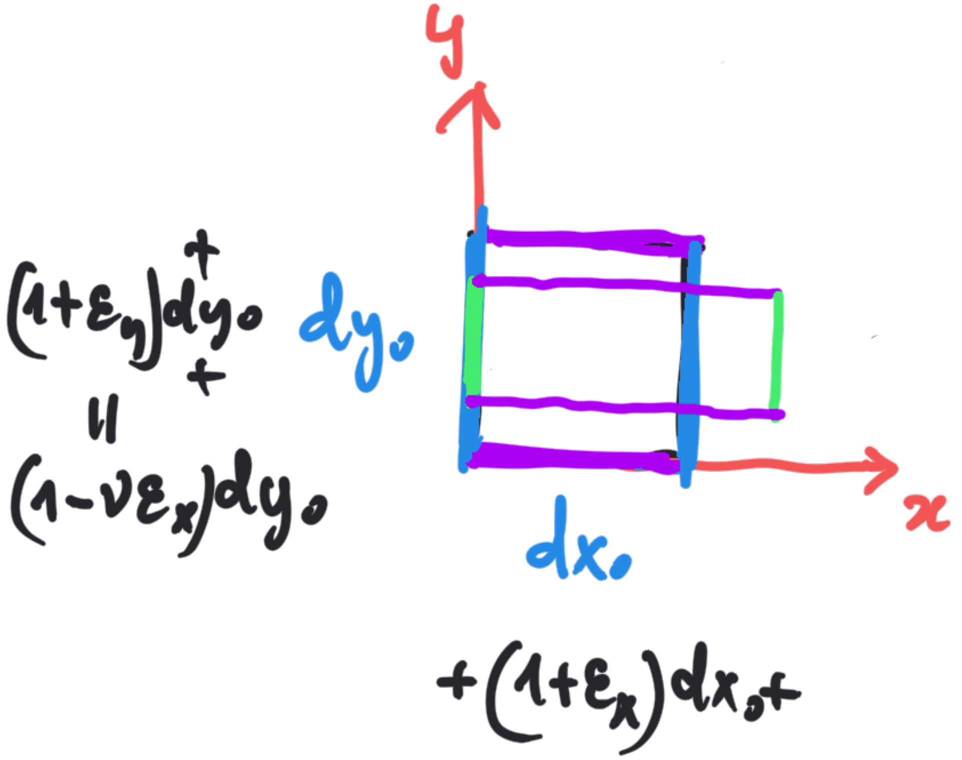
$$\underline{\underline{E}} = \begin{bmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix}$$

$$\varepsilon_y = \frac{a-a_0}{a} = -\nu \varepsilon_x$$

$$\varepsilon_z = \frac{b-b_0}{b_0} = -\nu \varepsilon_x$$

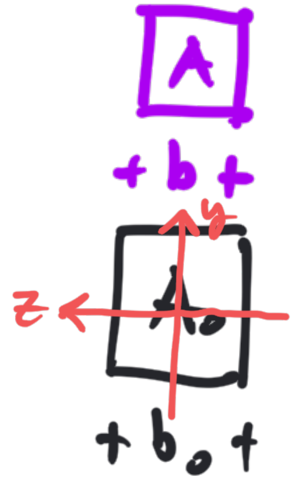
$$\varepsilon_x = \frac{\sigma_x}{E}$$

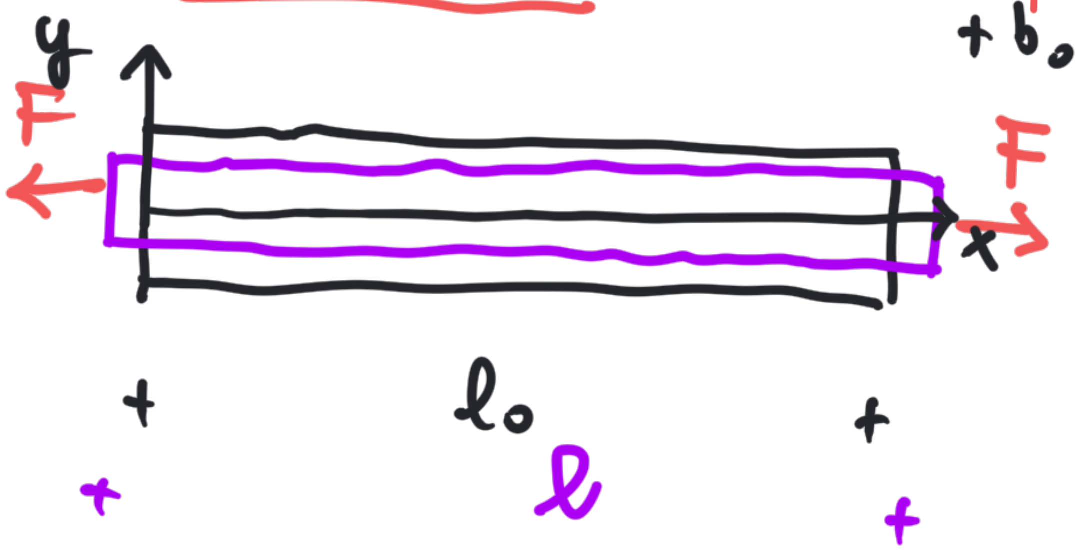
$$\sigma_x = E \varepsilon_x$$

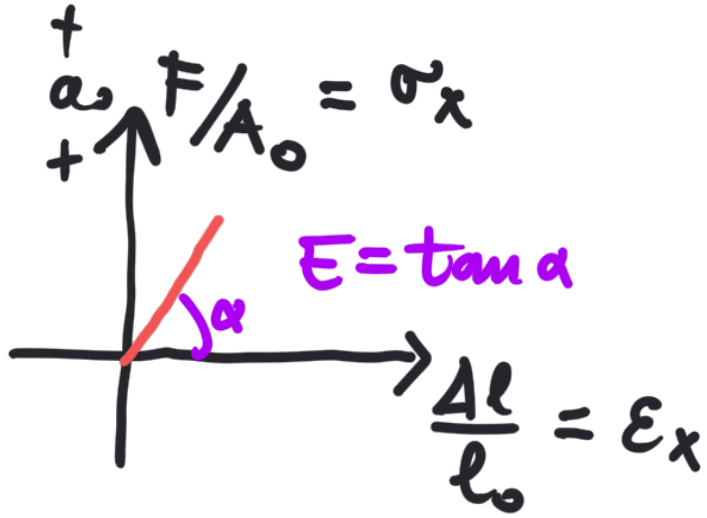


Misura sperimentale

Prova uniassiale







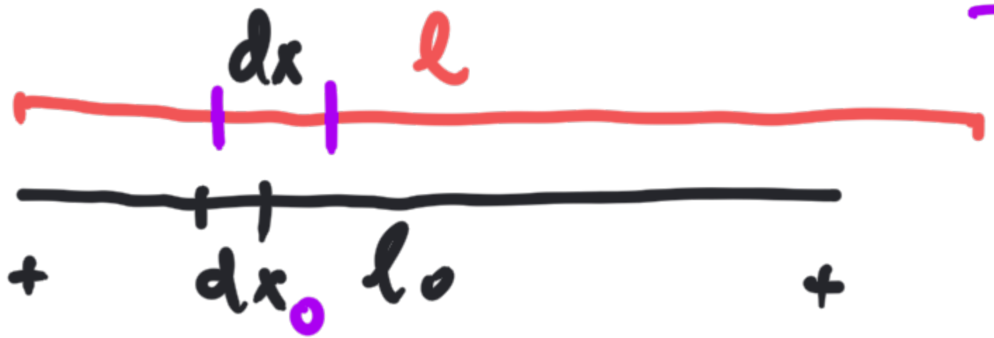
$$\Delta l = l - l_0$$

$\underline{T}$ cost. $\underline{E}$ cost.

$$\sigma_x = F/A_0 \qquad \varepsilon_x = \frac{\Delta l}{l}$$

Risultato indipendente da l_0 e dall'area della sezione trasversale del provino.

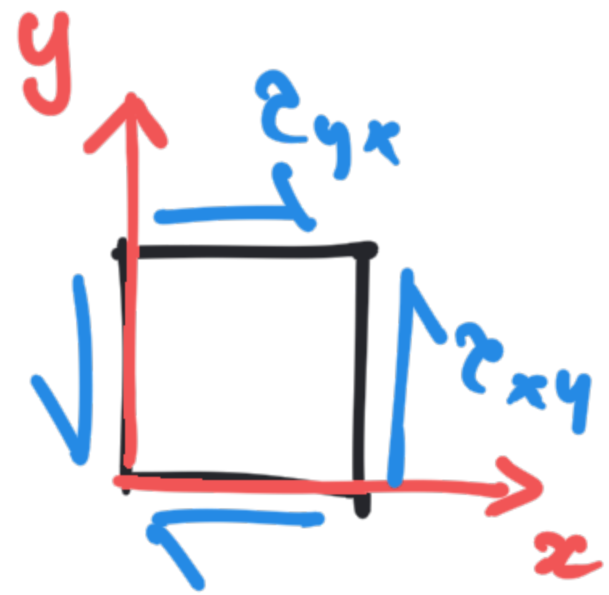
Solo dal materiale



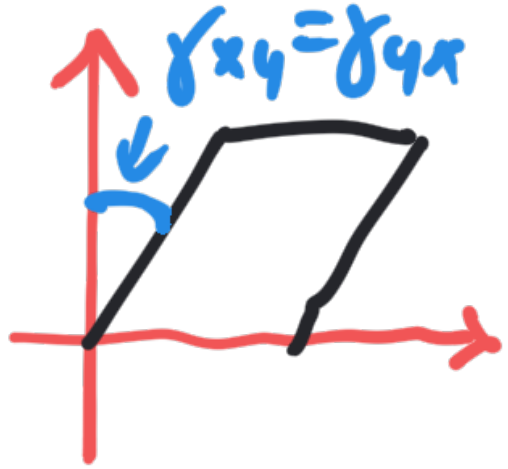
$$\frac{dx}{dx_0} - 1 = \frac{l}{l_0} - 1 \Rightarrow \overbrace{\frac{dx-dx_0}{dx_0}}^{\varepsilon_x} = \frac{l-l_0}{l_0}$$

Modulo di scorrimento G

$$\underline{T} = \begin{bmatrix} 0 & \tau_{yx} & 0 \\ \tau_{xy} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$ (taglio puro)



$$\underline{E} = \begin{bmatrix} 0 & \frac{\gamma_{yx}}{2} & 0 \\ \gamma_{xy}/2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \gamma_{xy} = \frac{\tau_{xy}}{G}$$



Prova di torsione

Caso generale

$$\underline{T} = \begin{bmatrix} \sigma_x & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix} \equiv$$ sovrapposizione di 3 stati uniassiali e 3 stati di taglio puro

$$\begin{bmatrix} \sigma_x & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 0 \\ 0 & \sigma_y & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma_z \end{bmatrix}$$

$$\begin{bmatrix} 0 & \tau_{xy} & 0 \\ \tau_{xy} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & \tau_{xz} \\ 0 & 0 & 0 \\ \tau_{xz} & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \tau_{yz} \\ 0 & \tau_{zy} & 0 \end{bmatrix}$$

(15.16)

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right] \qquad \varepsilon_z = \frac{1}{E}\left(\sigma_z - \nu(\sigma_x + \sigma_y)\right)$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right]$$

Caso generale

$$\underline{T} = \begin{bmatrix} \sigma_x & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix}$$

$\equiv$ sovrapposizione di 3 stati uniassiali e 3 stati di taglio puro

(15.16)

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right] \qquad \gamma_{xy} = \frac{\tau_{xy}}{G}$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right] \qquad \gamma_{xz} = \frac{\tau_{xz}}{G}$$

$$\varepsilon_z = \frac{1}{E}\left(\sigma_z - \nu(\sigma_x + \sigma_y)\right) \qquad \gamma_{yz} = \frac{\tau_{yz}}{G}$$