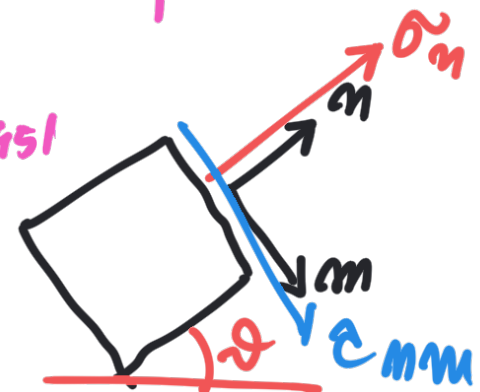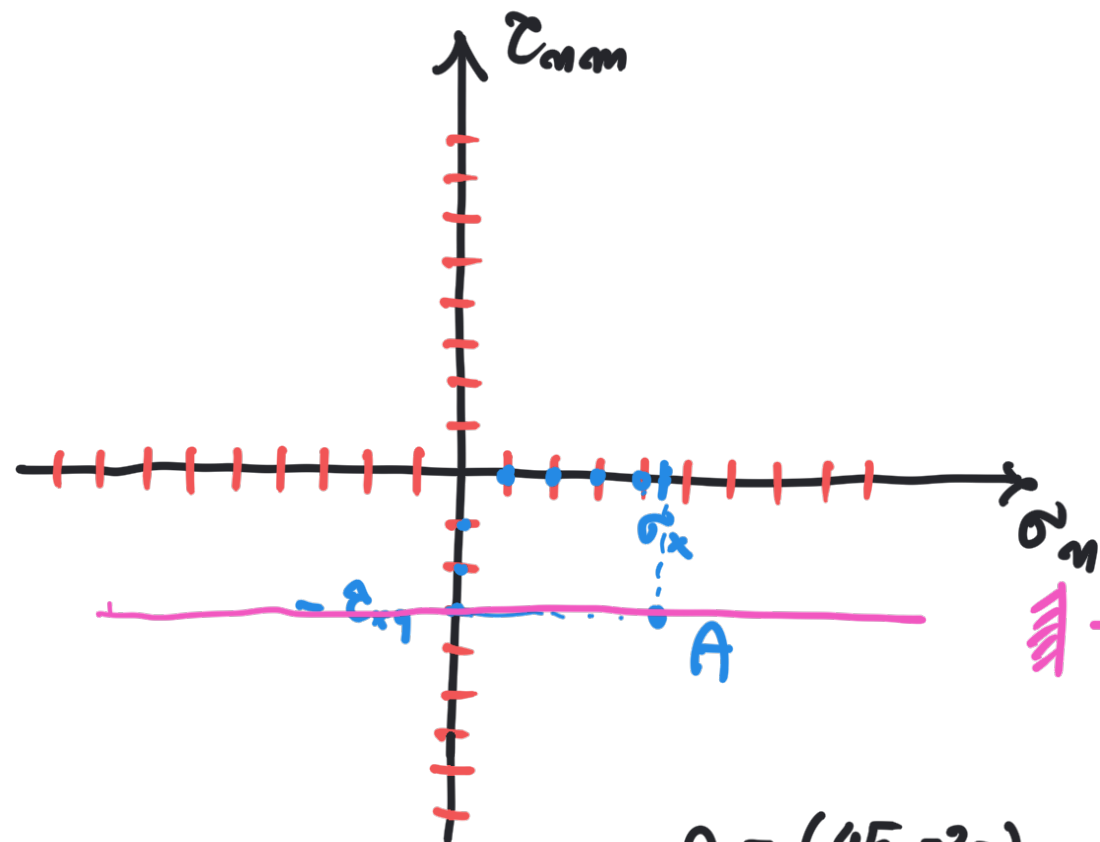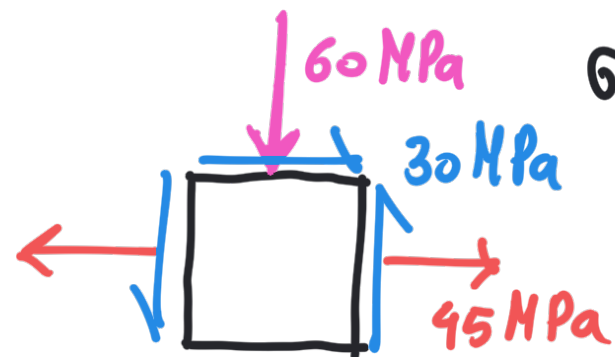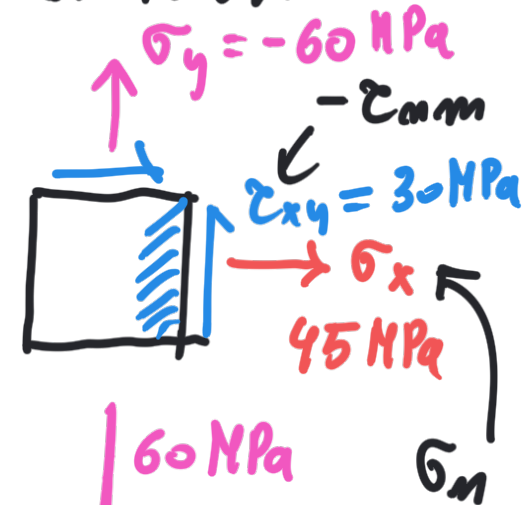Tensioni e direzioni principali e cerchio di Mohr.

$$T = \begin{pmatrix} 45 & 30 \\ 30 & -60 \end{pmatrix} \quad (MPa)$$

$\sigma_y = -60 \, MPa$

$-\tau_{nm}$

$\tau_{xy} = 30 \, MPa$

σ_x

$45 \, MPa$

σ_n

$60 \, MPa$

$30 \, MPa$

$45 \, MPa$

τ_{nm}

σ_{ix}

$-\tau_{xy}$

A

σ_n

$45°$

σ_n

n

m

τ_{nm}

ϑ

$\vartheta = 0$

n

m
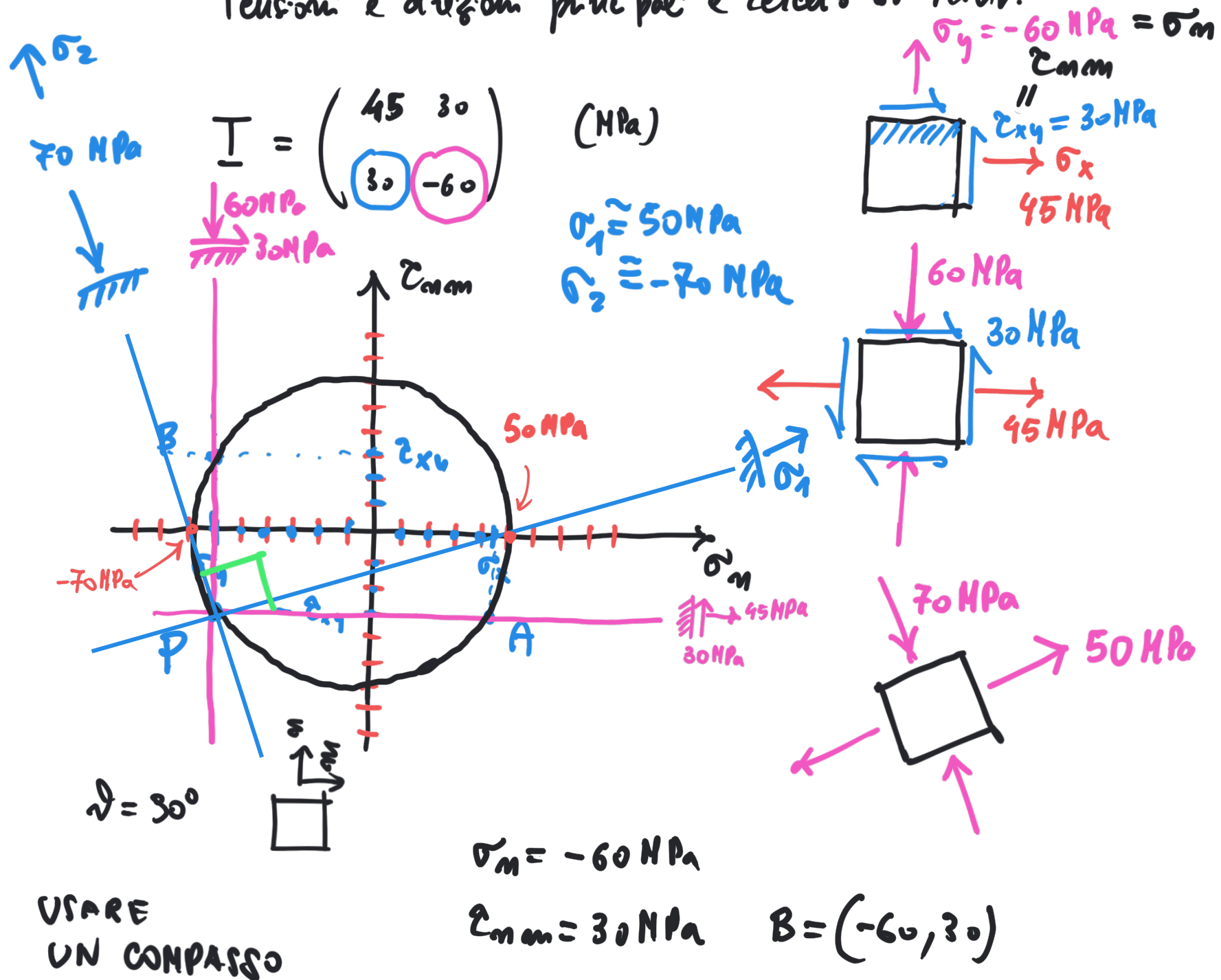
$A \equiv (45, -30)$

$\sigma_n = \sigma_x = 45 \, MPa$

$\tau_{nm} = -\tau_{xy} = -30 \, MPa$

Tensioni e direzioni principali e cerchio di Mohr.

$$\underline{\underline{T}} = \begin{pmatrix} 45 & 30 \\ 30 & -60 \end{pmatrix} \quad (MPa)$$

$\sigma_1 \cong 50\,MPa$
$\sigma_2 \cong -70\,MPa$

σ_2
70 MPa

60 MPa
30 MPa

$\sigma_y = -60\,MPa = \sigma_m$
τ_{mm}
$\tau_{xy} = 30\,MPa$
σ_x
45 MPa

60 MPa
30 MPa
45 MPa

τ_{mm}
τ_{xu}
50 MPa
B
−70 MPa
σ_1
σ_m
A
P

45 MPa
30 MPa

70 MPa
50 MPa

$\vartheta = 90°$

USARE
UN COMPASSO

$\sigma_m = -60\,MPa$
$\tau_{mm} = 30\,MPa$ $\quad B = (-60, 30)$

Tensioni e direzioni principali e cerchio di Mohr.

$$\underline{\underline{T}} = \begin{pmatrix} 45 & 30 \\ 30 & -60 \end{pmatrix} \quad (MPa)$$

$(\sigma_1, \underline{n}_1) \qquad \underline{\underline{T}}\,\underline{n}_1 = \sigma_1 \underline{n}_1$

$(\sigma_2, \underline{n}_2) \qquad \underline{\underline{T}}\,\underline{n}_2 = \sigma_2 \underline{n}_2$
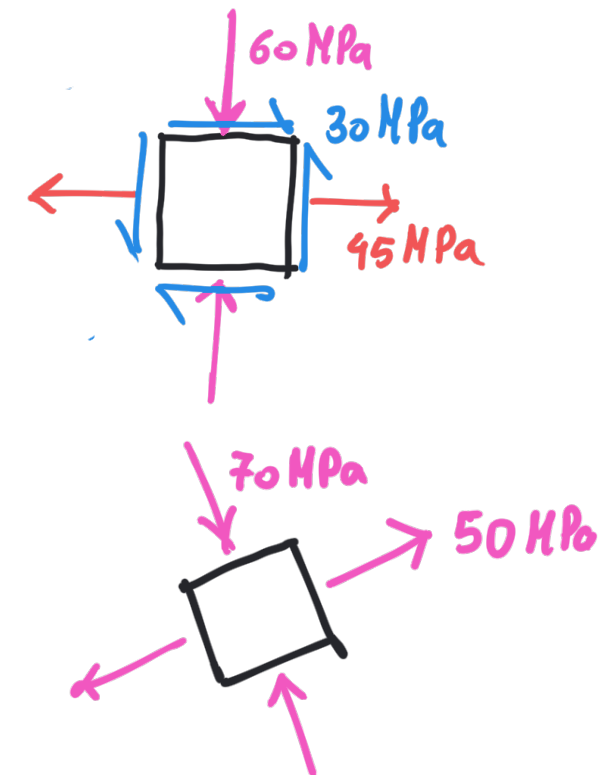
$$0 = \det\left(\underline{\underline{T}} - \sigma\underline{\underline{I}}\right) = \det\begin{pmatrix} 45-\sigma & 30 \\ 30 & -60-\sigma \end{pmatrix}$$

$$= -(45-\sigma)(60+\sigma) - 30\cdot 30$$

$$= \sigma^2 - 45\sigma + 60\sigma - 2700 - 900$$

$$= \sigma^2 + 15\sigma - 3600$$

$$\sigma = \frac{-15 \pm \sqrt{225 + 4\cdot 3600}}{2} = \frac{-15 \pm 120}{2} = \begin{cases} 105/2 = 52.5 \\[4pt] -\dfrac{135}{2} = -67.5 \end{cases}$$

Tensioni e direzioni principali e cerchio di Mohr.

$$T = \begin{pmatrix} 45 & 30 \\ 30 & -60 \end{pmatrix} \quad (MPa)$$

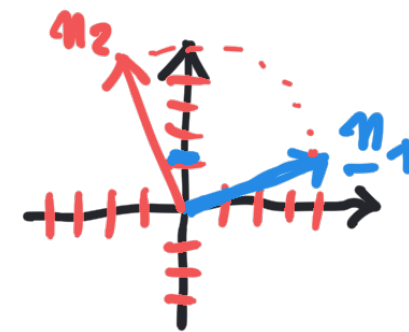$(\sigma_1, \underline{n}_1) \qquad T\underline{n}_1 = \sigma_1 \underline{n}_1$

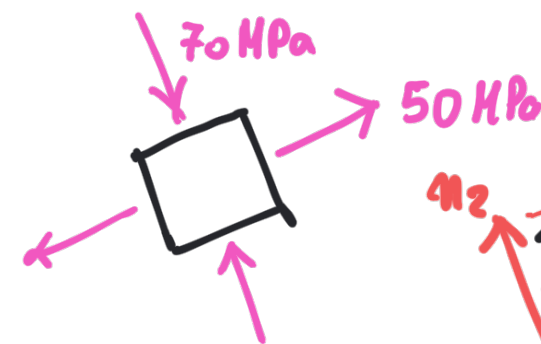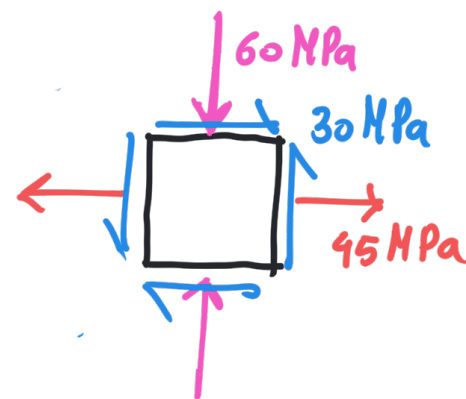$(\sigma_2, \underline{n}_2) \qquad T\underline{n}_2 = \sigma_2 \underline{n}_2$

$$\sigma = \begin{cases} 52.5 = \sigma_1 \\ \\ -67.5 = \sigma_2 \end{cases}$$

$$T - \sigma_1 I = \begin{pmatrix} 45-52.5 & 30 \\ 30 & -60-52.5 \end{pmatrix} = \begin{pmatrix} -7.5 & 30 \\ 30 & -112.5 \end{pmatrix}$$

$$\underline{n}_1 = \begin{pmatrix} \alpha_1 \\ \beta_1 \end{pmatrix} = \begin{pmatrix} 30 \\ 7.5 \end{pmatrix} \Big/ \sqrt{30^2 + 7.5^2} = \begin{pmatrix} 0.96 \\ 0.24 \end{pmatrix}$$

$$\begin{pmatrix} -7.5 & 30 \\ 30 & -112.5 \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \beta_1 \end{pmatrix} = \begin{pmatrix} -7.5\alpha_1 + 30\beta_1 \\ 30\alpha_1 - 112.5\beta_1 \end{pmatrix}$$

$$\underline{n}_2 = \begin{pmatrix} -0.24 \\ 0.96 \end{pmatrix}$$