Project 3 – The Football Manager

You have been hired by Farcelona, a very legitimate football team.

For the next $T = 8$ years, you are in charge of buying and selling football players to make your team as rich as possible. Remember, though, that you are trading players and your team must have at least 2 players for each of 10 different positions:

- goalkeepers (G),
- left defenders (DL), center defenders (CD), right defenders (DR),
- left midfielders (LM), center midfielders (CM), right midfielders (RM),
- left attackers (LA), center attackers (CA), and right attackers (RA).

For simplicity, assume you are required to have 2 players per role at all times, for a total of 20 players. To first experiment with a model, you are given time series data of the market prices of about 500 players of different positions (the price of player i at time t is denoted by $\bar{p}_t(i)$) - see Brightspace. In addition, consider you start with some budget $b_0 > 0$. Each year t, you can sell the players you have to obtain their current market price, you can buy new players by paying their current market price, or you can keep the players you have. In the final year T, you can assume you sell all the players you have, and your final "reward" is the market value of the players you sold added to the final budget b_T.

1. Mathematically describe a model to tractably determine the yearly optimal buy/sell strategy over T years until 2020 given this data, the budget constraint, and the 2 player per position constraint.
2. Determine the optimal buy/sell strategy. What would be your optimal team and how much money would you make after the T years?
3. Study how your decisions change when you add a utility function. Experiment with a few utility functions mentioned in the lectures.
4. Assume now that the data you were given is noisy,

$$p_t(i) = \underbrace{\bar{p}_t(i)}_{\text{data}} + \underbrace{\Delta p_t(i)}_{\text{random noise}} .$$

   Can you expand your model to deal with this uncertainty?