Solutions to Exercise set 4 – MPC (SC42125)

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1 Infinite horizon cost and constraints

1. We have to check the invariance of the given set with respect to the dynamics. Clearly, $\mathbb{X}_\mathrm{f}$ is feasible with respect to the state constraints and has vertices $V = \{v_1, \dots, v_4\}$, with $v_1 = [1 \quad 0]^\top$, $v_2 = [-1 \quad 0]^\top$, $v_3 = [0 \quad 1]^\top$ and $v_4 = [0 \quad -1]^\top$. Note that $A_K = A + BK$ is diagonal. Hence, $\mathbb{X}_\mathrm{f}$ is invariant for $x(k+1) = A_K x(k)$ because $A_K = \mathrm{diag}(0.5, -0.3)$ so the vertices of $\mathbb{X}_\mathrm{f}$ satisfy $A_K v_i \in \mathcal{X}$ for all $i = 1, \dots, 4$, i.e.,

$$\begin{aligned} A_K v_1 &= [0.5 \quad 0]^\top, & A_K v_2 &= [-0.5 \quad 0]^\top, \\ A_K v_3 &= [0 \quad -0.3]^\top, & A_K v_4 &= [0 \quad 0.3]^\top. \end{aligned}$$

2. In general, the maximal terminal set is obtained by checking the system constraints over a sufficiently long horizon N_c, i.e., for some finite N_c:

$$\mathcal{S}_{\max}(N_c) \coloneqq \left\{ x \in \mathbb{R}^2 \mid (A+BK)^i x \in \mathbb{X},\, i = 1, 2, \dots, N_c - 1 \right\}.$$

The minimum allowable value for N_c can be found by checking whether $A_K^{M+1} x \in \mathbb{X}_\mathrm{f}$ for all x such that $A_K^i x \in \mathbb{X}$ for $i = 1, 2, \dots, M$. If it is true, then $N_c = M$, otherwise $N_c > M$. In this example, for a given M, this condition can be checked by solving the following two linear programs

$$\begin{aligned} P_1 = \max_{x \in \mathbb{R}^2} \quad & [1 \quad 1] A_K^{M+1} x \\ \text{s.t.} \quad & A_K^i x \in \mathbb{X},\, \forall i = 0, 1, \dots, M, \end{aligned}$$

$$\begin{aligned} P_2 = \max_{x \in \mathbb{R}^2} \quad & [1 \quad -1] A_K^{M+1} x \\ \text{s.t.} \quad & A_K^i x \in \mathbb{X},\, \forall i = 0, 1, \dots, M, \end{aligned}$$

and then check

$$z = \max\{P_1, P_2\} \leq 1.$$

2 Infinite horizon cost to go as terminal penalty

See MATLAB code.

3 Terminal penalty with and without terminal constraint

See MATLAB code.

4 Unreachable setpoints in constrained versus unconstrained linear systems

See MATLAB code.

5 Terminal penalty and input constraints

See MATLAB code.