Exercises on Bayesian Modeling and Reasoning

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Instructions   These exercises are designed to help you practice Bayesian reasoning using real-world style problems. They should take approximately 30 minutes to complete in total. Show your calculations clearly and write a brief explanation for each answer.

1   Exercise: Medical Test Puzzle

A rare disease affects 1% of the population. A test for the disease has:

- Sensitivity (true positive rate), i.e., the probability of testing positive given that you have the disease: 99%.

- Specificity (true negative rate), i.e., the probability of testing negative given that you do not have the disease: 95%.

If a randomly selected person tests positive, what is the probability that they actually have the disease? Before making your calculations, make an intuitive guess about the answer. Whould you expect it to be over or under 50%? Afterwards, compare your guess with the result of your calculations. Do you think the result is surprising? Why or why not?

2   Exercise: The Two-Card Problem

You have two cards (i) One card is red on both sides (RR) and (ii) One card is red on one side and black on the other (RB). You pick a card at random and place it on the table. The side facing up is red.
Question: What is the probability that the other side of the card is also red?

3   Exercise: The Goalkeeper Problem

A football manager compares two goalkeepers, goalkeeper A and goalkeeper B, based on their save percentages against penalty shooters.

- Goalkeeper A has a better save percentage than goalkeeper B when the penalty shooter is **right-footed**.

- Goalkeeper A has a better save percentage than goalkeeper B when the penalty shooter is **left-footed**.

Question: Is it possible for goalkeeper B to have a higher overall save percentage than goalkeeper A?
Before calculating or reasoning, make an intuitive guess: Does it seem reasonable that someone worse in both subgroups could be better overall? Afterwards, compare your intuition with the result.

4 Exercise: The Monty Hall Problem

You are on a game show and presented with three doors: behind one door is a car (the prize you want), and behind the other two doors are goats (which you do not want). You choose one door, say Door 1. The host, who knows what is behind each door, opens another door, say Door 3, which has a goat behind it. Then, the host gives you the option to either stick with your original choice (Door 1) or switch to the remaining unopened door (Door 2). **Question:** Should you stick with your original choice or switch to the other door? What is the probability of winning the car if you switch versus if you stick?

5 Exercise: Blue Cab Problem

In a city, 85% of the taxis are green and 15% are blue. A hit-and-run accident occurs at night. A witness identifies the cab involved as blue. The witness is 80% reliable in distinguishing blue from green in such conditions. **Question:** What is the probability that the cab involved in the accident was actually blue?