Probabilistic and Statistical Modeling

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Contents

- **Primer on Probabilistic Modeling**
  https://www.inf.ed.ac.uk/teaching/courses/pmr/22-23/
  assets/notes/probabilistic-modelling-primer.pdf

Session 1 – Recap

- **What we covered:**
  - ▶ **Probabilistic Modeling:** Model real-world phenomena using probabilities.
  - ▶ **Probabilistic Reasoning (Inference):** Use known probabilities to infer unknowns.
  - ▶ **Bayesian Analysis:** Modeling and Reasoning with Bayes' rule.
  - ▶ **Core Rules of Probability:** The sum, product and Bayes' rule.
  - ▶ *Example: Alzheimer's diagnostic test.*
- **What's still to explore:**
  - ▶ Our example was simple; two $1D$ random variables were enough:
    - ★ X: the test result
    - ★ Y: the disease status
  - ▶ Real-world problems are much more complex
    - ★ Involve more random variables and of higher dimensions.

Session 2 – Recap

- **What we covered:**
  - ▶ **Multivariate Random Variables and Distributions:**
    - ★ PDFs, PMFs and CDFs
    - ★ Key properties: expectation and variance.
    - ★ How to sample from these distributions.
    - ★ Key-distributions: Bernoulli, Normal, Poisson.
  - ▶ We now have powerful tools to model complexity!
- **What's still to explore:**
  - ▶ How to use RVs and their properties to draw conclusions about real-world phenomena in a *principled* and *unified* way?
  - ▶ The **Bayesian framework** is a good way to do that.

Session 3 – Overview

- **What we'll explore:**
  - ▶ **The Bayesian Framework**
  - ▶ **Key Ingredients:**
    - ★ **Prior Distribution:** Our belief before seeing the data.
    - ★ **Likelihood:** How compatible is the observed data is with different parameter values.
    - ★ **Posterior Distribution:** Our updated beliefs after observing the data.
    - ★ **Predictive Distribution:** Make predictions about new, unseen data.

- **What you should already know:**
  - ▶ The intuition behind probabilistic modeling — Alzheimer's test case (Session 1)
  - ▶ Core probability rules: sum, product, and Bayes' rule (Session 1)
  - ▶ Handle multivariate distributions and apply their properties (Session 2)