1 Techniques of Integration

Integration by Parts

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

Partial Fractions

$$\frac{f(x)}{(ax-b)(cx-d)} = \frac{\frac{f(\frac{b}{a})}{c(\frac{b}{a})-d}}{ax-b} + \frac{\frac{f(\frac{d}{c})}{a(\frac{d}{c})-b}}{cx-d}$$

Definition of the Definite integral

$$\int_a^b f(x)dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(a+i\frac{b-a}{n})\frac{b-a}{n}$$

2 Differential Equations

Exponential Equations

$$\frac{dm}{dt} = km \qquad m = m_0e^{kt}$$

Logistic Growth

$$\frac{dP}{dt} = kP(M-P) \qquad P = \frac{M}{1+Ae^{-(Mk)t}}$$

Euler's Method

$$\Delta y = \frac{dy}{dx}\Delta x$$

Arc Length

$$L = \int_a^b \sqrt{1+(\frac{dy}{dx})^2}dx$$

$$L = \int_a^b \sqrt{1+(\frac{dx}{dy})^2}dy$$

3 Parametric, Polar, Vector

Parametric Equations

when $\frac{dx}{dt} \neq 0$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}(\frac{dy}{dx})}{\frac{dx}{dt}}$$

(length of curve)

$$L = \int_a^b \sqrt{(\frac{dx}{dt})^2+(\frac{dy}{dt})^2}dt$$

Vectors

$$\langle a,b\rangle = a\hat{\imath} + b\hat{\jmath}$$

$$r(t) = f(t)\hat{\imath} + g(t)\hat{\jmath}$$

$$\|\vec{v}\| = \sqrt{(\frac{dx}{dt})^2+(\frac{dy}{dt})^2}$$

Polar Graphing

$$x = r\cos\theta \qquad y = r\sin\theta$$

Polar Calculus

$$\frac{dy}{dx} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$$

$$A = \int_\alpha^\beta \frac{1}{2}r^2d\theta$$

$$A = \frac{1}{2}\int_\alpha^\beta R^2 - r^2d\theta$$

$$L = \int_\alpha^\beta \sqrt{r^2+(\frac{dr}{d\theta})^2}d\theta$$

$$\frac{dr}{dt} = \frac{dr}{d\theta}\frac{d\theta}{dt}$$