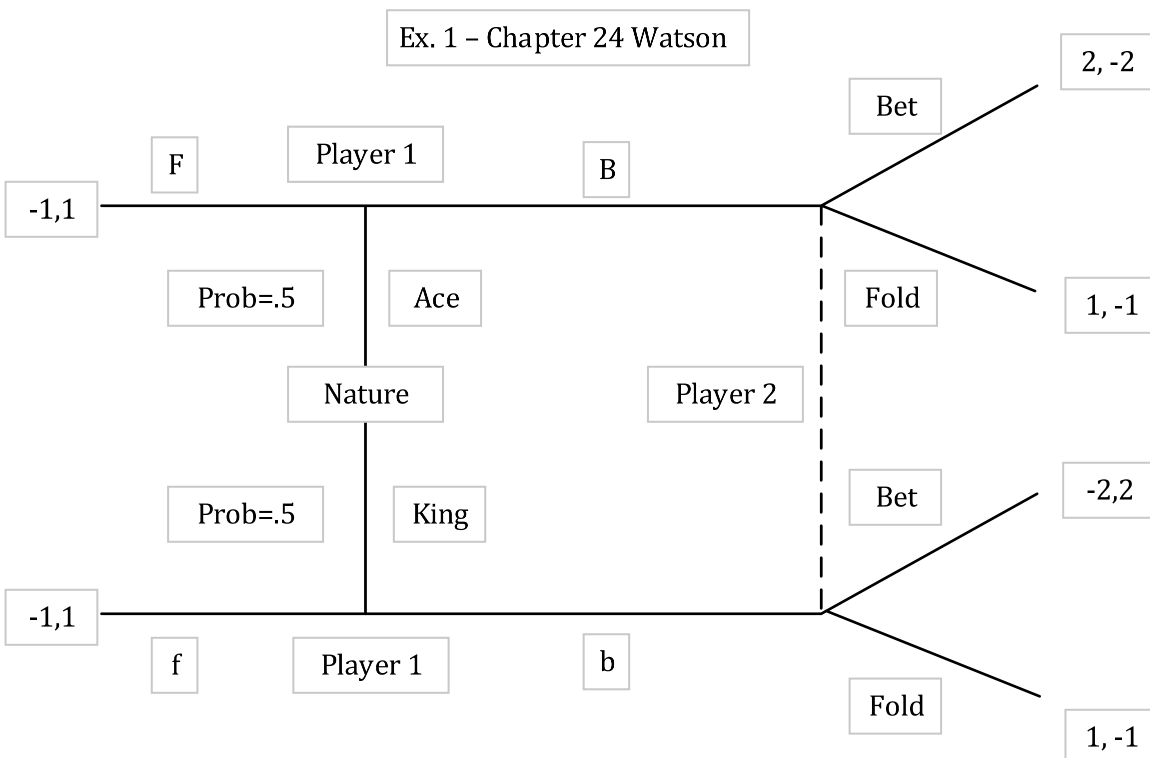
ECONS 424 – STRATEGY AND GAME THEORY

HOMEWORK #6 – ANSWER KEY

<u>WATSON CHAPTER 24 –EXERCISE 1</u>



Player 2 has only two available strategies $S_2 = \{Bet, Fold\}$

But player 1 has four available strategies $S_1 = \{Bb, Bf, Fb, Ff\}$

This implies that the Bayesian normal form representation of the game is

Player 2

Player 1

	Bet	Fold
Bb		
Bf		
Fb		
Ff		

In order to find the expected payoffs for strategy profile (Bb, Bet) top left-hand side cell of the matrix, we proceed as follows:

$$EU_1 = \frac{1}{2} * 2 + \frac{1}{2} * (-2) = 0$$

$$EU_2 = \frac{1}{2} * (-2) + \frac{1}{2} * (2) = 0$$

$$\rightarrow (0,0)$$

Similarly for strategy profile (Bf, Bet),

$$EU_1 = \frac{1}{2} * (-1) + \frac{1}{2} * (-2) = -\frac{3}{2}$$

$$EU_2 = \frac{1}{2} * 1 + \frac{1}{2} * 2 = \frac{3}{2}$$

For strategy profile (Ff, Bet),

$$EU_1 = \frac{1}{2}(-1) + \frac{1}{2}(-1) = -1$$

$$EU_2 = \frac{1}{2} * 1 + \frac{1}{2} * 1 = 1$$

For strategy profile (Bb, Fold),

$$EU_1 = \frac{1}{2} * 1 + \frac{1}{2} * 1 = 1$$

$$EU_2 = \frac{1}{2}(-1) + \frac{1}{2}(-1) = -1$$

For strategy profile (Bf, Fold),

$$EU_1 = \frac{1}{2} * 1 + \frac{1}{2}(-1) = 0$$

$$EU_2 = \frac{1}{2}(-1) + \frac{1}{2} * 1 = 0$$

Similarly for (Fb, Fold),

$$EU_1 = \frac{1}{2} * (-1) + \frac{1}{2} * 1 = 0$$

$$EU_2 = \frac{1}{2} * 1 + \frac{1}{2}(-1) = 0$$

Finally, for (Ff, Fold),

$$EU_1 = \frac{1}{2} * (-1) + \frac{1}{2}(-1) = -1$$

$$EU_2 = \frac{1}{2} * 1 + \frac{1}{2} * 1 = 1$$

We can now insert these expected payoffs into the Bayesian normal form,

Player 2

Player 1

	Bet	Fold
Bb	0, 0	1, -1
Bf	$\frac{1}{2}, -\frac{1}{2}$	0, 0
Fb	$-\frac{3}{2}, \frac{3}{2}$	0, 0
Ff	-1, 1	-1, 1

WATSON CHAPTER 26 –EXERCISE 6

Following the methods used above to convert this game into normal form, we see that players 1&2 have the following strategies spaces:

Player 2 has only two available strategies $S_2 = \{U, D\}$

But player 1 has four available strategies $S_1 = \{LL', LR', RL', RR'\}$

The Bayesian normal form game would be constructed as such:

Player 2

Player 1

	U	D
LL’		
LR’		
RL’		
RR’		

Making similar calculations as before, we may find the expected values of the different strategy profiles for each person and fill in the normal form table as such:

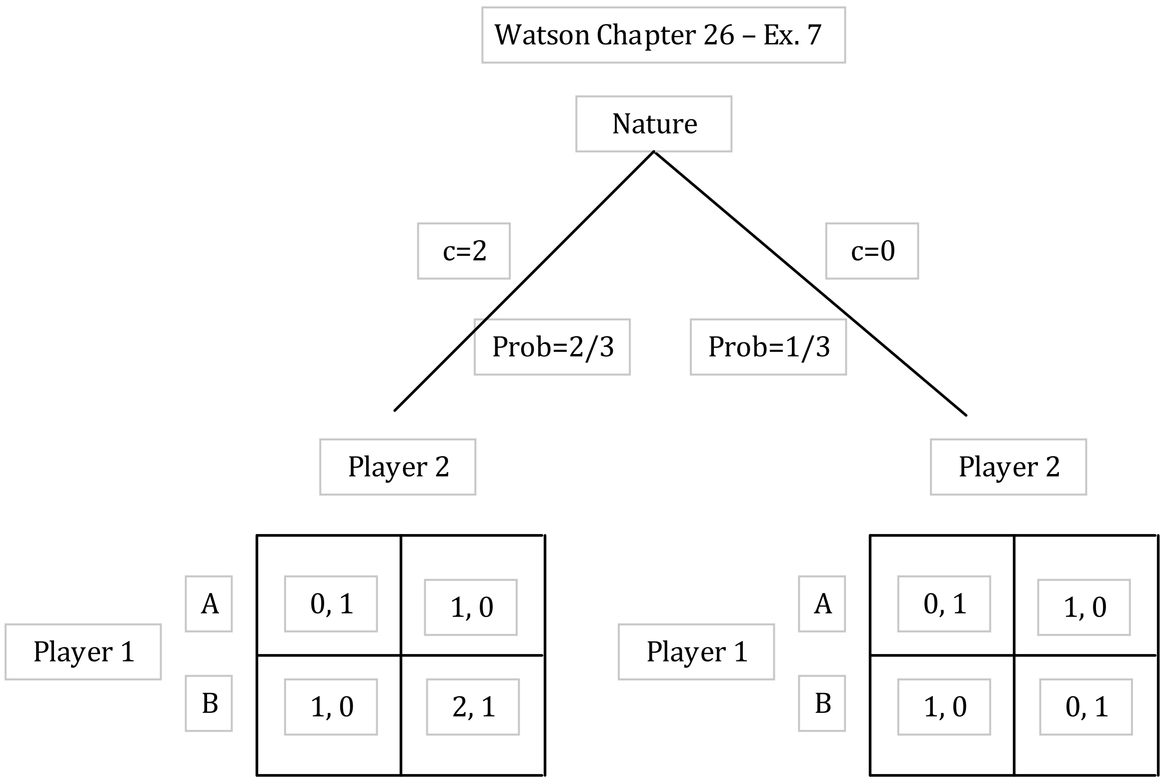
Player 2

Player 1

	U	D
LL’	$\underline{2}$, $\underline{0}$	2, $\underline{0}$
LR’	1, 0	3, $\underline{1}$
RL’	1, $\underline{2}$	3, 0
RR’	0, $\underline{2}$	$\underline{4}$, 1

It is evident by finding each player's Best Responses that the BNE is at {LL', U}

WATSON CHAPTER 26 –EXERCISE 7



A) Player 2 (uninformed) for only two strategies $S_2 = \{X, Y\}$
Player 1 (informed) has four strategies, depending on his type,
$$S_1 = \{AA', AB', BA', BB'\}$$
Where the first component of every strategy pair denotes what player 1 does when his type is c=2 while the second component reflects his choice when his type is c=0.

The Bayesian normal form representation is:

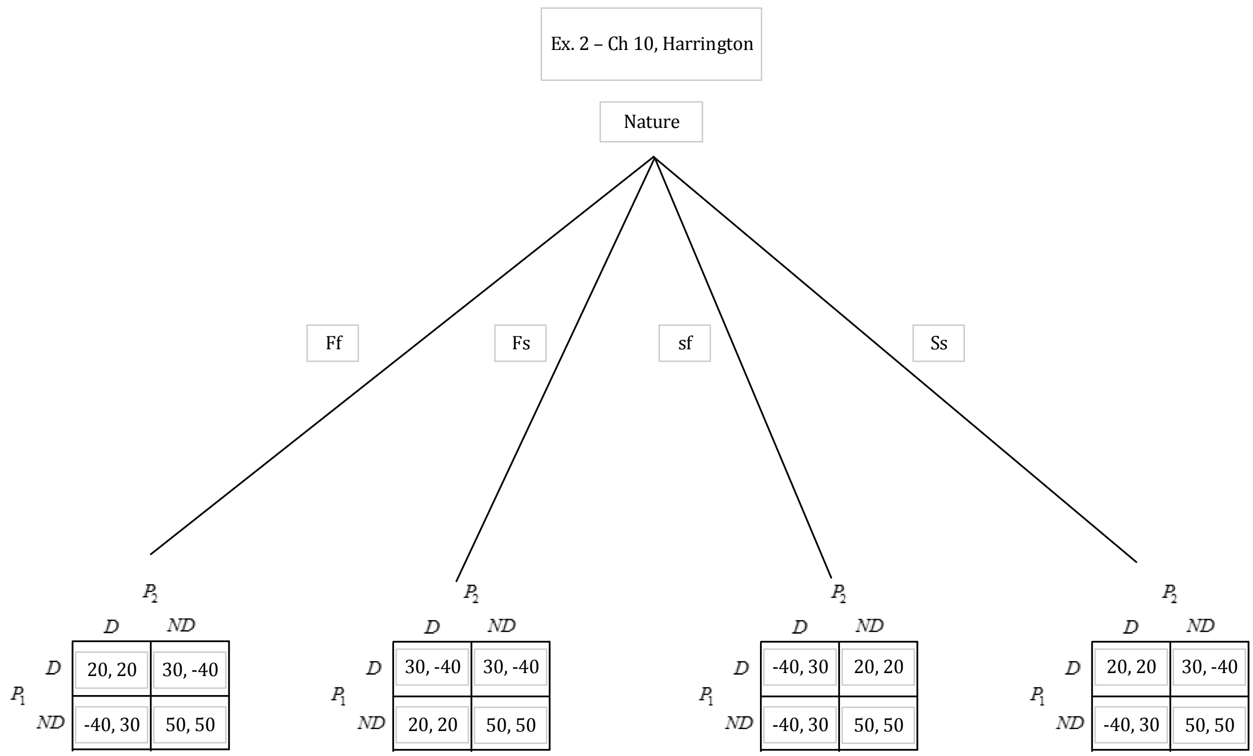
Player 2

Player 1

	X	Y
AA’	0, $\underline{1}$	1, 0
AB’	1/3, $\underline{2/3}$	2/3, 1/3
BA’	2/3, 1/3	$\underline{5/3}$, $\underline{2/3}$
BB’	$\underline{1}$, 0	4/3, $\underline{1}$

B) Underlining players’ best responses as usual, we find a unique BNE: {BA’, Y}

HARRINGTON CHAPTER 10 –EXERCISE 2



Where F/S denotes Fast/Slow for player 1 (Bat)

f/s denotes fast/slow for player 2 (Curly Bill)

Where D represents Draw

ND represents Not Draw

fast, then each has a payoff of 20. If at least one chooses draw, then there is a gunfight.

a. Is it consistent with Bayes–Nash equilibrium for there to be a gunfight for sure? (That is, both gunfighters draw, regardless of their type.)

ANSWER: Yes, as it is an equilibrium for each gunfighter to use a strategy that has him draw regardless of his type. The equilibrium conditions for Bat are

$$\text{Fast Type (Draw): } .6 \times 20 + .4 \times 30 \geq .6 \times (-40) + .4 \times 20 \Rightarrow 24 \geq -16$$

$$\text{Slow Type (Draw): } .6 \times (-40) + .4 \times 20 \geq .6 \times (-40) + .4 \times (-40) \Rightarrow -32 \geq -40.$$

The equilibrium conditions for Curly Bill are

$$\text{Fast Type (Draw): } .65 \times 20 + .35 \times 30 \geq .65 \times (-40) + .35 \times 20 \Rightarrow 23.5 \geq -19$$

$$\text{Slow Type (Draw): } .65 \times (-40) + .35 \times 20 \geq .65 \times (-40) + .35 \times (-40) \Rightarrow -19 \geq -40.$$

b. Is it consistent with Bayes–Nash equilibrium for there to be no gunfight for sure? (That is, both gunfighters wait, regardless of their type.)

ANSWER: Yes, as it is an equilibrium for each gunfighter to use a strategy that has him wait regardless of his type. Doing so realizes a payoff of 50—as a gunfight is avoided—and all other outcomes yield a lower payoff, so the expected payoff from any drawing must be less. More explicitly, the equilibrium conditions for Bat are

$$\text{Fast Type (Wait): } .6 \times 50 + .4 \times 50 \geq .6 \times 30 + .4 \times 30 \Rightarrow 50 \geq 30$$

$$\text{Slow Type (Wait): } .6 \times 50 + .4 \times 50 \geq .6 \times 20 + .4 \times 30 \Rightarrow 50 \geq 24.$$

The equilibrium conditions for Curly Bill are

$$\text{Fast Type (Draw): } .65 \times 50 + .35 \times 50 \geq .65 \times 30 + .35 \times 30 \Rightarrow 50 \geq 30$$

$$\text{Slow Type (Draw): } .65 \times 50 + .35 \times 50 \geq .65 \times 20 + .35 \times 30 \Rightarrow 50 \geq 23.5.$$

c. Is it consistent with Bayes–Nash equilibrium for a gunfighter to draw only if he is slow?

ANSWER: Yes. Consider a strategy profile in which each draws when slow and waits when fast. The equilibrium conditions for Bat are

Fast Type (Wait): $.6 \times 50 + .4 \times 20 \geq .6 \times 30 + .4 \times 30 \Rightarrow 38 \geq 30$

Slow Type (Draw): $.6 \times 20 + .4 \times 20 \geq .6 \times 50 + .4 \times (-40) \Rightarrow 20 \geq 14.$

The equilibrium conditions for Curly Bill are

Fast Type (Wait): $.65 \times 50 + .35 \times 20 \geq .65 \times 30 + .35 \times 30 \Leftrightarrow 39.5 \geq 30$

Slow Type (Draw): $.65 \times 20 + .35 \times 20 \geq .65 \times 50 + .35 \times (-40) \Leftrightarrow 20 \geq 18.5.$

HARRINGTON CHAPTER 10 –EXERCISE 5

Player 2 has only two strategies $S_2 = \{a, b\}$

Player 1 has four strategies $S_1 = \{xx', xy', yx', yy'\}$ where the first component of every strategy pair denotes what player 1 chooses when his type is H and the second component is what he selects when his type is L.

Here, the Bayesian normal form representation of the game is:

Player 2

Player 1

	a	b
xx’		
xy’		
yx’		
yy’		

Let's find the EU from strategy profile (xx', a),

$$EU_1 = p * 3 + (1 - p) * 2 = 2 + p$$

$$EU_2 = p * 1 + (1 - p) * 3 = 3 - 2 * p$$

$$\rightarrow (2 + p, 3 - 2p)$$

For Strategy profile (xy', a),

$$EU_1 = p * 3 + (1 - p) * 3 = 3$$

$$EU_2 = p * 1 + (1 - p) * 1 = 1$$

$$\rightarrow (3, 1)$$

Proceeding in this fashion, we find the complete normal form game to be:

Player 2

Player 1

	a	b
xx'	2+p, 3-2p	1, 2+p
xy'	3, 1	4-3p, 3p
yx'	2, 3-2p	1+4p, 2
yy'	3-p, 1	4+p, 2p

A) When p= 0.75 the above matrix becomes:

Player 2

Player 1

	a	b
xx’	2.75, 1.5	1, <u>2.75</u>
xy’	<u>3</u>, 1	1.75, <u>2.25</u>
yx’	2, 1.5	4, <u>2</u>
yy’	2.25, 1	<u>4.75</u>, <u>1.5</u>

Doing the usual underlining to find best responses for each player, we find that there is a unique BNE: (yy’, b).

B) Player 1:

When player 2 selects a, he prefers: 3>2+p for all p

3>2

3>3-p Hence, he selects xx’

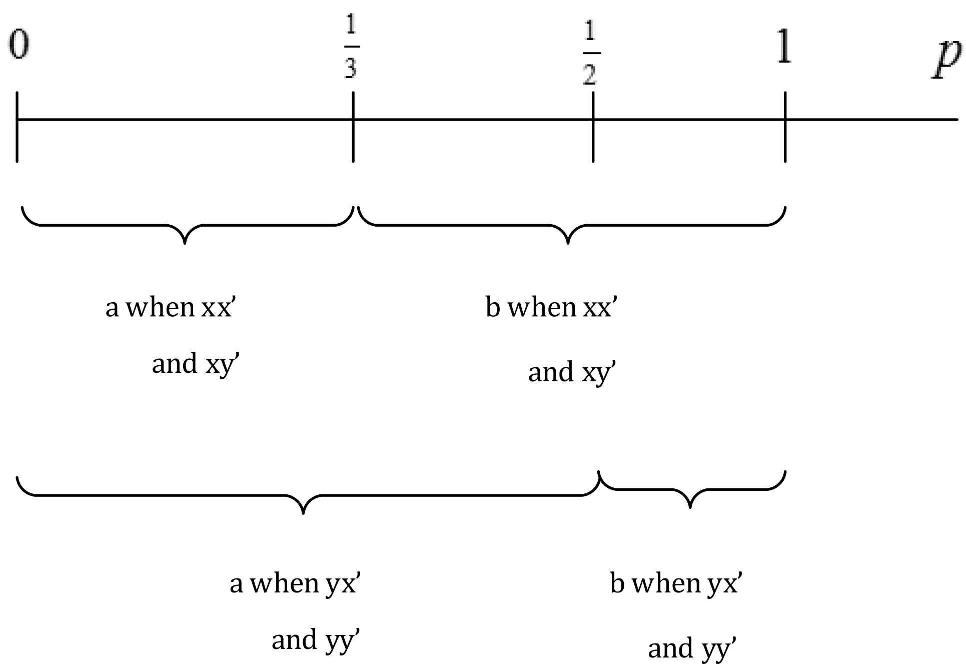
Player 2:

When player 1 selects xx’, he prefers a if 3-2p>2+p→ $1 > 3p \rightarrow \frac{1}{3} > p$

When player 1 selects xy’, he prefers a if $1 > 3p \rightarrow \frac{1}{3} > p$ (otherwise he prefers b)

When player 1 selects yx’, he prefers a if $3 - 2p > 2 \rightarrow 1 > 2p \rightarrow \frac{1}{2} > p$

When player 1 selects yy’, he prefers a if $1 > 2p \rightarrow \frac{1}{2} > p$ (otherwise he prefers b)



We can then divide our analysis into three different matrices

- One matrix for $p < \frac{1}{3}$
- Another matrix for $p \in \left[\frac{1}{3}, \frac{1}{2}\right]$
- Another matrix for $p > \frac{1}{2}$

First Case: $p < \frac{1}{3}$

Player 2

		a	b
	xx'	$2+p$, $\underline{3-2p}$	1, $\underline{2+p}$
	xy'	$\underline{3}$, $\underline{1}$	$4-3p$, $3p$
Player 1	yx'	2, $\underline{3-2p}$	$1+4p$, 2
	yy'	$3-p$, $\underline{1}$	$\underline{4+p}$, $2p$

Unique BNE: (xy', a)

Second Case: $p \in \left[\frac{1}{3}, \frac{1}{2}\right]$

Player 2

Player 1		a	b
	xx'	2+p, 3-2p	1, <u>2+p</u>
	xy'	<u>3</u>, 1	4-3p, <u>3p</u>
	yx’	2, <u>3-2p</u>	1+4p, 2
	yy'	3-p, <u>1</u>	<u>4+p</u>, 2p

Third Case: $p > \frac{1}{2}$

Player 2

Player 1		a	b
	xx'	2+p, 3-2p	1, $\underline{2+p}$
	xy'	$\underline{3}$, 1	4-3p, $\underline{3p}$
	yx'	2, 3-2p	1+4p, $\underline{2}$
	yy'	3-p, 1	$\underline{4+p}$, $\underline{2p}$

Unique BNE: (yy', b)

(This BNE is consistent with part (a) of this exercise, where $= 0.75 > \frac{1}{2}$).