ECON 40050

Game Theory

Exam 1 - Answer Key

Instructions:

1) You may use a pen or pencil, a hand-held nonprogrammable calculator, and a ruler. No other materials may be at or near your desk. Books, coats, backpacks, etc... must be placed against the wall. No electronic communication devices may be used.

2) As soon as the instruction to begin the test is given, please check that you have 10 numbered pages.

3) Be sure to show all of your work. Answers without supporting calculations will receive zero credit. You will receive credit only for the answers and supporting calculations that appear in this test packet.

4) All exams must be turned in by 1:45 pm. No extensions will be granted.

5) Be sure to read each question in its entirety before beginning your analysis.

6) The time estimates at the beginning of each question are only suggestions to help you manage your time.

NAME ____

Question 1 (10 minutes)	_______	(15 points)
Question 2 (10 minutes)	_______	(15 points)
Question 3 (10 minutes)	_______	(15 points)
Question 4 (15 minutes)	_______	(15 points)
Question 5 (20 minutes)	_______	(20 points)
Total: (65 minutes)	_______	(80 points)

1. Define each of the terms in a-f. Do not use equations. Rather explain what each of these concepts means in words.

a. Strategy profile - **A list consisting of one strategy for each player. (2)**

b. Nash equilibrium - **A strategy profile for which each player's strategy is a best response to the profile of all other players' strategies. (2)**

c. Dominant strategy - **A strategy is dominant for a player if it is the player's best strategy no matter what strategy choices all other players make. (2)**

d. Dominated strategy - **A strategy is dominated for a player if there is a second strategy that does at least as well no matter what strategy choices other players make and, for some profile of strategies for all other players, the second strategy does strictly better. (2)**

e. Mixed strategy - **A mixed strategy involves a player randomizing over 2 or more pure strategies. (2)**

f. Reaction function - **A reaction function describes a player's optimal strategy as a function of the strategy choices of all other players. (2)**

g. What are the components of a strategic form game? **The set of players, the space of all possible strategy profiles, and a description of payoffs for each player as a function of the strategy profile. (3)**

2. The questions in this problem refer to the following game.

		Player 2		
		L	M	R
	U	1,2	3,5	2,1
Player 1	M	0,4	2,1	3,0
	D	-1,1	4,3	0,2

a. Determine if either player has any dominated strategies. If so, identify them.
R for player 2 is dominated by M. For each player 1 strategy, M gives player 2 a higher payoff than does R. (2)

b. Does either player have a dominant strategy? Why or why not?
No. For either player to have a dominant strategy, 2 of her 3 strategies would need to be dominated. (2)

c. Use iterated elimination of dominated strategies to solve this game. Be clear about the order in which you are eliminating strategies. Also specify whether you are eliminating strictly or weakly dominated strategies.
1. Eliminate R as above. (Strict) 2. In the 3 x 2 game, U strictly dominates M. 3. In the 2 x 2 game, M strictly dominates L. 4. In the 2 x 1 game, D strictly dominates U.
IEDS Solution = (D,M). (5)

d. Is your solution a Nash equilibrium? Why or why not?
Yes, all IEDS solutions are Nash equilibria. If starting at (U,M), one player had an alternative strategy that gave her a higher payoff, IEDS would not have eliminated that alternative strategy. (3)

e. Give an example of a game that has a pure strategy Nash equilibrium that cannot be found using iterated elimination of dominated strategies.
One example would be the Battle of the Sexes game. (3)

		Player 2	
		B	O
Player 1	B	2,1	0,0
	O	0,0	1,2

3. Consider the following strategic-form game.

		Player 2 L	M	R
Player 1	U	2,4	3,0	1,-1
	D	3,2	10,3	0,4

a. Find all pure- and mixed-strategy Nash equilibria of this game. Be sure to show all of your steps.

1. For any λ between 1/5 and 1/2, a strategy of playing L with probability λ and R with probability $1-\lambda$ strictly dominates M.

2. Show that the 2x2 game has no pure strategy Nash equilibrium.

2. Let p denote the probability with which player 1 plays U and let q denote the probability with which player 2 plays L.

3.

$$Eu_1(U) = 2q + 1 - q = 1+q$$
$$Eu_1(D) = 3q$$

$$Eu_2(L) = 4p + 2(1-p) = 2 + 2p$$
$$Eu_2(R) = -p + 4(1-p) = 4 - 5p$$

4. Player 1 will be willing to mix if $Eu_1(U) = Eu_1(D)$ or when q = 1/2.
Player 2 will be willing to mix if $Eu_2(L) = Eu_2(R)$ or when p=2/7.

5. There is one Nash equilibrium of this game: (p,q) = (2/7,1/2).

b. What is the strategic advantage of using a mixed strategy?

It keeps the other player(s) off balance. They cannot anticipate how you will play the game.

c. Show that one of player 2's strategies is never a best response to any of player 1's pure strategies. Show for this game that this strategy is dominated by a mixed strategy.

L is the best response to U and R is the best response to D. Thus, M is never a best response. See part (a), step 1.

Scoring

Set-up	**(2)**
No Pure Nash	**(2)**
Eu_1 and Eu_2 calculations	**(4)**
Solution	**(2)**
M dominated - $q_2 = 0$	**(1)**
part (b)	**(2)**
part (c)	**(2)**

4. If 2 spiders find a dead insect at the same time, each spider will make menacing gestures to scare off the other. If one spider backs down, that spider gets nothing and the other spider gets the insect to itself. If both spiders back down, they can share the insect. If neither backs down, the spiders will fight. The payoffs resulting from the fight depend on the sizes of the spiders and are described below.

		Spider 2	
		Back down	Fight
Spider 1	Back down	5,5	0,10
	Fight	10,0	x,y

a. Suppose the spiders are the same size so that x=y. For what values of x, will each spider have a dominant strategy? What is the dominant strategy? (Show your work)

Since Fight is a best response Back Down. Fight is the only candidate for a dominant strategy. Fight will be a best response to Fight as long as x=y > 0. (5)

b. Again, suppose the spiders are the same size. For what values of x, will this game be a Prisoners' Dilemma? (Show your work)

To be a Prisoners' Dilemma, the game must have a dominant strategy equilibrium and this equilibrium must be Pareto inferior to some other set of payoffs. If 0 < x=y < 5, then the dominant strategy equilibrium will be (Fight, Fight) but both spiders would be better off if they both played Back Down. (5)

c. Suppose when spider 1 is smaller than spider 2 that $x < 0 < y$. Show that this game does not have a dominant strategy equilibrium but that it can be solved using IEDS.

For spider 1, Back down is a best response to Fight so spider 1 does not have a dominant or a dominated strategy.

For spider 2, Fight will be a dominant strategy so we can eliminate Back Down for the larger spider.

In the resulting 2x1 game, Fight is strictly dominated by Back Down for the smaller spider.

Thus, IEDS yields the unique solution (Back Down, Fight). (5)

5. There are two firms that produce identical products. These firms compete in each of two periods. Each firm starts out with 100 units of capital. In period 1, each firm must decide how much capital to invest in research (k_1 for firm 1 and k_2 for firm 2). In period 2, the firm will use the rest of its capital to produce its product. One unit of capital generates one unit of the firm's product. If firm 1 produces q_1 units in period 2 and firm 2 produces q_2 units in period 2, then the market price will be $200 - q_1 - q_2$.

Period 1 research reduces a firm's second period production cost. Without any research, the firm will be able to produce at a constant marginal cost of $50/unit. With research, the firm 1's constant marginal cost of production will equal

$$50(200 - k_1 - .001k_1k_2)/200$$

and firm 2's constant marginal cost of production will equal

$$50(200 - k_2 - .001k_1k_2)/200.$$

a. Write down the strategic form of this game. **(5)**
N = {1,2}; $S_i = \{k_i \mid 0 \leq k_i \leq 100\}$

$$\pi_1(k_1,k_2)=(k_1+k_2)(100-k_1)-(100-k_1)(200-k_1-.001k_1k_2)/4$$

$$\pi_2(k_1,k_2)=(k_1+k_2)(100-k_2)-(100-k_2)(200-k_2-.001k_1k_2)/4$$

b. Calculate each firm's reaction function. **(4)** Plot these reaction functions on the same graph. **(3)**

$$\partial\pi_1(k_1,k_2)/\partial k_1 = 100-2k_1-k_2+(300-2k_1+.1k_2-.002k_1k_2)/4$$

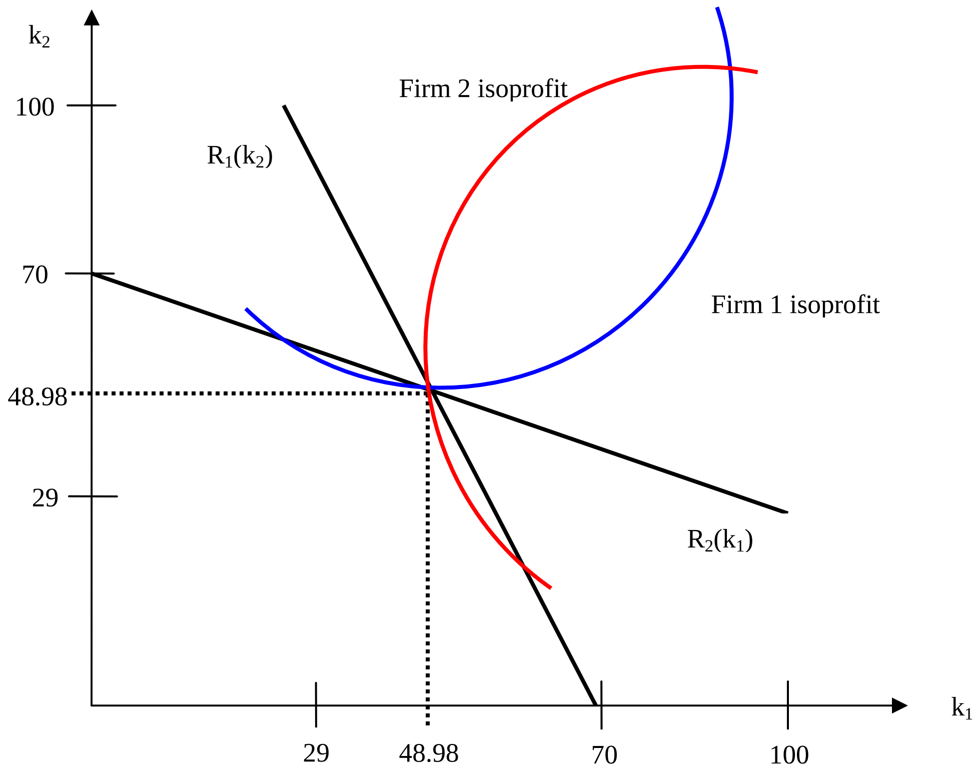
$$= 175-2.5k_1-.975k_2-.002k_1k_2$$

Setting this derivative equal to 0 implies $R_1(k_2)=(175-.975k_2)/(2.5+.002k_2)$**.**
By symmetry $R_2(k_1)=(175-.975k_1)/(2.5+.002k_1)$

c. Calculate the Nash equilibrium and locate it on your graph from part (b). **(3)**
The Nash equilibrium must satisfy $175-2.5k-.975k-.002k^2=0$ **or** $175-3.475k-.002k^2=0$.
Using the quadratic formula, the symmetric Nash equilibrium is $k_1 = k_2 = 48.98$.

d. Without calculating the cooperative solution directly, determine if the Nash equilibrium will result in too much or too little research in period 1?**(3)** Add the appropriate isoprofit curves to your graph from part (b) to support your answer. **(2)**

Since $\partial\pi_1/\partial k_2 = (100 - k_1)(1 + .001k_1/4) > 0$**, each firm's investment in research generates a positive externality for the other firm. This positive externality is not accounted for in each firm's profit-maximization calculations. Thus, the Nash equilibrium will result in too little research relative to the amounts that would maximize the firms' joint profit.**



k2
100
70
48.98
29
R1(k2)
Firm 2 isoprofit
Firm 1 isoprofit
R2(k1)
29
48.98
70
100
k1