Algorithmic Game Theory: Practice Midterm

Aaron Roth

The exam is closed book. Remember to write your name.

Solution Concepts

Order the following sets of solution concepts from smallest to largest: Correlated equilibria (CE), Pure strategy Nash equilibria (PSNE), Dominant strategy equilibria (DSE), Coarse correlated equilibria (CCE), Mixed strategy Nash equilibria (MSNE). Indicate which solution concepts are guaranteed to exist in every game.

Find a Nash equilibrium

1. Does the following game have any pure Nash equilibrium? If yes, find one. If no, find a mixed strategy Nash equilibrium.

	a	b	c	d
A	$3, 9$	$4, 8$	$3, 3$	$10, 2$
B	$4, 3$	$5, 4$	$4, 5$	$3, 4$
C	$3, 3$	$4, 5$	$5, 4$	$2, 3$
D	$2, 7$	$2, 8$	$4, 6$	$1, 4$

2. Describe all of the Nash equilibria in the following two player game, "Choose the Larger Number". $A_1 = A_2 = \{1, 2, 3, 4, 5\}$. $u_1(s_1, s_2) = 1$ if $s_1 > s_2$ and $u_1(s_1, s_2) = 0$ otherwise. Similarly, $u_2(s_1, s_2) = 1$ if $s_2 > s_1$ and $u_2(s_1, s_2) = 0$ otherwise.

Optimality

Consider the following statement:

> In a Nash equilibrium of a game, each player is simultaneously playing an optimal strategy, and so in any Nash equilibrium, the social welfare (sum of player utilities) is always maximized.

Is the statement correct or incorrect? If it is correct, provide a proof. If it is incorrect, provide a counterexample.

Correlated Equilibrium

Give an example of a 2 player game in which each player has 3 actions, that has a coarse correlated equilibrium that is not a correlated equilibrium. Describe the coarse correlated equilibrium.

Zero Sum Games

Suppose a 2-player zero-sum game has two distinct Nash equilibria: (s_1, s_2) and (s'_1, s'_2). Prove that (s'_1, s_2) and (s_1, s'_2) are also Nash equilibria of this game. *Hint: think about the minimax theorem.* Is this "exchange property" true in games that are not zero sum? Prove it or give a counter-example.