Econ 400, Final Exam

Name: ____

There are three questions taken from the material covered so far in the course. All questions are equally weighted. If you have a question, please raise your hand and I will come to your desk.

Make sure that you defend your answers with economic reasoning or mathematical arguments, and show that you are using the correct game theoretic concepts by identifying the equilibria explicitly.

Good luck.

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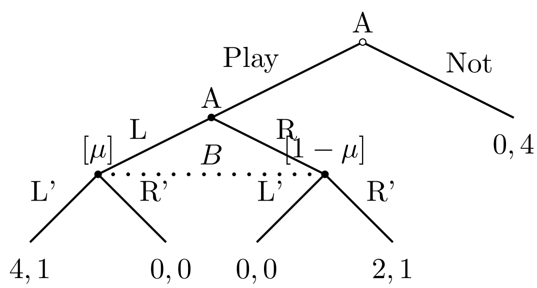
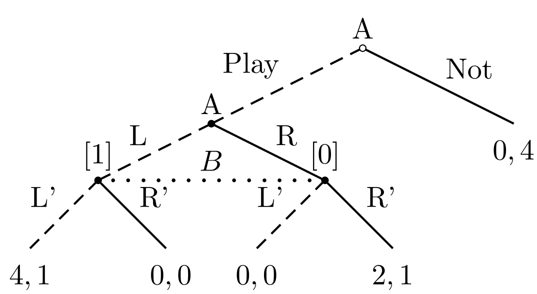
1. Consider the following game: Player A chooses whether to play or not; if A chooses to play, A then decides between action L and action R; B observes whether player A choose to play, but not what action A took, and then chooses between L' and R'.



i. Show there is a perfect Bayesian equilibrium where player A chooses to play, player A chooses L, and player B chooses L'.

Consider



Then we need to show the proposed strategies and beliefs satisfy Bayesian Beliefs and Sequential Rationality.

- Bayesian Beliefs:

$$\mu = pr[\text{A chose } L | \text{A chose Play}] = \frac{pr[\text{A chose } L \cap \text{A chose Play}]}{pr[\text{A chose Play}]} = \frac{1}{1} = 1$$

  So the proposed beliefs satisfy the Bayesian Beliefs criterion, given the strategies.

- Sequential Rationality:
  - Player B believes the expected payoff to L' is $\mu(1)+(1-\mu)(0) = 1$, and the expected payoff to R' is $\mu(0)+(1-\mu)(1) = 0$. So B has no profitable deviation from playing L'.
  - The expected payoff to player A from L is 4, while the expected payoff to player A from R is zero. So A has no profitable deviation from playing L
  - The expected payoff to player A from "Play" is 4, while the expected payoff to player A from "Out" is 0. So A has no profitable deviation from playing.

  So the proposed strategies are sequentially rational, given the beliefs.

ii. Show there is a perfect Bayesian equilibrium where player A chooses to player, player A mixes over L and R, and player B mixes over L' and R'.

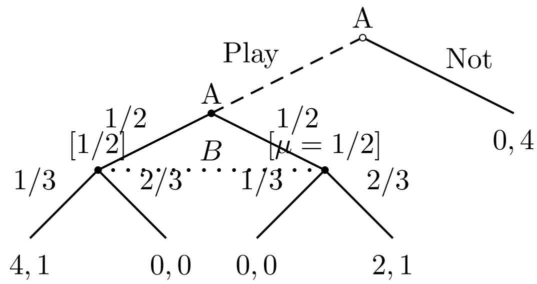
If we consider the subgame following A deciding to play as a simultaneous-move game, we can solve for the mixed-strategy equilibrium pretty easily:

		B	
		L'	R'
A	L	4,1	0,0
	R	0,0	2,1

Let σ_L be the probability that A uses L, and $\sigma_{L'}$ be the probability that B uses L'.

- A's strategy: The expected payoff to player B of L' is $\sigma_L(1)+(1-\sigma_L)(0)$, and the expected payoff to player B of R' is $\sigma_L(0)+(1-\sigma_L)(1)$. Equating and solving for σ_L gives $\sigma_L^* = 1/2$
- B's strategy: The expected payoff to player A of L is $\sigma_{L'}(4) + (1-\sigma_{L'})(0)$, and the expected payoff to player A of R is $\sigma_{L'}(0) + (1-\sigma_{L'})(2)$. Equating and solving for $\sigma_{L'}$ gives $\sigma_{L'}^* = 1/3$

Then we can propose strategies and beliefs



Now, to show this is a perfect Bayesian equilibrium, we check Bayesian Beliefs and Sequential Rationality:

- Bayesian Beliefs:

$$\mu = pr[\text{A chose } L|\text{A chose to play}] = \frac{pr[\text{A chose } L \cap \text{A chose to play}]}{pr[\text{A chose to play}]} = \frac{\sigma_L^*}{1} = \frac{1}{2}$$

  so the proposed beliefs are consistent with Bayes' rule.

- Sequential Rationality:
  - For player B the expected payoff to L' is $1/2$ and the expected payoff to R' is $1/2$. Since he is indifferent between his two pure strategies, any mixed strategy will give the same payoff. So player B has no profitable deviation from $\sigma_{L'} = 1/3$.
  - For player A the expected payoff to L is $4/3$ and the expected payoff to R is $4/3$. Since he is indifferent between his two pure strategies, any mixed strategy will give the same payoff. So player A has no profitable deviation from $\sigma_L = 1/2$.
  - Player A's expected payoff from entering is

$$\frac{1}{2}\frac{1}{3}4 + \frac{1}{2}\frac{2}{3}0 + \frac{1}{2}\frac{1}{3}0 + \frac{1}{2}\frac{2}{3}2 = \frac{4}{3} > 0$$

    so playing is better than not, and player A has no profitable deviation from playing.

So $\mu = 1/2$, player A chooses to play, player A mixes half-half over L and R, and player B plays L' with probability $1/3$ and R' with probability $2/3$ is a perfect Bayesian equilibrium of the game.

iii. Briefly explain which equilibrium you think is more likely to arise in practice, and why.

Some reasonable answers:

- Since play$/L/L'/\mu = 1$ is Pareto dominant ($4 + 1$ is greater than the payoffs from any other outcome), the players might coordinate on that.
- Since player B doesn't know what player A chose and gets the same payoff either way, the mixed equilibrium is more likely.

- It depends on the context. If the players have a past history of playing one equilibrium or the other (learning), or society has coordinated on a particular outcome (norms and mores), one equilibrium might be selected over the other, even if its less socially efficient.

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2. Consider the following strategic form game:

			B	
		L	C	R
	u	4,4	-1,5	-1,-1
A	m	5,-1	0,0	2,1
	d	-1,-1	1,2	0,0

i. Find all pure-strategy equilibria of the game. Find a mixed strategy equilibrium in which player A mixes over m and d, and the column player mixes over C and R .

The pure strategy Nash equilibria are (m, R) and (d, C) (do underlining). The mixed equilibrium has $\sigma_m = 2/3$, $\sigma_d = 1/3$, $\sigma_C = 2/3$ and $\sigma_R = 1/3$ (do the work on the exam).

ii. Consider the infinitely repeated game where the stage game is given above and players have a common discount factor $0 < \delta < 1$. Use the Nash threats folk theorem to show that, for δ sufficiently close to 1, there exists a Subgame Perfect Nash Equilibrium in which players use u and L in every period.

Consider the following trigger strategies:

- If the history is $(u, L), (u, L), ..., (u, L)$, play (u, L)
- After any other history, play (d, C)

Then, for the row player, the discounted payoff to cooperating is

$$4 + \delta 4 + \delta^2 4 + ... = \frac{4}{1-\delta}$$

and the expected payoff to *optimally* deviating is

$$5 + \delta 1 + \delta^2 1 + ... = 5 + \frac{\delta}{1-\delta}$$

Then cooperating is better than deviating if

$$\frac{4}{1-\delta} \geq 5 + \frac{\delta}{1-\delta}$$

So the minimum patience level that supports cooperation for the row player is $\delta \geq 1/4$

For the column player, the discounted payoff to cooperating is

$$4 + \delta 4 + \delta^2 4 + ... = \frac{4}{1-\delta}$$

and the expected payoff to *optimally* deviating is

$$5 + \delta 2 + \delta^2 2 + ... = 5 + \frac{\delta 2}{1-\delta}$$

Then cooperating is better than deviating if

$$\frac{4}{1-\delta} \geq 5 + 2\frac{\delta}{1-\delta}$$

So the minimum patience level that supports cooperation for the column player is $\delta \geq 1/3$

So as long as $\delta \geq 1/3$, the Nash threats folk theorem implies that the trigger strategies are a subgame Perfect Nash equilibrium of the infinitely repeated game.

iii. Can you construct another equilibrium using the Nash threats folk theorem that achieves a lower minimum δ to support cooperation? If so, briefly explain how it works and why it would achieve a lower δ. If not, explain why yours outperforms the others.

Yes. By using the mixed equilibrium, the players get a symmetric payoff of 2/3 after a deviation, rather than 1 or 2. Recall that the mixed eqm payoffs are

$$\frac{2}{3}\frac{1}{3}2 + \frac{1}{3}\frac{2}{3}1 + 0 = \frac{6}{9} = \frac{2}{3}$$

Since this punishment is worse, the payoff to deviating is

$$5 + \delta\frac{2}{3} + \delta^2\frac{2}{3} + \delta^3\frac{2}{3} + ... = 5 + \frac{2}{3}\frac{\delta}{1-\delta}$$

and the minimum δ is $3/13 < 1/4$.

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3. A new firm has recently entered a market. The incumbent, whose costs are known by all players to be $C(D) = cD$, is unsure of whether the new firm is fit or not. The entrant knows that it is fit — has costs $C(D) = c_L D$ with probability $c_L < c$— or unfit — has costs $C(D) = c_H D$ with $c_H > c$. The incumbent only knows that the entrant is fit with probability p and unfit with probability $1-p$. The two firms simultaneously choose prices, giving them demands

$$D_i(p_i, p_e) = 1 - p_i + \frac{1}{2}p_e$$

$$D_e(p_e, p_i) = 1 - p_e + \frac{1}{2}p_i$$

where

i. What are the firms' types? What is a Bayesian Nash Equilibrium of this game?

The incumbent has a single type, i, and the entrant has two types, H (for high cost) and L (for low cost). A Bayesian Nash equilibrium of the game is a set of strategies, p_i, p_H, p_L so that none of the player-types have an incentive to deviate.

ii. Solve for a Bayesian Nash equilibrium of the game.

The payoff to the high-cost entrant is

$$\pi_H = (1 - p_H + .5p_i)(p_H - c_H)$$

the payoff to the low-cost entrant is

$$\pi_L = (1 - p_L + .5p_i)(p_L - c_L)$$

and the payoff to the incumbent is

$$\pi_i = p\left\{(1 - p_i + .5p_H)(p_i - c)\right\} + (1 - p)\left\{(1 - p_i + .5p_L)(p_i - c)\right\}$$

Then a necessary condition for equilibrium is that each player-type be maximizing its payoff, given the strategies of the other player-types:

$$\frac{\partial \pi_H}{\partial p_H} = 1 - p_H + .5p_i - p_H + c_H = 0 \longleftarrow p_H = \frac{1 + .5p_i + c_H}{2}$$

$$\frac{\partial \pi_L}{\partial p_L} = 1 - p_L + .5p_i - p_L + c_L = 0 \longleftarrow p_L = \frac{1 + .5p_i + c_L}{2}$$

$$\frac{\partial \pi_i}{\partial p_i} = p\left\{1 - 2p_i + .5p_H + c\right\} + (1 - p)\left\{1 - 2p_i + .5p_L + c\right\} = 0$$

If we simplify the above equation,

$$\frac{\partial \pi_i}{\partial p_i} = 1 - 2p_i + c + \frac{p}{2}p_H + \frac{1-p}{2}p_L = 0$$

and substituting yields

$$\frac{\partial \pi_i}{\partial p_i} = 1 - 2p_i + c + \frac{p}{2}\frac{1 + .5p_i + c_H}{2} + \frac{1-p}{2}\frac{1 + .5p_i + c_L}{2} = 0$$

$$\frac{\partial \pi_i}{\partial p_i} = 1 - 2p_i + c + \frac{1}{4}p_i + \frac{p}{2}\frac{1 + c_H}{2} + \frac{1-p}{2}\frac{1 + c_L}{2} = 0$$

Now we solve the above equation for p_i, and we're essentially done:

$$p_i^* = \frac{7}{8}\left(1 + c + \frac{p}{4}(1 + c_H) + \frac{1-p}{4}(1 + c_L)\right)$$

And

$$p_H^* = \frac{1 + .5\frac{7}{8}\left(1 + c + \frac{p}{4}(1 + c_H) + \frac{1-p}{4}(1 + c_L)\right) + c_H}{2}$$

and

$$p_L^* = \frac{1 + .5\frac{7}{8}\left(1 + c + \frac{p}{4}(1 + c_H) + \frac{1-p}{4}(1 + c_L)\right) + c_L}{2}$$

That's the Bayesian Nash equilibrium of the game.

iii. How do the firm-type's strategies depend on p?

Well, notice that

$$p_H^* = \frac{1 + .5p_i^* + c_H}{2}$$
$$p_L^* = \frac{1 + .5p_i^* + c_L}{2}$$

so that if p_i^* is increasing or decreasing in p, so are p_H and p_L.

$$\frac{\partial p_i^*}{\partial p} = \frac{7}{8}\left(\frac{1+c_H}{2} - \frac{1+c_L}{2}\right) = \frac{7}{8}\left(\frac{c_H - c_L}{2}\right) \geq 0$$

so that if $p \uparrow$, $p_i^* \uparrow$, implying that $p_H^* \uparrow$ and $p_L^* \uparrow$.

iv. If the entrant had to pay a fixed cost F to enter the market, explain how we could model the entry decision in a two-stage game, where the entrant first decides whether to enter or not, and then the two firms compete as above. How would the entrant's decision of whether to enter or not depend on p?

I give two reasonable answers below. You might have done something else. The goal is for your answer to be well-reasoned and make sense with respect to what we've worked on in class.

The profits to the two entrant types, conditional on entry, is

$$\pi_H^* = (1 - p_H^* + .5p_i^*)(p_H^* - c_H) - F$$

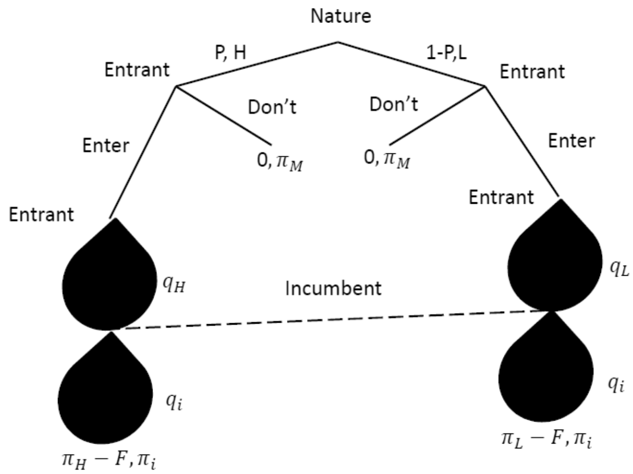
for the high cost entrant and

$$\pi_L^* = (1 - p_L^* + .5p_i^*)(p_L^* - c_L) - F$$

for the low cost entrant. Let π_m^* be the monopoly profits to the incumbent if the entrant does not enter.

Then you basically have an important choice to make: Does the entrant learn its type before deciding to enter, or after?

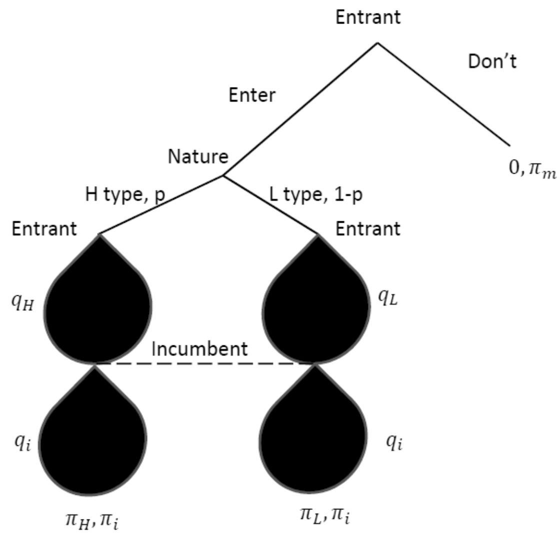
- If you choose before, then the firm's decision to enter acts as a signal that might reveal its type. Consider the extensive form:



Entry works likes a signal! If the low type enters but the high type doesn't, this ends up being a separating equilibrium. If both types enter, it's a pooling equilibrium.

So we might have two equilibria: the entrant-types pool and pay F to enter, and then the Bayesian game we analyzed above occurs (since the choice to enter doesn't reveal anything about the firm). But if the equilibrium is separating, so that only low-cost firms enter and high-cost firms stay out, the incumbent will realize that any entrant must be fit, and the game will become complete information Cournot with asymmetric costs. So in the separating equilibrium, nothing depends on p, while in the pooling equilibrium, p will play the same role as in the work in part i-iii.

- If you choose after, the decision about whether to enter or not doesn't reveal the firm's type, since



Here, since the entrant doesn't learn his type until after he's entered, so the decision of whether to enter or not doesn't act as a signal.

So the profits generated by entry — before the entrant learns its type — must be greater than F. As p goes up, the entrant is more likely to be high-cost and unfit, and it is more likely that the entrant will choose to stay out and avoid the fixed cost followed by being clobbered by the incumbent.

If you think that a "type" is the innovation that the entrant brings to the table (perhaps a better product design or manufacturing process), the first model is more appropriate. If you think that the "type" is realized by the entrant building its factory or hiring the best workers it can find (something that happens after it has committed to entry), then the second model is more appropriate.