Econ 414, Exam 1

Name: ____

There are three questions taken from the material covered so far in the course. All questions are equally weighted. If you have a question, please raise your hand and I will come to your desk.

Make sure that you defend your answers with economic reasoning or mathematical arguments, and show that you are using the correct game theoretic concepts by identifying the equilibria explicitly.

Good luck.

1.
Consider the following game in strategic form:

				B	
		w	x	y	z
	a	4,2	-3,3	0,4	2,-1
	b	2,0	2,-1	-1,3	8,2
A	c	5,-2	-2,2	1,0	7,-4
	d	4,10	-5,4	0,5	3,3

i. Perform iterated deletion of *strictly* dominated strategies (Strike the dominated strategy out with a line, writing the letter of the dominant strategy beside it).

- y strictly dominates z
- c strictly dominates d, c strictly dominates a
- y strictly dominates w

ii. Find all pure-strategy Nash equilibria, or explain why one doesn't exist.
There are no pure-strategy Nash equilibria: For all the remaining profiles, there isn't a profile where players use mutual best-responses:

	x	y
b	$\underline{2}$,-1	-1,$\underline{3}$
c	-2,$\underline{2}$	$\underline{1}$,0

iii. If there is a mixed-strategy Nash equilibrium, find the equilibrium strategy for the row player.
The row player chooses her strategy to make the column player indifferent over her expected pay-offs:

$$\mathbf{E}u_{col}(x) = \mathbf{E}u_{col}(y)$$

$$\sigma_b(-1) + \sigma_c(2) = \sigma_b(3) + \sigma_c(0)$$

$$\sigma_c = 2\sigma_b$$

Then we can use the equations $\sigma_c = 2\sigma_b$ and $\sigma_b + \sigma_c = 1$ to solve for σ_c and σ_b independently:

$$2\sigma_b + \sigma_b = 1 \longrightarrow \sigma_b^* = \frac{1}{3}$$

And $\sigma_c^* = \dfrac{2}{3}$

2. Price Competition

There are consumers located uniformly on $[0, 1]$, so between any two points $0 < a < 1$ and $0 < b < 1$ with $b > a$, there are $b - a$ consumers there. There is a firm located at 0, who charges price p_0, and a firm located at 1, who charges price p_1. Consumers have preferences as

$$u(x,p) = \begin{cases} v - tx - p_0 & \text{, Bought at 0} \\ v - t(1-x) - p_1 & \text{, Bought at 1} \end{cases}$$

The firm located at zero has total cost $C_0(q) = c_0 q$, while the firm located at one has total cost $C_1(q) = c_1 q$, with $c_1 > c_0$.

i. Find the demand for each firm's product as a function of the prices they charge.
The consumer indifferent between firm 0 and firm 1 is the customer who splits the market; everyone to his left goes to zero and everyone to his right goes to one. Let's call him $\hat{x}$. Then

$$v - t\hat{x} - p_0 = v - t(1-\hat{x}) - p_1$$

$$p_1 - p_0 + t = 2t\hat{x}$$
$$\hat{x} = \frac{1}{2} + \frac{p_1 - p_0}{2t}$$

So the demand for firm zero is

$$D_0(p_0, p_1) = \frac{1}{2} + \frac{p_1 - p_0}{2t}$$

and the demand for firm one is

$$D_1(p_1, p_0) = \frac{1}{2} + \frac{p_0 - p_1}{2t}$$

ii. Write the firms' profit maximization problems, solve for best-response functions, and graph the best-response functions. Does the game have strategic complements or substitutes?

Profit maximization problems

$$\pi_0(p_0, p_1) = \text{Total Revenue} - \text{ Total cost} = D_0 p_0 - c_0 D_0 = D_0(p_0 - c_0)$$

$$\pi_0(p_0, p_1) = \left[\frac{1}{2} + \frac{p_1 - p_0}{2t}\right](p_0 - c_0)$$

and the profit function for firm one is

$$\pi_1(p_1, p_0) = \left[\frac{1}{2} + \frac{p_0 - p_1}{2t}\right](p_1 - c_1)$$

Best Response Functions
Let's focus on firm zero. Maximize their profit with respect to p_0, taking p_1 as given:

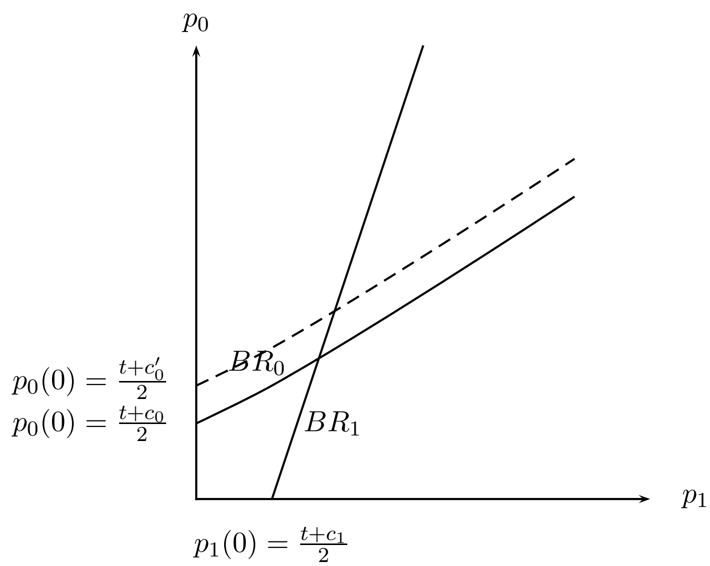
$$\frac{\partial \pi_0}{\partial p_0} = \frac{1}{2} + \frac{p_1 - p_0}{2t} - \frac{p_0 - c_0}{2t}$$

Set this equal to zero and solve for the BR function:

$$0 = \frac{1}{2} + \frac{p_1 - p_0}{2t} + \frac{-p_0 + c_0}{2t} \longrightarrow p_0(p_1) = \frac{t + p_1 + c_0}{2}$$

Now, notice that $p_0(p_1)$ is increasing in p_1 (and $dp_0/dp_1 = 1/2 > 0$). That means the game has strategic complements: If my opponent increases her strategy, I increase mine.

A good graph looks like



iii. Solve for the Nash equilibrium in prices.
Plugging $p_0(p_1)$ into $p_1(p_0)$ gives:

$$p_1(p_0(p_1)) = \frac{t + p_0(p_1) + c_1}{2} = \frac{t + c_1}{2} + \frac{p_0(p_1)}{2}$$

$$p_1 = \frac{t + c_1}{2} + \frac{1}{2}\left[\frac{t + c_0}{2} + \frac{p_1}{2}\right]$$

Solve this equation for p_1^*:

$$p_1(1 - \frac{1}{4}) = \frac{2(t + c_1) + t + c_0}{4}$$

$$p_1\frac{3}{4} = \frac{3t + 2c_1 + c_0}{4} \longrightarrow p_1^* = \frac{2c_1 + c_0}{3} + t$$

Then $p_0^* = \dfrac{2c_0 + c_1}{3} + t$.

iv. How does a change in c_0 affect the price that the firm located at 1 charges?
You can see that in the graph, when I raised c_0 to c_0', the best response curve shifted up, and the prices of both players increased. Doing this graphical analysis correctly is enough to get full credit on this part. Also, you can use calculus or make arguments about the equilibrium prices:

$$\frac{dp_1^*}{dc_0} = \frac{1}{3} > 0$$

3. Mixed Strategy Equilibrium

There are two firms deciding whether to enter a market or not. The market for their good is extremely small, so it can only profitably support one firm in the long run. Here are costs and benefits:

- π_d — The discounted profits from being a duopolist
- π_m — The discounted profits from being a monopolist
- E — The cost of entry

Here's a strategic form:

			Firm B
		Enter	Don't
	Enter	$\pi_d - E, \pi_d - E$	$\pi_m - E, 0$
Firm A	Don't	$0, \pi_m - E$	$0, 0$

i. What do you have to assume about π_d, π_m and E to ensure that (a) firms prefer to enter as monopolists rather than duopolists and (b) firms prefer not to enter if their opponent does.
For this model to make sense, it's got to be true that entering as a monopolist is better than as a duopolist: $\pi_m - E > \pi_d - E \to \pi_m > \pi_d$, and firms prefer not to enter if their opponent does, so $0 > \pi_d - E$, or $E > \pi_d$.

ii. Solve for all pure or mixed Nash equilibria.
Let's write in our best responses:

			Firm B
		Enter	Don't
	Enter	$\pi_d - E, \pi_d - E$	$\underline{\pi_m - E}, \underline{0}$
Firm A	Don't	$\underline{0}, \underline{\pi_m - E}$	$0, 0$

So there are two pure-strategy Nash equilibria: $(don't, enter)$ and $(enter, don't)$. If $\sigma_E = Pr[\text{Firm enters}]$ and $\sigma_D = Pr[\text{Firm doesn't enter}]$, then

$$\sigma_E(\pi_d - E) + \sigma_D(\pi_m - E) = 0$$

$$\sigma_D = \sigma_E \frac{E - \pi_d}{\pi_m - E}$$

This equation and $\sigma_D + \sigma_E = 1$ yields

$$\sigma_E \frac{E - \pi_d}{\pi_m - E} + \sigma_E = 1 \longrightarrow \sigma_E^* = \frac{\pi_m - E}{\pi_m - \pi_d}$$

iii. How does an increase in E affect the probability that either of the firms enter?
From the last step, if E goes up, σ_E^* goes down, since it enters the numerator negatively. Or,

$$\frac{d\sigma_E^*}{dE} = \frac{-1}{\pi_m - \pi_d} < 0$$