14.12 Game Theory
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Homework 6

Due on 12/7/2005 (in class)

1. (This question asks you to compute the equilibrium of a game that you played in the class.) There are two players, namely 1 and 2, who commonly own an object. The value of the object for each player i is v_i, where v_1 and v_2 are independently and identically distributed with uniform distribution on $[0,1]$. The value v_i is privately known by player i. Simultaneously, each player i bids a real number b_i. The player who bids the higher number "wins" the object; if the bids are identical, we toss a coin to determine the "winner". The winner i gets the object and pays the other player b_i. (If i wins, then $u_i = v_i - b_i$ and $u_j = b_i$.)

   (a) Find a symmetric, linear Bayesian Nash equilibrium, where $b_i(v_i) = a + cv_i$ for some constants a and c.

   (b) Consider any symmetric Bayesian Nash equilibrium, where the bid of v_i is $b(v_i)$ for a strictly increasing, differentiable function b. Find a differential equation that must be satisfied by b.

   (c) (Bonus, 5 points) Solve the differential equation.

   (d) (optional[1]) What did you bid in the quiz and what was your value?

2. (This question is also about a game you played in the class.) There are n students in the class. We have a certificate, whose value for each student i is v_i, where v_i is privately known by student i and $(v_1, \ldots, v_n)$ are independently and identically distributed with uniform distribution on $[0, 100]$. Simultaneously, each student i bids a real number b_i. The player who bids the highest number "wins" the certificate; if there are more than one highest bids, then we determine the "winner" randomly among the highest bidders. The winner i gets the certificate and pays b_i to the professor. [Hint: $\Pr(\max_{j\neq i} v_j \leq x) = (x/100)^{n-1}$ for any $x \in [0, 100]$.]

   (a) Find a symmetric, linear Bayesian Nash equilibrium, where $b_i(v_i) = a + cv_i$ for some constants a and c.

   (b) What is the equilibrium payoff of a student with value v_i?

   (c) Assume that $n = 80$. How much would a student with value v_i be willing to pay (in terms of lost opportunities and pain of sitting in the class) in order to play this game? What is the payoff difference between the luckiest student and the least lucky student?

3. A soda company, XC, introduces a new soda and wants to sell it to a representative consumer. The soda may be either Good or Bad. The prior probability that it is

[1] "Optional" means that you won't get any point for your answer.

Good is 0.6. Knowing whether the soda is Good or Bad, the soda company chooses an advertisement level for the product, which can be either an Ad Blitz, which costs the company c, or No Advertisement, which does not cost anything. Observing how strongly the company advertises the soda, but without knowing whether the soda is Good or Bad, the representative consumer decides whether or not to buy the product. After subtracting the price, the payoff of representative consumer from buying the soda is 1 if it is Good and -1 if it is Bad. His payoff is 0 if he does not buy the soda. If the soda is Good and representative consumer buys it (and therefore learns that the soda is Good), then the company sells the soda to other future consumers, enjoying a high revenue of R. If the soda is Bad and the representative consumer buys it, the company will have only a small revenue r. If the representative consumer does not buy the soda, the revenue of the company is 0. Assume that $0 < r < c < R$.

(a) Write this as a formal game. (It suffices to draw the game tree.)

(b) Find a "separating" perfect Bayesian Nash equilibrium, where different types of XC play different actions. (Verify that it is a perfect Bayesian Nash equilibrium.)

(c) Find a "pooling" perfect Bayesian Nash equilibrium, where all types of XC play the same action. (Verify that it is a perfect Bayesian Nash equilibrium.)

(d) Find an equilibrium for the case that $0 < c < r < R$.

4. Gibbons Problem 4.10

5. (Optional—This question is not part of the problem set. Try to solve it if you like challenges!) In Question 2, suppose that each student has the option of getting C without participating the game, where $0 < C < 100$. In particular, we have the following order of events.

- Each student i learns his/her value v_i.
- Then, each student decides on whether to participate.
- Then, observing which students participate, they submit their bids.

Find a symmetric perfect Bayesian Nash Equilibrium of this game.