Summer 2007 Final – Solutions

1. *Extensive game of Imperfect Information.*
    a. The game has 2 subgames: the subgame following player 1's choice of D, and the whole game.
    b. Player 1 has four strategies (A,B,C,D) and player 2 also has four strategies: (QX,QY,RX,RY). The strategic form is as follows:

	QX	QY	RX	RY
A	0,0	0,0	0,1	0,1
B	0,1	0,1	0,0	0,0
C	0,4	0,4	0,0	0,0
D	0,1	0,3	0,1	0,3

So the game has 8 Nash Equilibria:

NE={(A,RX); (A,RY); (B,QX); (B,QY);
(C,QX); (C,QY); (D,QY); (D,RY)}

   c. Note the only subgame besides the game itself starts following player 1's choice of D. So player 2 must act optimally from that subgame – ie, player 2 must choose Y following D. Thus the set of Subgame Perfect NE is:

SPNE={ (A,RY); (B,QY); (C,QY); (D,QY); (D,RY) }
(Just eliminate all those equilibria that involve player 2 choosing X).

2. *Beer and Quiche.*
    a. See lectures notes.

    b. Pooling on Quiche means p=0.1. Dueling after seeing the Quiche signal yields a receiver an expected payoff of 1(0.1) + (-1)(0.9) = -0.8. Not Dueling after seeing Quiche yields zero in expectation so a receiver will Not Duel if he sees Quiche. A wimpy sender gets his highest payoff (3) from following this strategy so would never deviate. A surly sender requires that a receiver, having seen Beer, will choose to Duel (otherwise, a surly sender would want to deviate from choosing Quiche and getting 2 versus choosing Beer and getting 3). Thus we need E[Duel| Beer] >= E[Not Duel |Beer] or q-(1-q) >= 0. Thus q >= ½. So our PBE is
{ (Quiche, Quiche), (Not Duel, Duel), (p,1-p), (q,1-q) | p=0.1, q>= ½ }

    c. Separating (Quiche, Beer) is not a PBE because with p = 1 and q = 0, a receiver's optimal strategy will be (Duel, Not). With these strategies, a wimpy sender gets a payoff of 1, but can deviate and play Beer and get a payoff of 2. So **no PBE** of this type exists.

3. *Advertising.*

$$
\begin{array}{cccc}
 & & \text{B} & \\
 & & \text{advertise} & \text{not advertise} \\
\text{A} \quad \text{advertise} & & 5,5 & 10,3 \\
\text{not advertise} & & 3,10 & a,a \\
\end{array}
$$

a) Note , in the stage game above, advertise is a strictly dominant strategy for both A and B. So if the game is played just once both firms will advertise.

b) Suppose B is playing a trigger a strategy. Want to find range for _for which it is best for A to also play a trigger strategy.

A has two strategies (i) play trigger strategy

(ii) deviate, in which case she will play advertise forever

Expected payoff if she plays trigger strategy :

$$
\begin{aligned}
\pi_a^c &= a + a(.9) + a(.9)^2 + a(.9)^3 + \ldots\ldots \\
&= a(1 + (.9) + (.9)^2 + (.9)^3 + \ldots\ldots) \\
&= a\left(\frac{1}{1-.9}\right) \\
&= a\left(\frac{1}{.1}\right) \\
&= 10a
\end{aligned}
$$

If A deviates, she earns 10 in the first period and then 5 in all other periods. A's expected payoff from deviating :

$$
\begin{aligned}
\pi_a^d &= 10 + 5(.9) + 5(.9)^2 + 5(.9)^3 + \ldots. \\
&= 10 + 5(.9)[1 + (.9) + (.9)^2 + (.9)^3 + \ldots.] \\
&= 10 + 4.5\left(\frac{1}{1-.9}\right) \\
&= 10 + 4.5\left(\frac{1}{.1}\right) \\
&= 10 + 45 \\
&= 55
\end{aligned}
$$

Trigger strategy equilibrium requires

$$
\begin{aligned}
\pi_a^c &\geq \pi_a^d \\
10a &\geq 55 \\
a &\geq 5.5
\end{aligned}
$$

So for a> =5.5 these trigger strategies constitute a sub-game perfect equilibrium.

4. Repeated Bertrand

    a. Consider the monopolist's problem:

$\text{Max}(Q) \quad \{ Q(\alpha - Q - \beta) \}$

$\text{FOC}(Q): \alpha - 2Q - \beta = 0 \rightarrow Q^m = \frac{1}{2}(\alpha - \beta)$

$\rightarrow p^m = \alpha - \frac{1}{2}(\alpha - \beta) = \frac{1}{2}(\alpha + \beta)$

$\rightarrow \Pi^m = \frac{1}{2}(\alpha - \beta) * [\frac{1}{2}(\alpha + \beta) - \beta]$

$\qquad = \frac{1}{2}(\alpha - \beta) * [\frac{1}{2}(\alpha - \beta)] = \frac{1}{4}(\alpha - \beta)^2$

    b. Note that with $\alpha = 8$ and $\beta = 2$, then $\Pi^m = 9$ and $1/3 * \Pi^m = 3$. Thus, along the equilibrium path, ie when firms cooperate and charge the monopoly price and split the monopoly profit, they earn:

$$\Pi^e = 3*(1 + \delta + \delta^2 + \delta^3 + \dots) = 3/(1-\delta)$$

By deviating, they could earn (very close) to the monopoly profit for one period and then, once the punishment of Bertrand Nash pricing sets in, they each set price equal to marginal cost (β), and have zero profit forever. Therefore deviation path payoffs are:

$$\Pi^d = 9 + 0*(\delta + \delta^2 + \delta^3 + \dots) = 9$$

So we require $\Pi^e \geq \Pi^d \rightarrow 3/(1-\delta) \geq 9 \rightarrow \delta \geq 2/3$.

    c. In general, Bertrand competition is more fierce than Cournot competition because of the assumption that a firm will steal the entire market by charging the lowest price. In a duopoly, firms undercut each other down to their marginal costs and earn no profits (ie the competitive outcome) in equilibrium. This is not true in the Cournot world where firms make positive profits in equilibrium.

Thus, by switching from Bertrand to Cournot, the equilibrium path payoff stayed the same (the joint monopoly profits), and the punishment phase payoff would have increased from zero to something positive, we would expect the critical discount factor to RISE. We require more patience of players to prevent a deviation.

However, it is important to note that the one period deviation payoff also changed under the Cournot setting. Firms no longer

can undercut on price and steal the market so their one period deviation payoff is smaller than in the Bertrand world. This may have the effect of lowering delta. However, mathematically, the long-term increases in the punishment phase payoff dominates, and delta will be higher for the demand and cost functions specified.

Full credit was obtained for providing an explanation and justification for your prediction. Determining the correct direction of the change in delta was not necessary.