EconS 424 – Midterm #1 - Answer Key

Exercise 1 – IDSDS, psNE and msNE

ANSWER: Note that c strictly dominates a and y strictly dominates z Thus, any Nash equilibrium in mixed strategies must assign zero probability to those dominated strategies. We can then eliminate them, so the reduced game is as shown in FIGURE SOL7.6.1.

FIGURE SOL7.6.1

Player 2

Player 1		x	y
	b	5,1	2,3
	c	3,7	4,6
	d	4,2	1,3

For this reduced game, b strictly dominates d, so the latter can be deleted. The reduced game is as shown in FIGURE SOL7.6.2.

FIGURE SOL7.6.2

Player 2

Player 1		x	y
	b	5,1	2,3
	c	3,7	4,6

This game has no pure-strategy Nash equilibria. To find the mixed-strategy Nash equilibria, let p denote the probability that player 1 chooses b and q denote the probability that player 2 chooses x. The equilibrium conditions ensuring that players want to randomize are

The expected payoffs for player 1 are:

$$EU_1(b) = q(5) + (1-q)(2) = 2 + 3q$$

$$EU_1(c) = q(3) + (1-q)(4) = 4 - q$$

For player 1 to be indifferent between strategies b and c, must be true that:

$$EU_1(b) = EU_1(c)$$

Thus

$$2 + 3q = 4 - q$$

$$q = 1/2$$

The expected payoffs for player 2 are:

$$EU_2(x) = p(1) + (1 - p)(7) = 7 - 6p$$

$$EU_2(y) = p(3) + (1 - p)(6) = 6 - 3p$$

For player 2 to be indifferent between strategies *x* and *y*, must be true that:

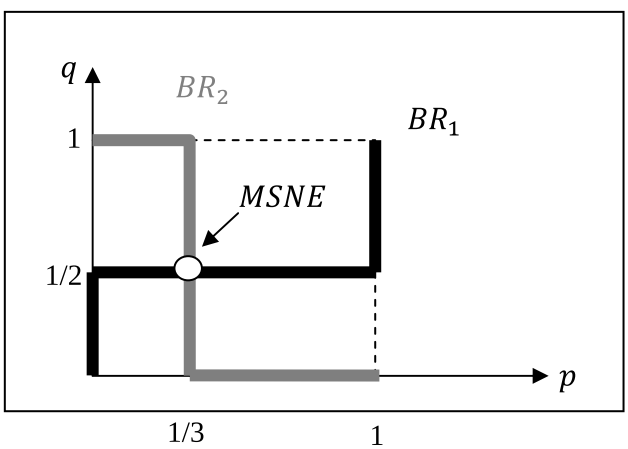
$$EU_2(x) = EU_2(y)$$

Thus

$$7 - 6p = 6 - 3p$$

$$p = 1/3$$

Graphically:



For player 1 when $q > 1/2$ is better to play b, thus $p = 1$, and play *c* otherwise.

Intuitively, if player 2 is very likely to play x, (which occurs when $q = 1$ as indicated in the first column of the matrix), player 1 is better off by responding selecting b (with a payoff of 5) rather than c (with a payoff of 3).

For player 2 when $p > 1/3$ is better to play y, thus $q = 0$, and play c otherwise.

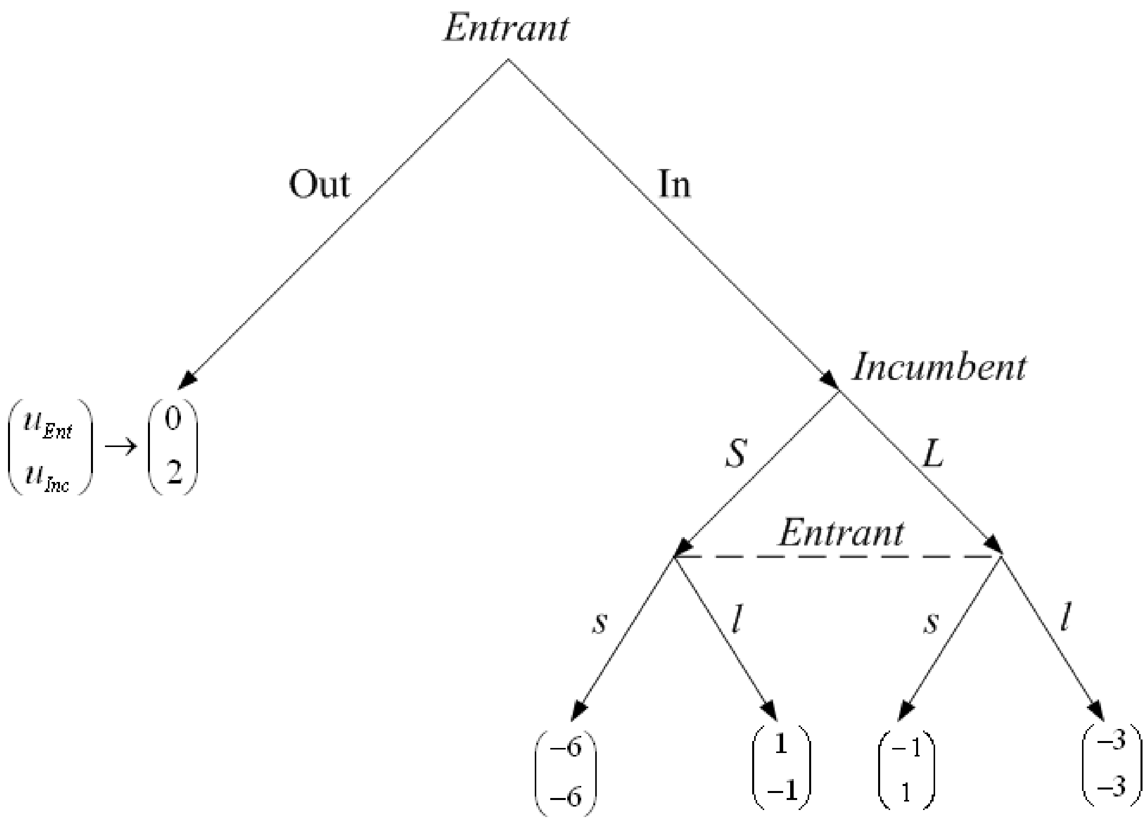
Intuitively, if player 1 is very likely to play b, (which occurs when $p = 1$ as indicated in the first row of the matrix), player 2 is better off by responding with y (with a payoff of 3) rather than x (with a payoff of 1).

Finally the Mixed Strategy Nash Equilibrium is given by:

$$MSNE = \{\big((0)a, (1/3)b, (2/3)c, (0)d\big), \big((1/2)x, (1/2)y, (0)z\big)\}$$

Exercise 2 - Entry deterrence.

Consider the following extensive form game. It represents the entry-deterrence game that we discussed in class, but with a slight modification. The entrant firm decides whether to enter the market where an incumbent is currently operating, or to remain out of the market. If the entrant decides to enter, then a simultaneous move game is played, where both the entrant and the incumbent must decide whether they will take either a small or large niche of the market.



a) Operating by backwards induction, firstly find all the Nash equilibria for the subgame initiated after the entrant firm decides to enter (simultaneous move game). Consider both the pure strategy Nash equilibria and the mixed strategy Nash equilibrium.

Notice that the subgame induced after the entrant decides to enter can be represented in its normal form representation (since it is a simultaneous move game) as follows:

		Entrant	
Incumbent		*Small, s*	*Large, l*
	Small, S	-6,-6	-1,1
	Large, L	1,-1	-3,-3

This game has two equilibria in pure strategies: (Large, Small) and (Small, Large).

In addition, there exists a mixed strategy Nash equilibrium, where the entrant randomizes with a probability q that makes the incumbent indifferent between choosing a Small or Large niche:

$$EU_I(\text{Small}) = EU_I(\text{Large}), \text{ that is}$$

-6q+(-1)(1-q)=1q+(-3)(1-q), and rearranging we obtain

2=9q

which implies that the entrant randomizes using a probability q=2/9

And similarly, the incumbent chooses a probability p such that the entrant is indifferent between selecting Small or Large. That is,

EU_E(Small)= EU_E(Large), that is

6p+(-1)(1-p)=1p+(-3)(1-p), and rearranging we obtain

2=9p

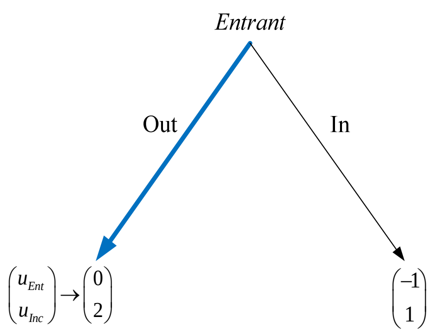
which implies that the incumbent randomizes with probability p=2/9

Then, the mixed strategy Nash equilibrium for this subgame is

{(2/9Small, 7/9Large),(2/9Small 7/9 Large)}

b) Once you have identified all the equilibria for the proper subgame where firms compete in what niche they will capture, find the subgame perfect Nash equilibria of the entire game. Notice that you just need to take into account the utility resulting from playing each of the possible Nash equilibria of the subgame, and then check what is optimal for the entrant to do (either In or Out). *Hint: there are three different SPNE.*
Let's analyze each of the different SPNE separately:

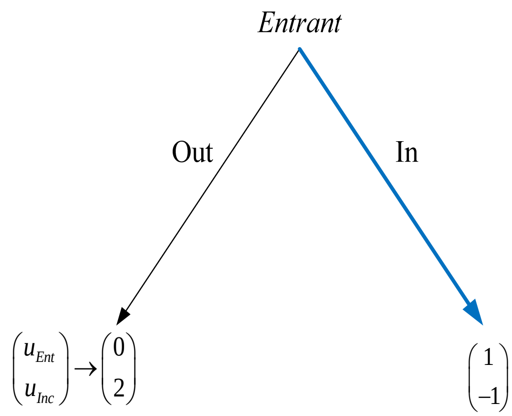
1. If the subgame is played using the pure strategy Nash equilibrium (Large, Small), then the corresponding payoff vector is (1,-1). That is, the Entrant is obtaining a payoff level from the psNE of this subgame of -1. Therefore, the Entrant prefers to remain out of the market (obtaining a payoff of 0) rather than entering and obtaining -1.



The first SPNE is then,

{Out, (Large, Small)}

2. If the subgame is played using the pure strategy Nash equilibrium (Small, Large), then the corresponding payoff vector is (-1,1). That is, the Entrant is obtaining a payoff level from the psNE of this subgame of 1. Therefore, the Entrant prefers to enter into the market (obtaining 1) rather than remaining outside (and obtaining a payoff of 0).



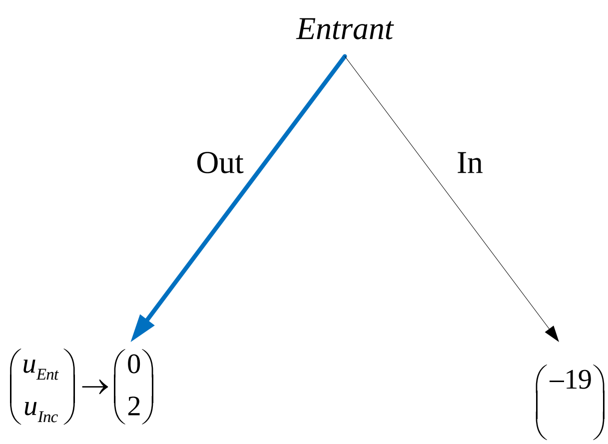
The second SPNE is then,

{In, (Small, Large)}

3. If the subgame induced after the Entrant decided to enter is played using the mixed strategy {(2/9Small, 7/9Large),(2/9Small 7/9 Large)}, then the corresponding expected utility level for the Entrant from playing such a mixed strategy Nash equilibrium in this subgame is

$$\frac{2}{9}\left((-6)\frac{2}{9}+1\frac{7}{9}\right)+\frac{7}{9}\left((-1)\frac{2}{9}+(-3)\frac{7}{9}\right)=-19$$

Therefore, the expected utility level from playing such mixed strategy Nash equilibrium for the entrant is so low, that it is better off by not entering into the market.



Then, the third SPNE is given by

{Out, {(2/9Small, 7/9Large),(2/9Small 7/9 Large)}}

Exercise 3 – Best response functions

(a) Fortune teller β takes A_α as given and chooses A_β to maximize his or her profit. The first-order condition with respect to A_β is given by $0 = \frac{\partial \pi_\beta}{\partial A_\beta} = \frac{30A_\alpha}{\left(A_\beta\right)^2} - 1$.

The second-order condition can be easily verified (i.e., differentiating with respect to A_β again, we obtain a negative expression, which confirms that the profit function is concave with respect to A_β).

Setting the above first order condition equal to zero we obtain $\frac{30A_\alpha}{\left(A_\beta\right)^2} = 1$

Rearranging, we have $30A_\alpha = \left(A_\beta\right)^2$

We can now solve for A_β to $A_\beta = R_\beta\left(A_\alpha\right) = \sqrt{30A_\alpha}$. Graphically, this is a concave function (see dashed curve below in blue color).

By symmetry, fortune teller α has a similar best-response function:

$$A_\alpha = R_\alpha\left(A_\beta\right) = \sqrt{30A_\beta} \ .$$

The two best-response functions are drawn in Figure below. Note that, since we are plotting fortune teller α 's best response function using the same axis, his best response function looks convex, but it is actually concave with respect to its argument, A_β, i.e., rotate the figure and you will see that $R_\alpha\left(A_\beta\right)$ is indeed concave with respect to A_β.

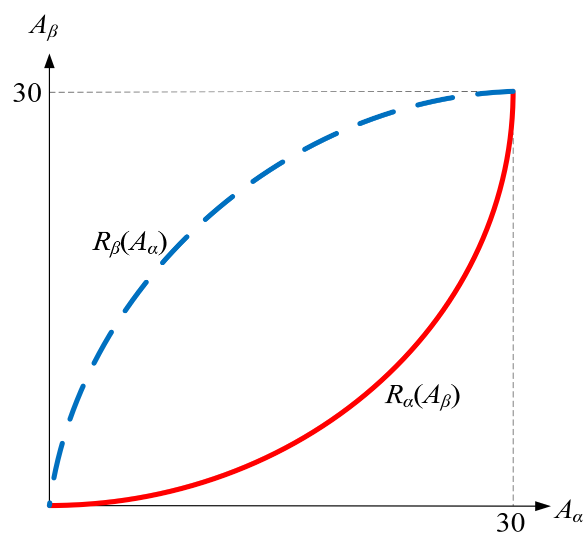


Figure: Fortune tellers' advertising best-response functions

(b) The figure above illustrates that the two best response functions intersect twice, reflecting the fact that there exist **<u>two</u>** Nash equilibria for this advertising game. $A_\alpha = A_\beta = 0$ constitutes one NE. To obtain the other NE, substituting $R_\alpha(A_\beta)$ into R_β yields $A_\beta = \sqrt{30\sqrt{30A_\beta}}$. Hence, we can now solve for A_β as follows $\left(A_\beta\right)^4 = 30^2\left(30A_\beta\right)$, which simplifies to $\left(A_\beta\right)^3 = 2,700$, and therefore $A_\beta = \sqrt[3]{2,700} = 30$. We hence obtain $A_\alpha^N = A_\beta^N = 30$ which is also illustrated in the Figure above.