Econ 400, Final Exam

Name: ____

There are three questions taken from the material covered so far in the course. All questions are equally weighted. If you have a question, please raise your hand and I will come to your desk.

Make sure that you defend your answers with economic reasoning or mathematical arguments, and show that you are using the correct game theoretic concepts by identifying the equilibria explicitly.

Good luck.

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1. Consider the following strategic form game:

		B	
		L	C
	u	4,4	-1,x
A	m	x,-1	0,0

where $x > 4$.

i. Find all pure-strategy equilibria of the game.

So m strictly dominated u and C strictly dominates C, and (m, C) is the only remaining profile. So by iterated deletion of strictly dominated strategies, (m, C) is the only pure-strategy Nash equilibrium.

ii. Consider the infinitely repeated game where the stage game is given above and players have a common discount factor $0 < \delta < 1$. Use the Nash threats folk theorem to show that, for δ sufficiently close to 1, there exists a Subgame Perfect Nash Equilibrium in which players use u and L in every period.

Consider the trigger strategies:

- If the history is $(u, L), (u, L), ..., (u, L)$, then play (u, L) this period
- After any other history, play (m, C).

Then the payoff to cooperating for both players is

$$4 + \delta 4 + \delta^2 4 + \delta^3 4 + ... = 4\frac{1}{1-\delta}$$

and the payoff to deviating is

$$x + \delta 0 + \delta^2 0 + .. = x$$

So cooperating is better than deviating if

$$4\frac{1}{1-\delta} \geq x$$

or

$$\delta \geq \frac{x-4}{x}$$

So by the Nash Threats Folk Theorem, as long as $\delta \geq (x-4)/x$, there is a subgame perfect Nash equilibrium in which players can cooperate and play (u, L) in every period.

iii. How does the minimum δ that achieves cooperation from part ii depend on x? Briefly explain the economic intuition behind this.

So

$$\frac{\partial \delta}{\partial x} = \frac{x-(x-4)}{x^2} = \frac{4}{x^2} > 0$$

so that if $x \uparrow$, $\delta \uparrow$. Another way to see this is

$$\delta(x) = \frac{x-4}{x} = \frac{1-4/x}{1}$$

so that $\delta(x)$ is an increasing function in x.

The economic intuition is that if the gain to backstabbing (x) increases, the players must be more patient and place more weight on the future (higher δ) to sustain cooperation.

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2. There are two players: The pitcher, P, and the batter, B. The pitcher warms up before the game, and privately learns whether $x_r = 1$ (he is having a "good day") or $x_r = 0$ (he is having a "bad day"). The batter does not observe this, but does know that this pitcher is "good" with probability p and "bad" with probability $1-p$. The pitcher can throw a fastball, Fb, or curveball, Cb, and the batter can swing fast, Fs, or slow Sw.

			B
		Fs	Ss
	Fb	$x_r, 1-x_r$	1,0
P	Cb	1, 0	0,1

i. Sketch all the strategic forms that might occur. What is a Bayesian Nash equilibrium for this game?

All strategic forms:

			B
		Fs	Ss
	Fb	0, 1	1,0
P	Cb	1, 0	0,1

			B
		Fs	Ss
	Fb	1, 0	1,0
P	Cb	1, 0	0,1

So sometimes it's matching pennies, and sometimes the row player has a dominant strategy.

A Bayesian Nash equilibrium is a strategy for the $x_r = 0$-type pitcher, $x_r = 1$-type pitcher, and batter so that no player-type has an incentive to deviate.

ii. Show that if $x_r = 1$, it is a weakly dominant strategy for the pitcher to throw a fastball.

The strategic form is

			B
		Fs	Ss
	Fb	$\underline{1}$, 0	$\underline{1}$,0
P	Cb	$\underline{1}$, 0	0,1

So the $x_r = 1$-type pitcher has a weakly dominant strategy to throw a fastball, since Fb gives a weakly higher payoff, no matter what B does.

iii. Is there a Bayesian Nash equilibrium where (a) the $x_r = 1$-type pitcher always throws fastballs, (b) the $x_r = 0$-type pitcher always throws curveballs, and (c) the batter swings slowly? Explain why or why not.

If the batter always swings slowly, the $x_r = 0$-type pitcher's payoff from Fb is 1 while his payoff to Cb is 0. So the $x_r = 0$-type pitcher has a profitable deviation.

iv. Is there a Bayesian Nash equilibrium where (a) the $x_r = 1$-type pitcher always throws fastballs, (b) the $x_r = 0$-type pitcher mixes over curveballs and fastballs, and (c) the batter mixes over swinging fast and slow? Explain why or why not.

If the batter and $x_r = 0$-type pitcher are mixing, they must be choosing their own mixed strategies to make their opponent indifferent in expectation over their pure strategies.

Let σ_{fb} be the probability that the $x_r = 0$-type pitcher throws a fastball. For the batter, the expected payoff to Fs is

$$p(0) + (1-p)(\sigma_{fb}(1) + (1-\sigma_{fb})(0))$$

and the expected payoff to Ss is

$$p(0) + (1-p)(\sigma_{fb}(0) + (1-\sigma_{fb})(1))$$

solving for σ_{fb} gives

$$\sigma^*_{fb} = \frac{1}{2}$$

Let σ_{fs} be the probability that the batter swings fast. For the $x_r = 0$-type pitcher, the expected payoff to Fb is

$$\sigma_{fs}(0) + (1-\sigma_{fs})(1)$$

and the expected payoff to Cb is

$$\sigma_{fs}(1) + (1-\sigma_{fs})(0)$$

solving for σ_{fs} gives

$$\sigma^*_{fs} = \frac{1}{2}$$

So it is a Bayesian Nash equilibrium for the $x_r = 1$-type pitcher to throw fastballs, the $x_r = 0$-type pitcher to mix half-half over curveballs and fastballs, and the batter to mix half-half over swinging fast and slow.

v. Explain how you answer in iv depends on p.

It doesn't. Wild, right? The behavioral reason is that since there is nothing that the batter can do about facing a strong pitcher, since that player-type is just going to throw fastballs. That leaves a "matching pennies" game between the weak pitcher and batter, and the optimal strategies for the players there is to mix evenly over their pure strategies, which has nothing to do with the prior belief p.

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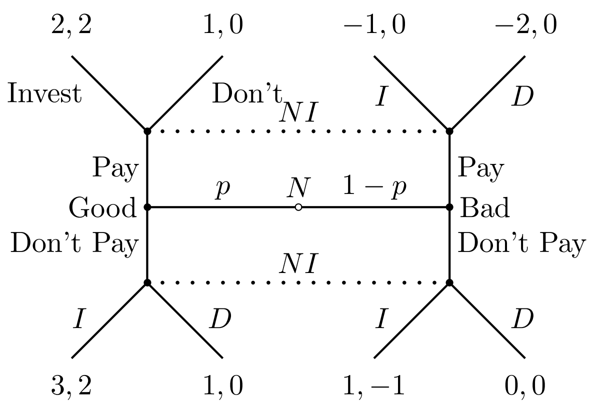
3. In general, it is a puzzle why firms pay dividends on their stock (if you own stock in a company, you sometimes get checks from them giving you a share of the profits). Why? Well, most of a mature firm's cash for investment comes from *retained earnings*, so you would think that firms would want to hold onto profits so they can reinvest in the company (since the wages for management are generally tied to the performance of the company). So firms don't have a clear incentive to given money away. The idea in this question is that firms pay dividends as a signal of their health to attract new investors.

There is a firm, who privately knows whether it is Good or Bad; good firms have higher earning potential than bad firms. The probability a firm is good is p, and the probability a firm is bad is $1-p$. There is a New Investors, NI, who is deciding whether to invest, I, or don't, D, in the firm. NI only observes whether the firm paid dividends or not, but not the firm's true type.

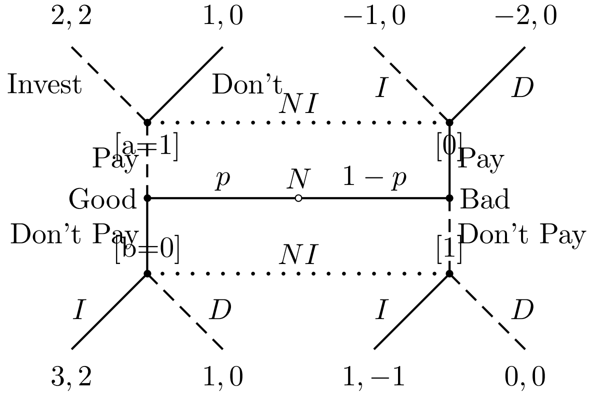


Here's the timing, just to make it explicit:

- Nature decides if the firm is good or bad.
- The firm decides whether to pay dividends or not.
- The potential investors decide whether to invest or not.

i. Is there a (separating) perfect Bayesian equilibrium where the good firm pays dividends, the bad firm doesn't, and the new investor invests in firms who pay dividends but does not in firms that don't?

That would look like



(I filled in the beliefs with "good guesses", that came by using Bayes' rule to see what beliefs would be compatible with the strategies.)

Now let's check sequential rationality and Bayesian beliefs:

- Sequential Rationality:
  - After the new investor gets a u, the expected payoff to I is $a2 + (1 - a)(-1) = 2$, and the expected payoff to D is $a0 + (1 - a)0 = 0$, so investing is better than not investing, and the new investor has no profitable deviation here.
  - After the new investor gets a d, the expected payoff to I is $b2 + (1 - b)(-1) = -1$, and the expected payoff to D is $b0 + (1 - b)0$, so not investing is better than investing, and the new investor has no profitable deviation here.
  - If the good firm pays dividends, it gets a payoff of 2, while refusing to pay dividends gives it a payoff of 1, so paying dividends is better than not paying dividends, and the good firm has no profitable deviation.
  - If the bad firm pays dividends, it gets a payoff of −1, while refusing to pay dividends gives it a payoff of 0, so not paying dividends is better than paying dividends, and the bad firm has no profitable deviation.

  Therefore, the strategies are sequentially rational given the beliefs.
- Bayesian beliefs:

  $$a = pr[\text{Good}|\text{Pay}] = \frac{pr[\text{Good} \cap \text{Pay}]}{pr[\text{Pay}]} = \frac{p}{p} = 1$$

  which matches the guess in the extensive form above.

  $$b = pr[\text{Good}|\text{Didn't pay}] = \frac{pr[\text{Good} \cap \text{Didn't pay}]}{pr[\text{Didn't pay}]} = \frac{0}{1-p} = 0$$

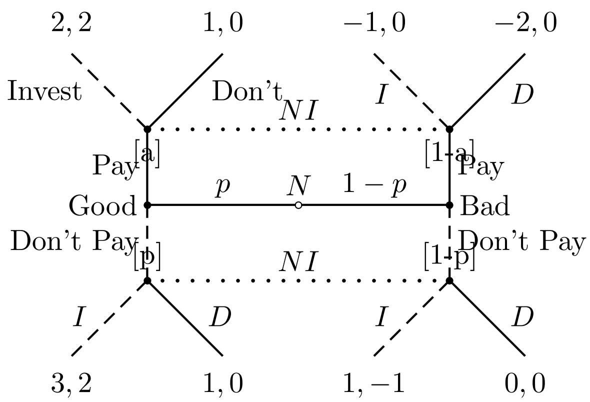
  which matches the guess in the extensive form above.

Therefore, $a = 1$, $b = 0$, the good type pays dividends, the bad type doesn't pay dividends, the new investor invests in firms who pay dividends, and the new investors does not invest in firms that don't is a perfect Bayesian equilibrium of the game.

Note that I checked sequential rationality *before* checking Bayesian beliefs above. See: There's no "special order" in which you do this, the beliefs and strategies are really mutually reinforcing.

ii. Is there a (pooling) perfect Bayesian equilibrium where neither firm type pays dividends, and the new investors always invests?

Consider



(where, again, I've filled in the beliefs by using Bayes' rule to come up with "good guesses" ahead of time)

Let's check Bayesian beliefs and sequential rationality:

- Bayesian beliefs:

$$a = pr[\text{Good}|\text{Pay}] = \frac{pr[\text{Good} \cap \text{Pay}]}{pr[\text{Pay}]} = \frac{0}{0}$$

  so Bayes' rule puts no restrictions on these beliefs.

$$b = pr[\text{Good}|\text{Didn't pay}] = \frac{pr[\text{Good}|\text{Didn't pay}]}{pr[\text{Didn't pay}]} = \frac{p}{1} = p$$

  So the beliefs given above are consistent with Bayes' rule.

- Sequential rationality:
  - If the new investor sees that the firm does not pay dividends, the expected payoff to investing is $2b + (1 - b)(-1) = 2p + (1 - p)(-1)$, while the expected payoff of not investing is zero. So if $2p + (1 - p)(-1) \geq 0$, or $p \geq 1/3$, the new investor has no profitable deviations here.
  - If the new investor sees that the firm does pay dividends, the expected payoff to investing is $2a + (1 - a)0$, while the expected payoff of note investing is zero. Then investing is better than not investing if $2a \geq 0$, or $a \geq 0$. So the new investor has no profitable deviations here.
  - If the good firm doesn't pay dividends, it gets a payoff of 3, while paying dividends gives a payoff of 2. So not paying is better than paying, and the good firm has no profitable deviations.
  - If the bad firm doesn't pay dividends, it gets a payoff of 1, while paying dividends gives a payoff of −2. So not paying is better than paying, and the bad firm has no profitable deviations.

Therefore, if $p \geq 1/3$ and for any a, the good and bad firms both not paying dividends and the new investor always investing is a perfect Bayesian equilibrium of the game.

iii. Briefly explain the difference between the equilibria in part i and part ii: How do they depend on p, and which do you think is more likely to occur in the real world?

In part i, it is a separating equilibrium, so behavior does not depend in any way on p. This is because the message or signal of paying dividends perfectly reveals the firm's type, so prior beliefs are made irrelevant.

In part ii, however, the firms pool and the investor is always supposed to invest. If dividends are paid, the new investor makes enough of a return on his investment so that it is worth it to gamble. If dividends are not paid, however, bad firms make a loss. So $p \geq 1/3$ really means that there are enough good investment opportunities out there so that, in expectation, it is worth it to gamble on investing and potentially making money on a good firm.

The model suggests that if there are many good investment opportunities, dividend payments are unnecessary. On the other hand, if there are few good firms around (p is low), the good firms can convince investors to back them by paying dividends. Since we see firms paying dividends in reality, we're probably in the second case. (Of course, many established firms don't pay dividends. However, this might be because the market has already figured out their type, and they don't need to keep throwing money away to attract interest and build up a reputation.)

iii. Briefly explain why a firm paying dividends is similar to an exotic bird having colorful feathers.

Birds dance and have colorful plumage to signal to other birds that they have calories to waste on ostentatious behavior. Since firms use retained earnings to fund new projects, paying dividends is very similar: The ostentatious behavior (plumage/dividends) can convince (other birds/investors) that the (bird/firm) is so good that it has (calories/profits) to spare.