Spring 2008 Final Solutions

1. (35%)
    a. A monopolist solves Max(Q) {Q(30-Q-6)}. The FOC of this problem is:
$$FOC(Q): 30 - 2Q - 6 = 0 \rightarrow Q^m = 12.$$
Therefore
$$P = 30 - Q^m = 30-12 = 18$$
Therefore
$$\pi^m = 12(18-6) = 12*12 = 144.$$

    b. An individual firm in the simultaneous Cournot game solves Max(q_i) { q_i (30- q_i – q_j – 6) }. The FOC of this problem is:
$$FOC(q_i) = 24-2\,q_i - q_j = 0 \rightarrow q_i = 12 - ½\,q_j$$
Plugging one BR function into the other gives us
$$q = 12 - ½(12 - ½\,q) = 6 + ¼\,q \rightarrow 3/4q = 6 \rightarrow q^c_i = 8.$$
Finally, evaluating the profit of each firm gives us:
$$\pi^c_i = 8(30-8-8-6) = 8*8 = 64.$$

    c. Note that ½ Q^m = 6. Therefore to find a firm's one period optimal deviation simply plug this quantity into their BR functions:
$$q^d_i = 12 - ½\,(6) = 9$$
Therefore:
$$\pi^d_i(\text{one period}) = 9*(30-9-6-6) = 9*9 = 81.$$

    d. Equilibrium path payoffs are $\pi^e_i = ½\,\pi^m\,(1+\delta+\delta^2+\ldots) = 72/(1-\delta)$. An optimal deviation followed by the punishment phase forever after yields:
$$\pi^d_i = 81 + 64(\delta+\delta^2+\ldots) = 81 + 64\delta/(1-\delta).$$
Therefore cooperation is optimal if
$$72/(1-\delta) >= 81 + 64\delta/(1-\delta)$$
$$72 >= 81 - 81\delta + 64\delta$$
$$17\,\delta >= 9$$
$$\delta^* >= 9/17 = 0.529 \text{ (approx)}.$$

    e. Less draconian strategies include limited punishment and tit-for-tat. Since in both of these, the deviating player is "given a second chance," it makes the deviation more attractive to begin with. This means that players will have to care about today (versus the future) a lot more (ie $\delta \rightarrow 1$) in order to not deviate.

2. (25%)
   a. See Osborne.

   b. 1 Subgame – the whole game starting with nature's choice.

   c. 2 information sets for each player. The sender has one following the strong signal and one following the weak signal. The receiver has one following the L action and one following the R action.

   d. Note, this is the same game from problem set 5, except here I ask for ALL PBEs. (There happens to only be 1, but full credit was achieved only by showing that other potential equilibria do not satisfy the requirements of a PBE.)
      i. Pooling on L. → $\sigma_s = (L,L)$ → $p = ½$. So if a receiver sees L, he should play U since the expected payoff from U (1) is greater than the expected payoff from D (½). On the right side, we look for strategies (and beliefs) of the receiver to make the sender (of each type) NOT want to deviate from (L,L). But notice that a weak sender ALWAYS deviates from playing L and getting 0 to playing R and getting 1 or 2. So NO PBE of this type exists.
      ii. Pooling on R. → $\sigma_s = (R,R)$ → $q = ½$. So if a receiver sees R, he should play D since the expected payoff from D (1) is greater than the expected payoff from U (½). On the left side, we look for strategies (and beliefs) of the receiver to make the sender (of each type) NOT want to deviate from (R,R). A strong sender never deviates since by playing R, he gets 3, his highest payoff. A weak sender can get 2 by playing R, or he could get either 0 or 3 by playing L. So we want him to get 0 → require that the receiver play U on the left. So we need:

$$E_{Rec}[U|L] >= E_{Rec}[D|L]$$
$$2p+0(1-p) >= 0p + 1(1-p)$$
$$3p >= 1$$
$$p >= 1/3$$

So our PBE is

$$\{(R,R),(U,D),(p,1-p),(q,1-q) \mid p >= 1/3, q = ½\}$$

      iii. Separating on $\sigma_s = (L,R)$ → $p=1$, $q=0$ → $\sigma_{rec} = (U,D)$. BUT a strong sender would deviate and get 3 instead of 1. So NO PBE of this type exists.

iv. Separating on σ_s = (R,L) → p=0, q=1 →σ_{rec} = (D,U).
BUT a strong sender would deviate and get 2 instead of 0.
So NO PBE of this type exists.

3. (20%)

   a. $\sigma_1 = \{AC,AD,BC,BD\}$. $\sigma_2 = \{L,R\}$. Note that player 2 cannot do different things at each node of his single information set!

   b. Consider the strategic form of the game:

Player 2

Player 1

	L	R
AC	$1 , \underline{4}$	$2 , 0$
AD	$1 , \underline{4}$	$4 , 2$
BC	$\underline{7} , \underline{0}$	$\underline{9} , \underline{0}$
BD	$\underline{7} , \underline{0}$	$\underline{9} , \underline{0}$

By the underlining method, there are 4 PSNE:

PSNE = { (BC, L), (BC, R), (BD, L), (BD, R) }

   c. To solve for the SPNE, consider the only subgame besides the game itself, the one starting with player 1's choice of C or D. Player 1 MUST choose D in this subgame! So eliminate any NE that involve player 1 NOT choosing D and we get:

      SPNE = { (BD,L), (BD, R) }

   d. Finally, to solve for the PBEs, consider the two SPNE's in turn and see if you can find beliefs to support those strategies as a PBE. Notice that player 2's information set is ON the equilibrium path for both candidate equilibria. Thus, player 2's belief is determined by player 1's strategy. Thus,

      PBE = { (BD, L), (p, 1-p) | p = 0 ;
      (BD, R), (p, 1-p) | p = 0 }.

4. (20%)
   a. (T,B,L,R) all survive IESDS.

   b. 2 Pure: (B,L) and (T,R) and 1 Mixed: [(0.5, 0.0, 0.5), (0.6, 0.0, 0.4)].

   The mixed strategy is found by equalizing the expected conditional payoffs for each player.
   For player 1:

   $$-10q + 15(1-q) = 0 \rightarrow 15 = 25q \rightarrow q = 3/5$$

   For player 2 :

   $$-10p + 10(1-p) = 0 \rightarrow 10 = 20p \rightarrow p = \tfrac{1}{2}$$

   You could also maximize the **unconditional** expected payoff to get to this solution. This takes longer and if you KNOW the solution is an interior solution, equalizing the expected payoffs will get you to your solution faster.