Applied Game Theory - Exam Fall 2010

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Rules and suggestions

- The idea is to complete first the questions and then the exercises.
- Having **the good** answer for all questions will guarantee to pass the exam with 10/20.
- Exercises are difficult if you are not familiar with the course contents. Suggestion: pick one exercise that you think is more feasible for you and focus on that one.

Questions [`10 points`]

State whether each of the following claims is true or false (or can not be determined). For each, explain your answer in (at most) one short paragraph. Explaining an example or a counter-example is sufficient. Absent this, a nice concise intuition is sufficient: you do not need to provide a formal proof. Points will be deducted for incorrect explanations.

1. A strictly dominated strategy can never be a best response.
2. If $(\hat{s}, \hat{s})$ is a Nash equilibrium of a symmetric, two-player game then $\hat{s}$ is evolutionarily stable.
3. William the Conqueror burned his boats because his soldiers were afraid of the dark.
4. Consider the strategy profile (s_A, s_B). If player A has no strictly profitable pure-strategy deviation then she has no strictly profitable mixed-strategy deviation.
5. In the duel game if your probability of hitting if you shoot now plus the probability of your opponent hitting if she were to shoot next turn is greater than one, then it is a dominant strategy for you to shoot now.

Problems Set

Please, select whether you want to solve Problem 1 **and** Problem 2, **or** Problem 3. Problem 3 is more involved and is worth as much as solving Problems 1 and 2.

Problem 1 [5 points]

Two players, A and B play the following game. First A must choose IN or OUT. If A chooses OUT the game ends, and the payoffs are A gets 2, and B gets 0. If A chooses IN then B observes this and must then choose *in* or *out*. If B chooses *out* the game ends, and the payoffs are B gets 2, and A gets 0. If A chooses IN and B chooses *in* then they play the following simultaneous move game:

		B	
		left	*right*
A	*up*	(3,1)	(0,-2)
	down	(-1,2)	1,3

1. Draw the tree that represents this game.
2. Find all the *pure-strategy* SPE[1] of the game.

Problem 2 [5 points]

Consider the following *simultaneous* move game, with two players, expressed in the normal form:

		Player 2	
		C	D
Player 1	C	(α,β)	(0,0)
	D	(0,0)	(β,α)

Let's assume $\alpha > \beta$, e.g. $\alpha = 2$ and $\beta = 1$.

1. Which kind[2] of game is it?
2. Find all the *pure-strategy* NE of the game.
3. Find all the *mixed-strategy* NE of the game, and compute[3] the expected payoff to each player.
4. Assume a trusted "Oracle" is available to the players. The "Oracle" throws a **fair** coin, which the players can observe to decide which strategy to select. Compute[4] the expected payoff to the players and compare it to the one you obtained in the previous question.

[1] Sub-game Perfect Equilibrium.
[2] Explain the family of games it belongs to.
[3] The student should show the most significant mathematical expressions used to arrive at the result.
[4] The student should show the most significant mathematical expressions used to arrive at the result.

Problem 3 [10 points]

We will call this the *Car-pooling Game*. This problem requires you to think about and eventually complete a model, given the (sometimes loose) problem definition below. This is going to be a *simultaneous move game*.

First, to make things simple, we will assume there are only two players in the game. These two players are people that need to drive to a *common* destination. The strategies are, for both players, either Drive or Pool. In case player i selects to D, she will take her own car and travel from her current position to the destination. Instead, if player j selects strategy P, she will car-pool with player i and go together to the common destination: as such, player i will first have to drive to player j's initial location, then get back on the main road and go towards the destination. Note that it does not make any sense for both players to select strategy P, as they would both wait indefintely for someone to pass by with her car.

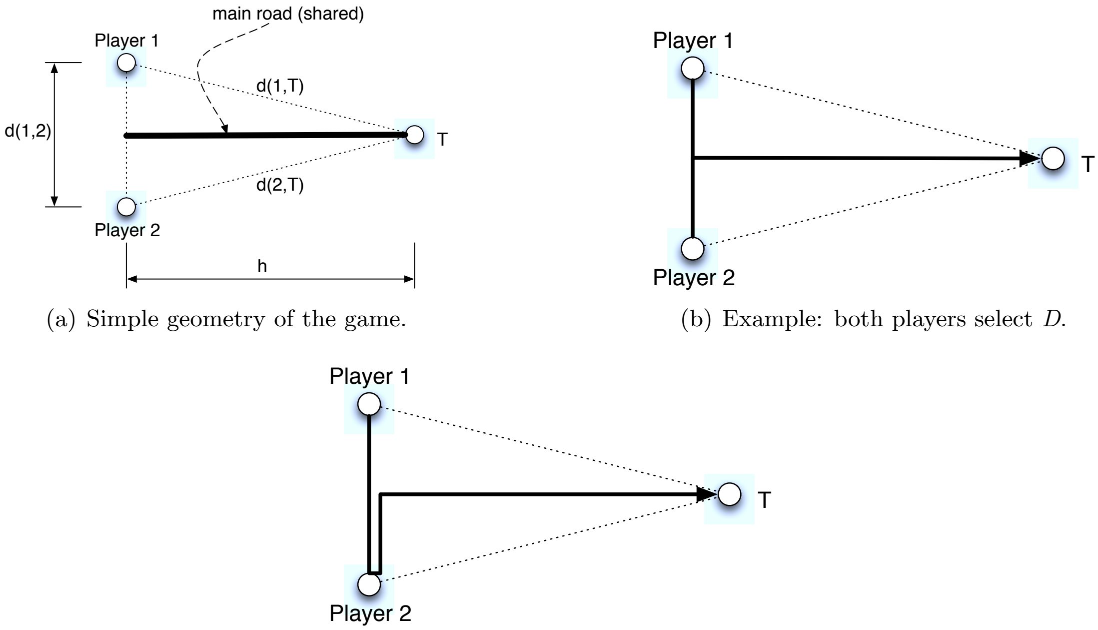
The payoffs are expressed in terms of delay, that is the time required to reach the destination from a the initial position, given a driving speed which is constant and equal for both player, *i.e.,* V. As we shall see later, the delay is obviously a function of the distance from the initial position(s) to the destination and will take into account the *congestion* that arises when too many cars take the same road at the same time. It is **important** to note that in this game, players seek at *minimizing* the delay they incur to reach their final destination.

It is easier to understand the game with a figure, which illustrates (on the plane), each players' initial position, the destination and the one available road to the destination.

Let's assume that $d()$ is a (euclidean) distance function. Also, let's label 1 and 2 the initial positions of player 1 and 2, respectively. Finally, assume that:

- The two players are at the same distance from their initial positions to the destination T
$$d(1,T) = d(2,T) \tag{1}$$
- The two players lie on a triangle whose height h is known, such that
$$d(1,2) < d(1,T) \tag{2}$$
- The *congestion* level α that is experienced by player i when travelling along the main road towards T is proportional to:
$$\alpha(x) = x \tag{3}$$
where x is the number of players concurrently travelling along the main road

In order to help you with the solution to this problem, let's start analyzing it. First, let's assume player i decides to select strategy D: then, her delay (refer to Fig. 1) will be as follows:
$$\frac{d(1,2)}{2V} + \frac{h}{V/\alpha(x)} \tag{4}$$



(a) Simple geometry of the game.

(b) Example: both players select D.

(c) Example: player 1 selects D and player 2 selects P.

Figure 1: The Car-pooling Game

Note that her driving speed has been divided by the congestion factor, such that if $x = 2$ players will decide to select strategy D, then the respective driving speeds will be $V/2$.

Instead, if player j selects strategy P, then player i will incur in a delay that amounts to:

$$\frac{3d(1,2)}{2V} + \frac{h}{V/\alpha(x)} \tag{5}$$

Note that this time (since we have two players only), $\alpha(x) = 1$, hence there will be no congestion on the main road.

Now, answer the following questions:

1. Briefly explain what are the important simplification assumptions that have been made. *Optionally*, you are allowed to make (and clearly state) your own simplifying assumptions.

2. Complete the following table, which represents the normal form of the game: Note

		Player 2	
		D	P
Player 1	D	(k_1, k_2)	(k_2, k_3)
	P	(k_3, k_2)	$(?, ?)$

that k_1 and k_2 are available from the discussions above (a little bit of algebra is required), while you have to derive k_3 and also to determine the value to assign to the payoff when both players selects strategy P.

3. Which kind of game is the Car-pooling game? Find all *pure-strategy* Nash equilibrium.

4. Find all *mixed-strategies* Nash equilibrium.

Optional Introduce a parameter to determine the congestion level on the main road and explain the impact of congestion with respect to the initial position of the players

Optional Generalize the model by taking into account $\{3, 4, 5, ..., n\}$ players.