

**Problem 1** (*Iterated Elimination of Strictly Dominated Strategies*)

Consider the iterated elimination of strictly dominated strategies in the strategic form game  $\langle \mathcal{I}, (S_i)_{i \in I}, (u_i)_{i \in I} \rangle$ . For all  $i \in \mathcal{I}$ , denote the set of strategies of player  $i$  at the  $k$ th step of the elimination by  $S_i^k$ . Suppose that each  $u_i(s_i, s_{-i})$  is continuous and each  $S_i$  is compact. Prove that  $S_i^\infty$  (for each  $i$ ) is nonempty.

**Hint:** You might use the fact that intersection of nested nonempty compact sets is nonempty, i.e.

Suppose  $\{A_j\}$  is a collection of sets such that each  $A_j$  is nonempty, compact, and  $A_{j+1} \subset A_j$ . Then  $A = \bigcap_j A_j$  is nonempty.

??? **Problem 2** (*Iterated Elimination of Strictly Dominated Strategies in Cournot Competition*)

Consider a market in which the price charged for quantity  $Q$  of some good is given by  $P(Q) = \alpha - \beta Q$  for some  $\alpha, \beta > 0$ . Assume that the cost of producing a unit of this good is  $c$ .

- Assume that there are two firms in the market. Using the iterated elimination of the strictly dominated strategies construct the sets of strategies  $S_1^k, S_2^k$  for any fixed  $k$ , and conclude that  $S_1^\infty$  is a singleton. (Use the definition of  $S_i^k$  given in question 1.)
- Assume that there are three firms. Show that  $S_1^\infty$  is not a singleton.

**Problem 3** Exercise 2.1(a) from Fudenberg and Tirole.

??? **Problem 4** (*Bertrand Competition with Different Marginal Costs*)

Suppose that two firms ( $A$  and  $B$ ) produce the same good and they have strictly positive marginal costs  $c_A$  and  $c_B$  such that  $c_B > c_A$ . Further assume that the firms can produce as many units as they wish at those marginal costs and consumers purchase the good only if the price  $p$  offered for the good satisfies  $p \leq R$  for a fixed  $R > 0$ .

- Assume that if the firms offer the same price, the demand is shared equally. Show that under this tiebreaking rule there exists no pure strategy Nash equilibrium.
- There exists a tiebreaking allocation under which the game has a unique equilibrium. Characterize this allocation and the corresponding equilibrium.

??? **Problem 5** (*Competition with Production Constraints*)

Consider a market with 2 firms which produce the same good. Assume that the demand for this good is  $Q$ , and the consumers in this market purchase the good only if its price satisfies  $p \leq R$ . Further assume that the production level  $K$  of each firm satisfies  $\frac{Q}{2} < K < Q$ .

- Assume that the demand is equally shared among the firms when they offer the same price. Under this tiebreaking rule write the payoff functions of the firms
- Show that there does not exist a pure strategy Nash equilibrium under this tiebreaking rule.
- Prove that this result does not depend on the tiebreaking rule.

??? **Problem 6** (*A war of attrition*) Two players are involved in a dispute over an object. The value of the object to player  $i$  is  $v_i > 0$ . Time is modeled as a continuous variable that starts at 0 and runs indefinitely. Each player chooses when to concede the object to the other player; if the first player to concede does so at time  $t$ , the other player obtains the object at that time. If both players concede simultaneously, the object is split equally between them, player  $i$  receiving a payoff of  $v_i/2$ . Time is valuable: until the first concession each player loses one unit of payoff per unit of time.

Formulate this situation as a strategic game and show that in all Nash equilibria one of the players concedes immediately.

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