Stat 154 Practice Final Exam Spring 2014

Name:

SID:

Attempt all questions and show your working - solutions without explanation will **not** receive full credit. Answer the questions in the space provided. Additional space is available on the final two pages. Two double sided sheets of notes are permitted. Answers can be left in numerical form without simplification except where specified.

Question 1

Find the optimal strategies in the following zero-sum games.

$$(a) \quad \begin{pmatrix} 2 & 4 \\ 7 & 1 \end{pmatrix} \qquad (b) \quad \begin{pmatrix} 3 & 4 & 4 \\ 5 & 1 & 4 \end{pmatrix}$$

Question 2

Answer the following:

(a) In the game of Nim if the piles are $17, 21$ and 22 find all winning moves.

(b) Recall that for Misere Nim the aim is to not take the last chip. If the piles are $1, 1, 1, 17, 1$ find all winning moves.

(c) An urn contains 2 yellow, 2 orange balls and two red balls. In a game players take turns to move and in each move may change a red ball into and orange or yellow ball or turn an orange ball into a yellow ball. The game terminates when all the balls are yellow. Is this a P or N position?

Question 3

Suppose two players share a common resource that needs repairing that will lead to a profit of v to both the players. If both players contribute to the repair it costs c_2 to each. If any of the players repair on their own, it costs c_1 where $c_2 < c_1 < v$. Write down the payoff matrix and find an evolutionary stable strategy in this game.

Question 4

In the following symmetric general sum game

$$\begin{pmatrix} (2,2) & (0,0) & (0,0) \\ (0,0) & (0,0) & (2,2) \\ (0,0) & (2,2) & (0,0) \end{pmatrix}$$

(i) Find all pure Nash equilibria.

(ii) Find all mixed Nash equilibria in which all probabilities are positive.

(vi) Which of these are evolutionary stable strategies?

Question 5 (i) There are 3 men, called A, B, C and 3 women, called X, Y, Z, with the following preference lists:

$$\begin{array}{rlcrl} \text{For } A: & X > Y > Z & \qquad & \text{For } X: & C > A > B \\ \text{For } B: & X > Y > Z & & \text{For } Y: & A > C > B \\ \text{For } C: & Y > X > Z & & \text{For } Z: & A > B > C \end{array}$$

Find the matchings given by the men proposing and women proposing Gale-Shapley algorithm for these preferences.

(ii) Find a set of preference list for 3 men and women such that in the men-proposing Gale-Shapley algorithm no man gets his first preference.

Question 6

Half of the cars a used car salesman gets cars to sell are lemons and the other half of them are good. The salesman knows which are which and always offers a warranty on new cars. He has the choice of offering a warranty on the lemons too. The salesmen sells all the cars for 15000 and unsold a good car is worth 12000 to him and a lemon is worth 9000. The expected cost of a warranty is 1000 for a good car and 5000 for a lemon. To a buyer a good car or a lemon with a warranty is worth 18000 while a lemon without a warranty is worth 12000. The salesman has the option of whether to offer warranties on lemons while the buyer has the option to buy or pass.

Determine the Nash equilibrium for the strategies of the salesman and buyer.

Question 7

Suppose in an election there are 3 candidates A, B and C. The voters have the following preferences. 10 voters have preference $A > C > B$, 8 voters have preference $B > C > A$ and 4 voters have preference $C > B > A$. Who wins the election if the voting rule used is

(i) Plurality.

(ii) Instant run-off.

(iii) Borda count.

(iv) In the instant run-off election do any voters have the an incentive to manipulate the vote individually. What about a collection of voters?

Question 8

A cake is represented by the unit interval $\Omega = [0, 1]$ and must be divided between three players. For $i = 1, 2, 3$, player i values the cake according to the measure

$$\mu_i([c, d]) = \int_c^d x^{i-1} dx.$$

Find a cutting of the cake into A_1, A_2 and A_3 such that each person feels like they got a $\frac{1}{3}$ share. That is for each i,

$$\mu_i(A_i) \geq \frac{1}{3}\mu_i[\Omega]$$

Additional Space

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