Stat 155 Homework # 8 Due April 14

Problems:

Q 1 Karlin-Peres Chapter 3 Q 3.4

First game

$$\begin{pmatrix} (4,4) & (2,5) \\ (5,2) & (3,3) \end{pmatrix}$$

Playing strategy B is dominant for both players and so the only Nash equilibrium is $x = y = (0,1)$. The only pure strategy not equal to x is $z = (1,0)$ and $z^T Ax < x^T Ax$ so x is evolutionary stable.

Second game:

$$\begin{pmatrix} (4,4) & (3,2) \\ (2,3) & (5,5) \end{pmatrix}$$

There are two symmetric pure strategies, $x = y = (1,0)$ and $x = y = (0,1)$. These are both evolutionary stable strategies since $(0,1)^T A(1,0) < (1,0)^T A(1,0)$ and $(1,0)^T A(0,1) < (0,1)^T A(0,1)$. There is also a mixed strategy which we find by equalizing payoffs

$$4x_1 + 3(1 - x_1) = 2x_1 + 5(1 - x_1)$$

which is solved by $x_1 = \frac{1}{2}$. So the mixed Nash equilibrium is $x = y = (1/2, 1/2)$. However, if $z = (1,0)$ then $x^T AZ < Z^T Az$ so this is not evolutionary stable.

Q 2 We introduce a third type to the hawk and dove game called bourgeois which will only fight if it got to the resource first. If we assume that the bird are equally likely to find the food first then the expected payoff matrix is

$$\begin{pmatrix} (\frac{v}{2} - c, \frac{v}{2} - c) & (v, 0) & (\frac{3v}{4} - \frac{c}{2}, \frac{v}{4} - \frac{c}{2}) \\ (0, v) & (\frac{v}{2}, \frac{v}{2}) & (\frac{v}{4}, \frac{3v}{4}) \\ (\frac{v}{4} - \frac{c}{2}, \frac{3v}{4} - \frac{c}{2}) & (\frac{3v}{4}, \frac{v}{4}) & (\frac{v}{2}, \frac{v}{2}) \end{pmatrix}.$$

Find the evolutionary stable strategies.

Historical Note: The game together with the label bourgeois was given by the famous biologist John Maynard Smith, politically a communist, who considered it "politically bourgeois" to value ownership

We first see that no symmetric strategy x can give positive probability to both hawks and doves. If it did then the payoff for hawks and doves against x must be the same. But if we compare the payoff of $(1/2, 1/2, 0)$ to that of $(0, 0, 1)$ we see that the bourgeois strategy dominates an equal combination of hawks and doves.

So suppose the probability of hawks is 0. Then we have the matrix

$$\begin{pmatrix} (\frac{v}{2}, \frac{v}{2}) & (\frac{v}{4}, \frac{3v}{4}) \\ (\frac{3v}{4}, \frac{v}{4}) & (\frac{v}{2}, \frac{v}{2}) \end{pmatrix}.$$

This is dominated by the bourgeois strategy so no equilibria has doves. Thus we only consider equilibria with hawks or bourgeois with the matrix,

$$\begin{pmatrix} (\frac{v}{2} - c, \frac{v}{2} - c) & (\frac{3v}{4} - \frac{c}{2}, \frac{v}{4} - \frac{c}{2}) \\ (\frac{v}{4} - \frac{c}{2}, \frac{3v}{4} - \frac{c}{2}) & (\frac{v}{2}, \frac{v}{2}) \end{pmatrix}.$$

Since $c > v/2$, we have that $\frac{v}{2} > \frac{3v}{4} - \frac{c}{2}$ and so the bourgeois dominates the hawk. Thus the only symmetric Nash equilibrium is pure bourgeois. Since it dominates separately against hawk and dove it is also evolutionary stable.

Q 3 Two wolves can each choose to hunt deer or rabbit. A wolf hunting rabbit will succeed and get payoff r. If a single wolf hunts the deer it will fail and have payoff 0 while if both hunt deer together by co-operating they may succeed and each get expected payoff $s/2$. Write down the expected payoff matrix and when $s > 2r$ find the evolutionary stable strategies.

The expected payoff matrix is

$$\begin{pmatrix} (s/2, s/2) & (0, r) \\ (r, 0) & (r, r) \end{pmatrix}$$

There are two pure equilibria, $x = y = (1,0)$ and $x = y = (0,1)$. Since $(0,1)^T A(1,0) < (1,0)^T A(1,0)$ and $(1,0)^T A(0,1) < (0,1)^T A(0,1)$ these are both evolutionary stable equilibria. By equalizing payoffs

$$x_1 s/2 = x_1 r + (1 - x_1) r$$

we have that $x = y = (2r/s, 1 - 2r/s)$ in a symmetric mixed Nash equilibrium. With $z = (0,1)$ we have that $z^T Ax = x^T Ax$ but that $z^T Az > x^T Az$ so it is not evolutionary stable.

Q 4 Two players each are each given an independent number uniform in $\{0,1,2\}$ which only they see. The first player may "pass" in which case the game ends and no money changes hands or may choose to "play". The second player may "pass" in which case he gives \$ 1 to player 1. If player 2 chooses to "play" then the player with the higher number wins \$ 2 from the player with the lower number. No money changes hands if both play and it is a tie. Find the Nash equilibria for the game.

For player 1 "play" dominates passing when she has a 2. Also playing with a 0 and passing with a 1 is dominated by playing with a 1 and passing with a 0. This she has three strategies

- Pass on 1 or 0.
- Pass on 0.
- Never pass.

Similarly player 2 has the same set of strategies.

- Pass on 1 or 0.

- Pass on 0.
- Never pass.

The expected payoff matrix is

$$\begin{pmatrix} \frac{2}{9} & \frac{3}{9} & \frac{4}{9} \\ \frac{2}{9} & \frac{2}{9} & \frac{4}{9} \\ \frac{2}{9} & -\frac{1}{9} & 0 \end{pmatrix}$$

We see that the top left entry is a saddle point so the equilibrium strategies are $x = y = (1, 0, 0)$. That is that both players only play when they have a 2.