

## Summer 2010 Final Solutions

1.

- a. By underlining, we see that there are two pure strategy NE at (B,L) and (B,R). T is dominated by B for player 1 so we need not consider T as part of any mixed strategy NE. Suppose player 1 plays B and player 2 mixes on (L,R) with probabilities (q,1-q). Given that player 1 plays B, player 2 gets a payoff of 1 if he plays either L or R. So player 2 is indifferent as required. For player 1 to play B, it must be that  $E[U_1|B] \geq E[U_1|T] \rightarrow 30q + 1(1-q) \geq 10q + 0(1-q) \rightarrow 29q + 1 \geq 10q \rightarrow 19q \geq -1$ . This holds for all q in [0,1]. So there are MSNE at:

$$\{ (0,1), (q,1-q) \mid q \text{ in } [0,1] \}$$

Note that the two pure strategy NE are subsumed in this set when  $q = 1$  and  $q = 0$  respectively.

- b. Along the equilibrium path, both players obtain:

$$\pi_i^e = 10(1 + \delta_i + \delta_i^2 + \delta_i^3 + \dots) = 10/(1 - \delta_i)$$

Deviation path payoffs are as follows:

$$\pi_1^d = 30 + 1(\delta_1 + \delta_1^2 + \delta_1^3 + \dots) = 30 + \delta_1/(1 - \delta_1)$$

$$\pi_2^d = 20 + 1(\delta_2 + \delta_2^2 + \delta_2^3 + \dots) = 20 + \delta_2/(1 - \delta_2)$$

For (T,L) to be played in all periods, neither player can have a profitable deviation. So, we require:

$$\pi_1^e \geq \pi_1^d$$

$$10/(1 - \delta_1) \geq 30 + \delta_1/(1 - \delta_1)$$

$$10 \geq 30 - 30\delta_1 + \delta_1$$

$$29\delta_1 \geq 20$$

$$\delta_1 \geq 20/29 \sim 0.69 = \delta_1^*$$

$$\pi_2^e \geq \pi_2^d$$

$$10/(1 - \delta_2) \geq 20 + \delta_2/(1 - \delta_2)$$

$$10 \geq 20 - 20\delta_2 + \delta_2$$

$$19\delta_2 \geq 10$$

$$\delta_2 \geq 10/19 \sim 0.53 = \delta_2^*$$

- c. Player 1 has a larger one period deviation than player 2 (30 vs 20), so in order for player 1 NOT to deviate, we require him to be more patient and care more about the future than player 2.

2.

- a. The game has 2 subgames. The subgame following player 1's choice of A and the entire game itself.
- b. Consider the following normal form specification:

		Player 2	
		R	Q
Player 1	AC	1, <u>3</u>	0,0
	AD	0,0	<u>3</u> , <u>1</u>
	BC	<u>2</u> , <u>10</u>	<u>2</u> , <u>10</u>
	BD	<u>2</u> , <u>10</u>	<u>2</u> , <u>10</u>

By underlining, the (pure strategy) NE are:

$$\{(AD,Q),(BC,R),(BD,R)\}$$

- c. Consider the normal form for the only subgame besides the game itself. We require that players play NE in every subgame.

		Player 2	
		R	Q
Player 1	C	<u>1</u> , <u>3</u>	0,0
	D	0,0	<u>3</u> , <u>1</u>

So all SPNEs where player 1 chooses C, player 2 must choose R. In all SPNEs where player 1 chooses D, player 2 must choose Q. Hence the SPNE are:

$$\{(AD,Q),(BC,R)\}$$

- d. Consider the two SPNEs we found in part (c). For (AD,Q), player 2's information set is ON the equilibrium path so player 2's belief is determined by player 1's equilibrium strategy and the only belief that is consistent with that strategy  $[0,1]$ , i.e., all the weight on player 2's right side node. So we have a PBE at:

$$\{(AD,Q), (p,1-p) \mid p = 0\}$$

For (BC,R), player 2's information set is OFF the equilibrium path so player 2's belief must be such that player R is optimal. We require  $E[U_2 \mid R] \geq E[U_2 \mid Q] \rightarrow 3p + 0(1-p) \geq 0p + 1(1-p) \rightarrow 3p \geq 1-p \rightarrow 4p \geq 1 \rightarrow p \geq \frac{1}{4}$ . So we have a PBE at:

$$\{(BC,R), (p,1-p) \mid p \geq \frac{1}{4}\}$$

3.

- a. Each player has two information sets. The sender's information sets are both singleton information sets, while the receiver's both include 2 nodes.
- b. Pooling on Quiche. Since both types of sender eat Quiche, the receiver sets  $p = 2/3$ . Then  $E_{\text{Rec}}[\text{Duel} | \text{Quiche}] = 2/3 \cdot 0 + 1/3 \cdot 3 = 1$  compared with  $E_{\text{Rec}}[\text{Not Duel} | \text{Quiche}] = 2/3 \cdot 3 + 1/3 \cdot 0 = 2$ . So the receiver will Not Duel if he see the sender eat Quiche. By following (Q,Q), the strong sender obtains a payoff of 1 while the weaker sender obtains a payoff of 2. The weak sender never wants to deviate and drink a beer because his payoffs (-1 or 1) are both strictly less than 2. A strong sender, if he deviates to Beer, receives a payoff of 1 if the Receiver duels and 2 if the receiver does not duel. So for the strong sender to NOT want to deviate, we require:

$$E_{\text{Rec}}[\text{Duel} | \text{Beer}] \geq E_{\text{Rec}}[\text{Not Duel} | \text{Beer}]$$

$$4q + 2(1-q) \geq 0q + 4(1-q)$$

$$2q + 2 \geq 4 - 4q$$

$$6q \geq 2$$

$$q \geq 1/3$$

So the complete PBE is:

$$\{ (Q,Q), (\text{Not Duel}, \text{Duel}), (p, 1-p), (q, 1-q) \mid p = 2/3, q \geq 1/3 \}$$

Note that the Strong sender is indifferent between eating Quiche and Beer, so both are a best response to the receiver's strategy.

- c. False. If the sender separates on (Beer, Quiche), or (B,Q), the receiver's information sets are both on the equilibrium path so he should set  $p=0$  and  $q=1$ . Given these beliefs, the receiver should play (Duel, Duel). Playing (B,Q), the strong sender gets payoff of 1 and a weak sender gets 4. A weak sender would never deviate since his payoffs from having a beer (-1 and 1) are both less than 4. However, a stronger sender could eat Quiche and get 3 ( $>1$ ). Hence, the PBE does not exist because the stronger sender would want to deviate.

4.

- a. Firm B has a monopoly over the demand for the drug so it solves:

$$\text{Max}(Q) \{ Q(120-Q) - 2200 \}$$

$$\text{FOC}(Q): 120 - 2Q = 0$$

$$Q = 60 \rightarrow P = 120 - 60 = \$60.$$

$$\text{Annual Profit (gross of fixed costs)} = 60 * \$60 = \$3,600$$

$$\text{2-Year Profit (net of fixed costs)} = \$7,200 - \$2,200 = \$5,000 > 0$$

So firm B should enter this market.

- b. If firms compete Cournot style, each solves:

$$\text{Max}(q_i) \{ q_i(120 - q_i - q_j) \}$$

$$\text{FOC}(q_i): 120 - 2q_i - q_j = 0$$

$$q_i(q_j) = \frac{1}{2}(120 - q_j)$$

Solve simultaneously:

$$q_i = \frac{1}{2}(120 - \frac{1}{2}(120 - q_i)) = 60 - 30 + \frac{1}{4} q_i$$

$$\frac{3}{4} q_i = 30$$

$$q_i = 30 * (\frac{4}{3}) = 120 / 3 = 40 = q_j$$

$$\text{So } Q = q_i + q_j = 80$$

$$P = 120 - 80 = \$40$$

$$\text{So profits of each firm} = 40 * \$40 = \$1600$$

- c. For the last year of the exclusivity period, firm B would obtain \$3600 if it operated as the sole monopolist over production of the drug. If it competed with firm G, each firm obtains a profit of \$1600. Thus, firm B would be willing to pay at most \$2000 to delay entry until the patent expired. However, it would only need to pay \$1600 (firm G's profit under Cournot) in order for firm G to accept the offer.

See <http://www.ftc.gov/opa/reporter/payfordelay.shtm> for more information on pay for delay.