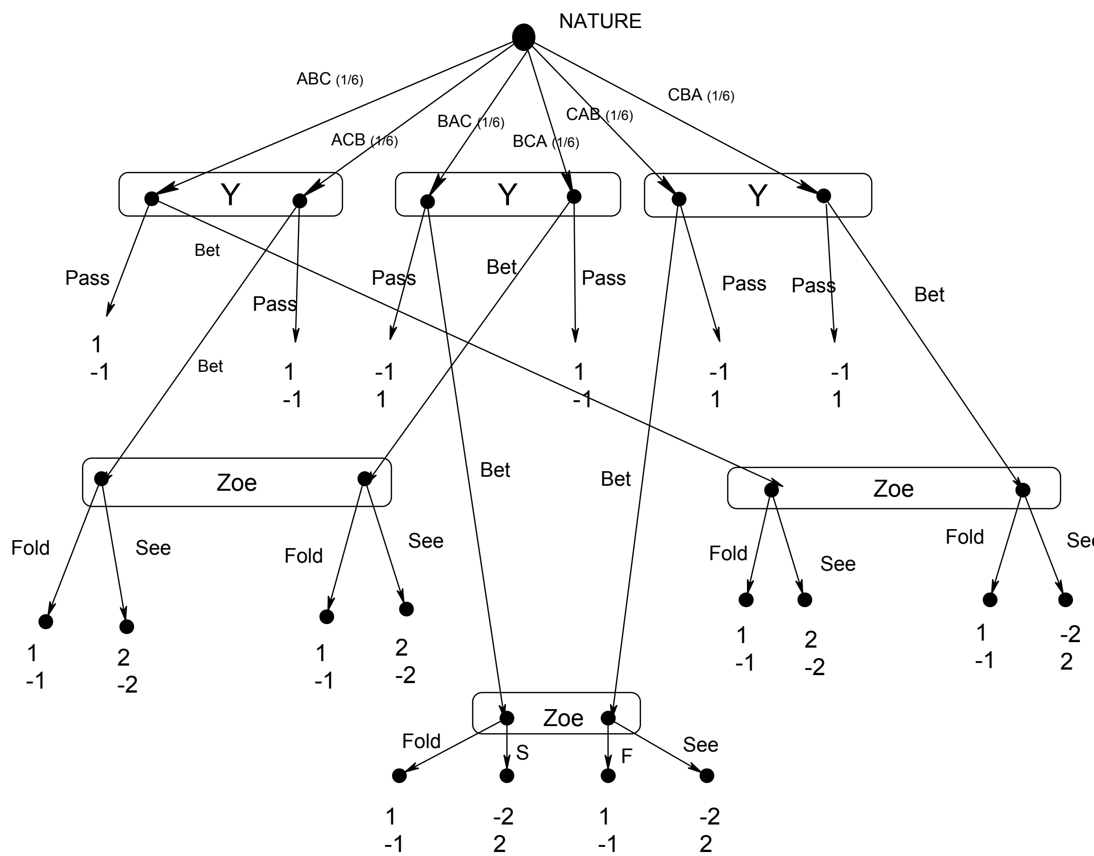
ANSWERS TO PRACTICE PROBLEMS $\boxed{10}$

1. **(a)** The extensive-form representation of the simplified poker game is as follows (the top number is Yvonne's net take in dollars and the bottom number is Zoe's net take).



(b) and (c) Each player has eight strategies (three possible situations, two choices in each: thus $2 \times 2 \times 2 = 8$ possibilities).

(d) Yvonne uses the strategy "If A pass, if B pass, if C bet" and Zoe uses the strategy "If A fold, if B fold, if C fold"). We calculate the expected net payoff for Yvonne. Zoe's expected net payoff is the negative of that.

Possible Cases	Top card is A Second is B	A C	B A	B C	C A	C B	Sum	Probability of each	Expected payoff
Y's payoff	1	1	−1	1	1	1	4	$\frac{1}{6}$	$\frac{4}{6}$
Explain	pass	pass	pass	pass	bet + fold	bet + fold			

(e) Yvonne uses the strategy "If A pass, if B pass, if C bet" and Zoe uses the strategy "see with any card". Once again, we calculate the expected net payoff for Yvonne. Zoe's expected net payoff is the negative of that.

Possible Cases	Top card is A Second is B	A C	B A	B C	C A	C B	Sum	Probability of each	Expected payoff
Y's payoff	1	1	−1	1	−2	−2	−2	$\frac{1}{6}$	$-\frac{2}{6}$
Explain	pass	pas s	pas s	pass	bet + see	bet + see			

(f) The game is given on the following page.

(g) Let $\searrow$ denote weak dominance, that is, a $\searrow$ b means that a weakly dominates b.

FOR YVETTE (row player): 3rd row $\searrow$ 1st row, 6th $\searrow$ 4th, 7th $\searrow$ 4th, 7th $\searrow$ 5th, 2nd $\searrow$ 8th.

FOR ZOE (column player): 3rd col $\searrow$ 1st col, 3rd $\searrow$ 4th, 3rd $\searrow$ 5th, 3rd $\searrow$ 7th, 3rd $\searrow$ 8th, 2nd $\searrow$ 8th, 4th $\searrow$ 5th, 4th $\searrow$ 8th, 6th $\searrow$ 2nd, 6th $\searrow$ 4th, 6th $\searrow$ 5th, 6th $\searrow$ 7th, 6th $\searrow$ 8th, 7th $\searrow$ 4th, 7th $\searrow$ 5th, 7th $\searrow$ 8th.

(h) Eliminating rows 1, 4, 5 and 8 and all columns except 3 and 6 we are left with:

		Zoe See only if A	**Zoe** See with A or B
Y	Bet always	**0, 0**	**−2/6, 2/6**
	If A, Bet, otherwise pass	**0, 0**	**1/6, −1/6**
	If A or B, Bet, if C Pass	**−1/6, 1/6**	**0, 0**
	If A or C, Bet, if B Pass	**1/6, −1/6**	**−1/6, 1/6**

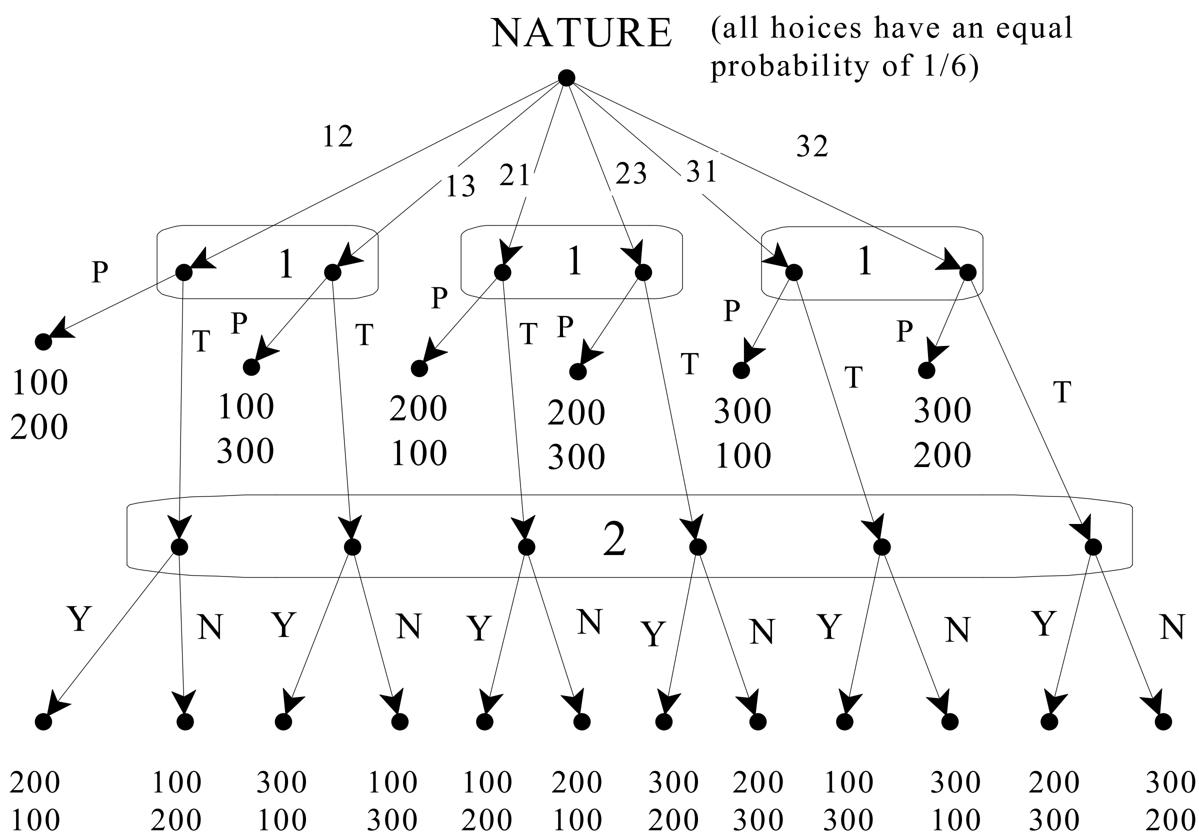
In the new matrix, second row dominates first and third. Eliminating them we have the following, which is a remarkable simplification of the original matrix:

		Zoe See only if A	**Zoe** See with A or B
Y	If A, Bet, otherwise pass	**0, 0**	**1/6, −1/6**
	If A or C, Bet, if B Pass	**1/6, −1/6**	**−1/6, 1/6**

ZOE

		If A fold, If B fold, If C fold	If A see, If B see, If C see	If A see, If B fold, If C fold	If A fold, If B see, If C fold	If A fold, If B fold, If C see	If A see, If B see, If C fold	If A see, If B fold, If C see	If A fold, If B see, If C see
Y	If A pass, if B pass if C pass	**0, 0**	**0, 0**	**0, 0**	**0, 0**	**0, 0**	**0, 0**	**0, 0**	**0, 0**
V	If A bet, if B bet if C bet	**1, −1**	**0, 0**	**0, 0**	**4/6, −4/6**	**8/6, −8/6**	**−2/6, 2/6**	**2/6, −2/6**	**1, −1**
O	If A bet, if B pass if C pass	**0, 0**	**2/6, −2/6**	**0, 0**	**1/6, −1/6**	**1/6, −1/6**	**1/6, −1/6**	**1/6, −1/6**	**2/6, −2/6**
N	If A pass, if B bet if C pass	**2/6, −2/6**	**0, 0**	**−1/6, 1/6**	**2/6, −2/6**	**3/6, −3/6**	**−1/6, 1/6**	**0, 0**	**3/6, −3/6**
N	If A pass, if B pass if C bet	**4/6, −4/6**	**−2/6, 2/6**	**1/6, −1/6**	**1/6, −1/6**	**4/6, −4/6**	**−2/6, 2/6**	**1/6, −1/6**	**1/6, −1/6**
E	If A bet, if B bet if C pass	**2/6, −2/6**	**2/6, −2/6**	**−1/6, 1/6**	**3/6, −3/6**	**4/6, −4/6**	**0, 0**	**1/6, −1/6**	**5/6, −5/6**
	If A bet, if B pass if C bet	**4/6, −4/6**	**0, 0**	**1/6, −1/6**	**2/6, −2/6**	**5/6, −5/6**	**−1/6, 1/6**	**2/6, −2/6**	**3/6, −3/6**
	if A Pass, If B or C, Bet,	**1, −1**	**−2/6, 2/6**	**0, 0**	**3/6, −3/6**	**7/6, −7/6**	**−3/6, 3/6**	**1/6, −1/6**	**4/6, −4/6**

2. **(a)** In the game below, 12 means "player 1 gets the envelope with \$100 and player 2 gets the envelope with \$200", 13 means "player 1 gets the envelope with \$100 and player 2 gets the envelope with \$300", etc. P stands for "pass" and T for "suggest a trade"; Y is "Yes" and N is "No".



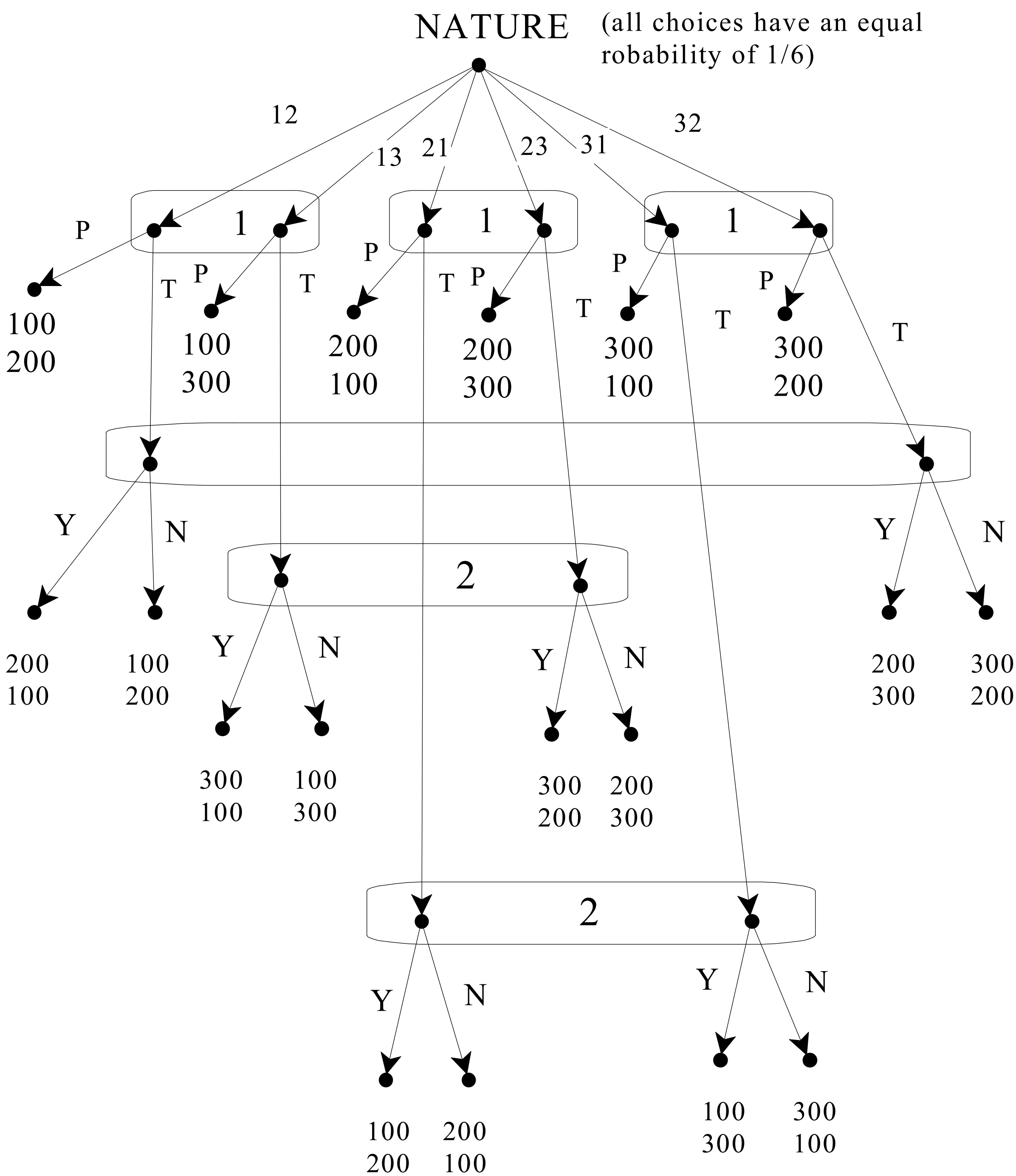
(b) Player 1 has eight strategies. One possible strategy is: "if get \$100 pass, if get \$200 propose trade, if get \$300 pass". We will use the following shorthand for the above strategy: PTP. Similarly for the others. Player 2 has only two strategies. How did we get those payoffs? Consider the first cell, for example. Given the strategies PPP and Y, the outcomes are (100,200) with probability 1/6, (100,300) with probability 1/6, (200,100) with probability 1/6, (200,300) with probability 1/6, (300,100) with probability 1/6, (300,200) with probability 1/6. Thus player 1's expected payoff is: (100 + 100 + 200 + 200 + 300 + 300) (1/6) = \$200. Similarly for the other player and for the other cells.

		Player 2	
		Y	N
Player 1	PPP	200 , 200	200 , 200
	PPT	150 , 250	200 , 200
	PTP	200 , 200	200 , 200
	PTT	150 , 250	200 , 200
	TPP	250 , 150	200 , 200
	TPT	200 , 200	200 , 200
	TTP	250 , 150	200 , 200
	TTT	200 , 200	200 , 200

The Nash equilibria are highlighted.

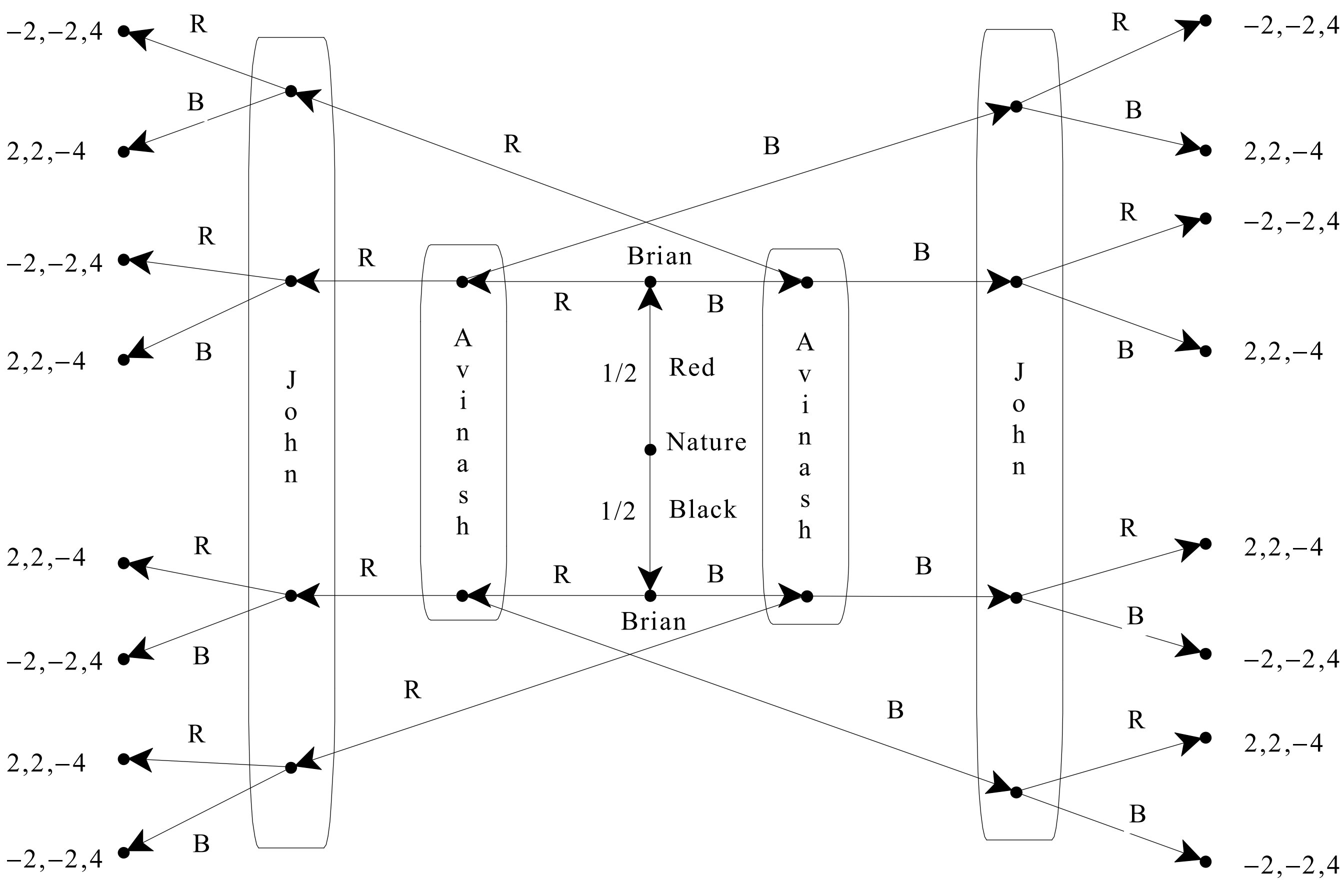
(c) For player 1 all the strategies are weakly dominated, except for TPP and TTP. Elimination of the weakly dominated strategies leads to a game where Y is strictly dominated for 2. Thus we are left with (TPP, N) and (TTP, N).

3. **(a)**



(b) Player 1's strategies are the same as before. Player 2 now has 8 strategies. Each strategy has to specify how to reply to player 1's proposal depending on the sum he (player 2) has. Thus one possible strategy is: if I have \$100 I say No, if I have \$200 I say Yes and if I have \$300 I say No.

4. **(a)**



(b) Each player has two information sets hence four strategies (one of two choices at each information set). The normal form is as follows.

Avinash

Brian	if B, B, if R, R	if B, B, if R, B	if B, R, if R, R	if B, R, if R, B
if B, B, if R, R	−2,−2, 4	0, 0, 0	0, 0, 0	2, 2, −4
if B, B, if R, B	0, 0, 0	0, 0, 0	0, 0, 0	0, 0, 0
if B, R, if R, R	0, 0, 0	0, 0, 0	0, 0, 0	0, 0, 0
if B, R, if R, B	2, 2, −4	0, 0, 0	0, 0, 0	−2,−2, 4

John chooses: if B, B and if R, R

Avinash

Brian		if B, B, if R, R	if B, B, if R, B	if B, R, if R, R	if B, R, if R, B
	if B, B, if R, R	0, 0, 0	0, 0, 0	0, 0, 0	0, 0, 0
	if B, B, if R, B	0, 0, 0	0, 0, 0	0, 0, 0	0, 0, 0
	if B, R, if R, R	0, 0, 0	0, 0, 0	0, 0, 0	0, 0, 0
	if B, R, if R, B	0, 0, 0	0, 0, 0	0, 0, 0	0, 0, 0

John chooses: if B, B and if R, B

Avinash

Brian		if B, B, if R, R	if B, B, if R, B	if B, R, if R, R	if B, R, if R, B
	if B, B, if R, R	0, 0, 0	0, 0, 0	0, 0, 0	0, 0, 0
	if B, B, if R, B	0, 0, 0	0, 0, 0	0, 0, 0	0, 0, 0
	if B, R, if R, R	0, 0, 0	0, 0, 0	0, 0, 0	0, 0, 0
	if B, R, if R, B	0, 0, 0	0, 0, 0	0, 0, 0	0, 0, 0

John chooses: if B, R and if R, R

		Avinash			
		if B, B, if R, R	if B, B, if R, B	if B, R, if R, R	if B, R, if R, B
Brian	if B, B, if R, R	2, 2, −4	0, 0, 0	0, 0, 0	−2,−2, 4
	if B, B, if R, B	0, 0, 0	0, 0, 0	0, 0, 0	0, 0, 0
	if B, R, if R, R	0, 0, 0	0, 0, 0	0, 0, 0	0, 0, 0
	if B, R, if R, B	−2,−2, 4	0, 0, 0	0, 0, 0	2, 2, −4

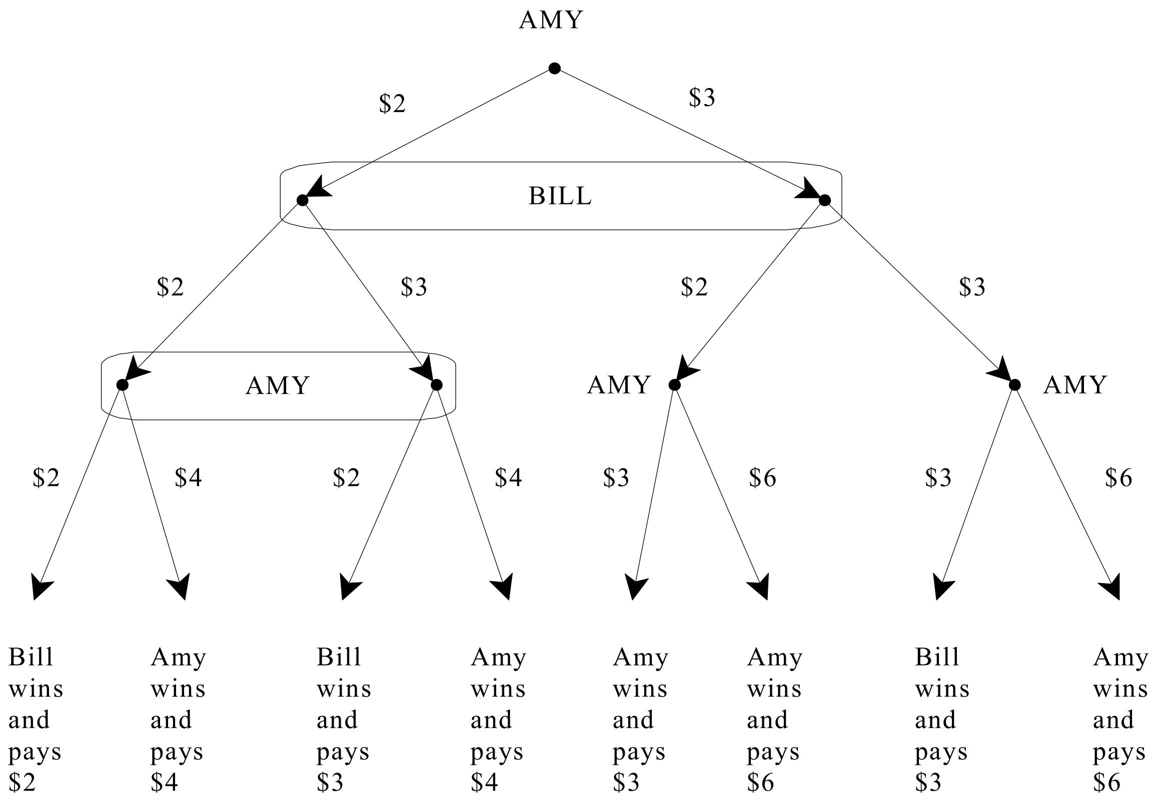
John chooses: if B, R and if R, B

How can we fill in the payoffs without spending more than 24 hours on this problem? There is a quick way of doing it. Start with the two colors, B and R. Under B write T (for true) if Brian's strategy says "if B then B" i.e. if Brian plans to tell the truth and write F (for false) if Brian's strategy says "if B then R" i.e. if Brian plans to lie; similarly, under R write T (for true) if Brian's strategy says "if R then R" and write F (for false) if Brian's strategy says "if R then B". In the next row, in the B column rewrite what is in the previous row if Avinash's strategy says "if B then B" and change a T into an F or an F into a T if Avinash's strategy says "if B then R". Similarly for the R column. Now repeat the same for John (in the B column a T remains a T and an F remains an F is John's strategy is "if B then B", while a T is changed into an F and an F is changed into a T if John's strategy is "if B then R"). Now in each column the payoffs are $(-2, -2, 40)$ if the last row has a T and $(2, 2, -4)$ if the last row has an F. The payoffs are then given by $\frac{1}{2}$ payoff in left column + $\frac{1}{2}$ payoff in right column.

For example, look at the cell in the second row, third column of the third matrix:

	B	R	
Brian's strategy: if B, B and if R, B	T	F	
Avinash's strategy: if B, R and if R, R	F	T	
John's strategy: if B, R and if R, R	F	T	Payoffs:
Payoffs	$(2, 2, -4)$	$(-2, -2, 4)$	$\frac{1}{2}(2, 2, -4) + \frac{1}{2}(-2, -2, 4) = (0, 0, 0)$

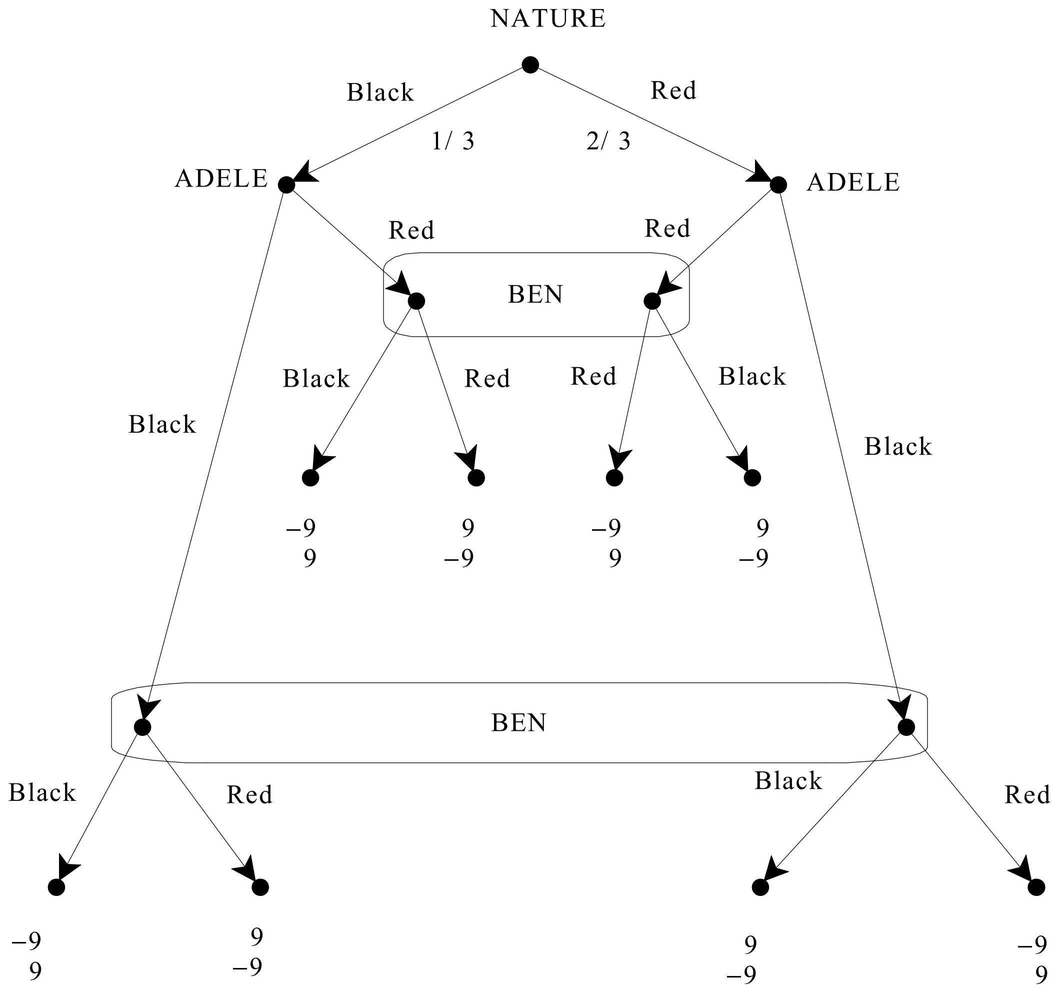
5. (a)


AMY
$2
$3
BILL
$2
$3
$2
$3
AMY
AMY
AMY
$2
$4
$2
$4
$3
$6
$3
$6
Bill wins and pays $2
Amy wins and pays $4
Bill wins and pays $3
Amy wins and pays $4
Amy wins and pays $3
Amy wins and pays $6
Bill wins and pays $3
Amy wins and pays $6

(b) The normal form is as follows:

		BILL	
		bid $2	bid $3
	$2, $2, $3, $3	Bill wins pays 2	Bill wins pays 3
	2 , 2, 3, 6	Bill wins pays 2	Bill wins pays 3
	2, 2, 6, 3	Bill wins pays 2	Bill wins pays 3
	2, 2, 6, 6	Bill wins pays 2	Bill wins pays 3
A	2, 4, 3, 3	Amy wins pays 4	Amy wins pays 4
M	2, 4, 3, 6	Amy wins pays 4	Amy wins pays 4
Y	2, 4, 6, 3	Amy wins pays 4	Amy wins pays 4
	2, 4, 6, 6	Amy wins pays 4	Amy wins pays 4
	3, 2, 3, 3	Amy wins pays 3	Bill wins pays 3
	3 , 2, 3, 6	Amy wins pays 3	Amy wins pays 6
	3, 2, 6, 3	Amy wins pays 6	Bill wins pays 3
	3, 2, 6, 6	Amy wins pays 6	Amy wins pays 6
	3, 4, 3, 3	Amy wins pays 3	Bill wins pays 3
	3, 4, 3, 6	Amy wins pays 3	Amy wins pays 6
	3, 4, 6, 3	Amy wins pays 6	Bill wins pays 3
	3, 4, 6, 6	Amy wins pays 6	Amy wins pays 6

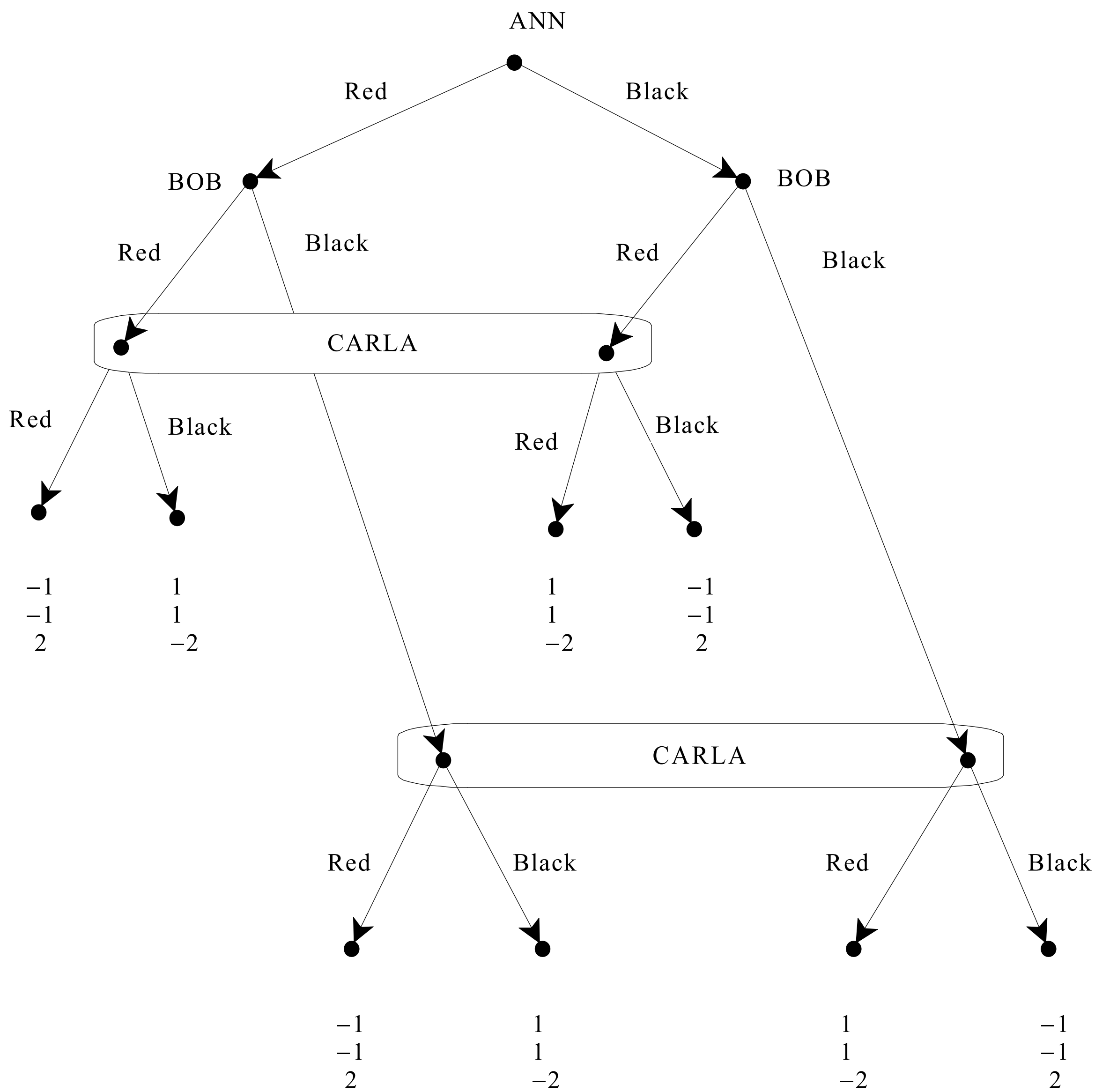
6. **(a)**



(b) The normal form is as follows:

Adele \ **Ben**	B always	B if B R if R	R if B B if R	R always
B always	3 , –3	3 , –3	–3 , 3	–3 , 3
B if B R if R	3 , –3	–9 , 9	9, –9	–3, 3
R if B B if R	3, –3	9 , –9	–9 , 9	–3 , 3
R always	3 , –3	–3 , 3	3 , –3	–3 , 3

7. **(a)**



(b) The normal form is as follows:

Ann's strategy set: {B, R},

Bob's strategy set: {(BB,RB), (BB, RR), (BR, RB), (BR, RR)} where, for example, the strategy (BR, RB) means: if Ann writes Black I will say Red and if she writes Red then I will say Black.

Carla's strategy set: {(BB,RB), (BB, RR), (BR, RB), (BR, RR)} where, for example, the strategy (BR, RR) means: if Bob says Black I will say Red and if he says Red then I will say Red.

Letting Ann choose rows, Bob columns and Carla matrices, the normal form will consist of four 2 × 4 matrices.

BOB

ANN	BB,RB	BR,RR	BB,RR	BR,RB
B	-1,-1,2	-1,-1,2	-1,-1,2	-1,-1,2
R	1,1,-2	1,1,-2	1,1,-2	1,1,-2

CARLA: BB, RB

BOB

ANN	BB,RB	BR,RR	BB,RR	BR,RB
B	1,1,-2	1,1,-2	1,1,-2	1,1,-2
R	-1,-1,2	-1,-1,2	-1,-1,2	-1,-1,2

CARLA: BR, RR

BOB

ANN	BB,RB	BR,RR	BB,RR	BR,RB
B	-1,-1,2	1,1,-2	-1,-1,2	1,1,-2
R	1,1,-2	-1,-1,2	-1,-1,2	1,1,-2

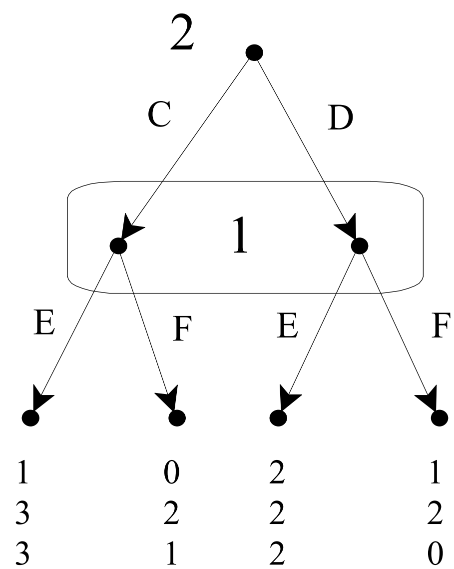
CARLA: BB, RR

BOB

ANN	BB,RB	BR,RR	BB,RR	BR,RB
B	1,1,-2	-1,-1,2	1,1,-2	-1,-1,2
R	-1,-1,2	1,1,-2	1,1,-2	-1,-1,2

CARLA: BR, RB

8. First we find the Nash equilibria of the subgame on the left:

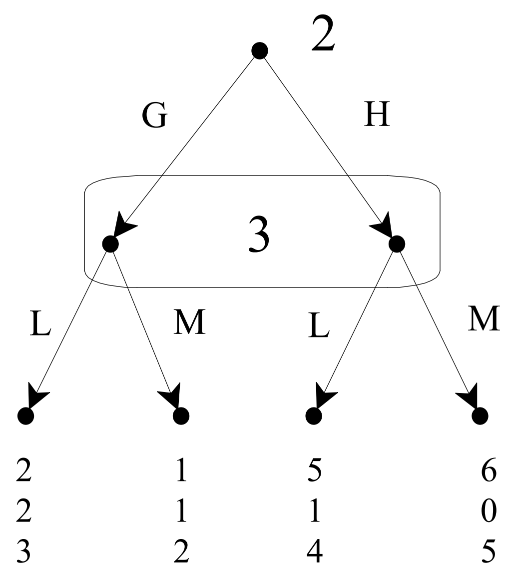


whose normal form is:

		Player 2	
		C	D
Player 1	E	1 , 3	2 , 2
	F	0 , 2	2 , 0

This game has a unique Nash equilibrium, where player 1 chooses E and player 2 chooses C.

Next we find the Nash equilibria of the subgame on the right:

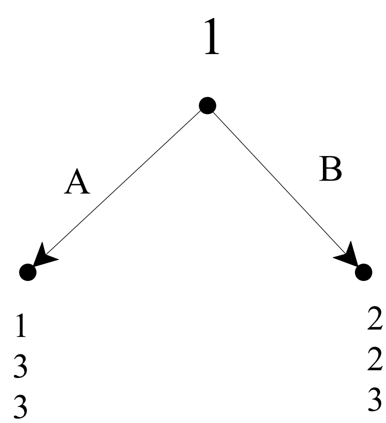


whose normal form is:

		Player 3: L	Player 3: M
Player 2	G	2 , 3	1 , 2
	H	1 , 4	0 , 5

This game has a unique Nash equilibrium, where player 2 chooses G and player 2 chooses L.

Thus the game reduces to:



and the optimal choice for player 1 is B. Thus the subgame-perfect equilibrium is given by:

strategy of player 1: BE

strategy of player 2: CG

strategy of player 3: L