Problem Set 1

1. Games in Strategic Form: Do questions 1-4 and one of 5a, 5b, or 5c

Underline all best responses, then perform iterated deletion of strictly dominated strategies. In each case, do you get a unique prediction for the outcome of the game? Find all pure-strategy Nash equilibria.

(a)

		B	
		L	R
	U	$2,3$	$4,2$
A	D	$-1,2$	$1,1$

(b)

		B		
		L	M	R
	T	$0,10$	$-2,-1$	$-2,-2$
	U	$1,1$	$0,3$	$2,2$
	D	$-1,-1$	$2,2$	$4,1$
A	B	$7,4$	$1,5$	$3,3$

(c)

		B		
		L	C	R
	U	$3,5$	$1,4$	$2,2$
	M	$2,4$	$3,3$	$-1,-3$
A	D	$1,1$	$1,11$	$5,0$

(d)

		B			
		L	C	O	R
	U	$2,2$	$1,3$	$6,2$	$1,1$
	T	$4,3$	$2,2$	$5,1$	$2,-3$
A	M	$1,9$	$1,5$	$-3,2$	$8,8$
	D	$5,8$	$-1,7$	$2,2$	$1,10$

2. Write the following games in strategic form. Find all pure-strategy Nash equilibria, if they exist.

(a). Two firms that make alcohol spend money trying to outdo one another in advertising. If they both advertise, they share industry profits of $10,000$ equally, but if one firm advertises and the other doesn't, the advertising firm gets $7,000$ and the non-advertiser gets $3,000$. If they fail to advertise, there are fewer sales but both save on the cost of the ad campaigns, and each makes $6,000$.

(b). Two firms work closely together and have to decide whether to buy Mac or Windows computers. If they both buy the same platform, they coordinate well together, and earn profits of 3 each. If they buy different platforms, they have trouble coordinating, and get a payoff of 1 each.

(c). You and an opponent both have a penny. Secretly, you choose Heads or Tails, then simultaneously reveal the strategies you picked. If the coins match (both heads or both tails), you get both pennies. If they are different (one head and one tail), your opponent gets both.

(d). Two firms are deciding whether or not to research a new technology. If neither firm researches, they both make zero. If both firms research, they get 5 each. If one firm researches and the other doesn't, the firm that does the research gets 10 and the firm that doesn't research gets 0.

3. Two firms can choose any whole dollar price from 0 to 4 to charge for their product, for which they have no production costs. The overall demand is for the good is $D(p) = 4-p$; however, since there are two firms, their prices determine what share of the market each firm gets. If the firms choose the same price, they split the market half-half. If one firm charges a strictly higher price, it gets nothing, while the firm that charged less serves the whole market. More formally, if p_i is firm i's price and p_j is its opponent's price, then profits for firm i are

$$\pi_i(p_i, p_j) = \begin{cases} (4-p_i)p_i & , p_i < p_j \\ (4-p_i)p_i/2 & , p_i = p_j \\ 0 & , p_i > p_j \end{cases}$$

Write out the strategic form and solve by iterated elimination of weakly dominated strategies. Find all Nash equilibria. Were any Nash equilibria removed by elimination of weakly dominated strategies? Is there an economic argument to focus on any of the eliminated strategies that you can think of?

4. (a). Consider the following *three* player simultaneous-move game: Player A chooses the strategic from $\{1, 2\}$, player B chooses a row, and C chooses a column. A gets the first number as payoff, B gets the second, C gets the third.

(1)

		C: L	C: R
B	U	$2,2,3$	$1,1,2$
B	D	$1,3,4$	$1,4,3$

(2)

		C: L	C: R
B	U	$1,1,3$	$2,1,2$
B	D	$-2,5,2$	$2,2,1$

Just to recap, Player A chooses either the left or the right strategic form above, not knowing what players B and C are doing; Player B chooses from U or D, not knowing what players A or C are doing; and Player C chooses from L or R, not knowing what Players A or B are doing. Find the dominant strategy equilibrium of the three-player game. (Explain all your reasoning).

(b). What are the pure-strategy Nash equilibria of the following three-player game:

(1)

		C: L	C: R
B	U	$2,1,1$	$0,5,4$
B	D	$1,2,4$	$1,2,3$

(2)

		C: L	C: R
B	U	$0,1,2$	$2,3,3$
B	D	$4,0,5$	$2,2,3$

5a. Our assumptions that players are intelligent, rational and have perfect information is pretty strong. It's easy to imagine players failing to complete a long string of iterated deletions of dominated strategies, or not being thoughtful enough to start this process in the first place. One suggestion to model learning in games is called "best-response dynamics", and it works like this: The game is repeated a large number of rounds, and players choose their strategy each new round to play a best-response to their opponent's *last* move.

So consider the game below. If the row player used b this round, the column player's best-response would have been z for a payoff of 3. If the column player's strategies are decided by best-response dynamics, then, we'd predict he should play z in the next round.

	x	y	z
a	3,3	1,2	1,0
b	0,1	2,2	3,3
c	1,2	-1,-1	4,4

(a). Find all Nash equilibria.
(b). Show that if you start at (b, y), play eventually reaches (c, z) and stays there permanently. Is this a Nash equilibrium?
(c). Show that if you start at (a, z), the learning process never reaches a Nash equilibrium.
(d). Is there any starting point that converges to the Nash equilibrium at (a, x)? Why is this a "fragile" Nash equilibrium under best-response dynamics, especially if players sometimes make mistakes?
(e). Show that if each player uses the same strategy twice, then their strategies form a pure-strategy Nash equilibrium, and they will play those strategies in all future periods.
(f). Is this a good model of learning or not? Explain your answer (particularly taking your answer to part (v) into account).

5b. The purpose of this question is to show that while the tools we have adopted (best-responses, strategy dominance, iterated deletion of dominated strategies) are useful, there are other ways to approach the material.

Rather than focus on dominance, let's try another idea, called "rationalizability". A strategy is *rationalizable* if it is a best-response to *some* profile of strategies your opponents could use.
(a). Write a mathematical definition for a rationalizable strategy similar to the definition of a dominant strategy, using the $u_i(s_i, s_{-i})$ notation. It's OK if you find this hard, or don't think your answer is correct; this is just an exercise in thinking in alternative ways about players, games and behavior.
(b). Write out a process similar to iterated deletion of strictly dominated strategies called 'iterated deletion of unrationalizable strategies'. Use it to solve the following game:

	L	C	R
U	3,2	1,1	3,3
M	5,3	6,6	2,3
D	2,2	-1,5	5,4

Is the solution a Nash equilibrium? Try solving the same game using IDDS; do you get a different prediction?
(c). For prisoners' dilemma, what are the rationalizable strategies? For battle of the sexes and matching pennies, show that all strategies for all players are rationalizable.
(d). Show that no strictly dominated strategy is rationalizable, using your definition from i. Are weakly dominated strategies rationalizable? Show that dominant strategies are always rationalizable. Are rationalizable strategies always dominant?
(e). Show that in a pure-strategy Nash equilibrium, both players are using rationalizable strategies.

5c. This question involves agents who care about how their payoff compares to that of their opponent.

We start from a particular strategy profile (a_i, a_j) in a strategic form, where the players have *material payoffs* $u_i(a_i, a_j)$, that capture the direct payoff to player i of a given strategy profile (a_i, a_j). The *kindness of player j to player i at* (a_i, a_j) is given by

$$k_{ji}(a_i, a_j) = u_i(a_i, a_j) - \frac{u_i(a_i, a_j) + u_j(a_i, a_j)}{2}$$

This compares the material payoff of player i to the average of the two player's material payoffs. If the average material payoff is larger than player i's material payoff, k_{ji} will be negative and player j is unkind to player i. If the average payoff is smaller than player i's material payoff, k_{ji} is positive and player j is being kind to player i. Note that if $k_{ji} > 0$ then $k_{ij} < 0$, so that if one player is being kind, the other player must be unkind.

Define the player's *kindness payoffs* as

$$U_i(a_i, a_j) = u_i(a_i, a_j) + ck_{ji}(a_i, a_j)$$

so that player i cares about his payoff per se, but also how kind his opponent is to him. The value c can be positive or negative, and measures how strongly the "kindness" motive affects the players' payoffs. A strategy profile $a^* = (a_i^*, a_j^*)$ is a pure-strategy Nash equilibrium if is a Nash equilibrium in the strategic form game with payoffs $(U_i(a_i, a_j), U_j(a_i, a_j))$.

(a). For $c = 3$, rewrite the strategic form for Battle of the Sexes with payoffs of $U_i(a_i, a_j)$ rather than $u_i(a_i, a_j)$, where the material payoffs are given by

		Column	
		L	R
Row	U	$2, 1$	$0, 0$
	D	$0, 0$	$1, 2$

Do the equilibrium predictions change? Explain the role that c plays in changing the agents' motives. Now, for $c = -1$, rewrite the strategic form for Prisoners' Dilemma with payoffs of $U_i(a_i, a_j)$ rather than $u_i(a_i, a_j)$, where the material payoffs are given by

		Column	
		L	R
Row	U	$1, 1$	$-1, 2$
	D	$2, -1$	$0, 0$

Do the equilibrium predictions change? Explain the role that c plays in changing the agents' motives.

(b). For what values of c are (F, B) or (B, F) pure-strategy Nash equilibria in the Battle of the Sexes game, and for what values of c are (F, F) and (B, B) pure-strategy Nash equilibria in the Battle of the Sexes game? Explain why different values of c lead to different equilibrium outcomes.

(c). Behavioral economists have noted that, "the same people who are altruistic to other altruistic people are also motivated to hurt those who hurt them" (*Advances in Behavioral Economics*, by Camerer, Loewenstein and Rabin, p. 296). We'll call this idea *reciprocal altruism.* Is the above model a good model of reciprocal altruism? Explain why or why not.

(d). Explain whether the following statement is true or false: "The contribution of behavioral economics is to explain why the payoffs take the values they do, because once we fix the players' payoffs we can then use standard game theory to analyze the strategic form. Therefore, it is without loss of generality to focus on self-regarding players."