Spring 2006 – Chesnes
120 Minutes – 500 Points

Economics 414 – Final Solutions

1. Matching Pennies

a. *(20%)* Dominant Strategy: a strategy that is best for a player in a game, regardless of the strategies chosen by the other players. Payoffs must be STRICTLY higher in the dominated strategy, not just weakly greater.

b. *(40%)* There is no pure strategy NE in this game. Thus, by Nash Existence, there must be a mixed strategy NE. Equalizing the expected payoffs from choosing Head and Tail yields a mixed strategy NE of

$$[\ (1/2,1/2),\ (1/2,1/2)\ ]$$

Expected payoffs to each player are 0.

c. *(40%)* Since player 2 moves second, he can simply choose the opposite of what player 1 chose and the payoff to the two players is always (-1,1). So working backwards, the SPNE involves player 2 choosing (T,H), ie, choose T if player 1 chooses H and choose H if player 1 chooses T. Player 1 can randomize between H and T with probabilities p and 1-p respectively such that $0 <= p <= 1$. A lot of you said that player 1 is INDIFFERENT between choosing H and T, but this implies that a mixed strategy will result. This game has an infinite number of SPNE; each yielding expected payoffs of –1 and +1 to players 1 and 2 respectively.

2. Stackelberg with n = 3

a. *(10%)* There are an infinite number of sub-games. One for each quantity choice of firm 1.

b. *(90%)* Firm 2 chooses q_2 to solve max $\{ q_2(a - q_1 - q_2 - q_3) \}$. Set the FOC equal to zero implies $q_2 = 0.5(a-q_1-q_3)$. Symmetrically for firm 3, $q_3 = 0.5(a-q_1-q_2)$. Solve these two equations simultaneously, yields:

$$q_2=q_3 = (1/3)(a-q_1)$$

Firm 1 then chooses q_1 to solve max $\{ q_1(a-q_1 - (2/3)(a-q_1)) \}$. FOC = 0 implies $q_1 = a/2$. Plug into above to find the SPNE:

$$(q_1 = a/2, \quad q_2 = q_3 = a/6)$$

Price is, $P = a-Q = a - a/2 - a/6 - a/6 = a / 6$. Profits for firm 1 are thus, $P^*q_1 = (a/6)^*(a/2) = a^2/12$. Profits for firms 2 and 3 are thus, $(a/6)^*(a/6) = a^2/36$.

3. **Brown/Smith from PS 4**
    a. *(10%)* Strategies for Brown = {L,R}. Strategies for Smith = {TT, TB, BT, BB}. Since Brown is uncertain who he is playing against, we model Smith has having strategies (X,Y) where X is what Smith does in case 1 and Y is what Smith does in case 2.

    b. *(70%)* See problem set 4 solutions. There are 3 BNE: (TT, L), (TB, R), and (BB,R).

    c. *(20%)* There are many answers to this question including sealed-bid auctions and firms competing without fully knowing the cost structure of their competitors. Prisoners' dilemma is NOT a game of imperfect information. Players both know the strategies and payoffs of their opponent. Imperfect information must be in regards to the payoff structure of your opponent. E.g., an opposing bidder's valuation or an opposing firm's cost function.

4. **Beer / Quiche**
    a. *(20%)* See lectures notes.

    b. *(40%)* Pooling on beer means q=0.1. Dueling after seeing the Beer signal yields a receiver an expected payoff of 1(0.1) + (-1)(0.9) = -0.8. Not dueling after seeing beer yields zero in expectation so a receiver will not duel if he sees beer. A wimpy sender requires that a receiver, having seen Quiche, will choose to duel (otherwise, a wimpy sender would want to deviate from choosing beer and getting 2 versus choosing quiche and getting 3). Thus we need E[Duel| Quiche] >= E[Not |Quiche] or p-(1-p) >= 0. Thus p >= ½. So our PBE is
    { (Beer, Beer), (Duel, Not), p >= 1/2 , q = 0.1 }

    c. *(40%)* Separating (Quiche, Beer) is not a PBE because with p = 1 and q = 0, a receiver's optimal strategy will be (Duel, Not). With these strategies, a wimpy sender gets a payoff of 1, but can deviate and play Beer and get a payoff of 2. So no PBE of this type exists.

5. **Grim Trigger**
   a. *(10%)* (T,L) is the unique pure strategy NE.

   b. *(40%)* Grim trigger strategies that would support (B,R) as the SPNE of the infinitely repeated game are as follows:
      i. For player 1, play B in the first period. For all other periods, as long as no player has deviated (not played (B,R)) in any previous period, play B. If any deviation by any player has occurred, play T forever.
      ii. For player 2, play R in the first period. For all other periods, as long as no player has deviated (not played (B,R)) in any previous period, play R. If any deviation by any player has occurred, play L forever.

   c. *(50%)* Equilibrium path payoffs are $4(1+\delta+\delta^2+\ldots) = 4/(1-\delta)$. Player 2 does NOT have a unilateral profitable deviation. Player 1, however, can deviate to T and get a payoff of 10, but that pulls the trigger. So player 1's deviation payoff is $10 + 3(\delta + \delta^2 + \ldots) = 10 + 3\delta/(1-\delta)$. Our trigger strategies form a SPNE where collusion is sustained if.

$$4/(1-\delta) >= 10 + 3\delta/(1-\delta) \qquad (*)$$
$$4 >= 10 - 10\delta + 3\delta$$
$$7\delta >= 6$$
$$\delta^* >= 6/7$$

   Note this is fairly close to 1. This is because the one period deviation payoffs to player 1 is relatively high compared to the collusive payoff (10 versus 4). As we increase the deviation payoff, the critical discount factor also would get closer to 1. *Full credit was obtained for saying that the critical discount factor satisfies equation (*).*