

1. Bruster's and Rita's both sell equally delicious ice cream, and compete for the same customers. Each can offer customers a rewards card (offering free ice cream after a certain number of purchases), or not. Suppose that profits at each firm are greater if neither offers a rewards card than if both do. If one firm offers a rewards card and the other doesn't, the one offering the rewards card earns higher profits than in the case where neither firm offers a reward card, while the firm that does not offer a rewards card earns lower profits than in the case where both firms offer a rewards card.
  - (a) Choose some profit numbers (payoffs) for this game that are consistent with the description above and write the game in normal form. Be sure to label the strategies and players in your game matrix.
  - (b) Does either player have a dominant strategy in this game? If so what is it?
  - (c) If Bruster's Sweet Rewards card offers 1 free ice cream for every 10 purchased, what would you expect Rita's Cool Card to offer in equilibrium and why?
2. A prisoner is trying to escape from prison. He can attempt to climb over the prison wall or dig a tunnel under the prison wall. The warden can prevent the prisoner from climbing over the wall by posting guards at the wall and he can prevent the prisoner from tunneling under the wall by having regular inspections of cells, but he has only enough guards to do one or the other and not both.
  - (a) Choose (and justify) some simple numerical payoffs for this game, and then write the game in normal form. Be sure to label the strategies and players in your game matrix. Is this game constant sum or non-constant sum? Explain/justify your answer.
  - (b) What is the mutual best response (Nash equilibrium) for the simultaneous move version of this game, as you have depicted in parts a or b? Give your reasoning.
3. In each of the following three games, each player can choose between two actions, cooperate or defect. Suppose that in all three games, higher payoff numbers are preferred to lower payoff numbers. For each game, find all of the pure and mixed strategy Nash equilibria. Show/explain how you found these equilibria.
  - (a) Prisoners Dilemma

		Player 2	
		Cooperate	Defect
		Cooperate	70,70
Player 1		Defect	80,10
			40,40

- (b) Stag Hunt

		Player 2	
		Cooperate	Defect
		Cooperate	70,70
Player 1	Cooperate	70,70	5,40
	Defect	40,5	40,40

(c) Chicken

		Player 2	
		Cooperate	Defect
		Cooperate	70,70
Player 1	Cooperate	70,70	50,80
	Defect	80,50	40,40

4. Suppose that Pat and Sam intended to communicate with each other about what to do tonight but the message never got through. Now each has to simultaneously and independently decide where to show up (communication is no longer possible). There are just two possibilities a ball game or a concert. Other things equal, Pat likes ball games better, and Sam likes concerts better. Both Pat and Sam agree that either event would be more fun if the other person were also there. However, Pat and Sam differ in their attitudes about how important it is that they be there together. Since Pat is choosing between the game and the concert and Sam is facing the same two choices, there are four possible outcomes. The table below shows how Sam and Pat rank these four outcomes.

Outcome	Pat's Ranking	Sam's Ranking
Best	Pat at game, Sam at game	Sam at concert, Pat at concert
Second Best	Pat at game, Sam at concert	Sam at game, Pat at game
Third Best	Pat at concert, Sam at concert	Sam at concert, Pat at game
Worst	Pat at concert, Sam at game	Sam at game, Pat at concert

- (a) Write down the normal form of this game. Choose payoffs that are consistent with the rankings given in the table above. Assume there are no ties (e.g. best is strictly better than second best, which is strictly better than third best which is strictly better than worst).
- (b) Find all pure strategy Nash equilibria for this game. Does either player have a dominant strategy? Explain.
5. Consider the game between a parent and a child. The child can choose to be good (G) or bad (B). The parent can punish the child (P) or not (N). The child gets enjoyment worth 1 from bad behavior and hurt worth -2 from punishment. The payoff to a child who behaves well and is not punished is 0; one who is bad and is not punished gets 1, one who is bad and gets punished earns  $1-2=-1$  and one who is good but is punished gets -2. The parent gets -2 from the child's bad behavior and 0 for good behavior. The parent also gets -1 if s/he inflicts punishment; thus if the child is good and the parent punishes the parent gets  $0-1=-1$ , if bad and

no punishment, the parent gets -2 and if bad and punishment the parent gets -3.

- (a) Set this up as a simultaneous move game and find all the Nash equilibria.
- (b) Next suppose the child chooses G or B first. After observing the child's behavior the parent chooses P or N. Draw the game tree and find the subgame perfect equilibrium.
- (c) Now suppose that before the child acts in the sequential move version of the game, the parent can decide whether or not to threaten to punish the child for bad behavior: Threat P if B (with the implicit promise of no punishment for good behavior) or No Threat. Suppose that, if made, the threat is perfectly credible. Illustrate the game in this case, find the subgame perfect equilibria and explain how and why your answers to parts b and c differ.