Stat 155 Homework # 9 Due April 21

Problems:

Q 1 Karlin-Peres Chapter 11 Q 11.1

The men proposing stable matching is A with Y, B with Z and C with X. In the women proposing algorithm is the same.

Q 2 Karlin-Peres Chapter 11 Q 11.2

Suppose that man m is matched to woman w in M and woman w' in M' and suppose that w is matched to man m' in M'. Suppose that m prefers w to w'. Then if w preferred m to m' then the matching M' would not be stable since m and w would prefere each other to their partners, hence w prefers m' to m.

Q 3 Karlin-Peres Chapter 11 Q 11.3

Suppose we have the following preferences with men A, B, C and women X, Y, Z:

$$\text{For } A: \quad Y > X > Z \qquad \text{For } X: \quad A > B > C \tag{1}$$

$$\text{For } B: \quad X > Y > Z \qquad \text{For } Y: \quad B > A > C \tag{2}$$

$$\text{For } C: \quad X > Z > Y \qquad \text{For } Z: \quad C > B > A \tag{3}$$

The men-proposing stable matching will be A with Y, B with X and C with Z. But if woman X lies and says her preferences are $C > A > B$ then in round 1 when both B and C propose to her she will reject B who will the propose to Y who will accept B. Then A will propose to X which is the first choice of X. The matching will be A with X, B with Y and C with Z so X improves her outcome by lying.

Q 4 There are 4 men, called A, B, C, D and 4 women, called W, X, Y, Z, with the following preference lists:

$$\text{For } A: \quad W > X > Y > Z \qquad \text{For } W: \quad B > A > C > D \tag{4}$$

$$\text{For } B: \quad X > W > Y > Z \qquad \text{For } X: \quad A > B > C > D \tag{5}$$

$$\text{For } C: \quad Y > Z > X > W \qquad \text{For } Y: \quad D > C > B > A \tag{6}$$

$$\text{For } D: \quad Z > Y > W > X \qquad \text{For } Z: \quad C > D > A > B \tag{7}$$

Find the total number of stable matchings.

The men proposing algorithm yields the matching A with W, B with X, C with Y and D with Z while the woman proposing algorithm yields A with X, B with W, C with Z and D with Y. We see that W and X can each be matched with either A or B and no other women since the men and women proposing algorithms give the best and worst outcomes

for the men. Thus there are two ways of matching these men with these women. Similarly men C and D can only be match with women Y and Z and again there are two possible matchings of these men and women. These can be done independently so there is a total of 4 matchings possible.