Econ 414, Exam 1

Name: ____

There are three questions taken from the material covered so far in the course. All questions are equally weighted. If you have a question, please raise your hand and I will come to your desk.

Make sure that you defend your answers with economic reasoning or mathematical arguments, and show that you are using the correct game theoretic concepts by identifying the equilibria explicitly.

Good luck.

1.
Consider the following game in strategic form:

				B	
		w	x	y	z
	a	4,4	3,3	5,1	2,2
	b	3,6	2,5	6,-3	1,4
A	c	-2,0	2,-1	0,0	2,1
	d	1,4	1,2	1,1	3,5

i. Perform iterated deletion of *strictly* dominated strategies (Strike the dominated strategy out with a line, writing the letter of the dominant strategy beside it).

- w strictly dominates x
- d strictly dominates c
- w strictly dominates y
- a strictly dominates b

This leaves strategies a, d, w, and z.

ii. Find all pure-strategy Nash equilibria, or explain why one doesn't exist.
There are two pure-strategy Nash equilibria: (a, w) and (d, z).

iii. If there is a mixed-strategy Nash equilibrium, find the equilibrium strategy for the row player.

For the row player to make the column player indifferent over her undominated pure strategies:

$$\mathbf{E}u_{col}(w) = \mathbf{E}u_{col}(z)$$

$$\sigma_a 4 + \sigma_d 4 = \sigma_a 2 + \sigma_d 5 \longrightarrow 2\sigma_a = \sigma_d$$

Then since $\sigma_a + \sigma_d = 1$ and $2\sigma_a = \sigma_d$, we can solve for the equilibrium strategies:

$$\sigma_a^* = \frac{1}{3}, \sigma_d^* = \frac{2}{3}$$

3. Cournot Competition with Different Costs

Assume there is a high cost firm, who has total costs $C_h(q_h) = c_h q_h$, and a lost cost firm, who has total costs $C_l(q_l) = c_l q_l$, with $c_h > c_l$. The market price is $p(q_h, q_l) = A - q_h - q_l$.

i. Write out the firms' profit maximization problems, and solve for their best-response functions and graph them. Does the game have strategic complements or strategic substitutes?
Profit functions:

$$\pi_h(q_h, q_l) = (A - q_h - q_l)q_h - c_h q_h$$

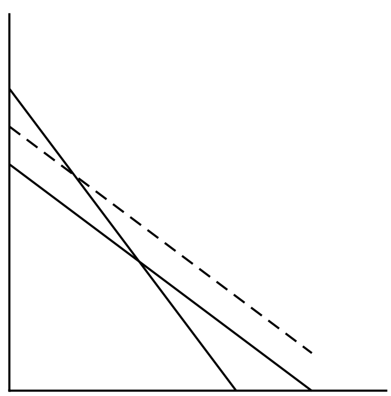
$$\pi_l(q_l, q_h) = (A - q_l - q_h)q_l - c_l q_l$$

Best response functions:

$$\frac{\partial \pi_h(q_h, q_l)}{\partial q_h} = A - 2q_h - q_l - c_h = 0 \longrightarrow q_h(q_l) = \frac{A - c_h - q_l}{2}$$

$$\frac{\partial \pi_l(q_l, q_h)}{\partial q_l} = A - 2q_l - q_h - c_l = 0 \longrightarrow q_l(q_h) = \frac{A - c_l - q_h}{2}$$

Since $dq_h(q_l)/dq_l = -1/2$ and $dq_l(q_h)/dq_h = -1/2$, the best response functions are downward sloping, so the game exhibits strategic substitutes. Here's a rough sketch of the correct graph (see the notes for a more detailed graph):



ii. Solve for the Nash equilibrium strategies.
We can substitute one best response function into the other, as I did in class, or use the first-order conditions as follows:
Substract the first-order condition for firm low from the first-order condition for firm high:

$$A - 2q_h - q_l - c_h - [A - 2q_l - q_h - c_l] = -q_h + q_l - c_h + c_l = 0 \longrightarrow q_h = q_l - c_h + c_l$$

Subsitute this simpler expression into the low cost firm's best-response function:

$$q_l(q_l - c_h + c_l) = \frac{A - c_l - [q_l - c_h + c_l]}{2} \longrightarrow q_l^* = \frac{A - 2c_l + c_h}{3}$$

Likewise, we can substitute $q_l = q_h + c_h - c_l$ into the high cost firm's best response function:

$$q_l(q_h + c_h - c_l) = \frac{A - c_h - [q_h - c_l + c_h]}{2} \longrightarrow q_h^* = \frac{A - 2c_h + c_l}{3}$$

These are the Nash equilibrium strategies.

iii. How does a change in c_l change the high cost firm's strategy?
If c_l goes up, q_h^* increases, since c_l enters positively in the numerator of q_h^*. Or,

$$\frac{dq_h^*}{dc_l} = \frac{1}{3} > 0$$

iv. How does a change in A affect the high cost firm's strategy? The low cost firm's strategy? Which firm's strategy is influenced more by a change in A?
If A goes up, both firms increase their strategies. However, the change affects both firms the same:

$$\frac{dq_h^*}{dA} = \frac{1}{3}$$

$$\frac{dq_l^*}{dA} = \frac{1}{3}$$

So, oddly enough, the low cost firm doesn't increase its production more than the high cost firm; they both increase their production the same amount. That's a counterintuitive result, and comes from the $P(q) = A - q$ assumption.

3. Mixed Strategy Equilibrium

Country A is planning an invasion of Country B. A can use its forces to attack beach North or beach South, and B's forces can defend North or South, but not both. If both countries choose to attack or defend the same beach, a battle occurs. Here are the pay-offs:

- W — Country A defeats B
- D — Country B is defeated
- p_n — The probability country A wins if a battle occurs on the North beach
- p_s — The probability country A wins if a battle occurs on the South beach

			B
		North	South
	North	$p_nW, -p_nD$	$W, -D$
A	South	$W, -D$	$p_sW, -p_sD$

i. Find all pure and mixed Nash equilibria.
There aren't any pure Nash equilibria, since $p_nW < W$, $p_sW < W$ and $-p_nD > -D$, $-p_sD > -D$.

So

			B
		North	South
	North	$p_nW, \underline{-p_nD}$	$\underline{W}, -D$
A	South	$\underline{W}, -D$	$p_sW, \underline{-p_sD}$

The mixed equilibrium is:
Row Strategy

$$\mathbf{E}u_{col}(N) = \mathbf{E}u_{col}(S)$$
$$-p_nD\sigma_{row}(N) + -D\sigma_{row}(S) = -D\sigma_{row}(N) + -p_sD\sigma_{row}(S)$$

This equation, and $\sigma_{row}(N) + \sigma_{row}(D) = 1$ yields

$$\sigma_{row}(N) = \frac{(1-p_s)}{(1-p_s)+(1-p_n)}$$

$$\sigma_{row}(S) = \frac{(1-p_n)}{(1-p_s)+(1-p_n)}$$

Column strategy
Similar work yields

$$\sigma_{col}(N) = \frac{(1-p_s)}{(1-p_s)+(1-p_n)}$$
$$\sigma_{col}(S) = \frac{(1-p_n)}{(1-p_s)+(1-p_n)}$$

ii. If W and D change, how are the players' strategies effected?
They're not— Note how the W's and D's all cancel, so the only relevant variable is p_n and p_s, which are the probabilities that the attacker wins the battle.

iii. Show mathematically that if p_n goes up, country A is more likely to attack the North Beach, and country B is more likely to defend it.

With Calculus:

$$\frac{d\sigma_{row}(N)}{dp_n} = \frac{(1-p_n)}{[(1-p_s)+(1-p_n)]^2} > 0$$

$$\frac{d\sigma_{col}(N)}{dp_n} = \frac{(1-p_s)}{[(1-p_s)+(1-p_n)]^2} > 0$$

Without Calculus: Notice that the probabilities of each country of choosing north are

$$\sigma_{col}(N) = \frac{(1-p_s)}{(1-p_s)+(1-p_n)}$$

Note that if p_n goes up, the denominator becomes smaller since $-p_n$ appears, so the whole quantity becomes larger.

$$\sigma_{row}(N) = \frac{(1-p_s)}{(1-p_s)+(1-p_n)}$$

The same argument is true here.

iv. Explain in words why the result in part iii is true.

In mixed strategy Nash equilibrium, I choose my strategy to keep my opponent indifferent over her pure strategies. If a strategy suddenly becomes more favorable to me and less favorable to my opponent, (i) she needs to play her best repsonse to that strategy more frequently to make it less attractive to me and (ii) that forces me to play the strategy more often, to keep her indifferent as well. Since p_n enters both players' utilities, it affects both their strategies.