

Problem 1 (*Iterated Elimination of Strictly Dominated Strategies*)

Consider the iterated elimination of strictly dominated strategies in the strategic form game $\langle \mathcal{I}, (S_i)_{i \in I}, (u_i)_{i \in I} \rangle$. For all $i \in \mathcal{I}$, denote the set of strategies of player i at the k th step of the elimination by S_i^k . Suppose that each $u_i(s_i, s_{-i})$ is continuous and each S_i is compact. Prove that S_i^∞ (for each i) is nonempty.

Hint: You might use the fact that intersection of nested nonempty compact sets is nonempty, i.e.

Suppose $\{A_j\}$ is a collection of sets such that each A_j is nonempty, compact, and $A_{j+1} \subset A_j$. Then $A = \cap_j A_j$ is nonempty.

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Problem 2 (*Iterated Elimination of Strictly Dominated Strategies in Cournot Competition*)

Consider a market in which the price charged for quantity Q of some good is given by $P(Q) = \alpha - \beta Q$ for some $\alpha, \beta > 0$. Assume that the cost of producing a unit of this good is c .

- Assume that there are two firms in the market. Using the iterated elimination of the strictly dominated strategies construct the sets of strategies S_1^k, S_2^k for any fixed k , and conclude that S_1^∞ is a singleton. (Use the definition of S_i^k given in question 1.)
- Assume that there are three firms. Show that S_1^∞ is not a singleton.

Problem 3 Exercise 2.1(a) from Fudenberg and Tirole.

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Problem 4 (*Bertrand Competition with Different Marginal Costs*)

Suppose that two firms (A and B) produce the same good and they have strictly positive marginal costs c_A and c_B such that $c_B > c_A$. Further assume that the firms can produce as many units as they wish at those marginal costs and consumers purchase the good only if the price p offered for the good satisfies $p \leq R$ for a fixed $R > 0$.

- Assume that if the firms offer the same price, the demand is shared equally. Show that under this tiebreaking rule there exists no pure strategy Nash equilibrium.
- There exists a tiebreaking allocation under which the game has a unique equilibrium. Characterize this allocation and the corresponding equilibrium.

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Problem 5 (*Competition with Production Constraints*)

Consider a market with 2 firms which produce the same good. Assume that the demand for this good is Q , and the consumers in this market purchase the good only if its price satisfies $p \leq R$. Further assume that the production level K of each firm satisfies $\frac{Q}{2} < K < Q$.

- Assume that the demand is equally shared among the firms when they offer the same price. Under this tiebreaking rule write the payoff functions of the firms
- Show that there does not exist a pure strategy Nash equilibrium under this tiebreaking rule.
- Prove that this result does not depend on the tiebreaking rule.

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Problem 6 (*A war of attrition*) Two players are involved in a dispute over an object. The value of the object to player i is $v_i > 0$. Time is modeled as a continuous variable that starts at 0 and runs indefinitely. Each player chooses when to concede the object to the other player; if the first player to concede does so at time t , the other player obtains the object at that time. If both players concede simultaneously, the object is split equally between them, player i receiving a payoff of $v_i/2$. Time is valuable: until the first concession each player loses one unit of payoff per unit of time.

Formulate this situation as a strategic game and show that in all Nash equilibria one of the players concedes immediately.

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