Fall 2007 – Chesnes
120 Minutes – 500 Points

Economics 414 – Final

Please answer ALL questions on this examination. Be sure to explain any non-standard notation that you use and JUSTIFY your answers. Each question's weight is shown in parentheses. Good Luck!

1. **(25%) Consider the following simultaneous move, stage game, G.**

Player 2

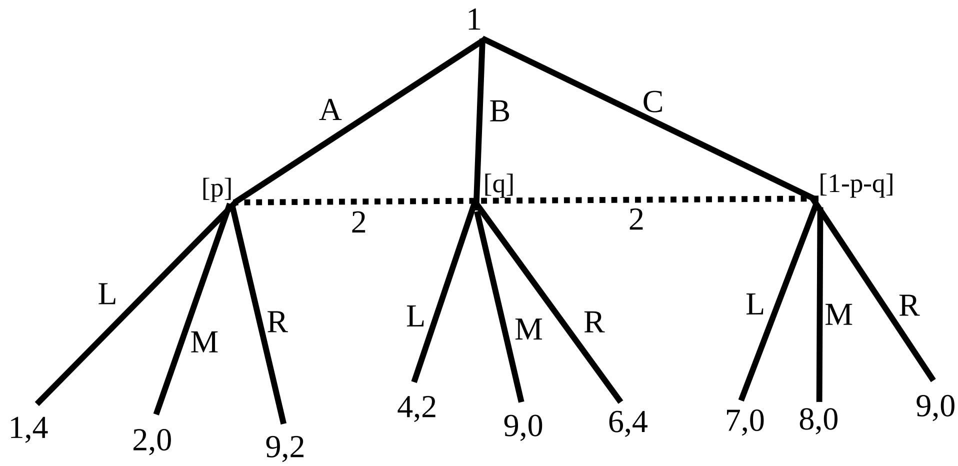
Player 1		L	C	R
	T	(3, 1)	(0, 0)	(20, 0)
	M	(2, 1)	(0, 2)	(3, 1)
	B	(1, 2)	(1, 1)	(4, 4)

   a. **Find the pure strategy Nash Equilibria of G.**

   b. **Consider the infinitely repeated game, $G(\infty,\delta)$. Formulate a grim trigger strategy for each player to sustain (B,R) as the Nash equilibrium of $G(\infty,\delta)$.**

   c. **Solve for the critical discount factor, δ^*, such that your strategies in (b) imply that players optimally choose (B,R) in every period of the game.**

2. **(25%) Consider the following dynamic game of incomplete information:**



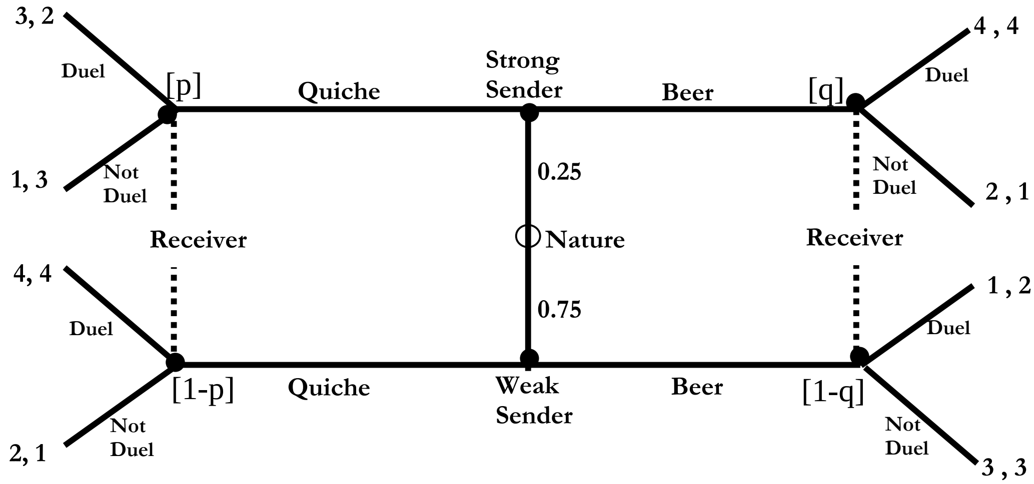
   a. **What are the strategies of each player?**

   b. **Write down the game in strategic (normal) form and solve for the pure strategy Nash Equilibria.**

   c. **Which Nash Equilibria (if any) that you found in part (b) are subgame perfect?**

   d. **Given beliefs, (p, q, 1-p-q) as shown in the game tree, solve for the Perfect Bayesian Equilibria of the game.**

3. (25%) Beer or Quiche?



Nature initially chooses the sender's type (according to the probabilities shown in the tree), which becomes known to the sender but not the receiver. The sender chooses to have beer or quiche for breakfast and the receiver chooses whether or not to duel with the sender.

a. State the 4 requirements necessary for a Perfect Bayesian Equilibrium (PBE).

b. Solve for a PBE involving "pooling on Quiche." (Ie, both types of sender have quiche for breakfast).

c. Solve for a PBE where strong sender has Beer for breakfast and a weak sender has Quiche.

4. **(25%) Repeated Bertrand vs. Repeated Cournot. Consider a duopoly with market (inverse) demand of $P(Q) = 12 - Q$ where each of the two firms have constant and equal marginal cost of production equal to zero.**

   Suppose initially that the game is played only once.

   a. **Show that if the firms could collude, they would set a price of $p^m = 6$ and each earn profits of $\frac{1}{2}\pi^m = 18$.**

   b. **Suppose firms cannot collude but play a Bertrand game. Find the Nash equilibrium profits of each firm.**

   c. **Suppose firms cannot collude but play a Cournot game. Show each firm has a best response function equal to: $q_i(q_j) = 6 - \frac{1}{2} q_j$ and in equilibrium, each firm produces q_i = 4 units and earns a profit of $\pi^c_i = 16$.**

   Now suppose firms cannot collude and they interact repeatedly. Consider the following grim trigger strategy for each firm:

   - ✓ **Choose p^m in the first period.**
   - ✓ **Choose p^m in each subsequent period if no firm has deviated in any prior period.**
   - ✓ **Choose the static Nash Equilibrium (Bertrand or Cournot) strategy in each subsequent period otherwise.**

   d. **Find the critical discount factor, δ^*, required to sustain cooperation (i.e. joint monopoly pricing) in all periods if firms are Bertrand competitors.**

   e. **Find the critical discount factor, δ^*, required to sustain cooperation (i.e. joint monopoly pricing) in all periods if firms are Cournot competitors. HINT: You may use the fact that the optimal one-period deviation by a firm is $q_i^d = 4.5$.**