

# Stat 155 Homework # 2 Due February 10

## Problems:

### Q 1

#### Solution

(a) The P states are the even integers. 100 is a P position

We can verify this by noting that, the terminal state is even, all move from N (odd) states move to P (even) states and all moves from P (even) states are to N (odd) states since  $S$  is entirely made of odd integers.

(b) The pattern PNPNNNNN repeats every nine integers. 100 is an N position

This can be checked by induction.

(c) P states are integers mod 3. 100 is an N position

We can check this by noting that firstly 0, the terminal position, is a P position. Secondly all moves from P positions move to N positions since none of the integers in  $S$  are divisible by 3. Finally from every N position we can move to a P position by subtracting 1 or 2.

### Q 2

(a) In binary 12, 19 and 27 are 1100, 10011 and 11011. The nim-sum is 100 in binary. Only 12 has a one in its third binary digit so the only winning move is to subtract 4 from the pile with 12.

(b) In binary 13, 17, 19, and 23 are 1101, 10001, 10011 and 10111. The nim-sum is 11000. Since 17, 19 and 23 have a one in position 5, the winning moves are to remove 8 from the pile with 17 or 8 from the pile with 19 or 8 from the pile with 23.

(c) In the misere version of Nim the winning strategy is to play the same as in standard nim until there is only one pile with more than one chip. This the winning moves are the same.

### Q 3

The Nim sum of 1 to 63 is exactly 0. Thus there are

### Q 4

By chopping the square  $(1, 2)$ , the one above the terminal square, leaves only a single square and force the opponent to play a losing move. Thus this is a winning move.