Stat 155 Homework # 6 Solutions

Problems:

Q 1 Ferguson Chapter III Section 2.5 Q 5

By marking the optimal value in each column for player one and the optimal value in each row for player two we can find the pure strategies.

(a) The only pure Nash equilibrium is Strategy 2 for player 1 and strategy 2 for player 2 with payoffs $(2, 2)$.

(b) There are 3 pure Nash equilibria: Strategy 5 for player 1 and strategy 1 for player 2 with payoffs $(1, 1)$. Strategy 1 for player 1 and strategy 3 for player 2 with payoffs $(1, 1)$. Strategy 4 for player 1 and strategy 4 for player 2 with payoffs $(0, 0)$.

Q 2 Karlin-Peres Chapter 3 Q 3.3

The matrix is

$$\begin{pmatrix} (8, 8) & (3, 10) \\ (3, 10) & (0, 0) \end{pmatrix}$$

There are two pure Nash equilibria corresponding to one student working and the other defecting. We then check for mixed equilibria which will be symmetric since the game is symmetric. Equalizing the utilities for player 1 given that player 2 uses strategy $(y, 1 - y)$ we have that

$$8y + 3(1 - y) = 10y$$

and hence $y = \frac{3}{5}$. So the optimal strategies are $x = y = (\frac{3}{5}, \frac{2}{5})$.

Q 3 Karlin-Peres Chapter 3 Q 3.8

The matrix is

$$\begin{pmatrix} (\ell/2, \ell/2) & (\ell*, s*) & (\ell*, t) \\ (s*, \ell*) & (s/2, s/2) & (s, t) \\ (t, \ell*) & (t, s) & (t/2, t/2) \end{pmatrix}$$

so the two pure Nash equilibria are one choosing ℓ and the other choosing s. In all the mixed strategies both players use all 3 strategies since if one player uses only two then the other players best response would be to use the third one leading to a pure strategy. We solve by equalize the expected payment under the 3 strategies. Thus if player 2 uses (y_1, y_2, y_3) then

$$y_1 \ell/2 + y_2 \ell + y_3 \ell = y_1 s + y_2 s/2 + y_3 s = y_1 s + y_2 s + y_3 s/2$$

and since $y_1 + y_2 + y_3 = 1$ then for some constant C

$$\ell(1 - y_1/2) = s(1 - y_2/2) = t(1 - y_3/2) = C.$$

Hence
$$y_1 = 2 - \frac{2C}{\ell}, \quad y_2 = 2 - \frac{2C}{s} \quad y_3 = 2 - \frac{2C}{t}.$$

Then
$$1 = y_1 + y_2 + y_3 = 6 - 2C\left(\frac{1}{\ell} + \frac{1}{s} + \frac{1}{t}\right)$$

and hence
$$C = \frac{5}{2}\left(\frac{1}{\ell} + \frac{1}{s} + \frac{1}{t}\right)^{-1}$$

and so
$$y_1 = 2 - \frac{5}{\ell}\frac{1}{\ell}\left(\frac{1}{\ell} + \frac{1}{s} + \frac{1}{t}\right)^{-1}$$
$$y_2 = 2 - \frac{5}{s}\frac{1}{\ell}\left(\frac{1}{\ell} + \frac{1}{s} + \frac{1}{t}\right)^{-1}$$
$$y_3 = 2 - \frac{5}{t}\frac{1}{\ell}\left(\frac{1}{\ell} + \frac{1}{s} + \frac{1}{t}\right)^{-1}.$$

Q 4 Karlin-Peres Chapter 3 Q 3.9 Since we are looking for symmetric equilibria let the strategies for all the players be $(p, 1-p)$ denoting probability p of advertising in the morning. Since the equilibria is mixed we equalise the expected payments and hence,

$$200(1 - p)^2 = 300p^2.$$

Hence $p^2 + 4p - 2 = 0$ and so $p = -2 \pm \sqrt{6}$. Since $-2 - \sqrt{6} < 0$ the only solution is $\sqrt{6} - 2$. Thus we have that the unique symmetric mixed Nash equilibrium is $(\sqrt{6} - 2, 3 - \sqrt{6})$.