Introduction

Game Theory

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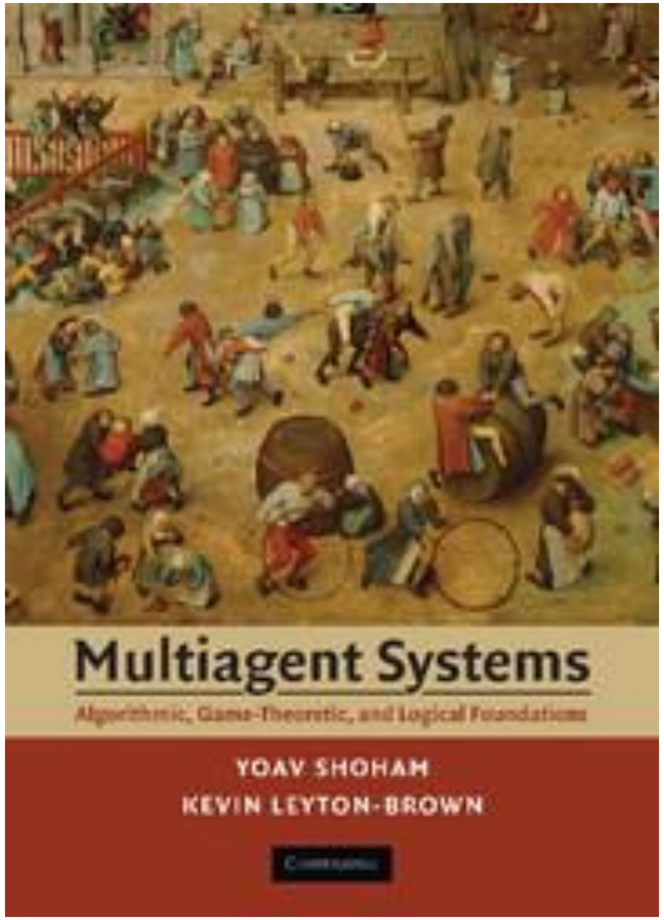
Why Study Games

- Game theory is the mathematical study of interaction among independent, self interested agents
- It has been applied to disciplines as diverse as economics (historically, its main area of application) such as
  - Political science
  - Biology
  - Psychology
  - Linguistics
  - Computer science.
- This Course: Studying different Game Theory Models

This Course



- Exercises $\approx 20\% - 30\%$
  - Theory + programming
- Midterm Exams $\approx 30\% - 40\%$
- Final Exam $\approx 30\% - 50\%$

What Will We Learn? (Part one)

- Non-Cooperative Game Theory
  - Normal Form Games
- Computing the Solutions
  - Computing Equilibria of Systems
- Games with Sequential Actions
  - Extensive Form Games
- Richer Representations
  - Repeated Games
  - Stochastic Games
  - Bayesian Games
  - And …

What Will We Learn? (Part 2)

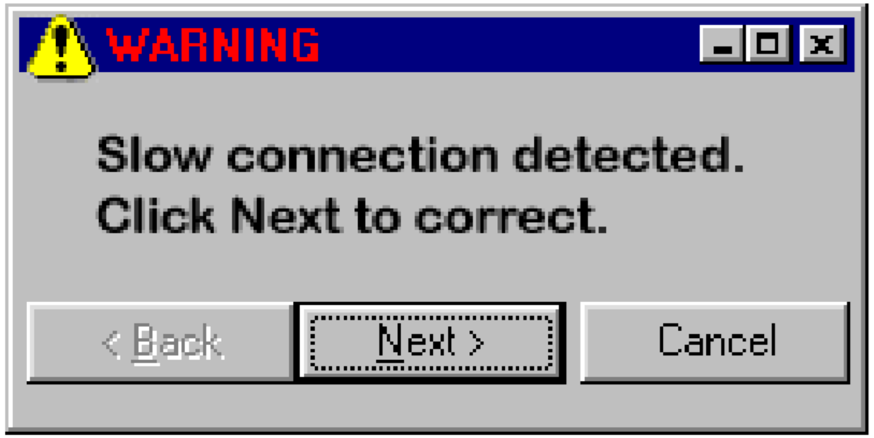
- Social Choice
- Mechanism Design
- Auctions
- Coalitional Game Theory

Non-Cooperative Game Theory

- Agents are self interested
  - Each agent has his own description of which states of the world he likes
- The dominant approach to modeling an agent's interests is *utility theory*:
  - Quantifying agents' degree of preference across a set of available alternatives
  - The theory also aims to understand how these preferences change when an agent faces uncertainty about which alternative he will receive.
- The Utility Function: mapping from states of the world to real numbers, which are interpreted as measures of an agent's level of happiness in the given states.

Non-Cooperative Game Theory

- Example:
  - One feature of TCP is the *backoff* mechanism; if the rates at which you and your colleague send information packets into the network causes congestion, you each back off and reduce the rate for a while until the congestion subsides (The correct implementation)
  - A defective one, however, will not back off when congestion occurs.
  - This problem is an example of what we call a two-player game:
    - both use a correct implementation: both get 1 ms delay
    - one correct, one defective: 4 ms for correct, 0 ms for defective
    - both defective: both get a 3 ms delay



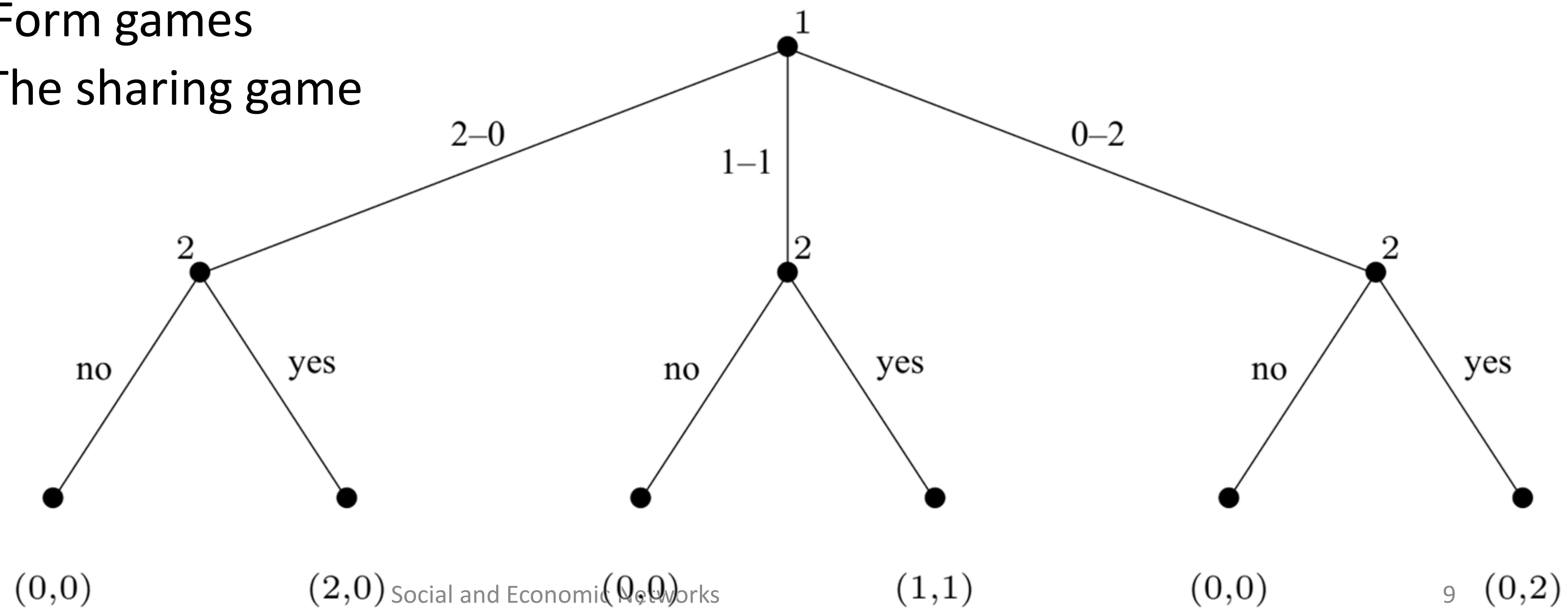
	C	D
C	$-1, -1$	$-4, 0$
D	$0, -4$	$-3, -3$

Non-Cooperative Game Theory

- What will happen assuming both players acts selfish?
  - Equilibria: The convergence states
    - Nash Equilibrium
- How much bad are Equilibria?
- How to analyze other types of strategies?
  - When action set is continuous or infinite?
- How much hard is it to compute the equilibria of games?
  - Computing the solution concepts
  - Sometimes it is NP-Hard and sometimes computable in polynomial time

Games with Sequential Actions

- Normal form games are static and don't consider any dynamism in analysis
  - What can we do if the game happens in a sequence of actions
  - Extensive Form games
  - Example: The sharing game



Richer Representations

- Repeated Games
- Stochastic Games
- Bayesian Games
- Congestion Games
- Graphical Games
- And …

Social Choice

- You are a babysitter for 3 babies, Will, Liam and Vic and you want to choose an activity. Their preferences are:

$$
\begin{aligned}
\text{Will:} &\quad a \succ b \succ c \\
\text{Liam:} &\quad b \succ c \succ a \\
\text{Vic:} &\quad c \succ a \succ b
\end{aligned}
$$

- How to choose an activity?
- Plurality Rule: Ask each kid to vote for his favorite activity and then pick the activity that received the largest number of votes (break the ties by alphabetical order)-Choose 'a'

Social Choice

$$\begin{array}{rl} \text{Will:} & a \succ b \succ c \\ \text{Liam:} & b \succ c \succ a \\ \text{Vic:} & c \succ a \succ b \end{array}$$

- It does not meet the Condorcet condition: If there exists a candidate x such that for all other candidates y at least half the voters prefer x to y, then x must be chosen-Choose ‘b’
- How about this preferences?

$$\begin{array}{rl} \text{Will:} & a \succ b \succ c \\ \text{Liam:} & b \succ c \succ a \\ \text{Vic:} & c \succ a \succ b \end{array}$$

- Social choice: Studying different aggregation methods

Mechanism Design

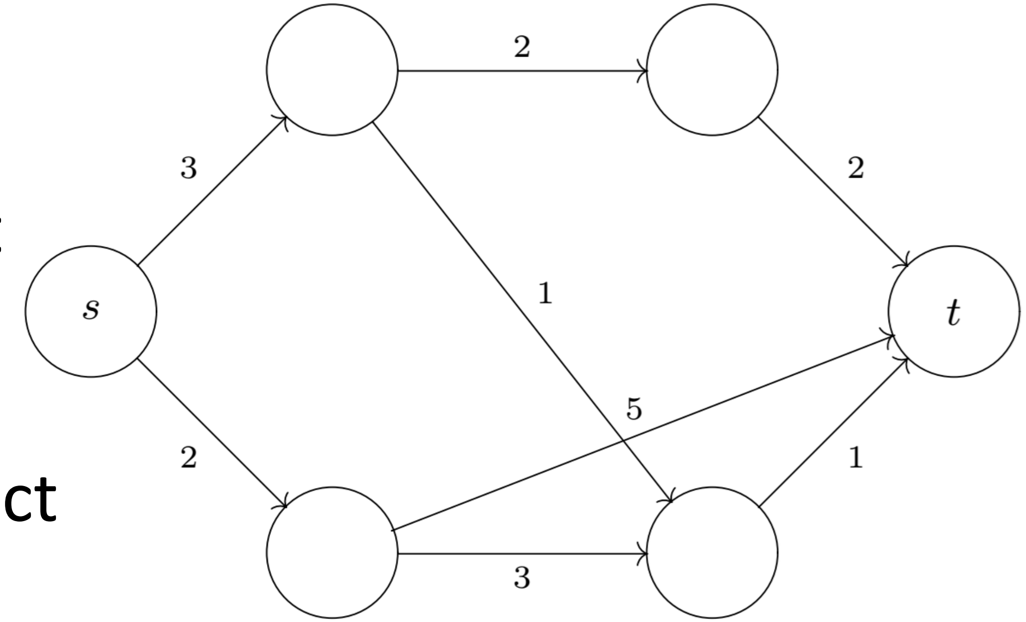
- Assume that in addition to Will, Liam, and Vic you must also babysit their devious new friend, Ray.

$$\begin{aligned} \text{Will:} \quad & b \succ a \succ c \\ \text{Liam:} \quad & b \succ a \succ c \\ \text{Vic:} \quad & a \succ c \succ b \\ \text{Ray:} \quad & c \succ a \succ b \end{aligned}$$

- Will, Liam, and Vic are sweet souls who always tell you their true preferences. But little Ray, he is always figuring things out.
- If we use plurality rule for aggregation, Ray may lie about his true preferences. How?
- How to deal with such issues?

Mechanism Design

- You want to find the least-cost path from S to T in a network
- Shippers may lie about their cost
- Your one advantage is that you know that they are interested in maximizing their revenue.
- How can you use that knowledge to extract from them the information needed to compute the desired path?



Auction Design

- The problem is to allocate (discrete) resources among selfish agents
- Single Good Auctions
  - Each buyer has his own valuation for the good, and each wishes to purchase it at the lowest possible price.
  - Our task is to design a protocol for this auction that satisfies certain desirable global criteria. For example, we might want an auction protocol that maximizes the expected revenue of the seller or we want a truthful auction

- Example: Which of the following auctions is truthful:
  - First Price Auction: The buyer with the highest bid wins the auction and must pay his bid.
  - Second Price Auction: The buyer with the highest bid wins the auction and pay the second bid.

Cooperative Game Theory

- A parliament is made up of four political parties, A, B, C, and D, which have 45, 25, 15, and 15 representatives, respectively.
- They are to vote on whether to pass a $100 million spending bill and how much of this amount should be controlled by each of the parties.
- A majority vote, that is, a minimum of 51 votes, is required in order to pass any legislation, and if the bill does not pass then every party gets zero to spend.
- Which coalitions may form?
- How should the formed coalition divide its payoff among its members in order to keep it safe?