200C – Micro Theory – Professor Giacomo Bonanno

PRACTICE PROBLEMS $\boxed{10}$

Topic: IMPERFECT-information games

VERY IMPORTANT: do **not** look at the answers until you have made a VERY serious effort to solve the problem. If you turn to the answers to get clues or help, you are wasting a chance to test how well you are prepared for the exams. I will **not** give you more practice problems later on.

1. Consider the following highly simplified version of **Poker**. There are three cards, marked A, B and C. A beats B and C, B beats C. There are two players, Yvonne and Zoe. Each player contributes $1 to the pot before the game starts. The cards are then shuffled and the top card is given, face down, to Yvonne, the second (face down) to Zoe. Each player looks at, and only at, her own card: **she does not see the card of the other player nor the remaining card.** Yvonne, the first player, may *pass*, or *bet* $1. If she passes, the game ends, the cards are turned and the pot goes to the high-card holder (recall that A beats B, B beats C). If Yvonne bets, then Zoe can *fold*, in which case the game ends and pot goes to Yvonne, or *see* by betting $1. If Yvonne sees the game ends, the cards are turned and the pot goes to the high-card holder.

(a) Represent this game as an extensive game with imperfect information. **Don't forget to write the payoffs!**

Hint # 1: put Nature at root of the tree and make her choose a possible ordering of the cards. Actually, the drawing of the game will be easier if you omit the root and simply start with the nodes that come after Nature's choice.

Hint # 2: think very carefully about what each player knows at the time when she has to move.

Recall that a strategy for a player is a complete plan of action drawn before the beginning of the game.

(b) How many strategies does Yvonne have?

(c) How many strategies does Zoe have?

Once each player has chosen her strategy, the outcome is a lottery, because the end-outcome depends on what cards will be drawn.

(d) Fix the following strategies.
For Yvonne: If A pass, if B pass, if C bet.
For Zoe: if Yvonne bets and I get an A I will fold, if Yvonne bets and I get a B I will fold, if Yvonne bets and I get a C I will fold.
Calculate the payoff of each player, assuming that **both players are risk neutral**.

(e) Redo the same with the following strategies.
For Yvonne: If A pass, if B pass, if C bet.
For Zoe: see always (i.e. no matter what card she get).

(f) Now that you have understood how to calculate the payoffs, represent the entire game as a normal form game (a matrix), assigning the rows to Yvonne and the columns to Zoe. [This will take you the entire night, but at your age sleep is not that important and also it will keep you out of trouble.]

(g) What strategies of Yvonne are weakly dominated? What strategies of Zoe are weakly dominated?

(h) What do you get when you apply the procedure of iterative elimination of weakly dominated strategies?

2. Consider the following two-person game. There are three unmarked envelopes. One contains \$100, one contains \$200 and the third contains \$300. They are shuffled very well and then one envelope is given to player 1 and another is given to player 2 (the third one remains on the table and will never be talked about again). Player 1 opens her envelop and secretly check its content. Then she either says “pass”, in which case each player gets to keep his/her envelope, or she asks player 2 to trade his envelope for hers. Player 2 does is not allowed to see the content of his envelope and has to say either Yes or No. If he says No, then the two players get to keep their original envelopes. If, on the other hand, player 2 says Yes, then they trade envelopes and keep what they are given. **Assume that both players are risk-neutral.**

(a) Represent this situation as an imperfect information game.

(b) Draw the corresponding normal-form game and find all the Nash equilibria.

(c) What do you get when you apply the iterative deletion of weakly dominated strategies?

3. Consider again the envelope game described above. Now, however, player 2 is allowed to secretly check the content of his envelope before he decides whether or not to accept player 1’s proposal.

(a) Represent this situation as an imperfect information game.

(b) List all the strategies of player 1 and all the strategies of player 2.

4. Three players, Avinash, Brian and John play the following game. Two cards, one red and the other black, are shuffled well and put face down on the table. Brian picks the top card, looks at it without showing it to the other players (Avinash and John) and puts it back face down. Then Brian whispers either “Black” or “Red” in Avinash’s ear, making sure that John doesn’t hear. Avinash then tells John either “Black” or “Red”. Finally John announces either “Black” or “Red” and this exciting game ends. The payoffs are as follows: if John’s final announcement matches the true color of the card Brian looked at, then Brian and Avinash give \$2 each to John. In every other case John gives \$2 each to Brian and Avinash.

(a) Represent this game as an extensive game of imperfect information.

(b) Write the corresponding normal form (or matrix form) assuming that the players are risk-neutral.

5. Two players: Amy and Bill. They simultaneously write a bid on a piece of paper. The bid can only be either \$2 or \$3. Then the referee looks at the bids, announces the lowest bid (without saying whose bid it is) and invites Amy to double her bid if she wants to. If Amy’s (final) bid is greater than Bill’s bid then Amy gets the object (and pays her bid), otherwise Bill gets the object (and pays his bid).

(c) Represent this game as an extensive game of imperfect information.

(d) Write the corresponding normal form (or matrix form) assuming that the players are risk-neutral.

6. There are three cards, one black and two red. They are shuffled well and put face down on the table. Adele picks the top card, looks at it without showing it to Ben and then tells Ben either "Black" or "Red". Ben then has to guess the true color of the card. If he guesses correctly he gets $9 from Adele, otherwise he gives her $9.

(a) Represent this game as an extensive game of imperfect information.

(b) Write the corresponding normal form (or matrix form) assuming that the players are **risk-neutral**.

7. Three players: Ann, Bob and Carla. Ann writes either "Red" or "Black" on a piece of paper and gives it to Bob. Bob looks at it, without showing it to Carla, and then tells Carla either "Red" of "Black". Finally Carla says either "Red" or "Black". If Carla guesses the color that Ann had written down, then Ann and Bob give $1 each to Carla, otherwise Carla gives $1 each to Ann and Bob.

(a) Represent this game as an extensive game of imperfect information.

(b) Write the corresponding normal form (or matrix form) assuming that the players are **risk-neutral**.

8. Find the subgame-perfect equilibrium of the following game: