Algorithmic Game Theory

Javad Salimi

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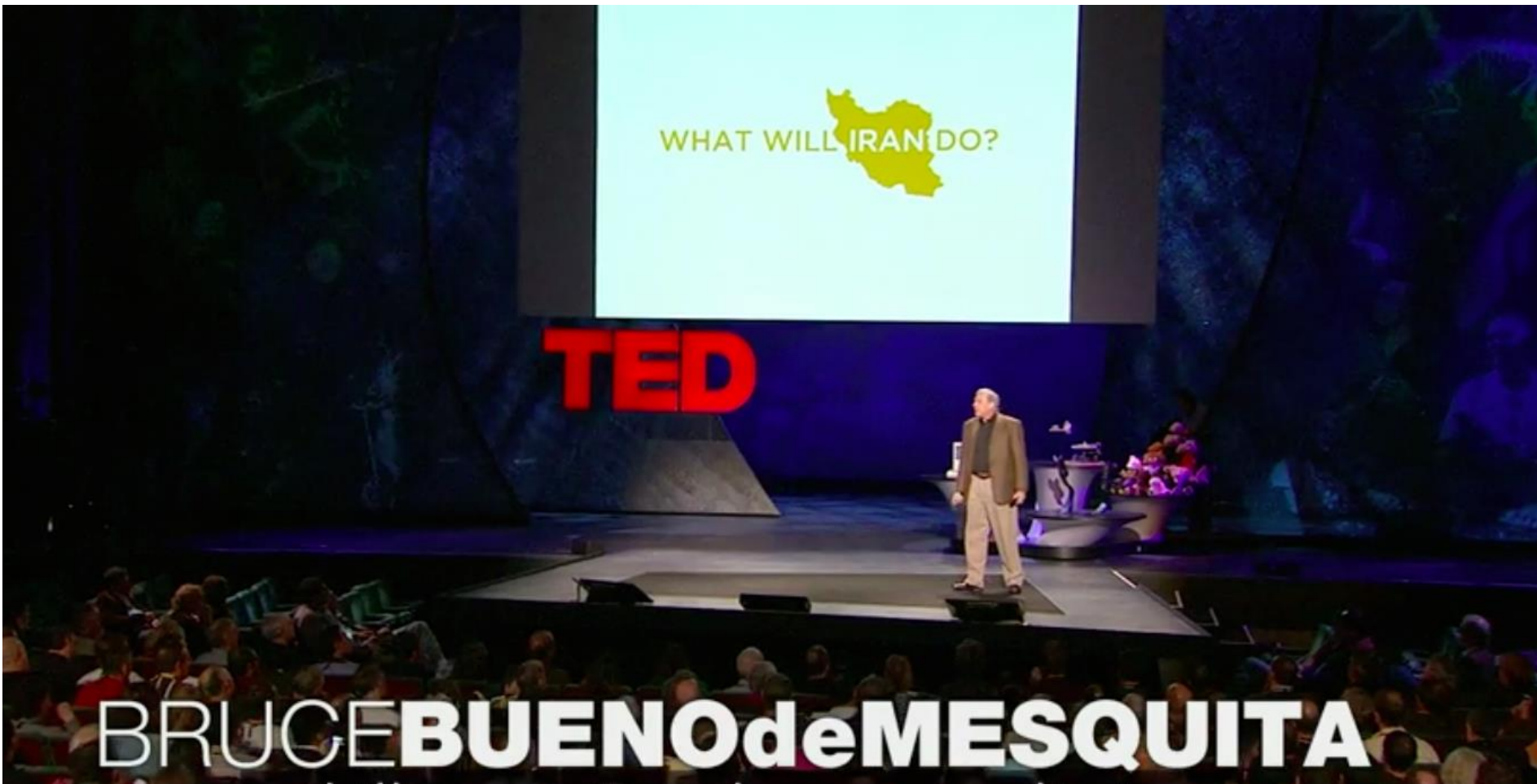
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  - Rationality, Values, Beliefs, and Limitations
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Game Theory:
A Simple Definition

Game Theory: Predict The Future

WE CAN
PREDICT THE
FUTURE

WE CAN ALSO
USE THIS
KNOWLEDGE TO

SCIENTIFICALLY

ENGINEER IT.

Game Theory: Main Assumption

People are looking out for what's good for them

Game Theory: Rationality (Self-Interested)

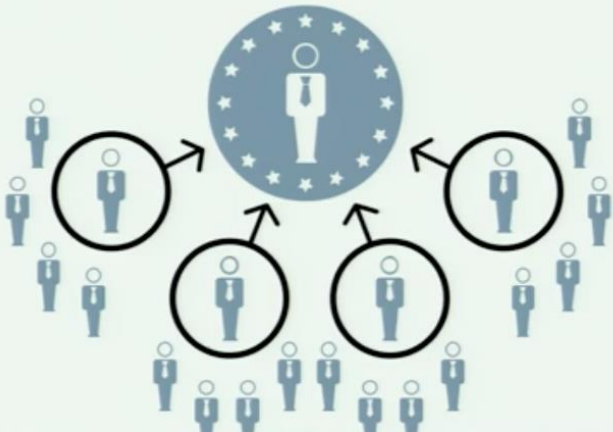RATIONAL PEOPLE

[DO WHAT THEY THINK IS IN THEIR BEST INTEREST]
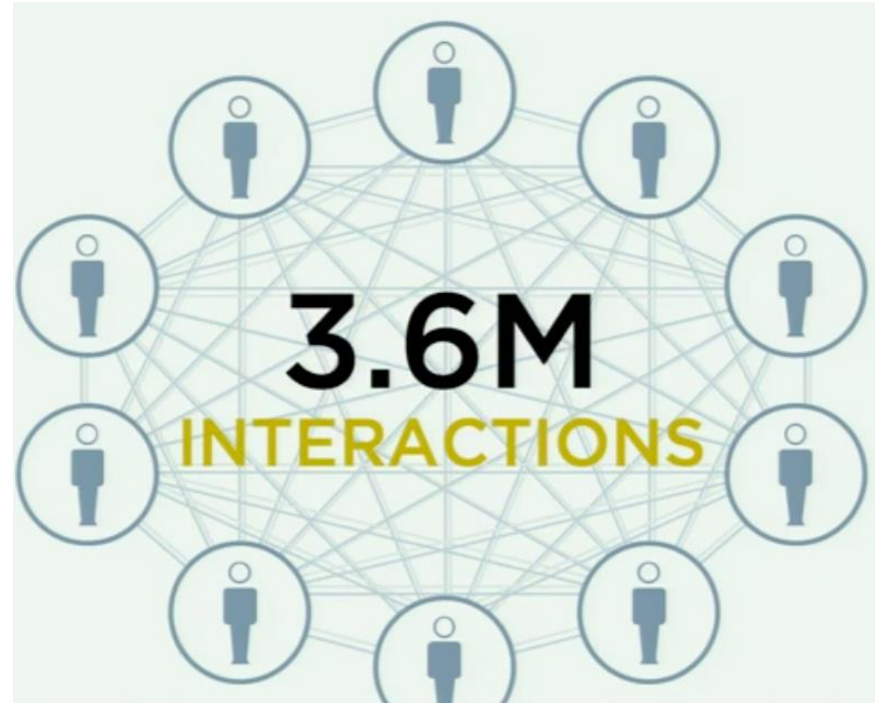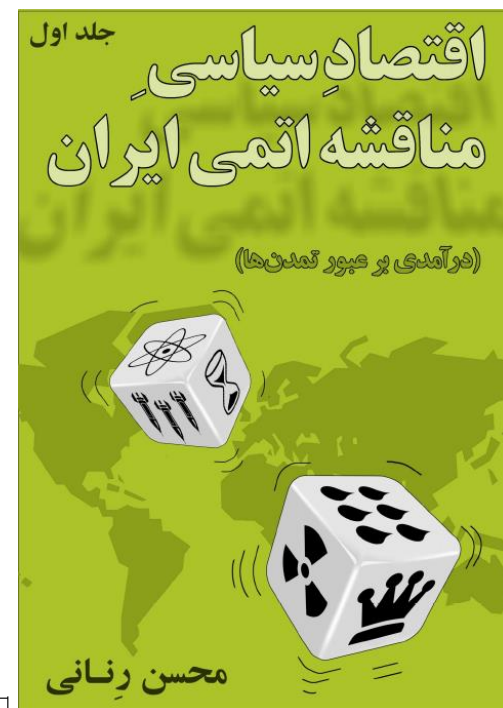
Game Theory: Values and Beliefs

More Recent Event:
5+1 and IRAN Talks

		غرب		
		سازش (مذاکره/تشویق)	پایداری (فشار همه جانبه)	کج دار و مریز
ایران	سازش (توقف غنی سازی)	W_1 ($H \leq W_1 \leq W_0$) $H \geq I_0$ ۱	$W_0 = \max_W$ $\min_I$ ۲	??? ($\min_I \leq B \leq H$) ۳
	پایداری (ادامه قاطع غنی سازی)	$W_0 \leq S$ I_1 ($S \leq I_1 \leq I_0$) ۴	$\min_W$ $I_0 = \max_I$ ۵	??? $C \leq I_1$ ۶
	کج دار و مریز	??? $a_1 \leq I_1$ ۷	??? $a_2 \geq p(I_0)$ ۸	??? $a_3 \geq C$ ۹

اقتصادِ سیاسیِ مناقشه اتمی ایران

جلد اول

(درآمدی بر عبور تمدن‌ها)

محسن رِنانی

		بازیگر مدافع (ب)	
		سازش یا پذیرش شکست	مقاومت تا پیروزی
بازیگر مهاجم (الف)	سازش یا پذیرش شکست	(صفر) ، (صفر)	(50+) ، (50−)
	مقاومت تا پیروزی	(100−) ، (100+)	(200−) ، (200−)

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Game Theory Definition
[Apple Dictionary]

The branch of mathematics concerned with the analysis of strategies for dealing with competitive situations where the outcome of a participant's choice of action depends critically on the actions of other participants. Game theory has been applied to contexts in war, business, and biology.

Game Theory Definition [Wikipedia]

Game theory is a mathematical method for analyzing calculated circumstances, such as in games, where a person's success is based upon the choices of others. More formally, it is "the study of mathematical models of conflict and cooperation between intelligent rational decision-makers." An alternative term suggested "as a more descriptive name for the discipline" is interactive decision theory.
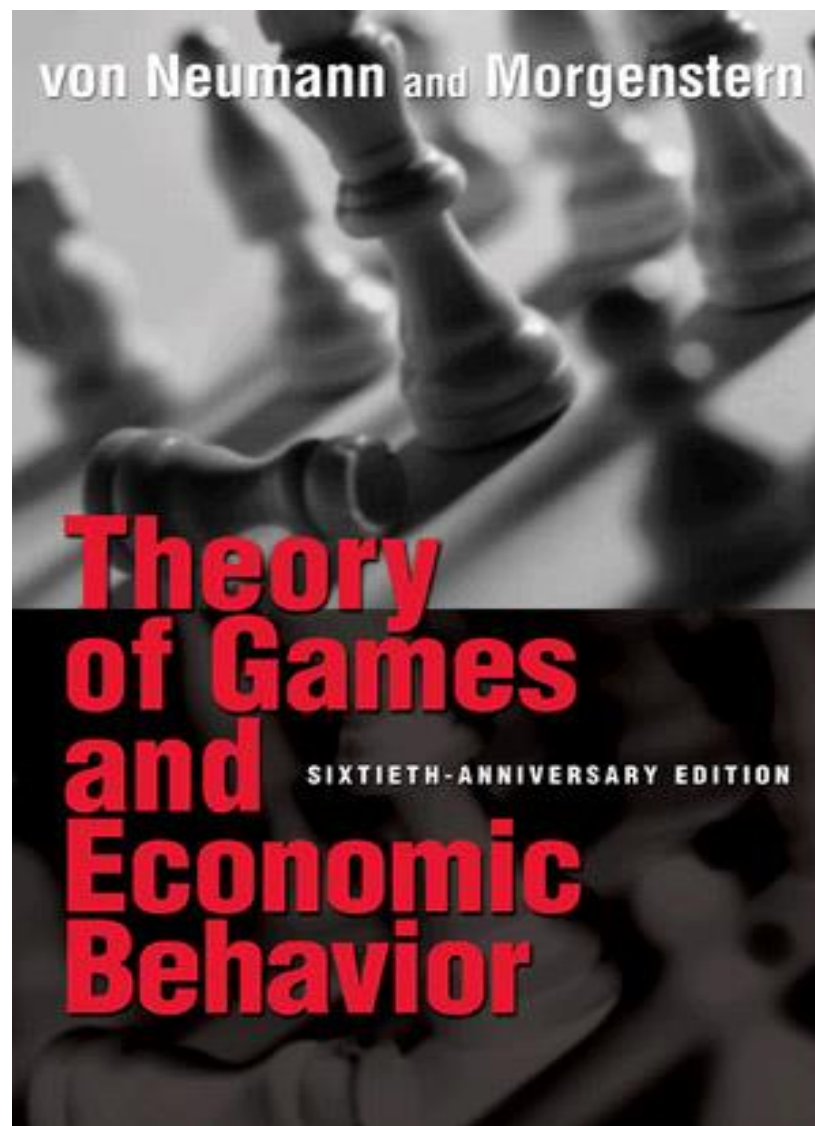
History of Game Theory

- **1713, James Waldegrave:** First known discussion of game theory occurred in a letter, provides a minimax mixed strategy solution to a two-person version of the **card game "le Her"**.

- **1787, James Madison:** A game-theoretic analysis of the ways states can be expected to behave under different **systems of taxation**.

- **1838, Antoine Augustin:** Considered duopoly in "Recherches sur les principes mathématiques de la théorie des richesses Cournot"

History of Game Theory (cont.)

- Early papers by
  - Zermelo (1912, perfect information),
  - Borel (1927, mixed strategies),
  - von Neumann (1928, minimax theorem)
- Von Neumann and Morgenstern, *Theory of Games and Economic Behavior (1944)*

von Neumann and Morgenstern

Theory of Games and Economic Behavior

SIXTIETH-ANNIVERSARY EDITION

History of Game Theory (cont.)

- General development of mathematical theory of games
  - At Fine Hall [Princeton Mathematics Department] and RAND Corporation
  - Late 1940s onwards
  - Work of John Nash on non-cooperative games and [Nash] equilibrium
  - Possible applications to Cold War nuclear strategy

MATHEMATICS
Princeton University

History of Game Theory (cont.)

- Game theory was brought to attention of **social scientists** in mid-1950s:
  - Luce and Raiffa, *Games and Decisions*, 1957
- Some **political** applications by non-political scientists at this time:
  - Arrow, *Social Choice and Individual Values*, 1951
  - *Black, Theory of Committees and Elections*, 1958
  - *Downs, An Economic Theory of Democracy*, 1957
  - *Schelling, The Strategy of Conflict*, 1960
  - *Buchanan & Tullock, The Calculus of Consent*, 1962

History of Game Theory (cont.)

- Earliest work by a **political scientist**:
  - Riker, *The Theory of Political Coalitions (1962)*
  - *Followed by much further work by Riker and his students who constituted the "Rochester school" (late 1960s to early 1970s)*
- Game theory enters mainline **political science** (late-1970s)
- Game theory (and "number crunching") allegedly dominates mainstream political science, provoking the "**Perestroika Movement**" within the discipline

Nobel Winners

1. John Nash, 1994
2. Reinhard Selten, 1994
3. John Harsanyi , 1994
4. Robert Aumann, 2005
5. Thomas Schelling, 2005
6. Leonid Hurwicz, 2007
7. Eric Maskin, 2007
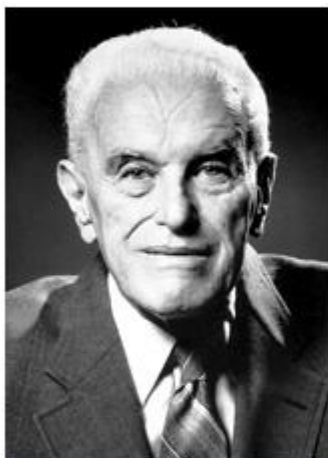8. Roger Myerson, 2007

More Noble Prizes on Game Theory

- **Eight game-theorists** have won the Nobel Memorial Prize in Economic Sciences

- **William Spencer Vickrey** was awarded the Nobel Memorial Prize in Economics with James Mirrlees for their research into the economic theory of incentives under asymmetric information.

- **John Maynard Smith** was awarded the Crafoord Prize for his application of game theory to biology

The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 1994

"for their pioneering analysis of equilibria in the theory of non-cooperative games"

John C. Harsanyi

🕐 1/3 of the prize

USA

University of California Berkeley, CA, USA

b. 1920
(in Budapest, Hungary)
d. 2000

John F. Nash Jr.

🕐 1/3 of the prize

USA

Princeton University Princeton, NJ, USA

b. 1928

Reinhard Selten

🕐 1/3 of the prize

Federal Republic of Germany

Rheinische Friedrich-Wilhelms-Universität Bonn, Federal Republic of Germany

b. 1930

Thomas Schelling: Nobel Winner in 2005

- Schelling worked on dynamic models, early examples of evolutionary game theory.

- Professor of *foreign affairs*, *national security*, *nuclear strategy*, and *arms control* at the School of Public Policy at University of Maryland, College Park
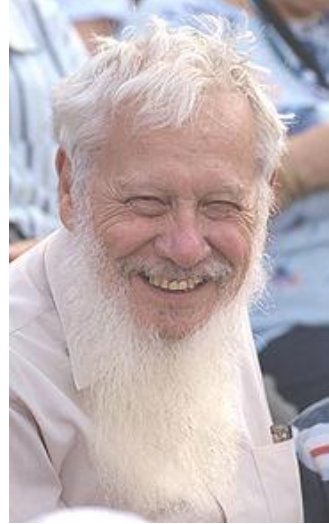
Cold War!

	Country 2	
Country 1	**Reduce military investment**	**Increase military investment**
Reduce military investment	(1, 1)	(-1, 2)
Increase military investment	(2, -1)	(0, 0)

Payoffs:

✧ **2:** I have weaponry superior to the one of the opponent
✧ **1:** We have equivalent weaponry and managed to reduce it on both sides
✧ **0:** We have equivalent weaponry and did not managed to reduce it on both sides
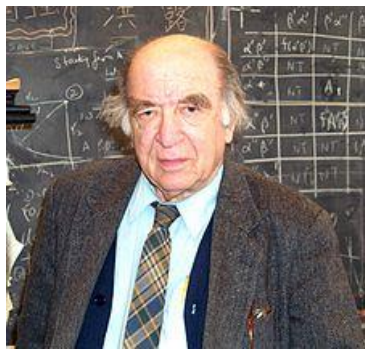✧ **-1:** My opponent has weaponry that is superior to mine
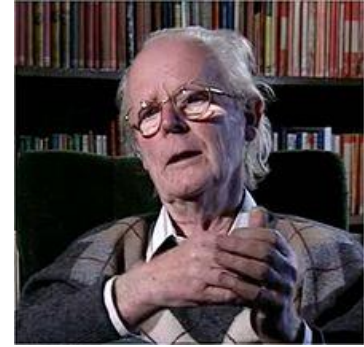
Robert Aumann
Nobel Winner in 2005

- Professor at the Center for the Study of Rationality

- Contributed more to the equilibrium school, introducing an equilibrium coarsening, correlated equilibrium, and developing an extensive formal analysis of the assumption of common knowledge and of its consequences.

Leonid **Hurwicz**, **Eric Maskin** and **Roger Myerson**: Nobel Winners in 2007

- For having laid the foundations of mechanism design theory

- Myerson's contributions include the notion of proper equilibrium

- Hurwicz introduced and formalized the concept of incentive compatibility
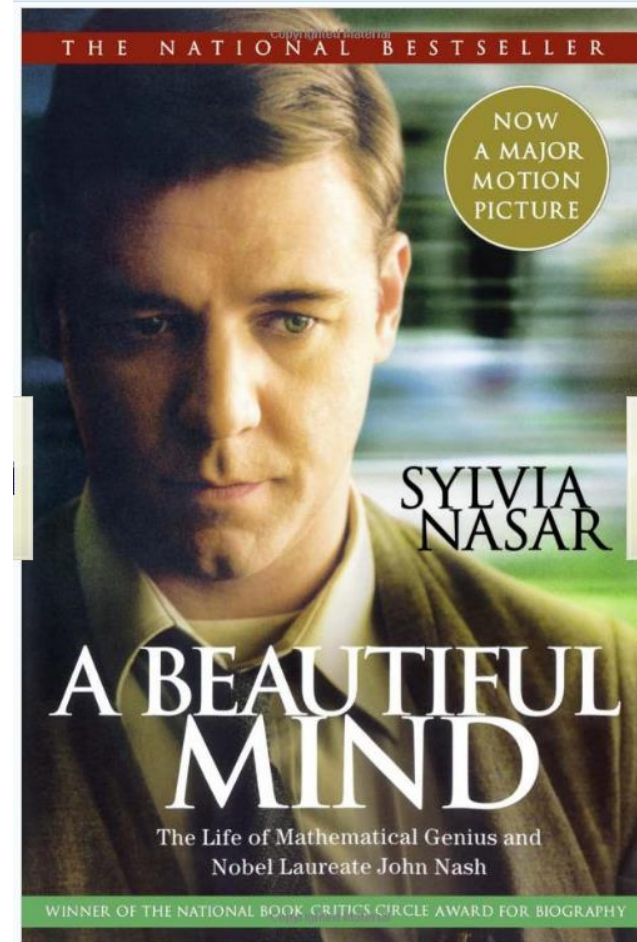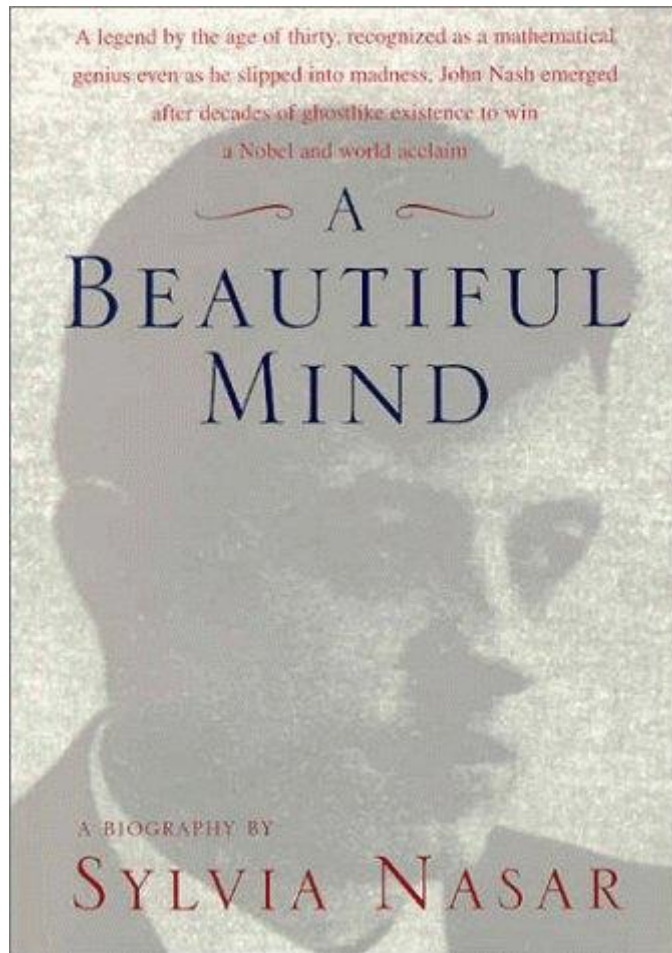
John Maynard Smith

- A British theoretical evolutionary biologist and geneticist

- In the 1970s, game theory was extensively applied in **biology**, largely as a result of the work of John Maynard Smith and his **evolutionarily stable strategy**.

- The ESS is to explain the emergence of animal communication

William Vickrey

- Vickrey was awarded the Nobel Memorial Prize in Economics with James Mirrlees for their research into **the economic theory of incentives under asymmetric information**.

- The announcement of the prize was made just three days prior to his death

Game theory publicized by *A Beautiful Mind* biography (and later movie) about John Nash

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Let's focus on Engineering Applications!

GAME THEORY IN ELECTRICAL AND COMPUTER ENGINEERING

Recent Engineering Applications

1. Networked-systems
   - Communication, Transportation networks, Electricity markets
   - Nagle, RFC 970, 1985 ("datagram networks as a multi-player game")
   - Paper in first volume of IEEE/ACM ToN (1993)
   - Wider interest starting around 2000

2. Large-scale networks (Internet)
   - Interconnections of smaller networks rely on various degrees of **competition** and **cooperation**.

3. Online advertising on the Internet: Sponsored search **auctions**.

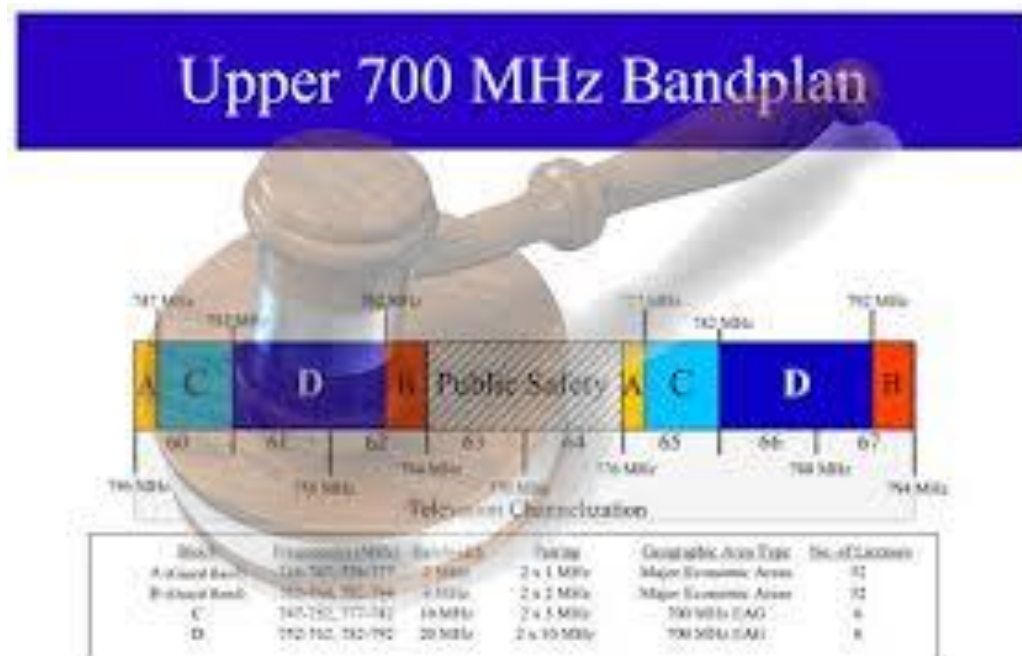4. Information evolution and belief propagation in **social networks**.

5. Sustainability and smart grids.

6. **Distributed control** of competing heterogeneous users.

Spectrum Auction

- The main application of game theory in communications

- Used by FCC to sell spectrum to operators

Game Theory For Security (2007 LAX)

Many Targets   **Few Resources**

How to assign limited resources to defend the targets?
Game Theory: Bayesian Stackelberg Games

Bit Torrent

- Bam Cohen (born October 12, 1975)

- Author of the peer-to-peer (P2P) BitTorrent protocol

- Use a simple Tit for Tat protocol to share chunk of files between peers

Other Engineering Applications

1. Communications
2. Wireless/Mobile
3. Spectrum/Channel Allocation
4. Control
5. Cloud Computing
6. Networking
7. Security/Privacy
8. Cryptography (MPC)
9. Cognitive Radios
10. AI
11. Bioinformatics

Limitations of Game Theory

- No unified solution to general conflict resolution

r Real-world conflicts are complex

   m models can at best capture important aspects

r Players are (usually) considered rational

   m Determine what is best for them given that others are doing the same

r No unique prescription

   m Not clear what players should do

r **But it can provide intuitions, suggestions and partial prescriptions**

   m **best mathematical tool we currently have**

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Intended Learning Outcome

1. Model your engineering problems with a game-theoretic approach

2. Analyze defined games

3. Use the game results to design protocols (i.e., mechanism design)

Main Concepts

1. Strategy, Payoffs, Actions
2. Strictly Dominance
3. Non-Cooperative Behavior
4. Dilemma
5. Coordination
6. Knowledge/Information
7. Rationality
8. Thinking Strategically
9. Nash Equilibrium
10. Pareto Optimality
11. Cournot Duopoly
12. Zero-Sum Games
13. Mixed Strategy
14. Dynamic Games
15. Backward Induction
16. Moral Hazard
17. Incentive Design
18. Imperfect Information
19. Subgame Perfect Nash Equilibrium
20. Repeated Games
21. Perfect Bayesian NE
22. Mechanism Design
23. Auction Theory
24. Evolutionary Stable Strategy

What about this course!?

COURSE ADMINISTRATIONS

IUT Web Course

Foundations of Game Theory

for Electrical and Computer Engineering

1. Slides

2. Homework

3. Project Lists

4. Any question, drop me an email
   (salami.sartakhti@gmail.com) please!

Assessment

- Written Midterm ~ 30%

- Written Final Exam ~ 50%

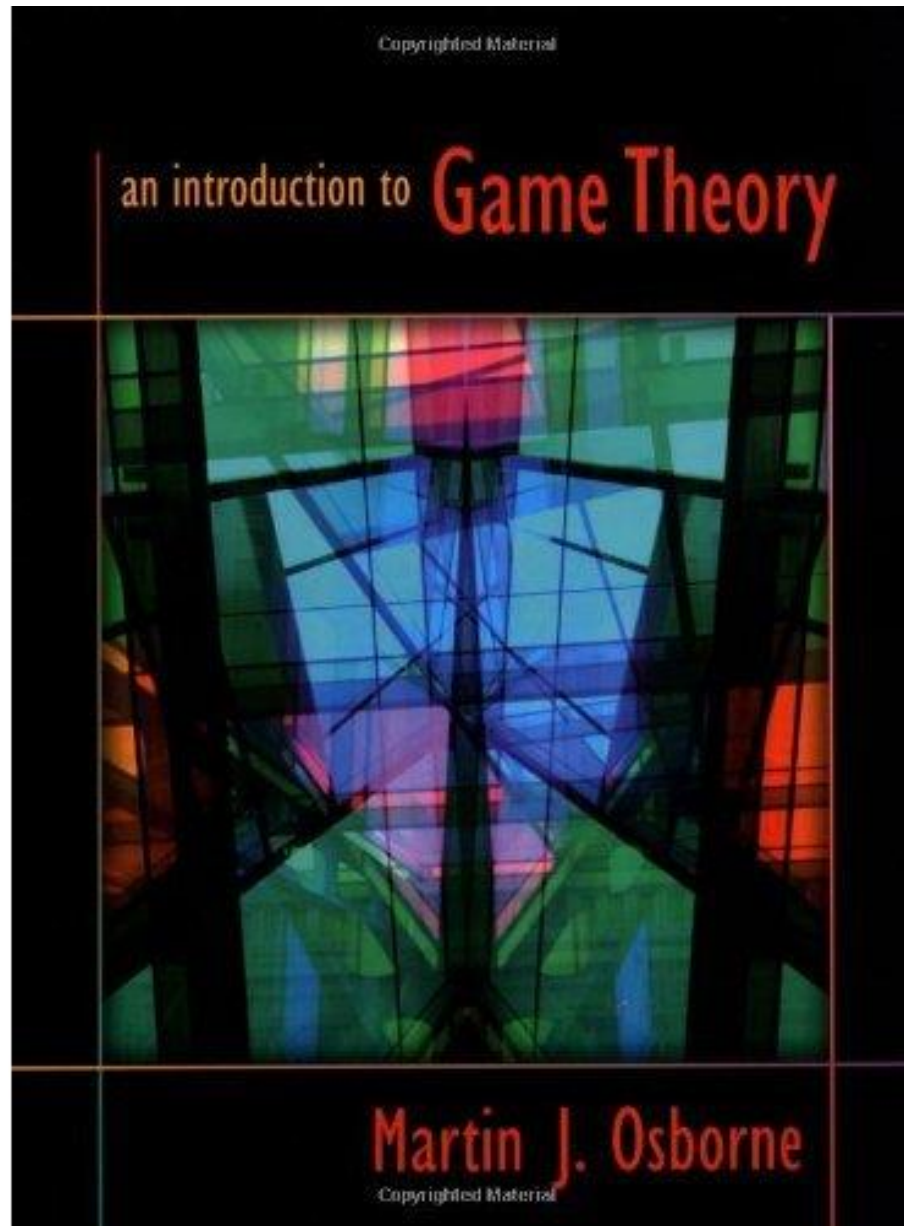- Homework ~ 10% (Around 3-6 Homework)

- Projects ~ 15%

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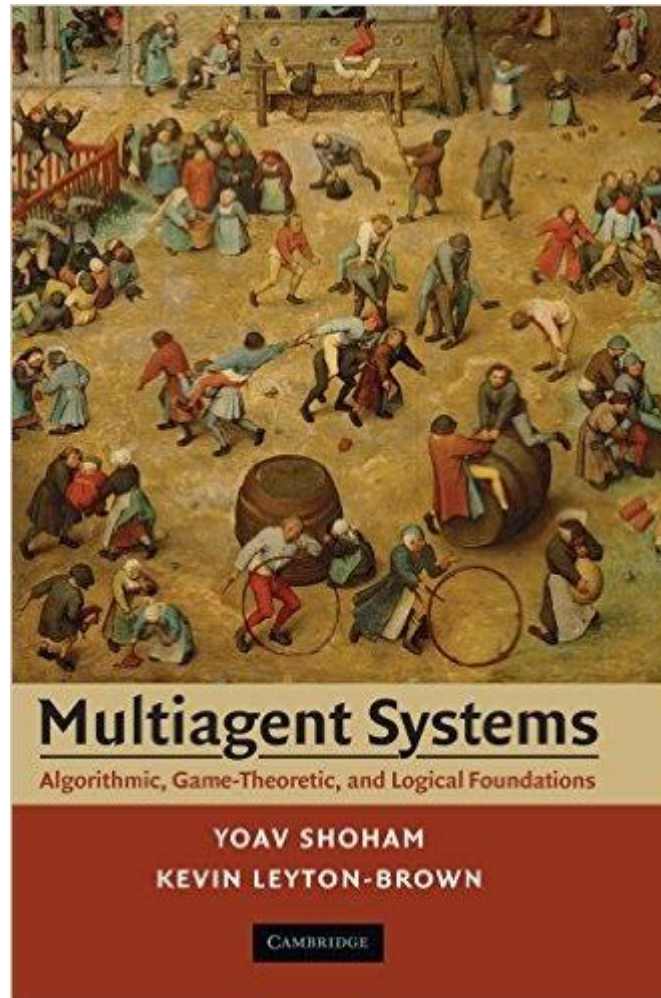- Course Outlines and Administrations
- References

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- **Course:**
  - **Dr. Benjamin Polak, "Game Theory", Open Yale Course (http://oyc.yale.edu)**
- **Books**
  - Martin J. Osborne, "An introduction to game theory" Oxford University Press, 2004.
  - Shoham and Brown, "Multiagent Systems: Algorithmic, Game-Theoretic, and Logical Foundations", 2008
  - G. Owen, *Game Theory,* 3rd edition, Academic Press, 1995
  - D. Fudenberg and J. Tirole, *Game Theory,* MIT Press, 1991
  - *"The Work of John Nash in Game Theory,"* Nobel Seminar, December 8, 1994
  - Tamer Başar and G.J. Olsder, *Dynamic Noncooperative Game Theory,* 2nd edition, Classics in Applied Mathematics, SIAM, Philadelphia, 1999.
  - L. Buttyan, J.-P. Hubaux, *Security and Cooperation in Wireless Networks,* Cambridge University Press , 2007
  - E. Altman, *The Theory of Network Engineering Games," 2011.*
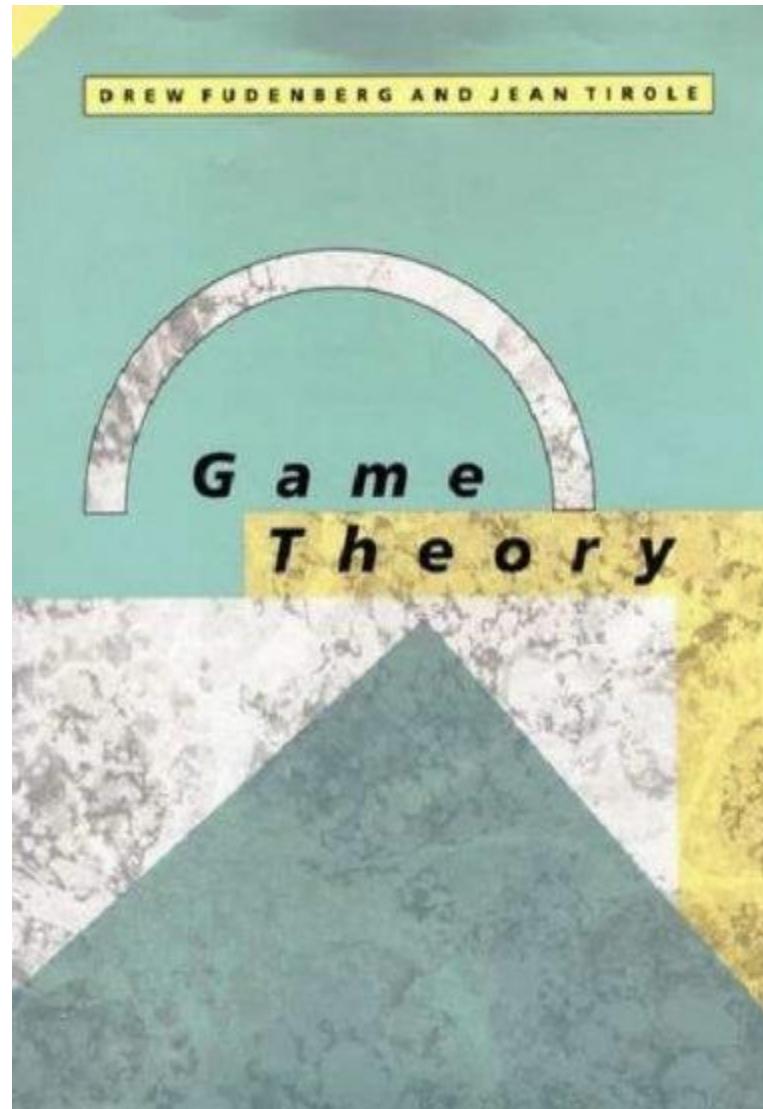
An Introduction to Game Theory
M. J. Osborne

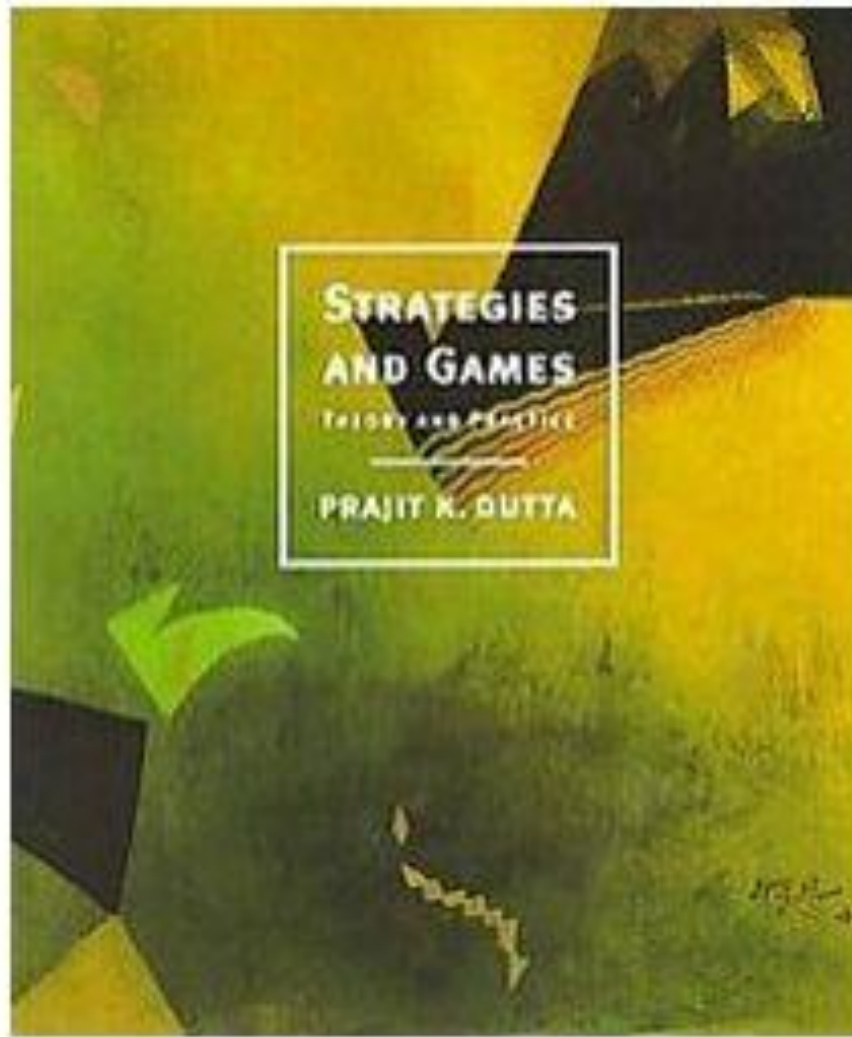# Multiagent Systems: Algorithmic, Game-Theoretic, and Logical Foundations
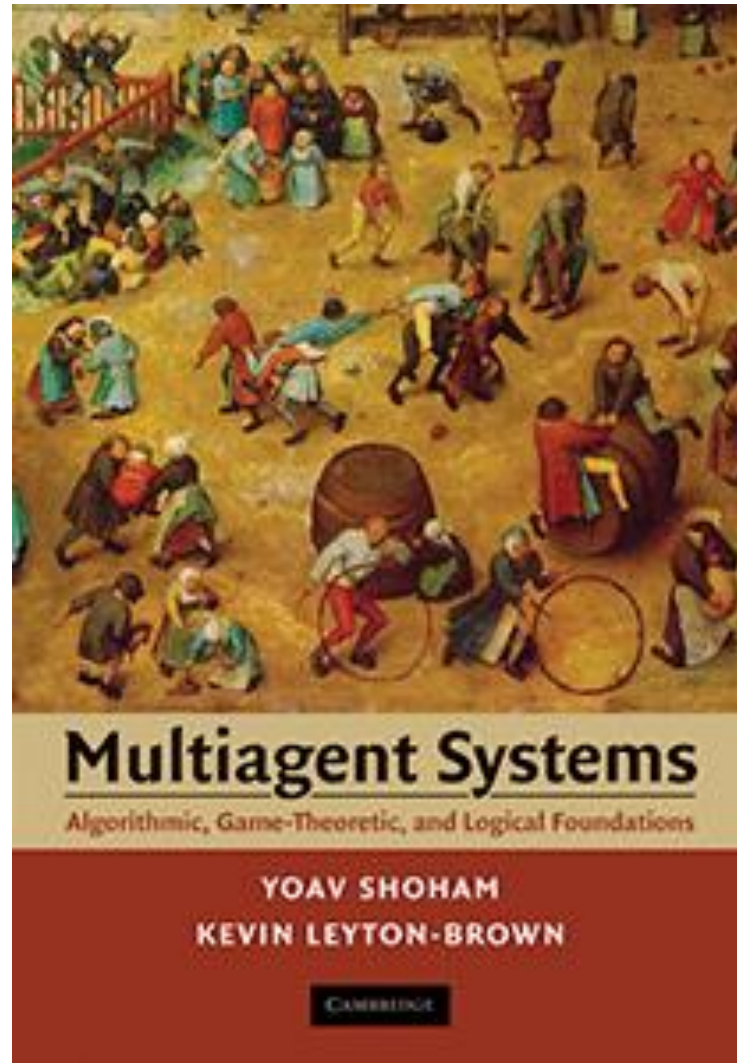
Game Theory
Drew Fudenberg and Jean Tirole
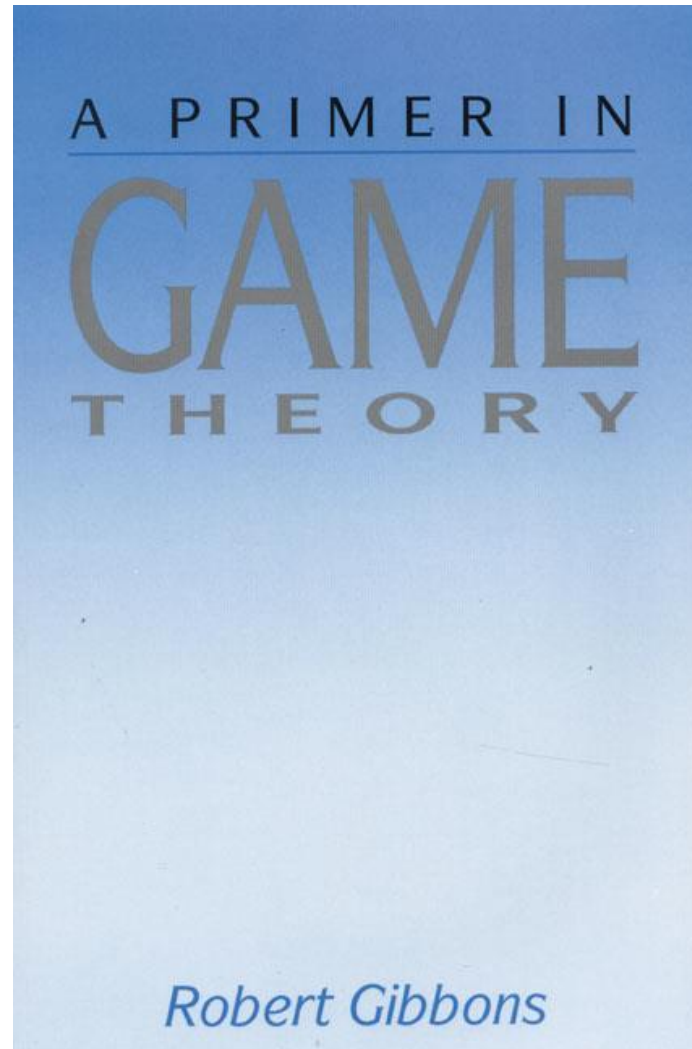
Strategies and Games
P. N. Dutta

Multiagent Systems
Y. Shoham and K. L. Brown

Multiagent Systems

Algorithmic, Game-Theoretic, and Logical Foundations

YOAV SHOHAM

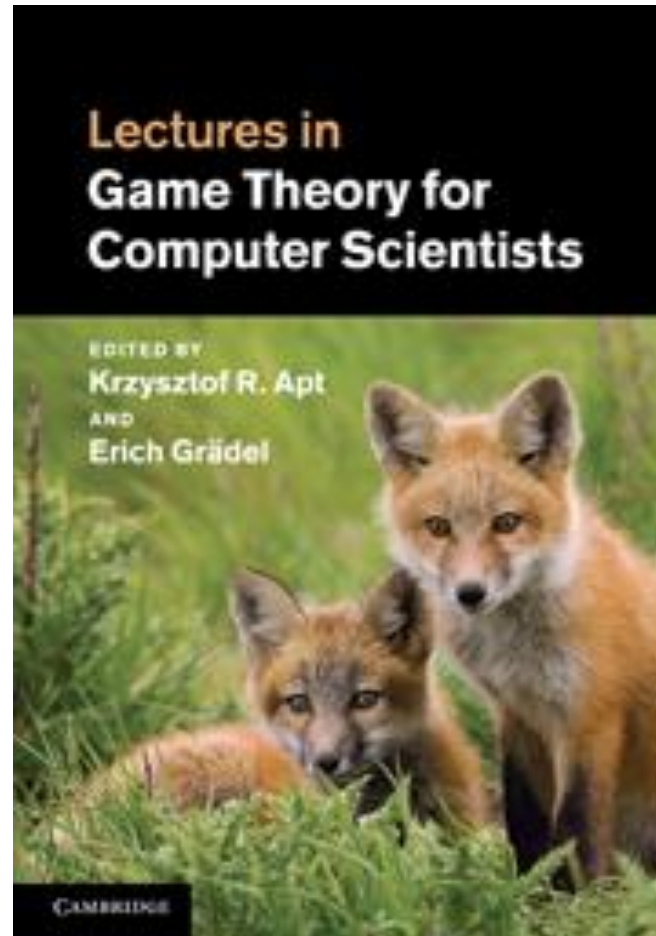KEVIN LEYTON-BROWN

CAMBRIDGE

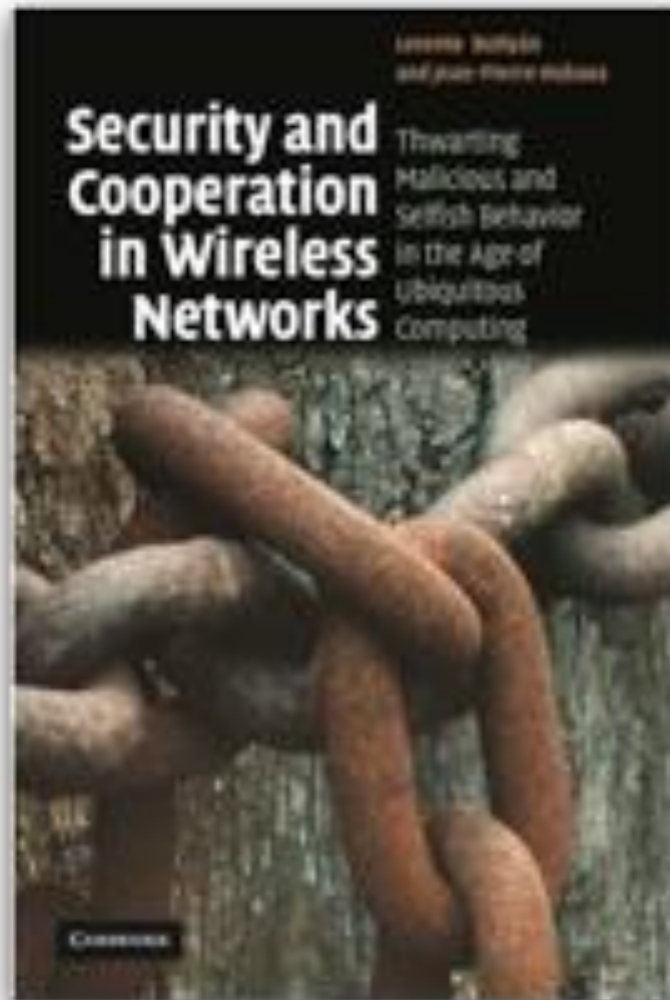# A Primer In Game Theory
R. Gibbons

Lectures in Game Theory for Computer Scientiest

Krzysztof R. Apt and Erich Gradel

Security and Cooperation in Wireless Networks
L. Buttyan and J.-P. Hubaux

References

- **Journal and Conferences**
  - Infocom (IEEE conference on International Conference on Computer Communications)
  - GameNets (Game Theory for Networks)
  - AAAI and AAMAS (Artificial Intelligenc)
  - WEIS (Workshop on the Economics of Information Security)
  - GameSec (Conference on Decision and Game Theory for Security)
  - IEEE Journal on Selected Areas in Communications (JSAC):
    - Cooperative Communications in MIMO Cellular Networks, December 2010
    - Game Theory in Communication Systems, September 2008
    - Non-Cooperative Behavior in Networking, August 2007
    - Cooperative Communications and Networking, February 2007