Algorithmic Game Theory

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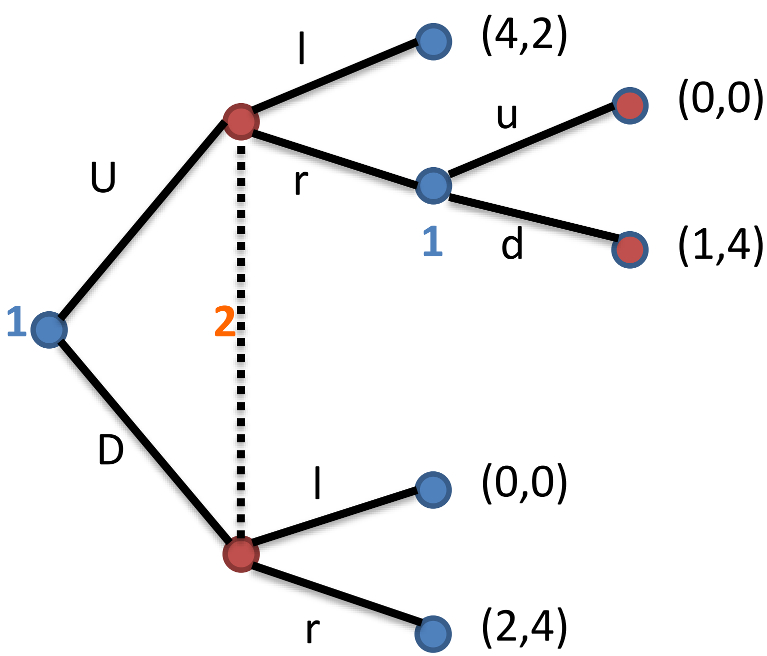
Dynamic Games III

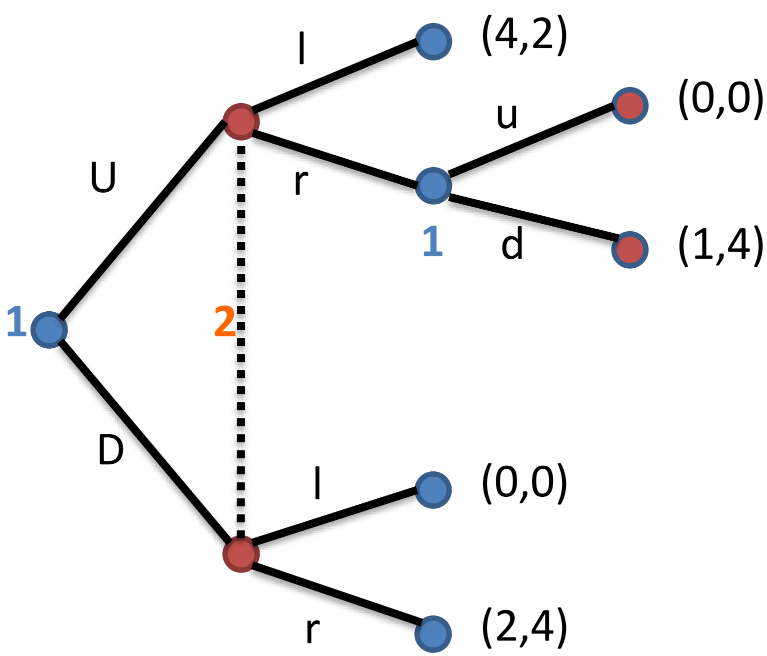
1. Solve Imperfect Information Games
2. Sub-game Definitions
3. Sub-game Perfect Nash Equilibrium (SPNE)
4. Don't Screw-Up Game
5. Match Maker Game
6. Applied Examples:
   a. Market Game
   b. Microsoft vs Mozilla
   c. TDMA Transmission

Solve Imperfect Information Games



- Before we analyze the game, let's figure out some basic facts
- How many information sets we have?
  - Player 2 has 1 information set
  - Player 1 has 2 information sets
- What are the strategies?
  - Player 1: **Uu, Ud, *Du, Dd***
  - Player 2: **l, r**

Static Game Solution



Player 2

Player 1		l	r
	Uu	4,2	0,0
	Ud	4,2	1,4
	Du	0,0	2,4
	Dd	0,0	2,4

•***Do you notice the redundancy here?***

•Let's find the NE of this game

Solutions: Nash Equilibria

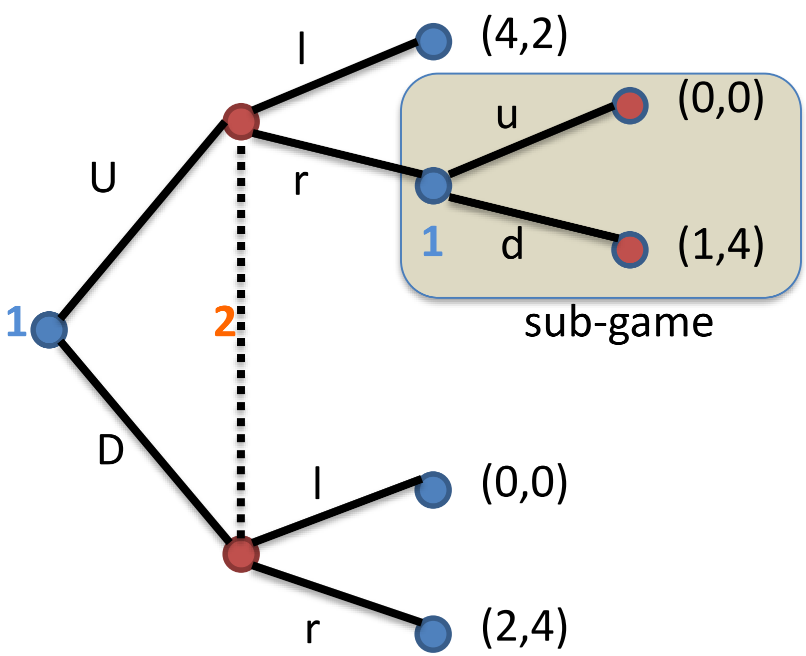
		Player 2	
		l	r
Player 1	Uu	4,2	0,0
	Ud	4,2	1,4
	Du	0,0	2,4
	Dd	0,0	2,4

(Uu,l)

(Du,r)

(Dd,r)

Dynamic Game Solution



- Let's try to use **BI**
- Starting from the end, player 1 will choose *down*
- Then, although player 2 doesn't know where she is on the tree, she will notice that she's always better-off choosing *right*
- This implies that player 1 will then choose *down*

NE of Static Games vs BI

- Nash Equilibria:
  - (Uu,l) → not compatible with BI
  - (Du,r) → not compatible with BI
  - (Dd,r) → **This is compatible with BI**

- We're not saying these are not NE, it's just that they are **inconsistent** with what we could predict with BI

→ We need a new notion of solution, that is able to treat games that have both sequential moves and simultaneous moves

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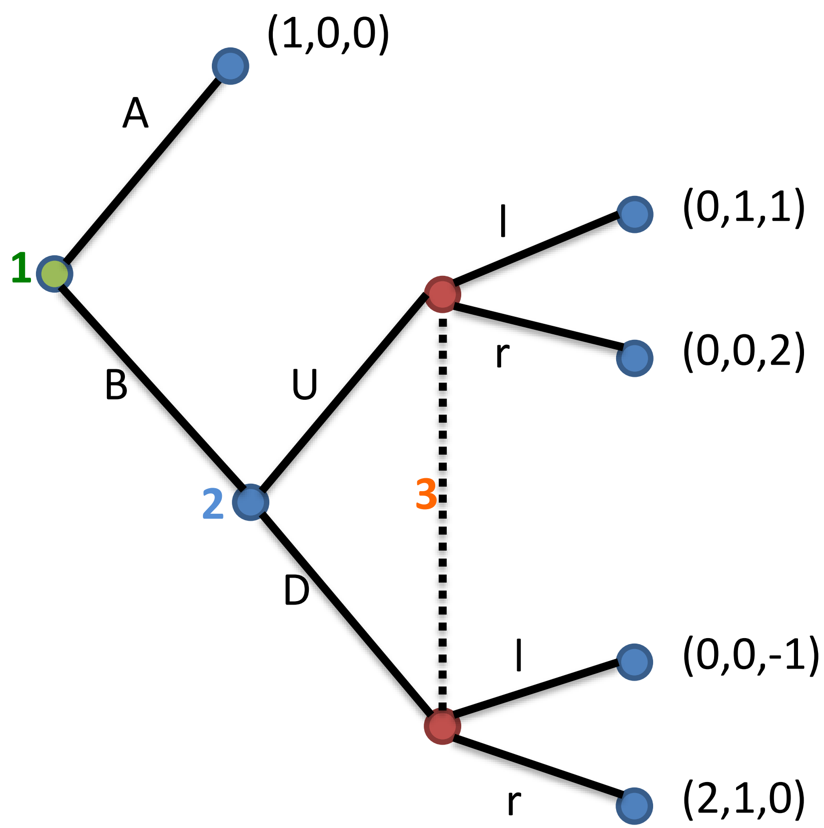
3-Player Game



- We will model, in the next slide, the game as follows:
  - Player 1 chooses a **matrix**
  - Player 2 and 3 will play the game player 1 chose

Static Game Model

Player 1: A

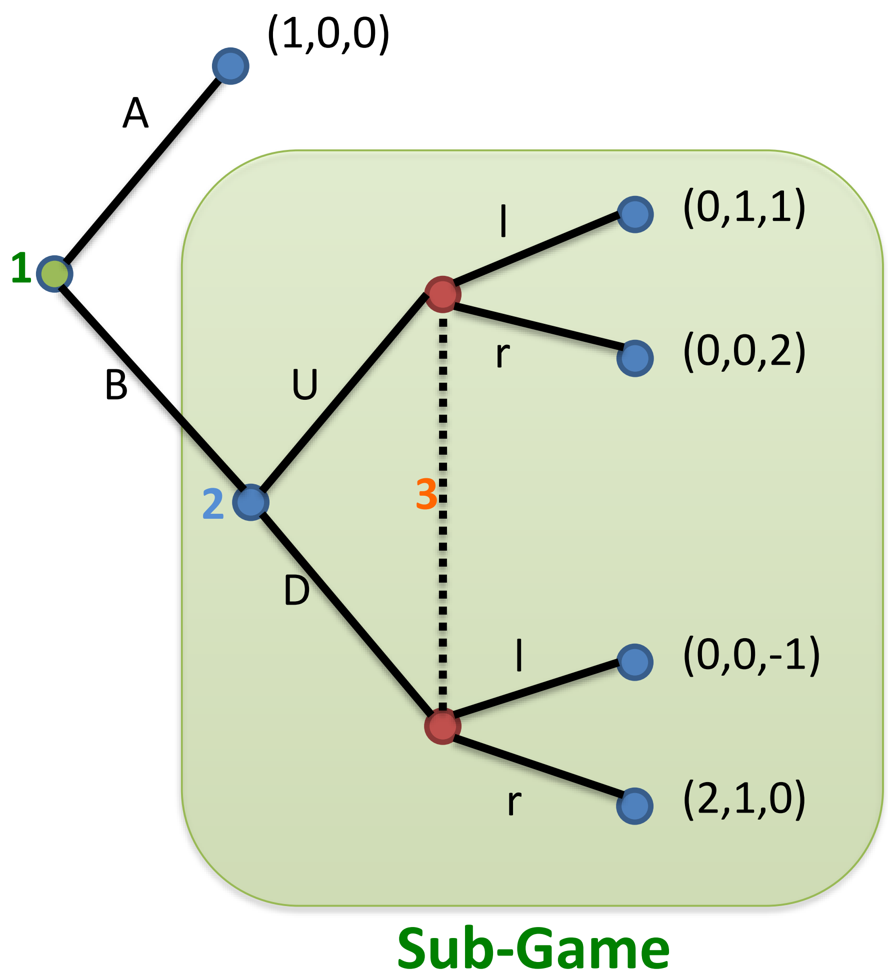
		3: l	3: r
2	U	1,0,0	1,0,0
2	D	1,0,0	1,0,0

Player 1: B

		3: l	3: r
2	U	0,1,1	0,0,2
2	D	0,0,-1	2,1,0

- There are a lots of NE in this game!
  - E.g.: [A,U,l]
- ***Question***: How can you check that it is a NE?
- ***Question***: Does this NE make sense?

3-Player Dynamic Game



- Let's have a look at the sub-game we identify in the game-tree
  - ***Observation***: it involves only two players

		Player 3	
		l	**r**
Player 2	**U**	1,1	0,2
	D	0,-1	1,0

NE of our Sub-Game

		Player 3: l	Player 3: r
Player 2	U	1,1	0,2
Player 2	D	0,-1	1,0

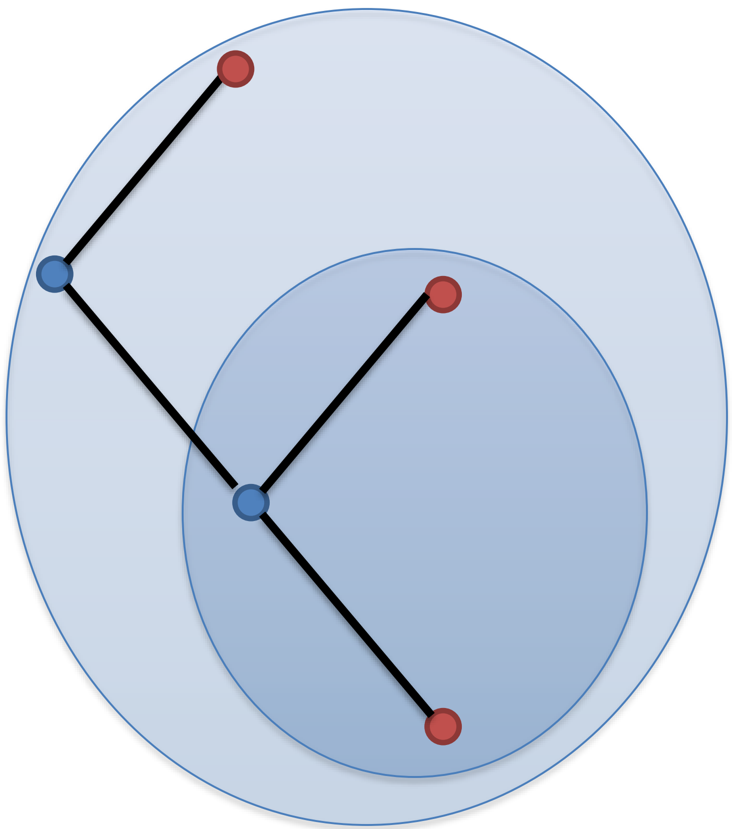
- What are the NE of this sub-game?
  - Notice that player 3 has a **dominant strategy**

  ➔NE = (D,r)
- This new equilibrium clashes with the equilibrium we just found before!

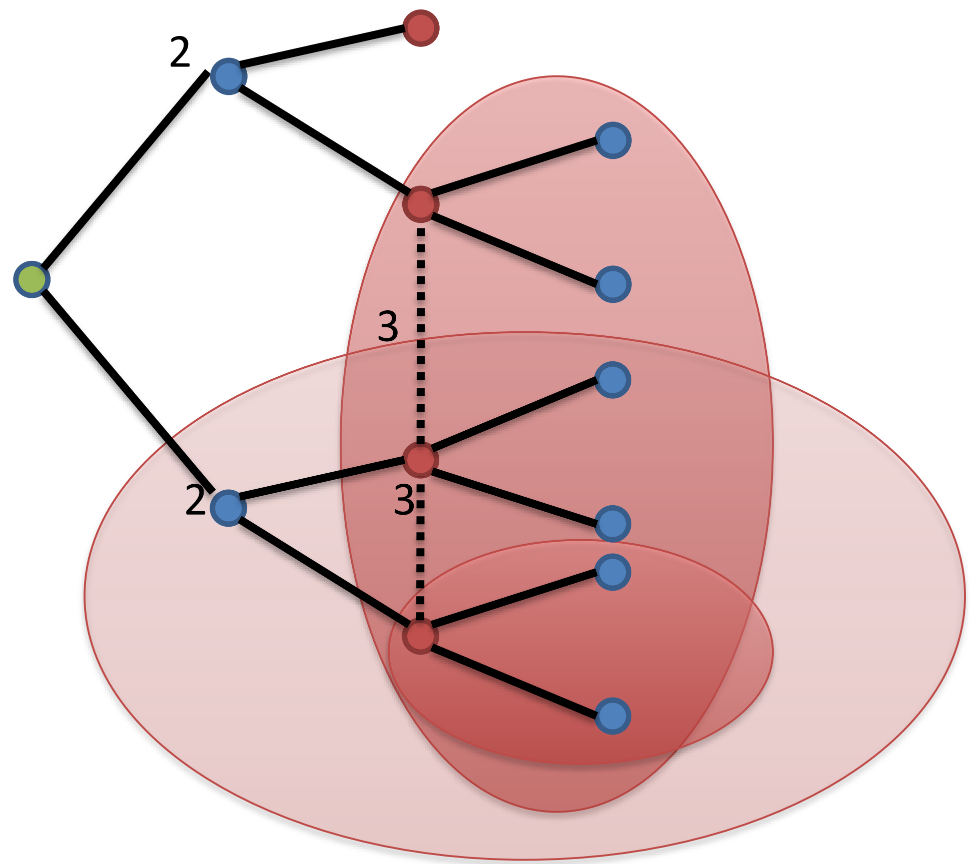
Definition: Sub-Games

- A **sub-game** is a part of the game that looks like a game within the tree. It satisfies the three following properties:
  1. It starts from a **single node**
  2. It **comprises all successors** to that node
  3. It **does not** break up any information set

Examples of Sub-Games



2 Sub-games



1 Sub-game

What do we want?

Let's find a way to rule out those Nash equilibria that instruct players down the tree to play in sub-games according to strategies that are not Nash equilibria.

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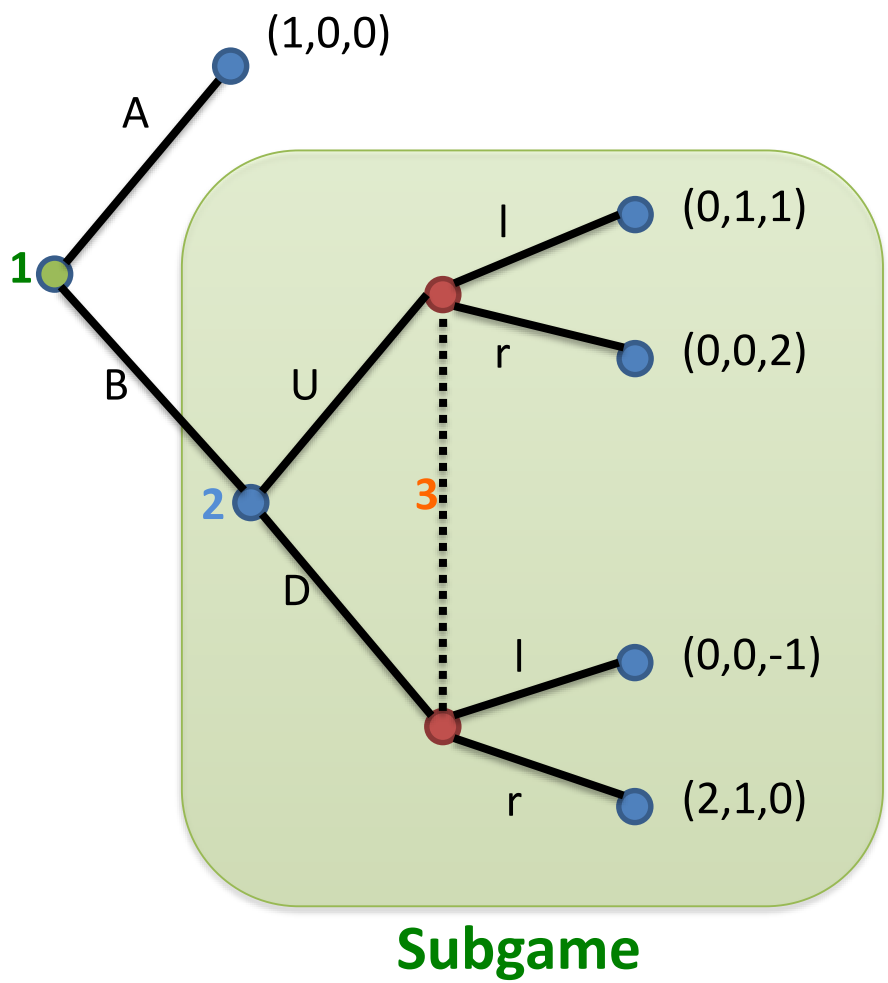
Definition: Sub-game Perfect Equilibrium

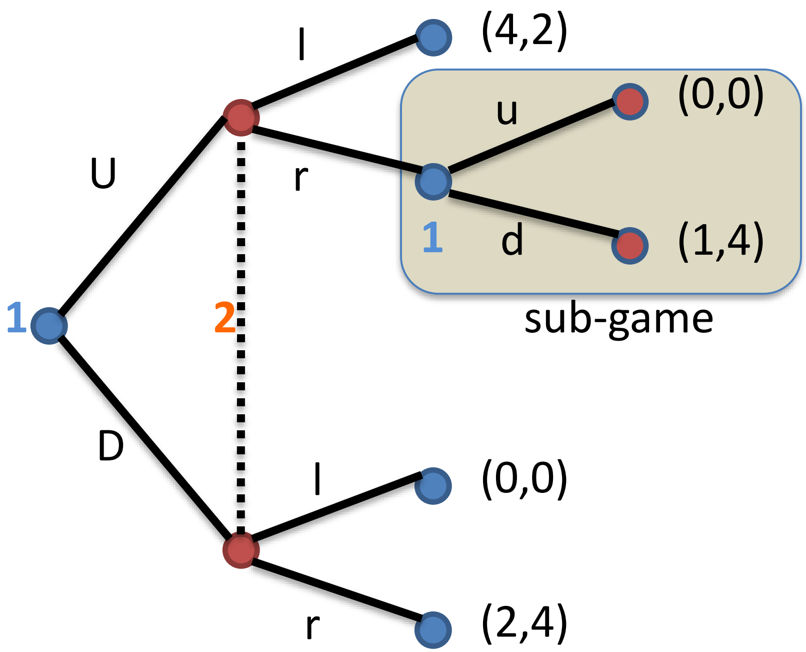
A Nash Equilibrium $(s_1^*, s_2^*, \ldots, s_N^*)$ is a **Sub-Game Perfect Nash Equilibrium (SPNE)** if it induces a Nash Equilibrium in every sub-game of the game

SPNE: Example I



The **SPNE is (B,D,r),** because (D,r) is the only NE of the only subgame

SPNE: Example II



The **SPNE is (Dd,r)** Because (d) is the only NE of the only sub-game

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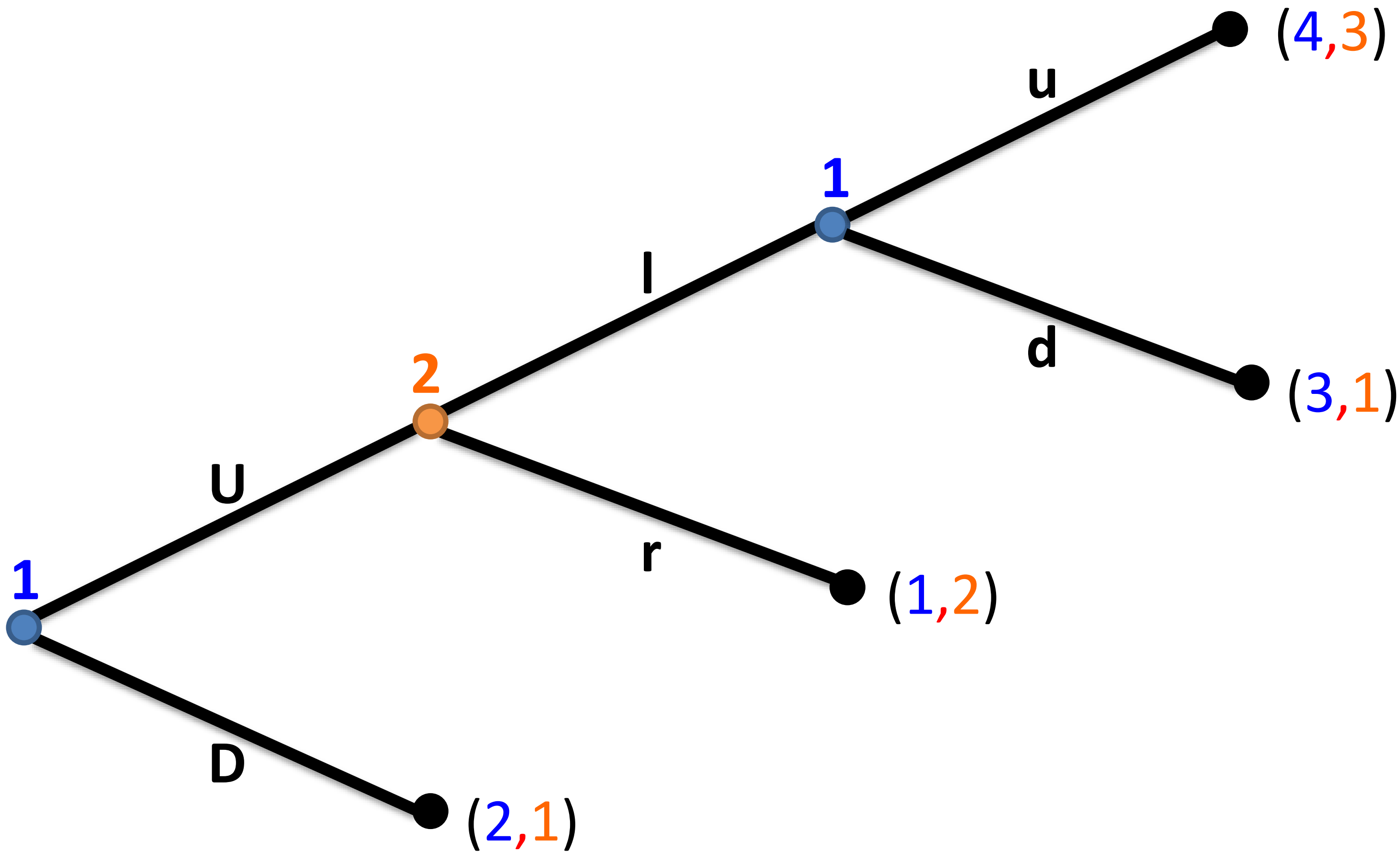
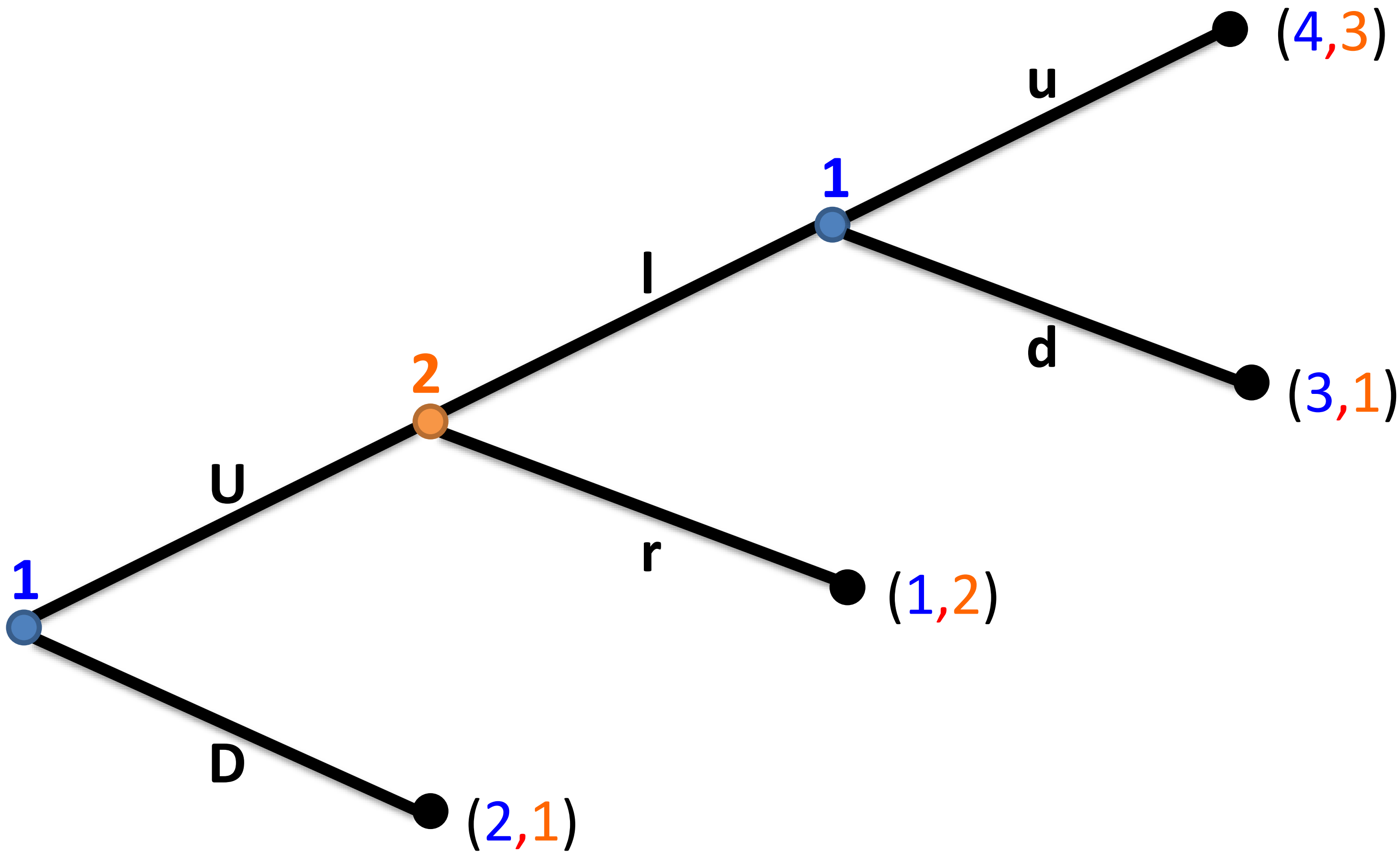
Don’t Screw Up



Let’s Play This!

Solution: Backward Induction



Solution: Nash Equilibria

Player 1		Player 2: l	r
	Uu	4,3	1,2
	Ud	3,1	1,2
	Du	2,1	2,1
	Dd	2,1	2,1

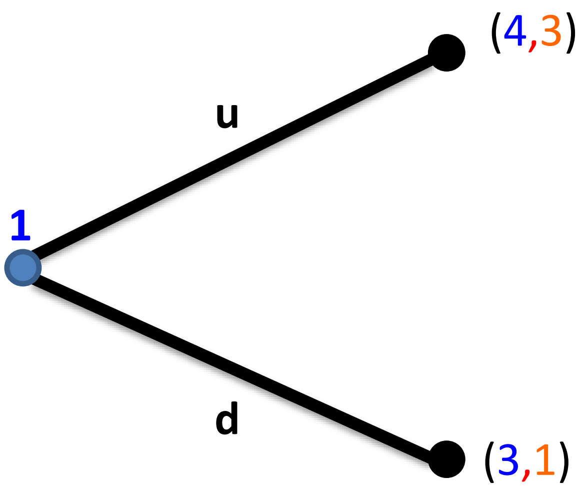
Nash Equilibria:

1. **(Uu,l) ➔ compatible with BI**
2. (Du,r) ➔ not compatible with BI
3. (Dd,r) ➔ not compatible with BI

Subgame I

Player 1	u	4,3
	d	3,1

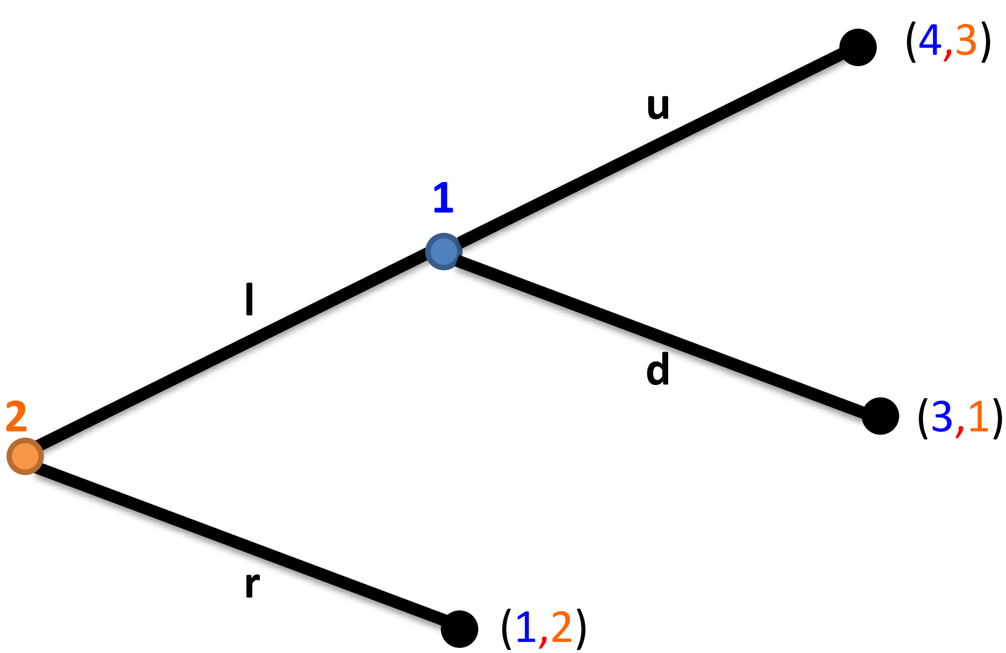
NE = (u)



(Uu,l)	(Du,r)	(Dd,r)
✓	✓	✗

So here (Dd,r) is eliminated since it induces non-Nash equilibrium play in this sub-game

Subgame 2



		2 l	r
1	u	4,3	1,2
	d	3,1	1,2

NE = (u,l) and (d,r)

(Uu,l)	(Du,r)	(Dd,r)
✓	✗	✓

So here (Du,r) is eliminated since it induces non-Nash equilibrium play in this sub-game

Solution: SPNE

- The only SPNE is (Uu,l)
  - This is the backward induction prediction

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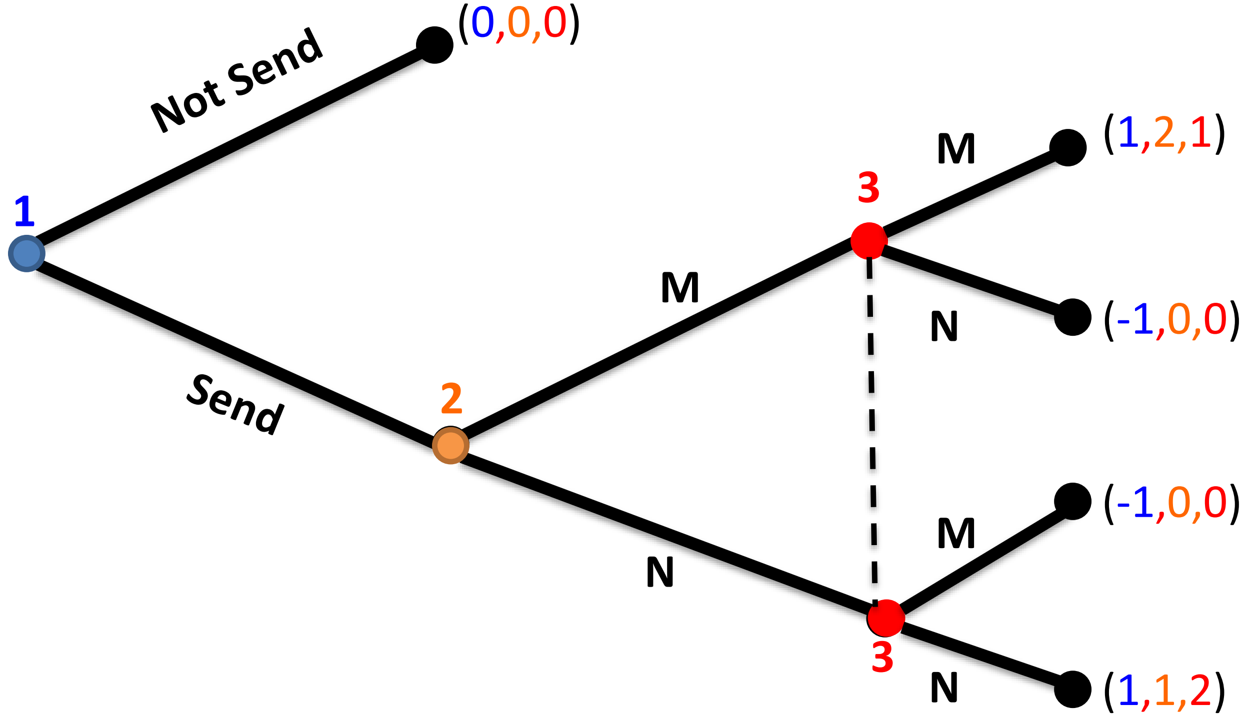
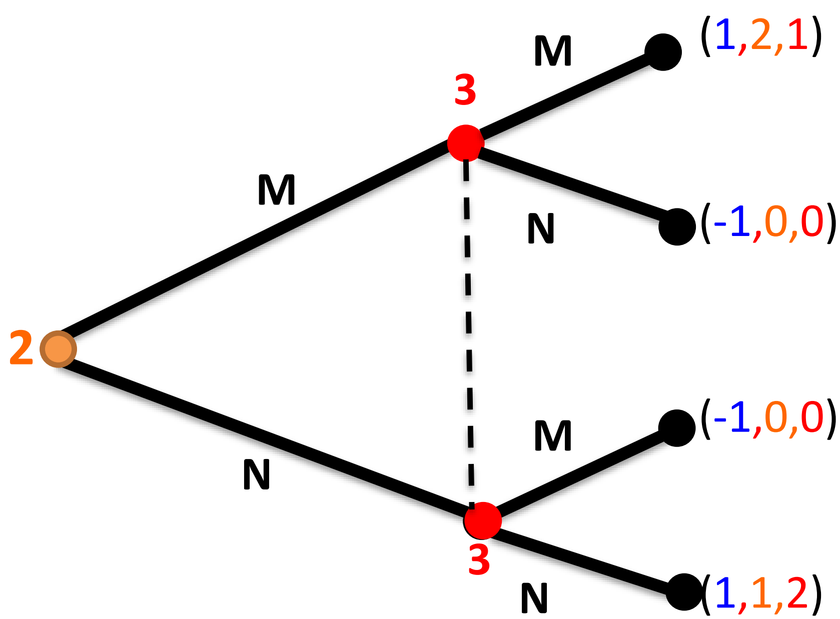
Matchmaker Game

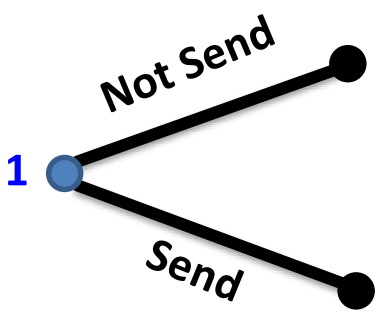


Sub-game Solution



		3	
		M	N
2	M	1,2,1	-1,0,0
	N	-1,0,0	1,1,2

NE = (M,M) and (N,N)



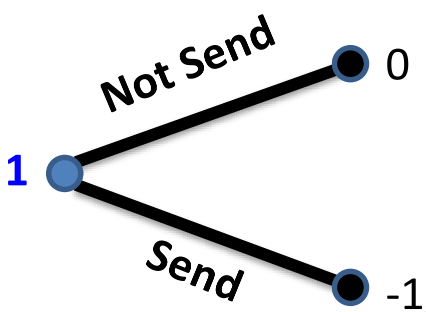
1-- value to "Player 1" of the Nash equilibria in this sub-game

Solution SPNE

- There are two SPNE
  1. (Send,M,M)
  2. (Send,N,N)

Mixed Strategy Solution

- There is a Mixed NE
  - [(2/3,1/3),(1/3,2/3)]
- If Player 1 sends 2 and 3, then they meet with probability 2/9+2/9=4/9 and fail to meet with probability 5/9
- Player 1 expected payoff at this equilibrium is: 4/9(1) + 5/9 (-1) = -1/9



➔ SPNE= (Not Send, Mix NE, Mix NE)

-1/9-- value to “Player 1” of the Nash equilibria in the sub-game

Conclusions

- **Sub-game perfect equilibrium** implies **backward induction**
- Look for the Nash equilibria in each of the sub-games, roll the payoffs back up, and then see what the optimal moves are higher up the tree

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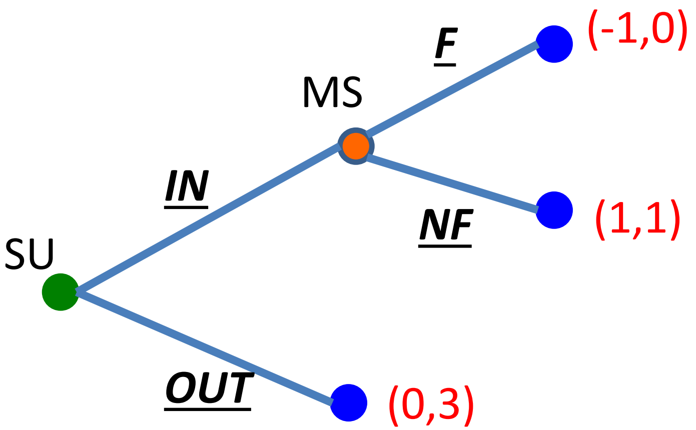
A Market Game

- Assume there are two players
  - An incumbent monopolist (MicroSoft, MS) of O.S.
  - A young start-up company (SU) with a new O.S.
- The strategies available to SU are:
  Enter the market (***<u>IN</u>***) or stay out (***<u>OUT</u>***)
- The strategies available to MS are:
  Lower prices and do marketing (***<u>FIGHT</u>***) or stay put (***<u>NOT FIGHT</u>***)

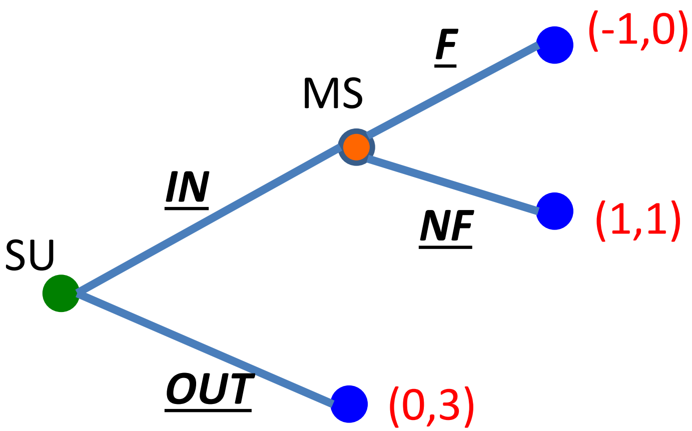
A Market Game



- What should you do?

- Analyze the game with BI
- Analyze the normal form equivalent and find NE

A Market Game



SU \ MS	F	NF
IN	-1,0	1,1
OUT	0,3	0,3

Backward Induction

(IN, NF)

?

Nash Equilibrium

(IN, NF)
(OUT, F)

This is a NE, but relies on an ***incredible*** threat

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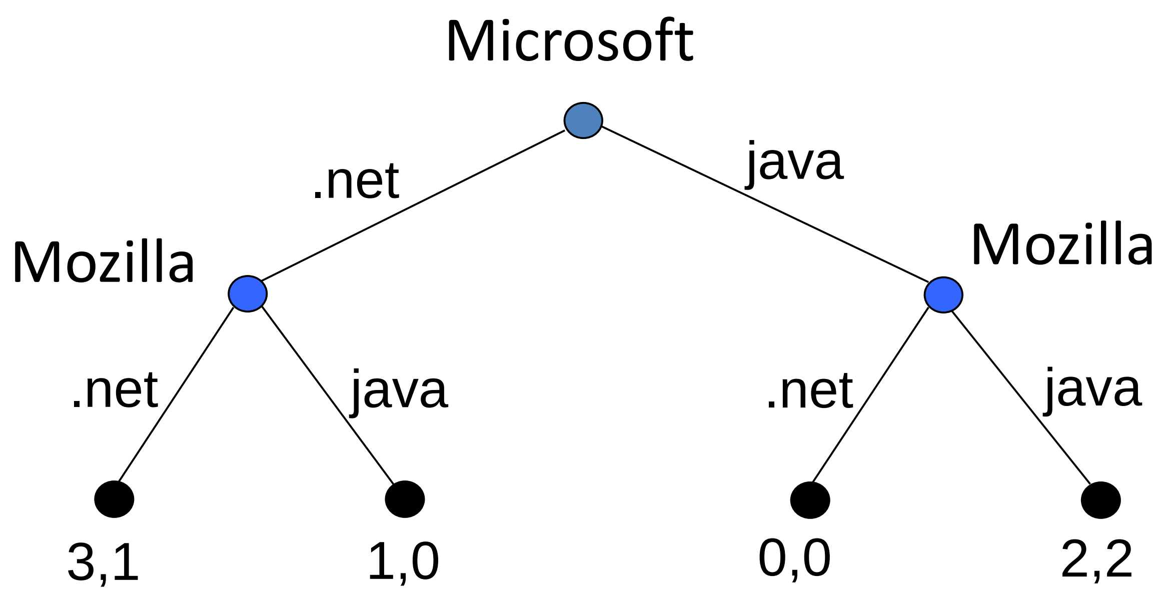
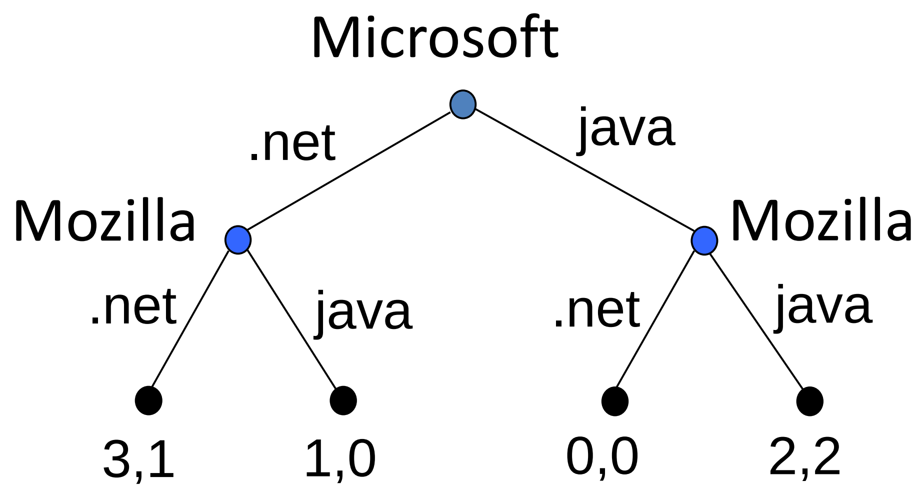
Microsoft vs Mozilla: Game Tree

- Microsoft and Mozilla are deciding on adopting new browser technology (.net or java)
  - Microsoft moves first, then Mozilla makes its move



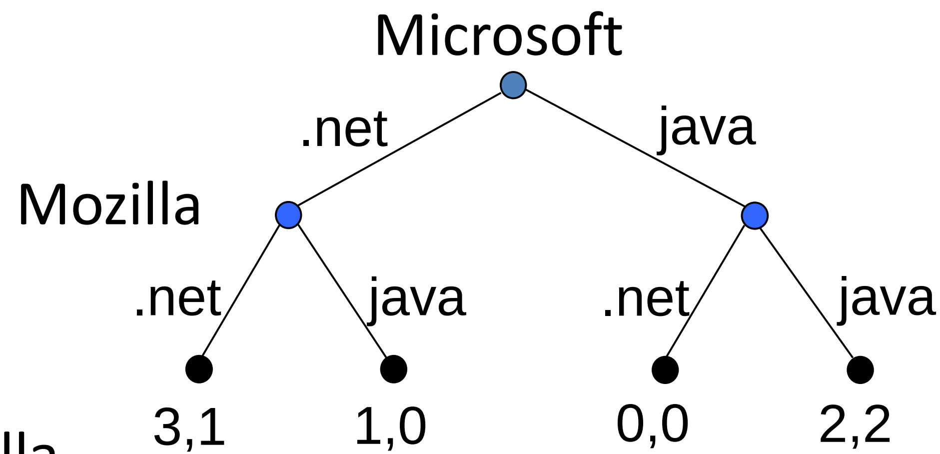
Converting to Matrix Game



Mozilla

Microsoft		.net, .net	.net, java	java,.net	java, java
	.net	3, 1	3, 1	1, 0	1, 0
	java	0, 0	2, 2	0, 0	2, 2

NEP and Incredible Threats



Mozilla

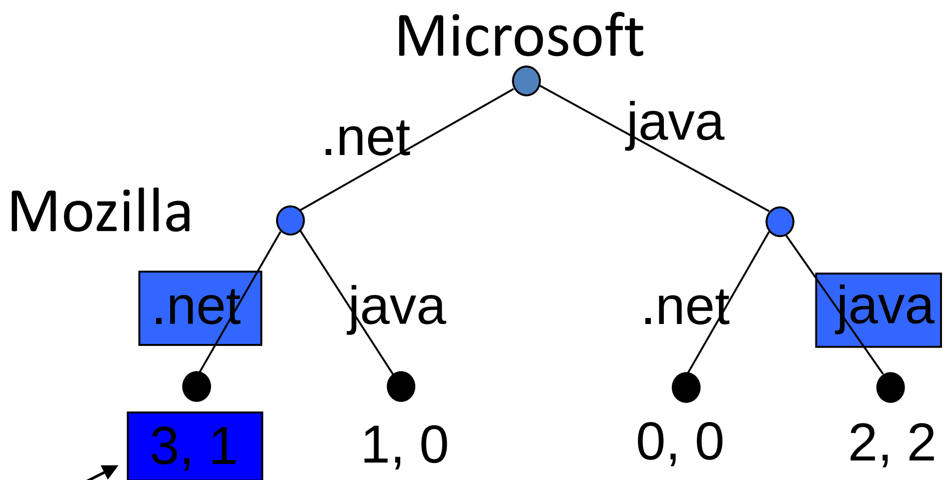
Microsoft		.net, .net	.net, java	java, .net	java, java
	.net	3, 1	3, 1	1, 0	1, 0
	java	0, 0	2, 2	0, 0	2, 2

NEP

incredible threat

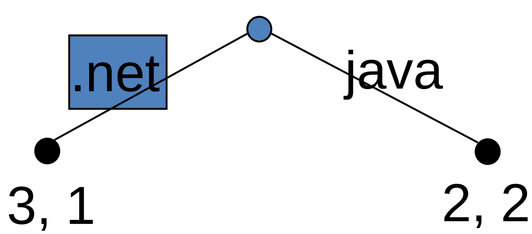
- Play "java no matter what" is not credible for Mozilla
  - if Microsoft plays .net then .net is better for Mozilla than java

Solving the Game (Backward Induction)



Best strategy for
Mozilla: .net, java
(follow Microsoft)

Equilibrium
outcome



Best strategy for
Microsoft: .net

- Single NEP

  Microsoft -> .net,   Mozilla -> .net, java

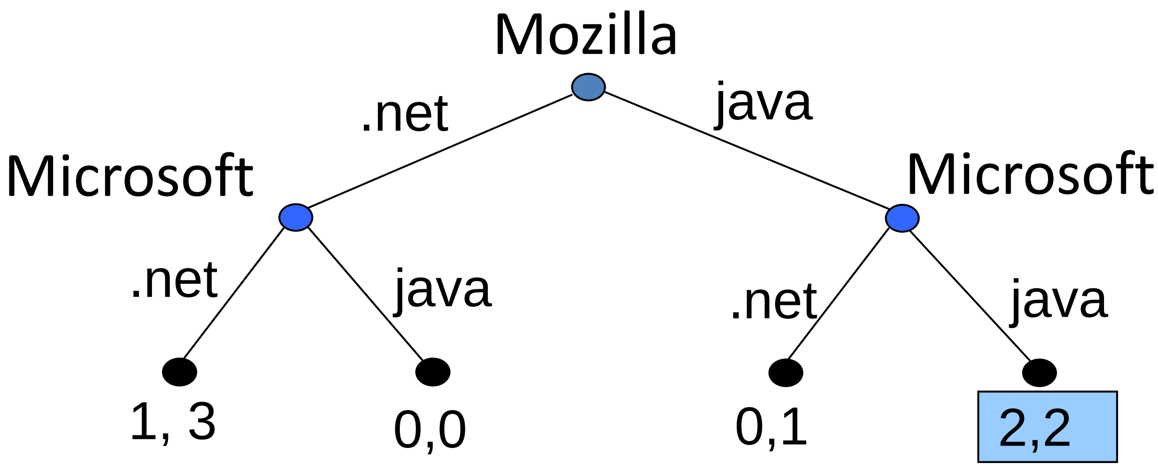
Kuhn's Theorem

Backward induction always leads to Nash Equilibrium in Sequential Games with Perfect Information

- Effective mechanism to remove "bad" NEP
  - incredible threats

Leaders and Followers

- What happens if Mozilla moves first?



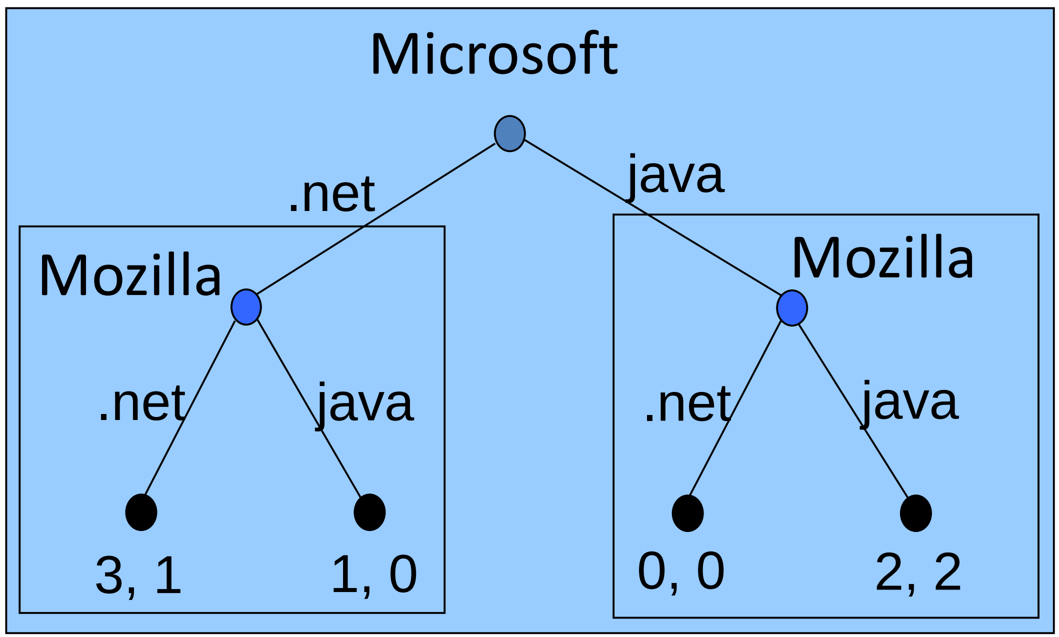
- NEP after backward induction:

  Mozilla: java
  Microsoft: .net, java

- Outcome is better for Mozilla, worst for Microsoft
  - **incredible threat becomes credible!**
- 1st mover advantage
  - Remember that it can also be a disadvantage…

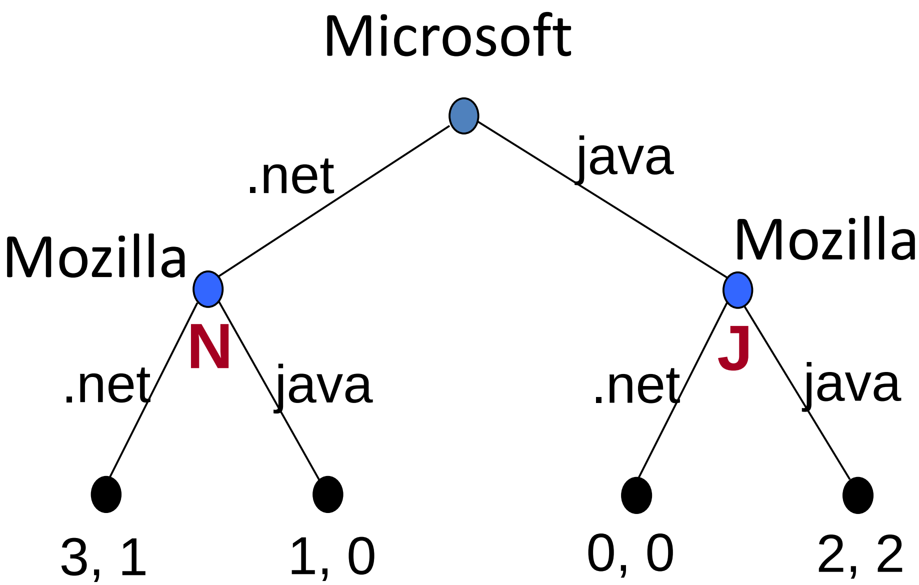
The Sub-Games

- Definition: A sub-game is any sub-tree of the original game that also defines a proper game
  - – includes all descendants of non-leaf root node



- 3 subtrees
  - full tree, left tree, right tree

Sub-game Perfect Nash Equilibrium



- (N, NN) is not a NEP when restricted to the subgame starting at **J**
- (J, JJ) is not a NEP when restricted to the subgame starting at **N**
- (N, NJ) is a subgame perfect Nash equilibrium

Mozilla

MS		NN	NJ	JN	JJ
	N	3,1	3,1	1,0	1,0
	J	0,0	2,2	0,0	2,2

Subgame Perfect NEP

Not subgame Perfect NEP

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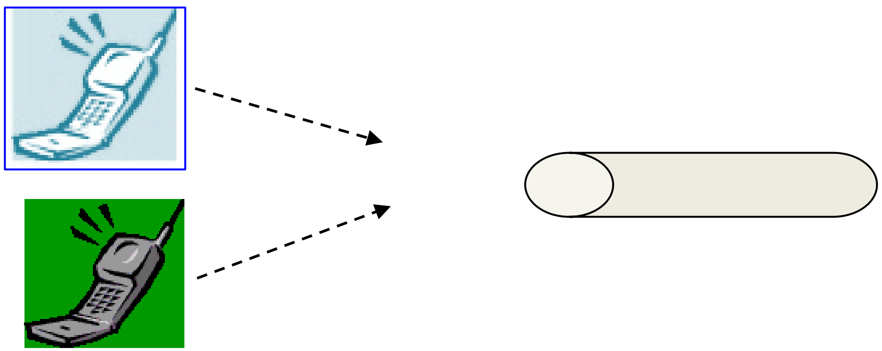
Extensive-form Games of TDMA

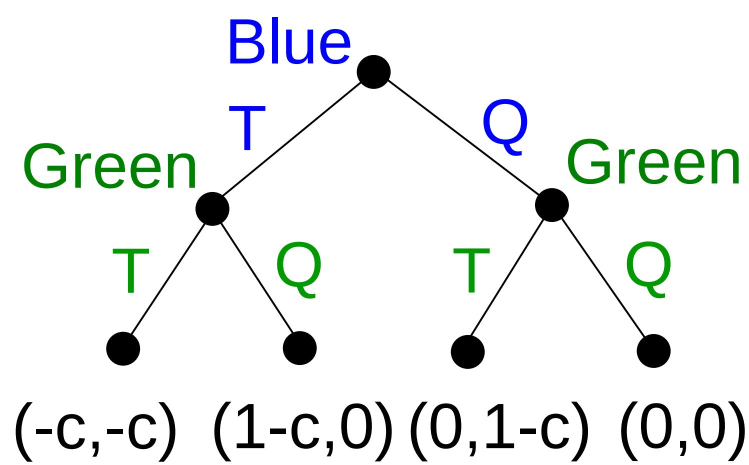
Blue plays first, then Green plays.



Time-division channel

Reward for successful transmission: 1

Cost of transmission: c
(0 < c << 1)



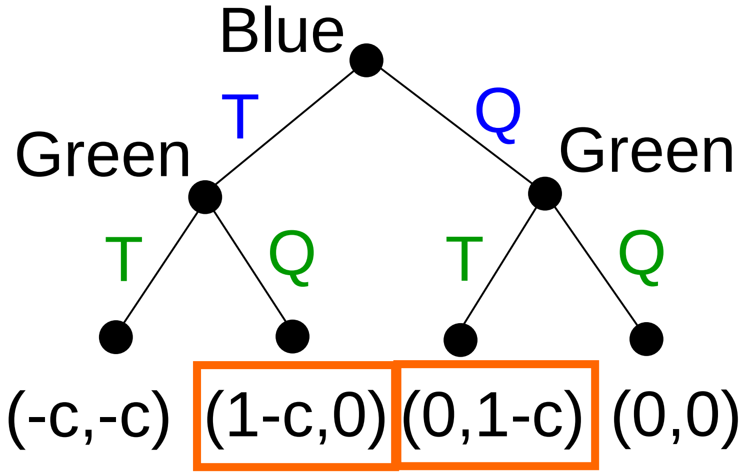
Strategies in Dynamic Games

- strategies for Blue:

  T, Q

- strategies for Green:

  TT, TQ, QT and QQ



If they have to decide independently: three Nash Equilibria

(T,QT), (T,QQ) and (Q,TT)

Extensive to Normal Form

Green

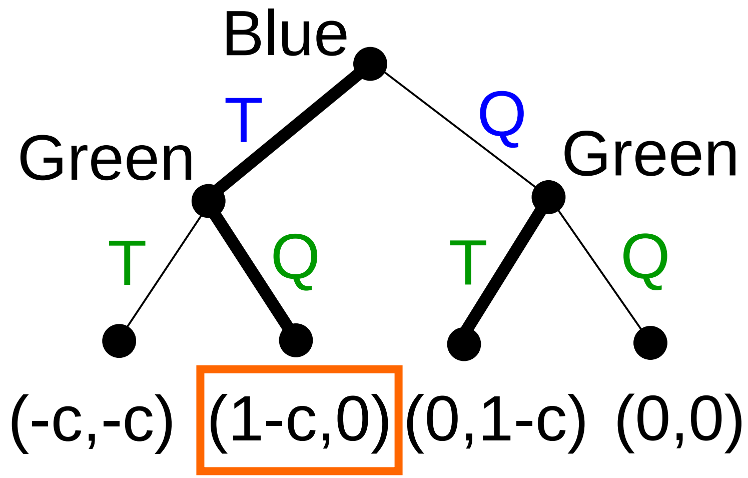
Blue

Blue vs. Green	TT	TQ	QT	QQ
T	(-c,-c)	(-c,-c)	$\boxed{(1-c,0)}$	$\boxed{(1-c,0)}$
Q	$\boxed{(0,1-c)}$	(0,0)	(0,1-c)	(0,0)

Backward Induction

- Solve the game by reducing from the final stage
- Eliminates Nash equilibria that are *incredible threats*

incredible threat: (Q, TT)



Sub-game Perfection

- Extends the notion of Nash equilibrium

One-deviation property: A strategy s_i conforms to the *one-deviation property* if there does not exist any node of the tree, in which a player i can gain by deviating from s_i and apply it otherwise.

Subgame perfect equilibrium: A strategy profile s constitutes a subgame perfect equilibrium if the one-deviation property holds for every strategy s_i in s.

Finding subgame perfect equilibria using backward induction

SPNE: (T, QT)