بسم الله الرحمن الرحیم

Algorithmic Game Theory

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دانشگاه کاشان

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Let's Play a Game!

The Grade Game

You should choose between α and β!

Note:

- *Do not show your neighbors what you are doing!*
- *Look this as a "grade bid".*
- *I will randomly pair your paper with one other paper.*
- *Neither you nor your pair will ever know with whom you were paired.*

I will grade like this:

- If you put α and your pair puts β, then you will get grade A, and your pair grade C;
- If both you and your pair put α, then you both will get the grade B-;
- If you put β and your pair puts α, then you will get the grade C and your pair grade A;
- If both you and your pair put β, then you will both get grade B+

Represent a Game

My Grade

Me \ My Pair	α	β
α	B -	A
β	C	B +

Pair's Grades

Me \ My Pair	α	β
α	B -	C
β	A	B +

Grade Game: Outcome Matrix

My Pair

		α	β
Me	α	B - , B -	A , C
	β	C , A	B + , B +

1st grade
Row player

2nd grade
Column player

We can find everything
that was in the game in one table!

Grade Game: Let's Discuss

What did you do?

–How many chose α?

–How many chose β?

–Why?

Grade Game: Our Answer

- Regardless of my partner choice, there would be better outcomes for me by choosing α rather than β;

- What we have examined is **not** a game yet

Grade Game: Payoff

- Right now we have:
  - The players
  - Strategies, that is the actions players can take
  - We know what the outcomes are

- We are missing **objectives**, i.e. payoffs
- Basically we don't know what players care about

Grade Game: Payoff Choices

- Two different payoffs:
  - We only care about our own grade
  - We might care about other people's grade

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Grade Game: Payoff Matrix

You only care about your own grades **(Selfishness)**

Payoffs:

(A,C) → (19,8)

(B-, B-) → (12,12)

B+ → 14

Hence the preference order is:

A > B+ > B- > C
19>14>12>8

		My pair α	**My pair** β
Me	α	**12 , 12**	**19 , 8**
Me	β	**8 , 19**	**14 , 14**

Grade Game: Selfishness

- What should you do, in this case?
  - Play α! Indeed, no matter what the pair does, by playing α you would obtain a higher payoff
- What do we call people who only care about their own grades?

Definition:

We say that my strategy α **strictly dominates** my strategy β, if my payoff from α is <u>strictly</u> greater than that from β, <u>regardless of what others do</u>.

First Lesson

Do Not Play
Strictly Dominat<u>ed</u> Strategies!

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Contracts and Collusion

- Why shouldn't you play strictly dominated strategies?
  - Because if I play a dominat<u>ing</u> strategy I'm doing better than what I could do regardless what the other does
- Let's look again at the payoff matrix
  - If we (me and my pair) reason selfishly, we will both select α, and get a payoff of 12;
  - But if we reasoned in a different way, we could end up <u>both</u> with a payoff of 14 (Make Contract)

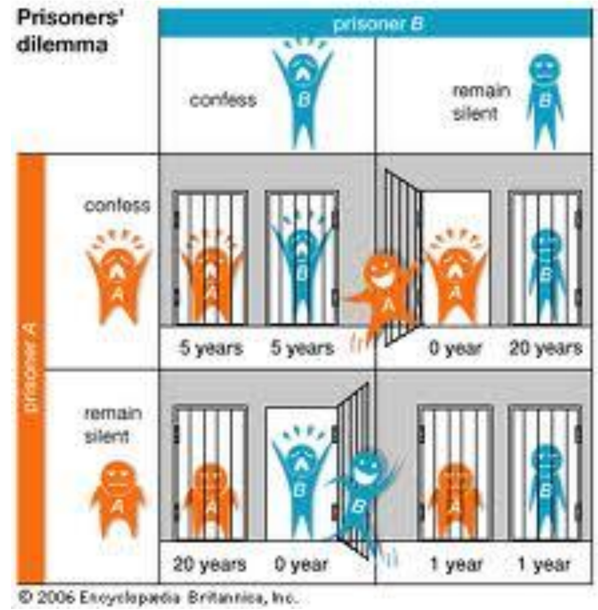
Failure of Collusion

- What's the problem with this latter reasoning?
- Suppose you have super mental power and oblige your partner to agree with you and chose β, so that you both would end up with a payoff of 14…
- Even with **communication**, it wouldn't work, because at this point, you'd be better of by choosing α, and get a payoff of 19

Second Lesson

Rational Choice
(i.e., Not Choosing a Dominated Strategy)
Can Lead to Outcomes that Suck!

The Prisoner's Dilemma



- Did you know it?
- Any other examples?
- What kind of remedies we have for such situations?
  - Repeted game/punishment/… (We will get back to this later)

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The Grade Game: Payoff Matrix

- Possible payoffs: This time people are more incline to be **altruistic**

Payoffs:

(A,C) → 19 – 6 = 13
my 'A' my guilt

(C, A) → 8 – 2 = 6
my 'C' my indignation

This is a **coordination problem**

My Pair

		α	β
Me	α	12 , 12	13, 6
	β	6,13	14,14

The Grade Game [Coordination]

- What would you do in this case?
  - By choosing α you may “minimize your losses”
  - By choosing β you may “maximize your profit”

- We have the same game structure, the same outcomes, but the payoffs are different
- Is there any dominated strategy in this game?

Third Lesson

Payoffs Matter!

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The Grade Game: Selfish vs Altruistic

In this case, α still **dominates**

The fact we (selfish player) are playing against an altruistic player doesn't change my strategy, even by changing the other Player's payoff

My pair (Altruistic)

		α	β
Me (Selfish)	α	12 , 12	19, 6
	β	8,13	14,14

The Grade Game:
Altruistic vs Selfish

- What happened here?
- Do I have a dominating strategy?
- Does the other player have a dominating strategy?

By thinking of what my "opponent" will do I can decide what to do.

		My pair (Selfish)	
		α	β
Me (Altruistic)	α	12 , 12	13, 8
	β	6,19	14,14

Observations

- In realistic settings:
  - It is often hard to determine what are the payoffs of your "opponent"
  - It is easier to figure out my own payoffs

- In general, we have to figure out what are the odds (probability) of my "opponent" being selfish or altruistic

Fourth Lesson

Put Yourself in Others' Shoes and Try to Figure Out What They Will Do!

"Think Strategically"

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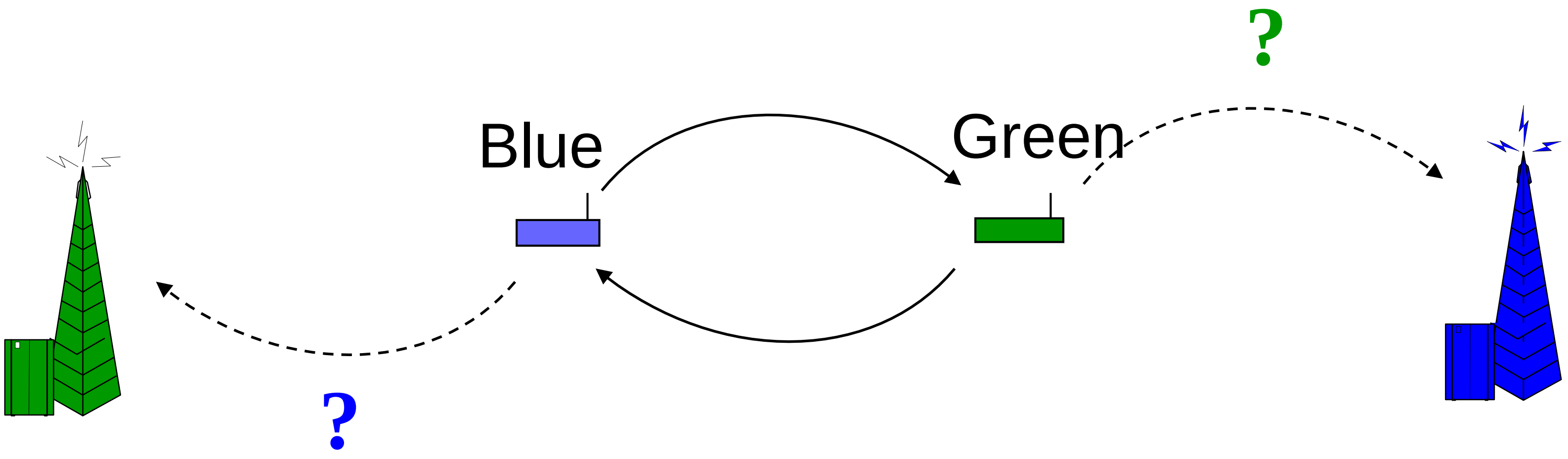
The Forwarder's Dilemma



Forwarder Game

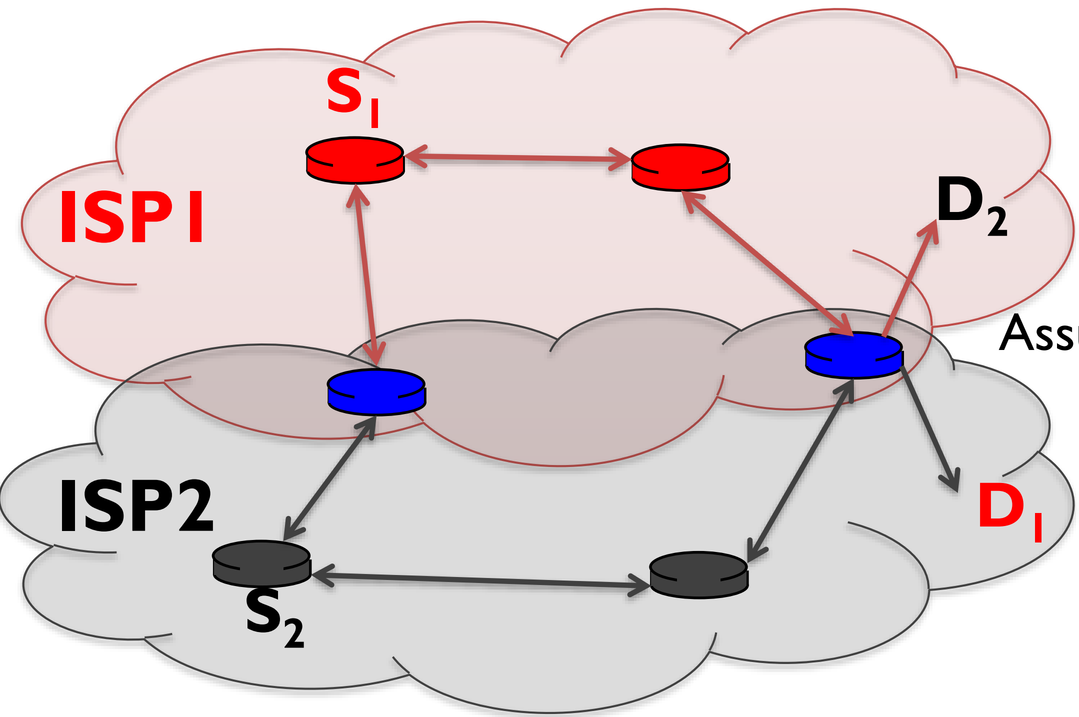
- users controlling the devices are ***rational*** = try to maximize their benefit

Blue \ Green	Forward	Drop
Forward	(1-c, 1-c)	(-c, 1)
Drop	(1, -c)	(0, 0)

- Reward for packet reaching the destination: 1
- Cost of packet forwarding: c (0 < c << 1)

strategy Drop ***strictly dominates*** strategy Forward

ISP Routing Games



Assume that the unit cost along a link is 1

ISP1 \ ISP2	Hot Potato	Cooperate
Hot Potato	(-5, -5)	(-2,-6)
Cooperate	(-6, -2)	(-3, -3)

Prisoner's Dilemma (Final Words)

- In each of the previous examples we end up with a bad outcome
- This is **not a failure of communication**
- Solutions:
  - Contracts ➔ change the payoffs
  - Repeated interaction

Summary

- We've seen a compact representation of games: this is called the **normal form**
- Lessons we learned:
  1. Do not play strictly dominated strategies
  2. Put yourself in others' shoes

- It doesn't just matter what your payoffs are
- It's also important what other people's payoff are, because you want to try and figure out what they're going to do and respond appropriately