Department of Economics, University of California, Davis
Ecn 200C – Game Theory – Professor Giacomo Bonanno

PRACTICE PROBLEMS $\boxed{1}$

Topic: modeling situations as games

VERY IMPORTANT: do **not** look at the answers until you have made a VERY serious effort to solve the problem. If you turn to the answers to get clues or help, you are wasting a chance to test how well you are prepared for the exams. I will **not** give you more practice problems later on.

1. How could they do that! They abducted little Barky, your favorite dog! They asked for $1,000 in unmarked bills and threatened to kill Barky if you don't pay. They said they would call back in four hours. That's when they will give you the details of where to leave the money and so forth. You've got four hours to decide what to do. Call the cops? Sit on your hands? Wake up the bank manager?

First, you have to consider how you feel about what might happen. Most of all, you would like not to pay and still get little Barky back in one piece. The worst that can happen is have him killed when you've paid anyway. Between the remaining two options you prefer paying and having him safe to not paying and having him killed.

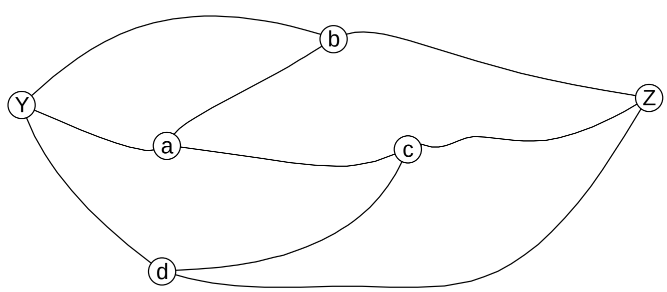
Next you must decide how the kidnappers feel about things. To do this, you need to decide whether they' re freelance or professional operators. Assume for the moment that they're freelances (this is going to be their only shot at the business of dog-napping). How likely are they to kill little Barky? There are two schools of thought on this matter. One dognapper will expect all of the aggravation and inconvenience caused by killing the hostage to outweigh by far any minor benefit arising from eliminating a key witness. For this type of dognapper, call him the ***Nice Freelance***, the best outcome, of course, is getting the money without having to kill the dog; the worst outcome is killing the dog without getting the money. Between the other two outcomes he will prefer the one where he gets the money. Another dognapper, call him the ***Nasty Freelance***, will think differently: killing the hostage and getting rid of all that evidence cuts down the risk of capture and conviction so markedly that it's the most sensible thing to do. Consider now the case of a ***Professional*** dognapper: he expects to be in this business for a long time and, therefore, reputation is an important issue for him: once a deal is offered, it must be honored. Pay/no kill and No pay/kill are the top two choices, with pay/no kill as the best possible outcome. If the deal falls through, it is obviously better to have the money (pay/kill) than not (no pay/no kill): not getting paid and not carrying out the death threat is the worst thing that can happen, representing the first step on the road to ruin.

The dreaded call has come! The dognapper has told you that you should leave the money at 10 pm sharp outside Professor Bonanno's office in the Social Science Building (what has he got to do with all this?). At precisely that time, and in another part of town, he will decide whether to kill Barky or set him free.

(a) Represent the situation you face as a normal-form game (you should write three different games, one for each type of dognapper you might be facing). Represent preferences over outcomes using an ordinal utility function whose possible values are 1, 2, 3 and 4 (the higher the utility, the better the outcome).

(b) Wait a minute! What about calling the police? Once you call them they will take the matter in their hands and decide what to do on your behalf (they will play the game instead of you). The police will take a tough line on dognapping. They want to discourage the others. They don't get any medals for helping distraught dog-owners pay to get their dogs home safe and sound. They get promoted for busting villains. That doesn't mean to say that they would rather see Barky dead than alive. They want to save his life, but they also have to think of all the other dognappers who will be encouraged if dognapping looks like an easy way to make money. Not paying is their top priority. Represent the situation where you call the police as a game.

2. Consider the following road map. Yvette is at point Y and Zoe is at point Z.
Zoe must go from point Z to point Y, **always moving West.**
Yvette, on the other hand, must travel from point Y to point Z, along the above system of roads, **without returning to point Y and without using the same segment twice.** (She does **not** have to travel East all the time, however).



(a) Write down all the possible routes that Yvette could take. Consider these as her strategies.

(b) Write down all the possible routes that Zoe could take. Consider these as her strategies.

(c) Represent this as a normal-form game, assuming the following: if Yvette and Zoe go through the same intersection (e.g. they both go through point *a*) then Zoe wins. Otherwise, Yvette wins.

3. Consider the following two-player games where each player is given a set of cards and each card has a number on it. The players are Antonia and Bob. Antonia's cards have the following numbers: 0, 1, 2, 3 and 4, whereas Bob's cards are: 0, 1 and 2. Each player chooses a card without knowing the other player's choice. The outcome depends on the sum of the points of the cards chosen.

For each of the following variations, represent the game in matrix form.

A) If the sum is greater than 3, Antonia gets $10 and Bob gets nothing. If the sum is less than or equal to 3, Bob gets $10 and Antonia gets nothing.

B) If the sum is greater than 2, and it turns out to be an even number, Antonia gets $10 and Bob gets nothing. If the sum is greater than 2 and it is an odd number Antonia pays $5 and Bob gets nothing. If the sum is less than or equal to 2, and it turns out to be an even number, Bob gets $5 and Antonia gets 0; if it is an odd number Bob gets $10 and Antonia gets nothing.

C) Now suppose that Antonia's cards are changed to: 2, 4 and 6. (Bob's cards are still 0, 1 and 2). If the sum of points is greater than or equal to 5, Antonia gets $10 minus the total number of points; otherwise (ie., if the sum is less than 5) she gets nothing. The outcome for Bob is as follows: if the sum of points is an odd number, Bob gets as many dollars as the total points; if the sum of points turns out to be an even number and is less than or equal to 6, Bob gets $2; otherwise he gets nothing.