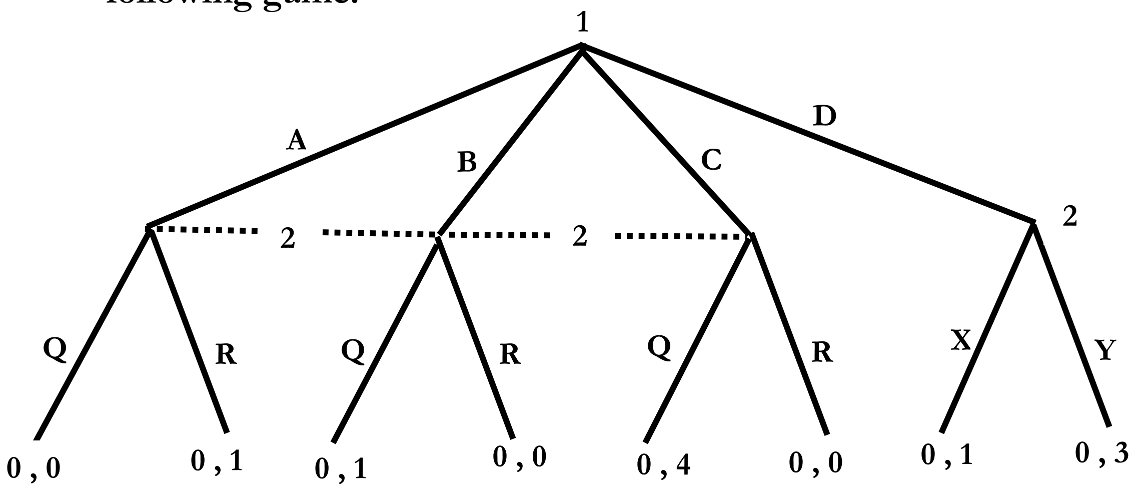
Summer 2007 – Chesnes
120 Minutes – 500 Points

Economics 414 – Final

Please answer ALL questions on this examination. Be sure to explain any non-standard notation that you use and JUSTIFY your answers. Each question's weight is shown in parentheses. Good Luck!

1. ***(25%) Extensive game of Imperfect Information.*** **Consider the following game:**

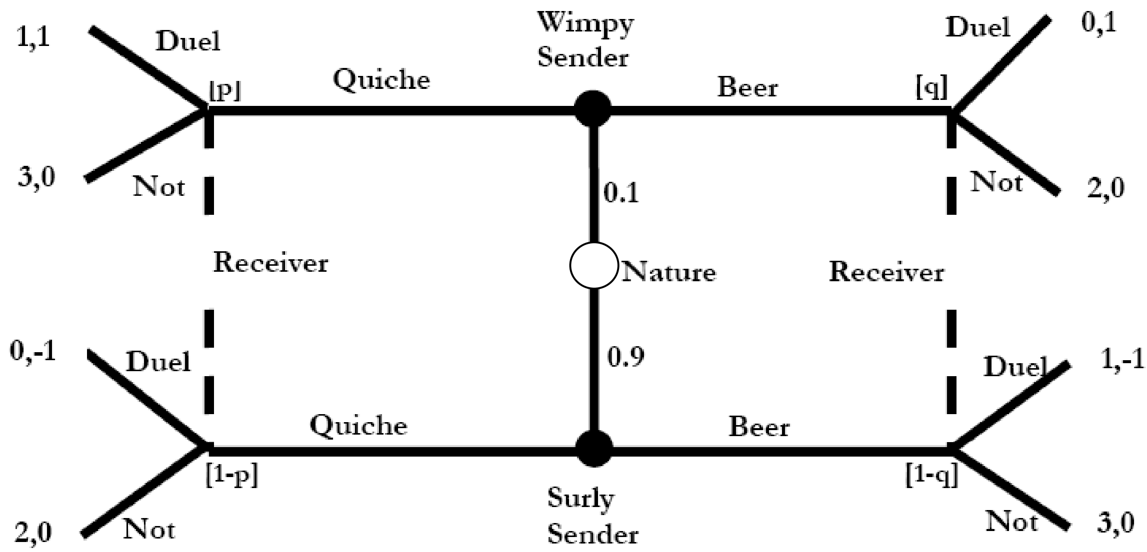


   a. **How many sub-games does the above game have?**

   b. **Write down the game in strategic (normal) form and solve for the *pure strategy* Nash Equilibria.**

   c. **Which of the NE you solved for in part (b) are Sub-Game Perfect?**

2. *(25%) Beer and Quiche anyone?* **Consider the following dynamic game of imperfect information:**



Nature initially chooses the sender's type (according to the probabilities shown in the tree) which becomes known to the sender but not the receiver. The sender chooses to have beer or quiche for breakfast and the receiver chooses whether or not to duel with the sender.

a. **State the 4 requirements necessary for a Perfect Bayesian Equilibrium.**

b. **Solve for a PBE which involves "pooling on quiche."**

c. **Show there does NOT exist a separating equilibrium in which a wimpy sender chooses quiche and a surly sender chooses beer.**

3. *(25%) Advertising*. **Consider two firms that can either advertise or not advertise. If they both advertise, they each earn a profit of 5. If one advertises and the other does not, then the firm that advertises earns 10 and the other firm earns 3. If neither advertises, then each earns an amount α, where $5 < \alpha < 10$. Firms make their decisions on advertising simultaneously.**

   a. **Suppose the game is played only once. Find all Nash Equilibria.**

   b. **Now suppose the game is to be played an unknown number of times. Both firms know there is a probability 0.1 that the current play of the game will be the last time the game is played. So there is a probability 1.0 that the game will be played at least once, a probability 0.9 that the game will be played at least twice, a probability 0.81 that it will be played at least three times, and so on.**

      Consider the following grim trigger strategy: A firm *does not* advertise the first time the game is played. In later periods, it *does not* advertise if no one has advertised up until that point; otherwise, it advertises. The discount factor is 1.0. For what range of values of α do these trigger strategies constitute a Sub-Game Perfect Nash Equilibrium where firms never advertise?

4. ***(25%) Repeated Bertrand.*** **Consider Bertrand's model of oligopoly. Suppose market (inverse) demand is $P(Q) = \alpha - Q$ and each of the three (3) firms have a *total* cost function $C_i(q_i) = \beta * q_i$. α and β are positive constants such that $\alpha > \beta$.**

   a. **Show that if the firms are able to collude and split the monopoly profit, they will each charge $p^m = ½ (\alpha+\beta)$ and split (equally) profits of $\Pi^m = 1/4 (\alpha-\beta)^2$.**

   b. **Now suppose $\alpha = 8$ and $\beta = 2$. Consider the following grim trigger strategy for each firm:**
      - ✓ **Choose p^m in the first period.**
      - ✓ **Choose p^m in each subsequent period if no firm has deviated in any prior period.**
      - ✓ **Choose the static Nash Equilibrium Bertrand price in each subsequent period otherwise.**

      Find the critical discount factor, δ^*, required to sustain cooperation (i.e. joint monopoly pricing) in all periods.

   c. **Suppose industry demand and cost functions are the same as above. What would happen to the critical discount factor you found in part (b) if the 3 firms competed *Cournot*-style instead *Bertrand*-style? (*Do not* solve for the critical value mathematically, but provide an intuition why δ^* may go up, go down, or stay the same. You MUST justify your answer to get full credit.)**