Spring 2006 – Chesnes
120 Minutes – 500 Points

Economics 414 – Final

Please answer ALL questions on this examination. Be sure to explain any non-standard notation that you use and JUSTIFY your answers. Each question is worth 20% of the total. Good Luck!

1. Consider the following simultaneous move game:

Player 2

Player 1		Head	Tail
	Head	(1 , -1)	(-1 , 1)
	Tail	(-1 , 1)	(1 , -1)

a. Define what is meant by a dominant strategy as it applies to simultaneous move games.

b. Find all Nash equilibria of the simultaneous move game and write down the expected payoff to each player.

c. Now suppose the game is changed so that player 1 moves first and player 2 moves second after observing player 1's move. Solve for all sub-game perfect Nash equilibria of this game and write down the expected payoff to each player.

2. Three oligopolists operate in a market with inverse demand given by $P(Q) = \alpha - Q$, where $Q = q_1 + q_2 + q_3$ and $q_i \geq 0$ is the quantity produced by firm i. Each firm has constant marginal costs equal to zero. There are no fixed costs. Firms choose their quantities as follows:

- **Firm 1 moves first and chooses q_1.**
- **Firms 2 and 3 observe q_1 and then choose q_2 and q_3 simultaneously.**

a. How many sub-games does this game have?

b. Solve for the sub-game perfect Nash equilibrium and show that in equilibrium, firm 1 earns profits of $(1/12)\alpha^2$ and firms 2 and 3 both earn profits of $(1/36)\alpha^2$.

3. Consider the following simultaneous move game of incomplete information. Nature first determines if the payoffs are as in case 1 or as in case 2. Both cases are equally likely. Then Smith learns whether nature has drawn case 1 or case 2, but Brown does not. Smith chooses T or B and Brown chooses L or R simultaneously.

Case 1

		Brown	
		L	R
Smith	T	1, 1	0, 0
	B	0, 0	0, 0

Case 2

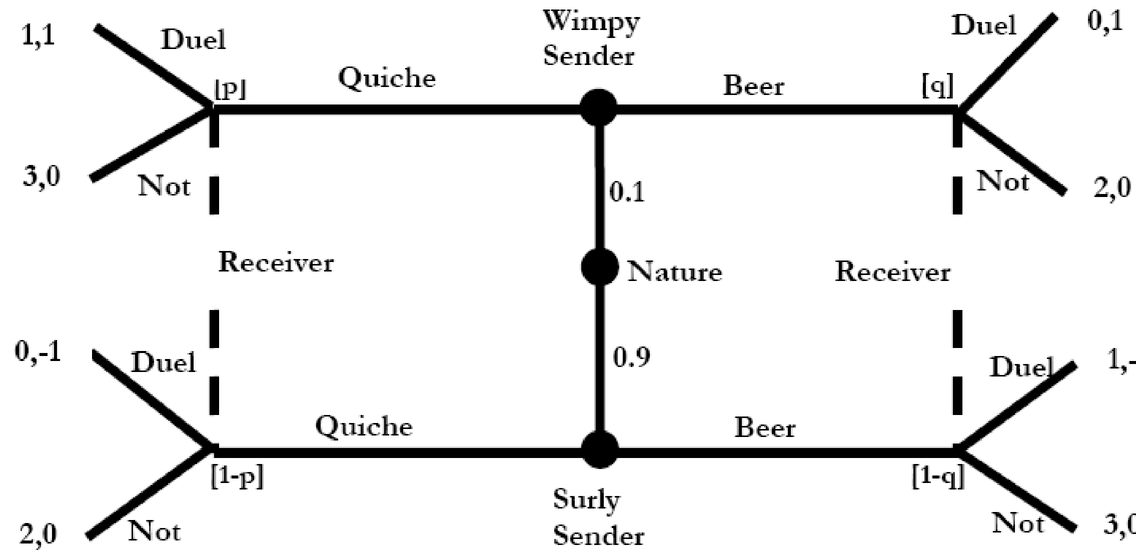
		Brown	
		L	R
Smith	T	0,0	0, 0
	B	0, 0	2, 2

a. What are the strategies of each player?

b. Find all pure-strategy Bayesian Nash equilibria. Justify your answer.

c. Give two examples of real-world situations which could be modeled as simultaneous move games of imperfect information.

4. Beer and Quiche anyone? Consider the following dynamic game of imperfect information:



Nature initially chooses the sender's type (according to the probabilities shown in the tree) which becomes known to the sender but not the receiver. The sender chooses to have beer or quiche for breakfast and the receiver chooses whether or not to duel with the sender.

a. State the 4 requirements necessary for a Perfect Bayesian Equilibrium.

b. Solve for a PBE which involves "pooling on beer."

c. Show there does NOT exist a separating equilibrium in which a wimpy sender chooses quiche and a surly sender chooses beer.

5. Consider the following simultaneous move, stage game, G.

		Player 2		
		L	C	R
	T	3,1	0,0	10,0
Player 1	M	2,1	0,2	3,1
	B	1,2	1,1	4,4

a. Find the unique pure strategy Nash equilibrium of G.
b. Define a grim trigger strategy for each player to sustain (B,R) as the Nash equilibrium of $G(\infty,\delta)$.
c. Solve for the critical discount factor, δ^*, such that your strategies in (b) imply that players optimally choose (B,R) in every period of the game.