Stat 155 Homework # 7 Due April 7

Problems:

Q 1 Karlin-Peres Chapter 3 Q 3.18

Player 1 can travel from A to C via either B or D. Player 2 can travel from B to D via either A or C. The payoff matrix of the game is

$$\begin{pmatrix} (7,7) & (5,4) \\ (7,8) & (5,5) \end{pmatrix}$$

Travelling through A is dominated by traveling through C for player 2. The pure Nash equilibria are Player 1 traveling through B and Player 2 travelling through C or Player 1 traveling through D and Player 2 travelling through C.

Q 2 On a TV show two contestants must choose between 4 with values d_1, d_2, d_3 and d_4 (you can assume they are all positive). If they choose different prizes they both get their choice but if they choose the same prize it is destroyed and they leave with nothing.

(a) Write down the payoff matrix.

(b) Describe the pure Nash equilibria of the game.

(c) Find the symmetric mixed Nash equilibria of the game.

For simplicity we will assume that $d_1 > d_2 > d_3 > d_4$. The payoff matrix is

$$\begin{pmatrix} (0,0) & (d_1,d_2) & (d_1,d_3) & (d_1,d_4) \\ (d_2,d_1) & (0,0) & (d_2,d_3) & (d_2,d_4) \\ (d_3,d_1) & (d_3,d_2) & (0,0) & (d_3,d_4) \\ (d_4,d_1) & (d_4,d_2) & (d_4,d_3) & (0,0) \end{pmatrix}$$

The two pure strategies correspond to the cells (d_1, d_2) and (d_2, d_1). Let (p_1, p_2, p_3, p_4) be the symmetric mixed strategy. Then by equalizing the payoffs we have that

$$d_1(1-p_1) = d_2(1-p_2) = d_2(1-p_3) = d_4(1-p_4) = C.$$

Then $p_i = 1 - C/d_i$ and so

$$1 = p_1 + p_2 + p_3 + p_4 = 4 - C(\frac{1}{d_1} + \frac{1}{d_2} + \frac{1}{d_3} + \frac{1}{d_4})$$

and so $C = 3(\frac{1}{d_1} + \frac{1}{d_2} + \frac{1}{d_3} + \frac{1}{d_4})^{-1}$ and

$$p_i = 1 - \frac{3}{d_i}(\frac{1}{d_1} + \frac{1}{d_2} + \frac{1}{d_3} + \frac{1}{d_4})^{-1}.$$

Q 3 Two players play a card game with a standard well shuffled deck of cards. Player 1 draws a card at random from the deck without showing it to player 2. Player 1 can either say "ace" or "pass". If player 1 says ace then player 2 must either "accept" and give \$ 1 to player 1 or "reject" and claim that player 1 is lying. If the card is an ace player 1 gets \$ 2 from player 2 while if it is not an ace player 1 pays player 2 a penalty or \$ R.

For each value of R find the Nash equilibrium for this game and the expected payment.

When player 1 has an ace, saying ace dominates saying pass. So the two strategies for player 1 are saying ace when it isn't an ace or passing when it isn't an ace. The two startegies for player 2 are saying accept or reject. The payoff matrix is.

$$\begin{pmatrix} 1 & \frac{2}{13} - \frac{12R}{13} \\ \frac{1}{13} & \frac{2}{13} \end{pmatrix}$$

Provided that $R > 0$ there are never any saddle points. Thus we may use the 2 player zero-sum game formula to find the optimal strategies $(x, 1-x)$ and $(y, 1-y)$ with

$$x = \frac{\frac{2}{13} - \frac{1}{13}}{1 - \frac{2}{13} + \frac{12R}{13} + \frac{2}{13} - \frac{1}{13}} = \frac{1}{12 + 12R}, \qquad y = \frac{\frac{2}{13} - \frac{2}{13} + \frac{12R}{13}}{1 - \frac{2}{13} + \frac{12R}{13} + \frac{2}{13} - \frac{1}{13}} = \frac{R}{1+R}$$

Using the formula the expected payoff is

$$V = \frac{\frac{2}{13} - \frac{1}{13}(\frac{2}{13} - \frac{12R}{13})}{1 - \frac{2}{13} + \frac{12R}{13} + \frac{2}{13} - \frac{1}{13}} = \frac{2 - (\frac{2}{13} - \frac{12R}{13})}{12 + 12R} = \frac{2+R}{13 + 13R}$$