Algorithmic Game Theory

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Cournot Duopoly

- Analyze through the eyes of Game Theory
- Reviewing the basic economic concepts
- Antoine Augustin Cournot (1801-1877) French philosopher and mathematician

Cournot Duopoly

- Why do we study it? [Game theory answer]
- So far we've seen two types of games
  - Those with few players and few (discrete) strategies
  - Those with a lot of players (e.g. the number game) and few strategies

- CD is a game with few players but a continuum of strategies

Cournot Duopoly

- Why do we study it? [Economics answer]
- This game lies between two extreme cases in economics, in situations where firms (e.g. two companies) are competing on the same market
  - Perfect competition
  - Monopoly
- We're interested to understand what happens in the middle

Cournot Duopoly

- Given a Cournot Duopoly model of a market, we want to understand what will happen in the market

- We want to understand, from the welfare point of view, if what happens is good or bad for producers/consumers

- There are many applied examples in the Economics of Information Technology (e.g., **EC, WEIS, and NetEcon** conferences)
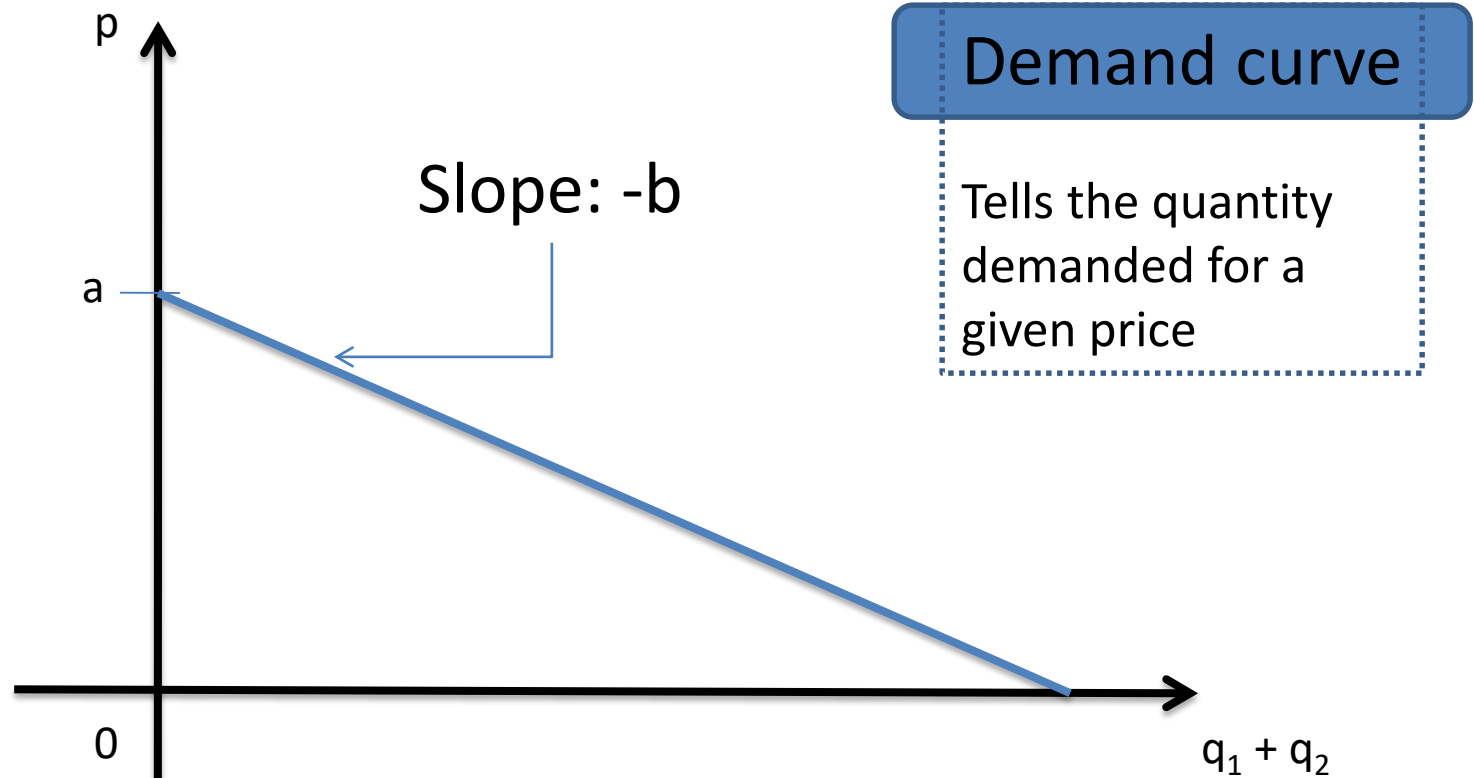
Cournot Duopoly: Game Model

- The players: 2 Firms, e.g. Coke and Pepsi

- Strategies: quantities players produce of **_identical_** products: q_i, q_{-i}
  - Products are perfect substitutes

Cournot Duopoly: Game Model

- **Cost of production:** $c \times q$
  - Simple model of ***constant marginal cost***
  - Marginal Cost:
    - Marginal cost is the change in total cost that arises when the quantity produced changes by one unit.

- **Prices:** $p = a - b\,(q_1 + q_2)$

Price in the Cournot Duopoly Game

p

Slope: -b

a

Demand curve

Tells the quantity demanded for a given price

0

$q_1 + q_2$

Cournot Duopoly: Game Model

- The payoffs: firms aim to ***maximize profit***

$$u_1(q_1,q_2) = p \times q_1 - c \times q_1$$

- Profits = Revenues (Benefit) − Costs
- Game vs. maximization problem

Cournot Duopoly: Game Model

$$u_1(q_1, q_2) = p \times q_1 - c \times q_1$$

$$p = a - b(q_1 + q_2)$$

➔ $u_1(q_1, q_2) = a \times q_1 - b \times q^2_1 - b \times q_1 q_2 - c \times q_1$

Cournot Duopoly: Game Model

- Now, we've defined the players, the strategies and the payoffs

- We want to find the NE of this game

- How do we do this?

Cournot Duopoly: Best Response

- First order condition

$$\frac{\partial u_1(q_1, q_2)}{\partial q_1} = 0$$

- Second order condition

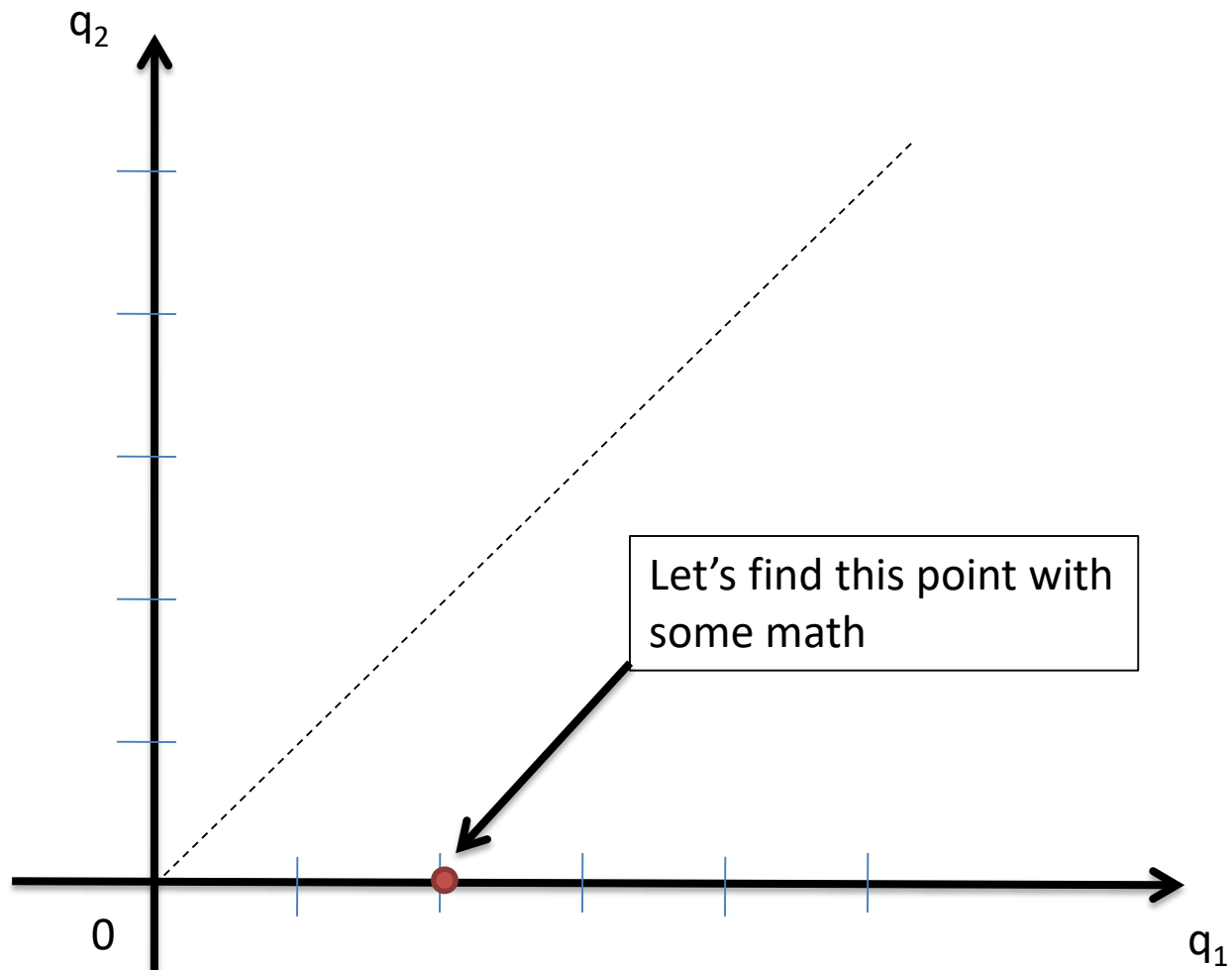$$\frac{\partial^2 u_1(q_1, q_2)}{\partial^2 q_1} < 0$$

Cournot Duopoly: Best Response

- First order condition $a - 2bq_1 - bq_2 - c = 0$

- Second order condition $-2b < 0$
  [make sure it's a max]

➜
$$\begin{cases} \hat{q}_1 = BR_1(q_2) = \dfrac{a-c}{2b} - \dfrac{q_2}{2} \\[4mm] \hat{q}_2 = BR_2(q_1) = \dfrac{a-c}{2b} - \dfrac{q_1}{2} \end{cases}$$

Cournot Duopoly: NE

- We could just find the NE now, right?

- How would you go for doing this?

- Instead, let's see things graphically

Let's find this point with some math

q_2

q_1

0

What if Firm 2 didn't produce at all?

➔ $q_2 = 0$

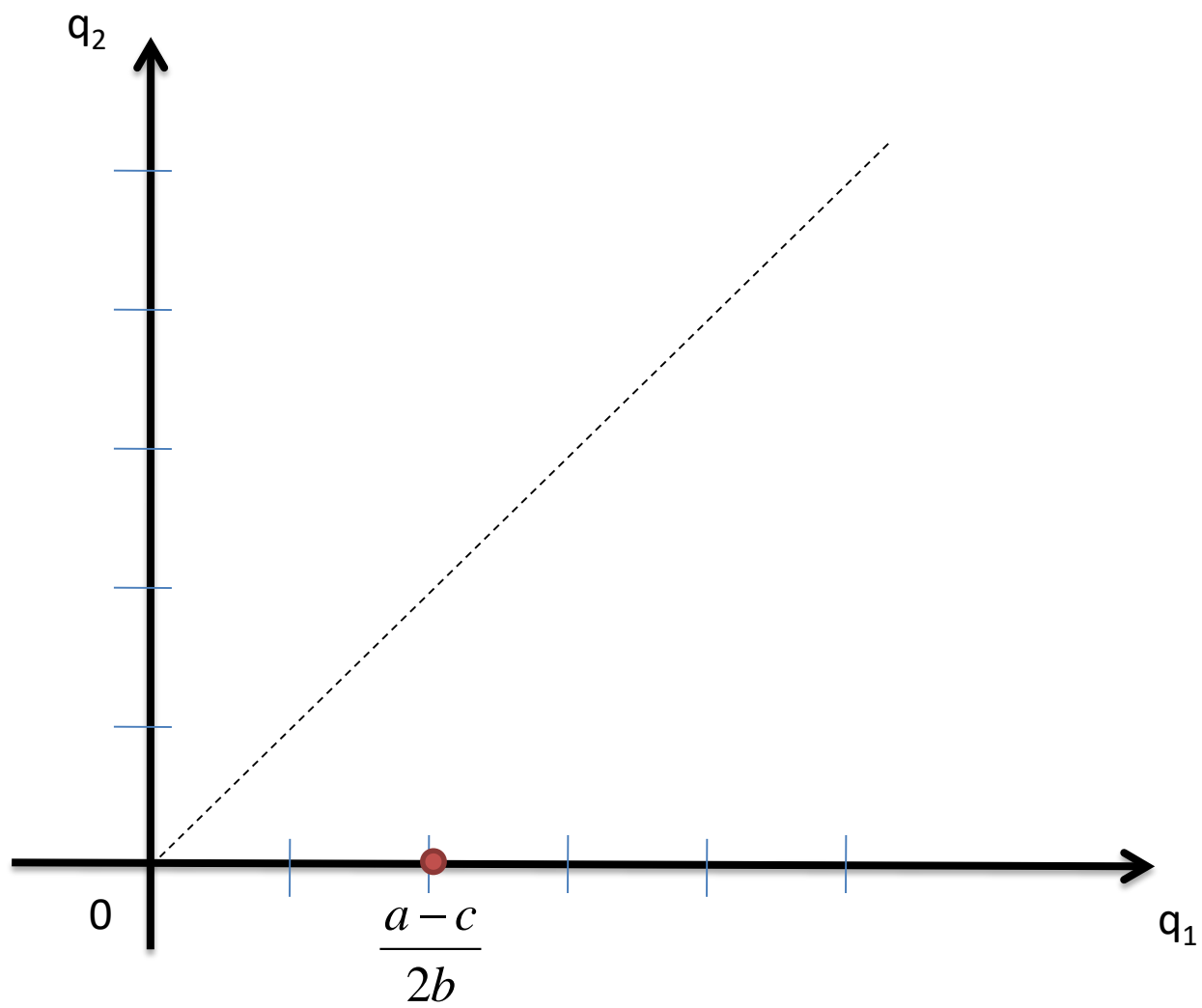➔ *What is the best response for Firm 1*

BR for Firm 1 when $q_2 = 0$

- What would be the BR for Firm 1 if Firm 2 didn't produce at all?

$$\hat{q}_1 = BR_1(q_2) = \frac{a-c}{2b} - \frac{q_2}{2}$$

$$\hat{q}_M = BR_1(q_2 = 0) = \frac{a-c}{2b}$$

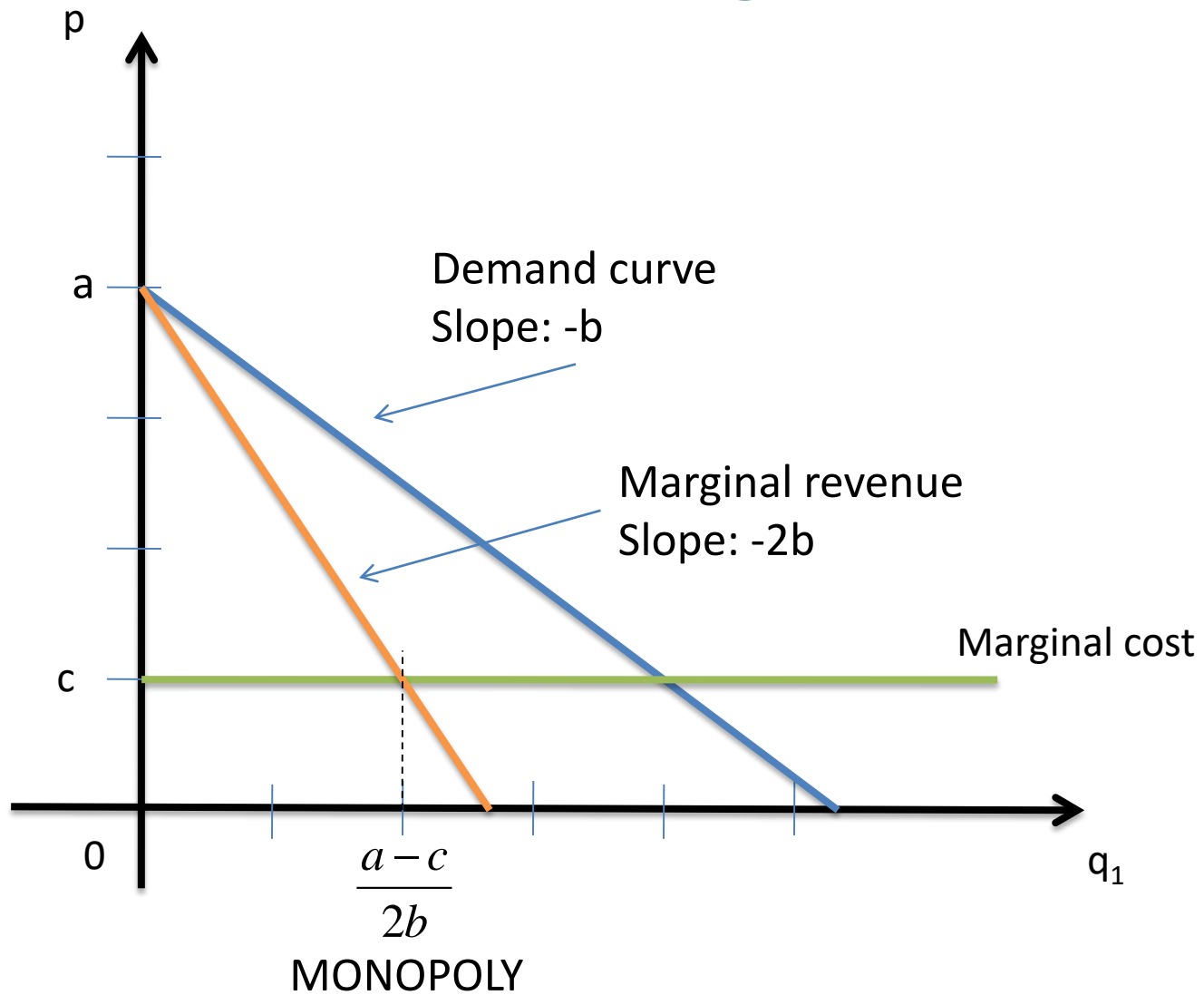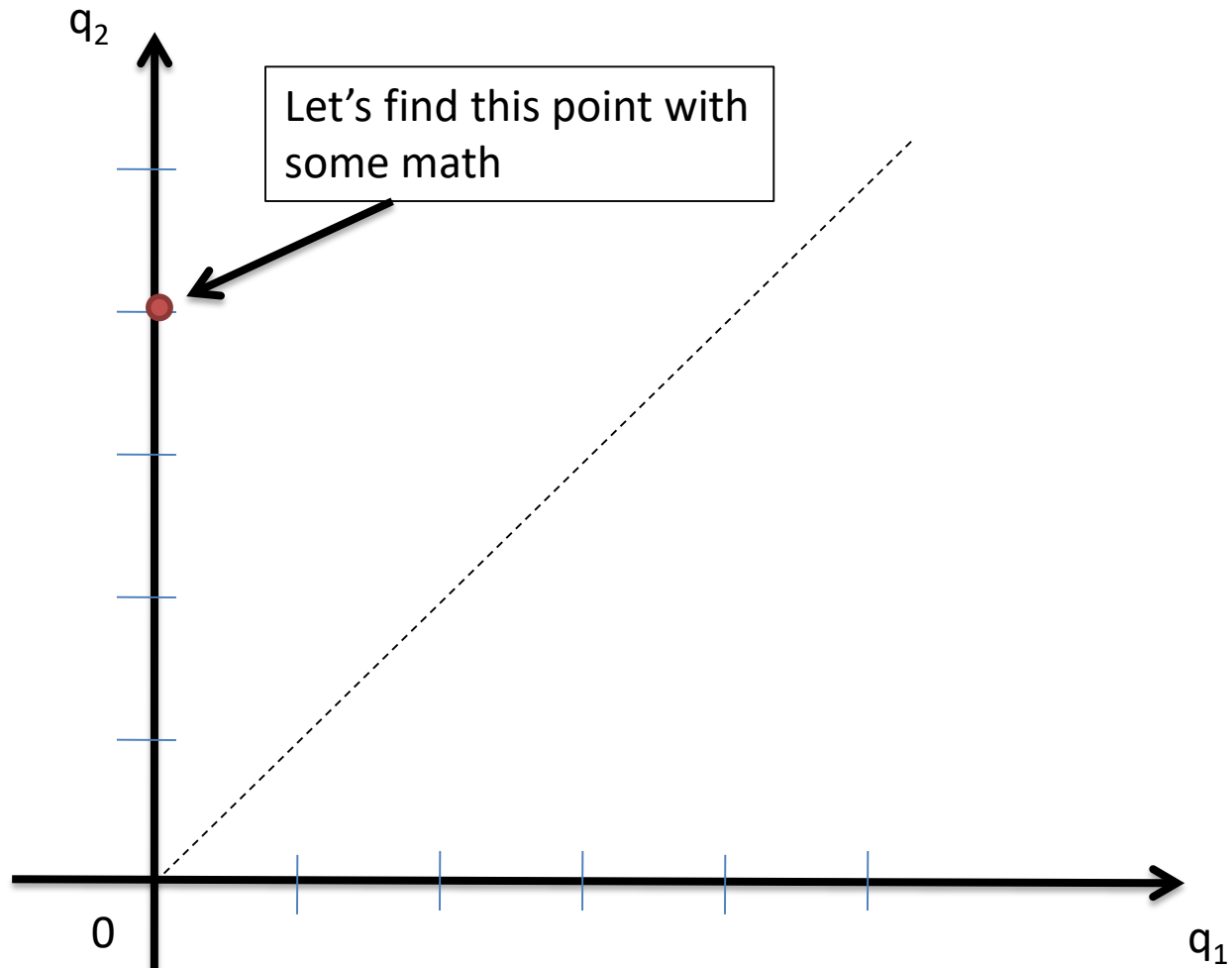- Let's put this quantity on the plot…

Economics Discussion

- What is this quantity we just found called?
- It is called the **_monopoly quantity_**

- When Firm 2 does not produce, then Firm 1 is a monopolist on the market

- Let's see economic representation!

Monopoly

p

a   Demand curve
     Slope: -b

     Marginal revenue
     Slope: -2b

                                    Marginal cost
c

0        $\frac{a-c}{2b}$ q_1

MONOPOLY

When marginal revenue = Marginal cost

How much quantity should Firm 2 produce for Firm 1 not to produce at all?

➔ *q₂ such as q₁ would be the best response for Firm 1*

How much quantity should Firm 2 produce for Firm 1 not to produce at all?

➔ *q_2 such as q_1 would be the best response for Firm 1*

When BR for Firm 1 is $q_1 = 0$?

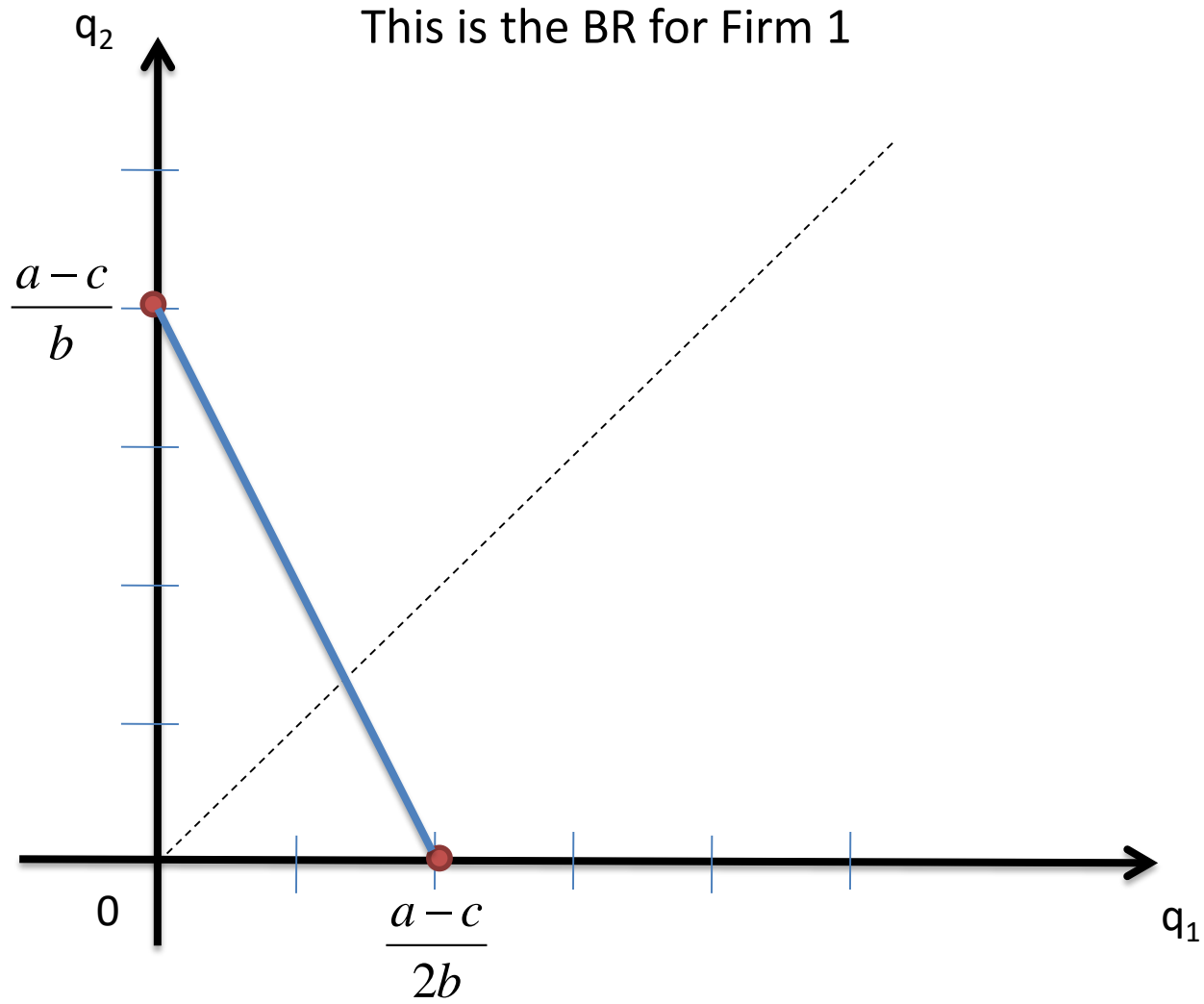- We simply take the BR expression and set it to zero

$$\hat{q}_1 = BR_1(q_2) = \frac{a-c}{2b} - \frac{q_2}{2}$$

$$BR_1(q_2) = 0$$

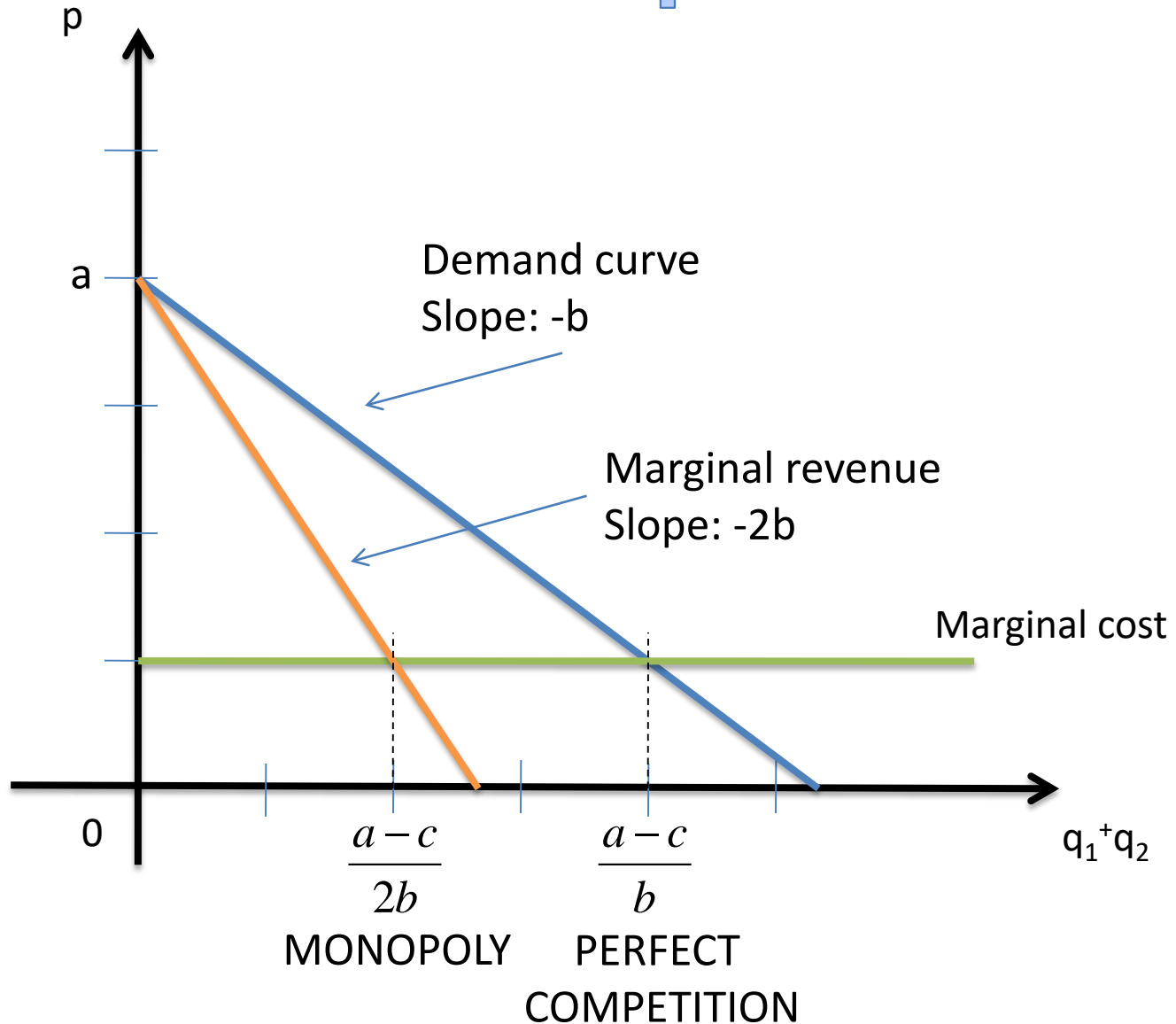$$\Rightarrow q_{PC} = \frac{a-c}{b}$$

- Let's put this quantity on the plot…

This is the BR for Firm 1

q_2

$\dfrac{a-c}{b}$

0

$\dfrac{a-c}{2b}$

q_1

Economics Interpretation

- What is this quantity we just found called?
- It is called the **_perfect competition quantity_**

- When Firm 2 produces this quantity, the best response for Firm 1 is not to produce
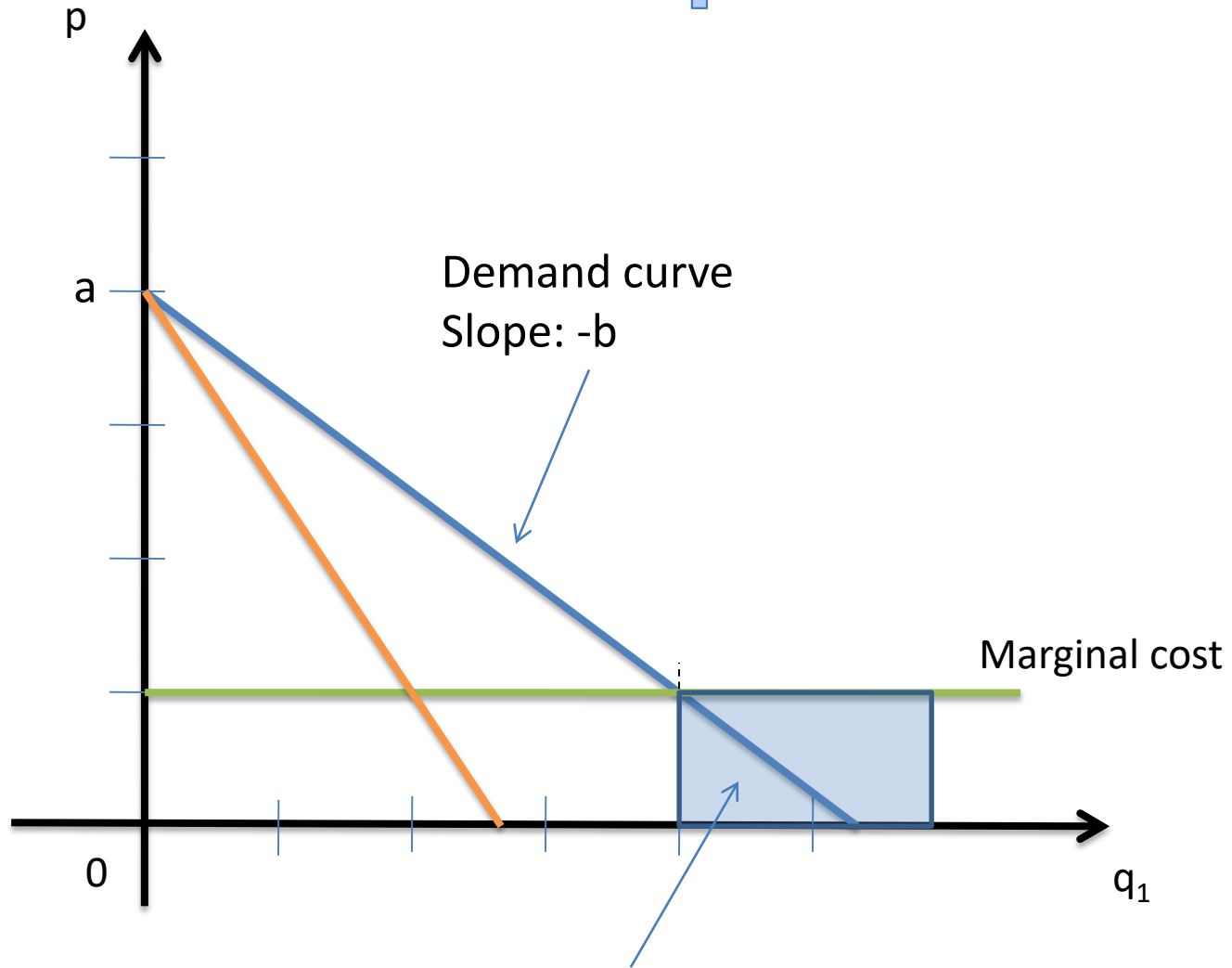 - Why?

Perfect Competition

p

a

Demand curve
Slope: -b

Marginal revenue
Slope: -2b

Marginal cost

0

$\dfrac{a-c}{2b}$

MONOPOLY

$\dfrac{a-c}{b}$

PERFECT
COMPETITION

$q_1{}^+q_2$

When demand = Marginal cost

Perfect Competition

p

a

Demand curve
Slope: $-b$

Marginal cost

0

q_1
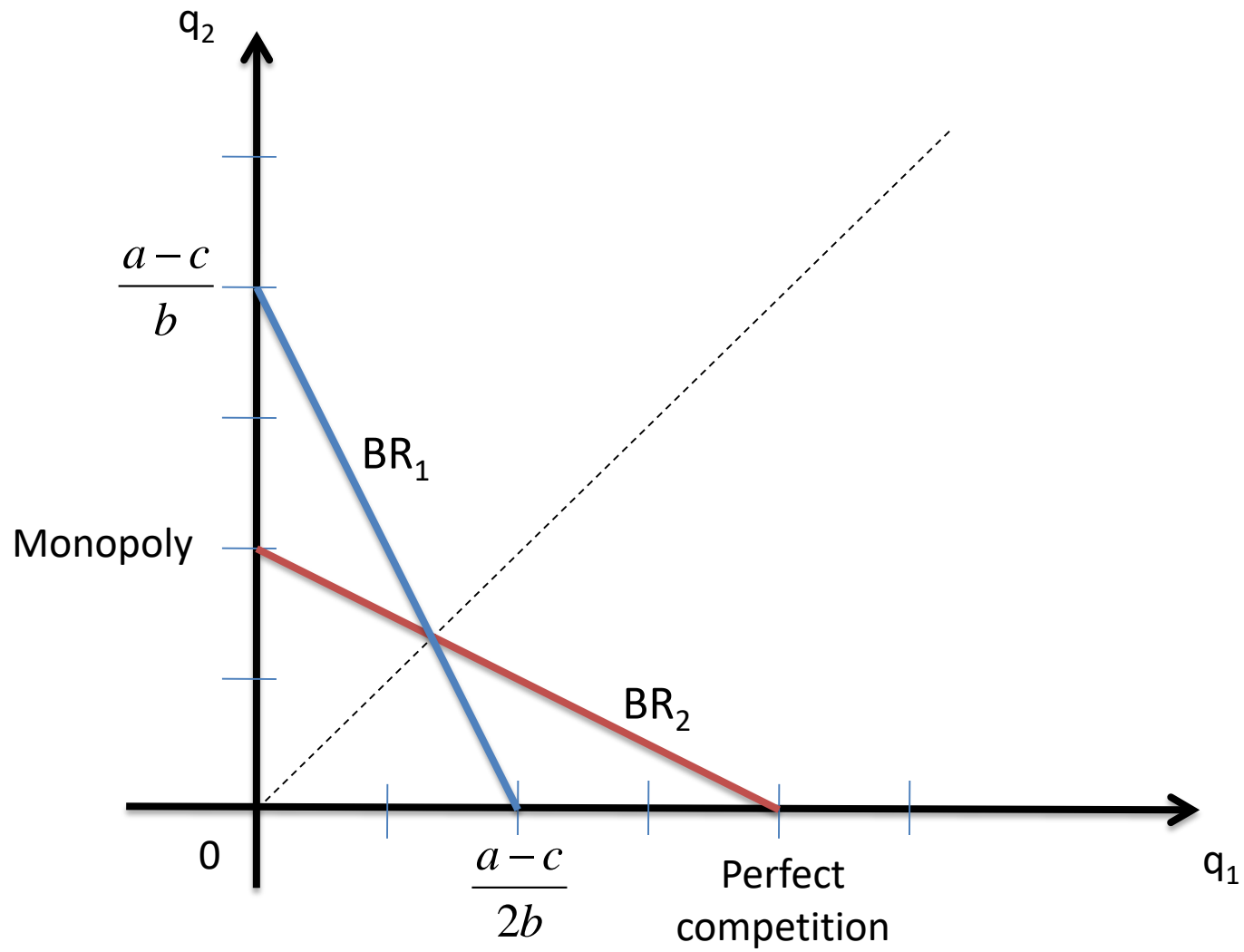
If Firm 1 would produce more, the
selling price would not cover her costs

When demand = Marginal cost

The game is symmetric

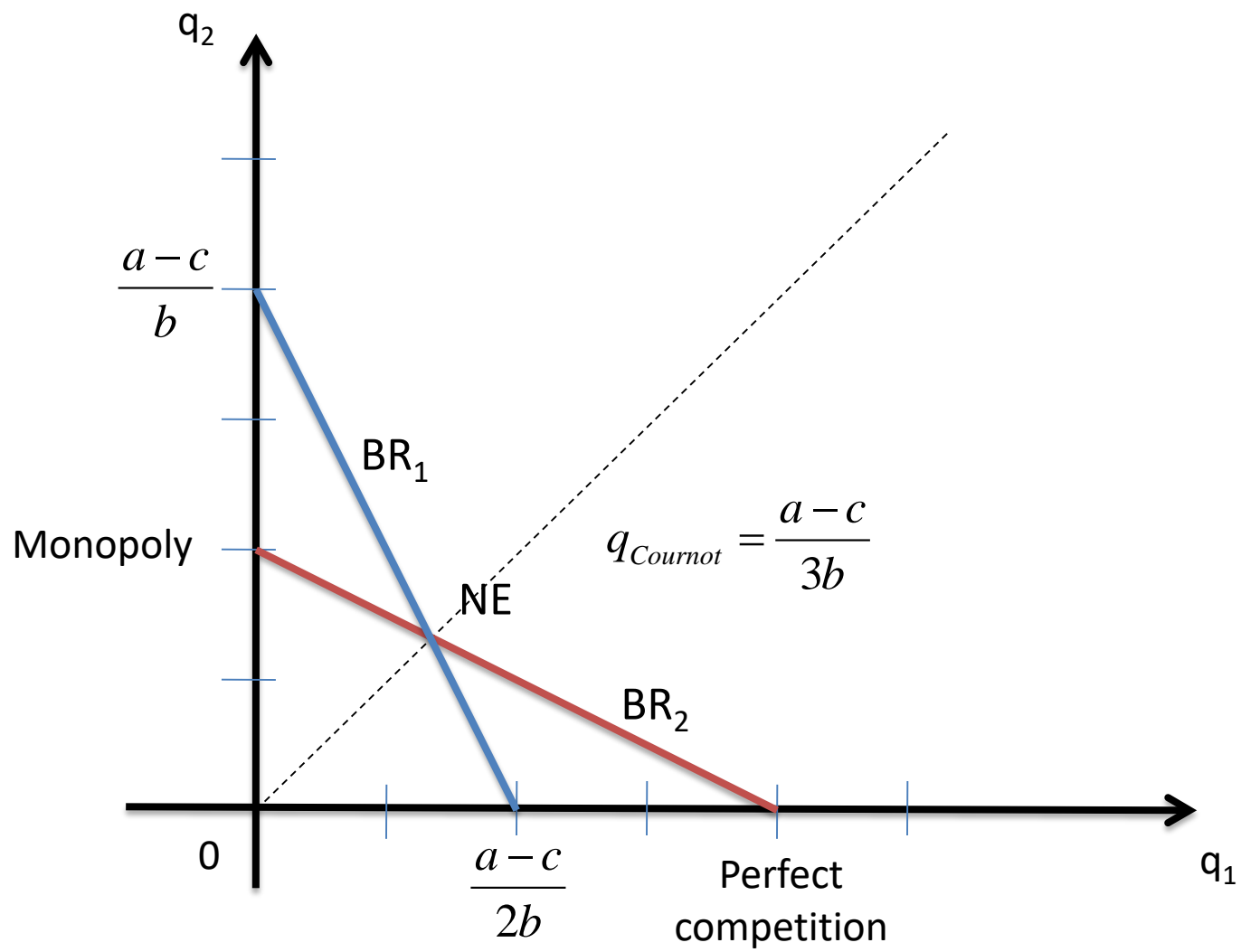# What is the NE of the Cournot Duopoly?

- Graphically we've seen it, formally we have:

$$BR_1(q_2) = BR_2(q_1) \Rightarrow q_1^* = q_2^*$$

$$\frac{a-c}{2b} - \frac{\hat{q}_2}{2} = \hat{q}_2$$

$$\Rightarrow q_1^* = q_2^* = \frac{a-c}{3b}$$

- We have found the *COURNOT QUANTITY*

q_2

$\dfrac{a-c}{b}$

BR$_1$

Monopoly

NE

$q_{Cournot} = \dfrac{a-c}{3b}$

BR$_2$

0

$\dfrac{a-c}{2b}$

Perfect
competition

q_1

Cournot Duopoly: Observations

- This game is different from the games we've seen so far:
  - Partnership game
  - Investment game
- In those games, the more the other player would "do" the more I would do

➔ **Strategic Complements**

Cournot Duopoly: Observations

- In this game, the more the other player do, the less I would do

➔ This is a game of ***Strategic Substitutes***

    – Note: of course the goods were substitutes

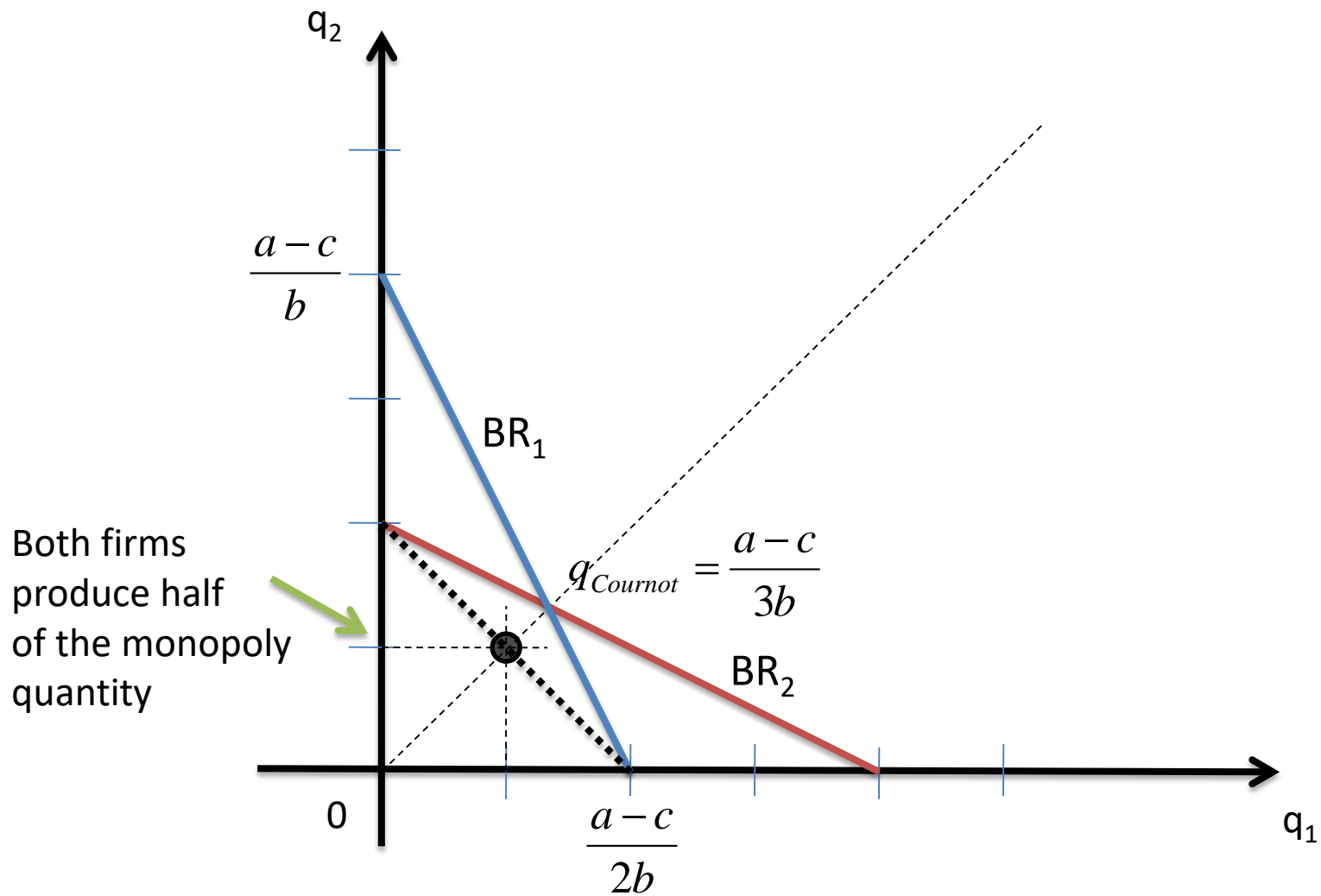   – We're talking about strategies here

Cournot Duopoly: What about the Market?

- Let's now take the perspective of the market and not of a single player

- What about the total industry profits?

  – Are they maximized?

  – Where on the plot we drew before, are industry profits maximized?

$$q_2$$

$$\frac{a-c}{b}$$

BR$_1$

Industry profits are maximized

$$q_{Cournot} = \frac{a-c}{3b}$$

BR$_2$

$$0$$

$$\frac{a-c}{2b}$$

$$q_1$$
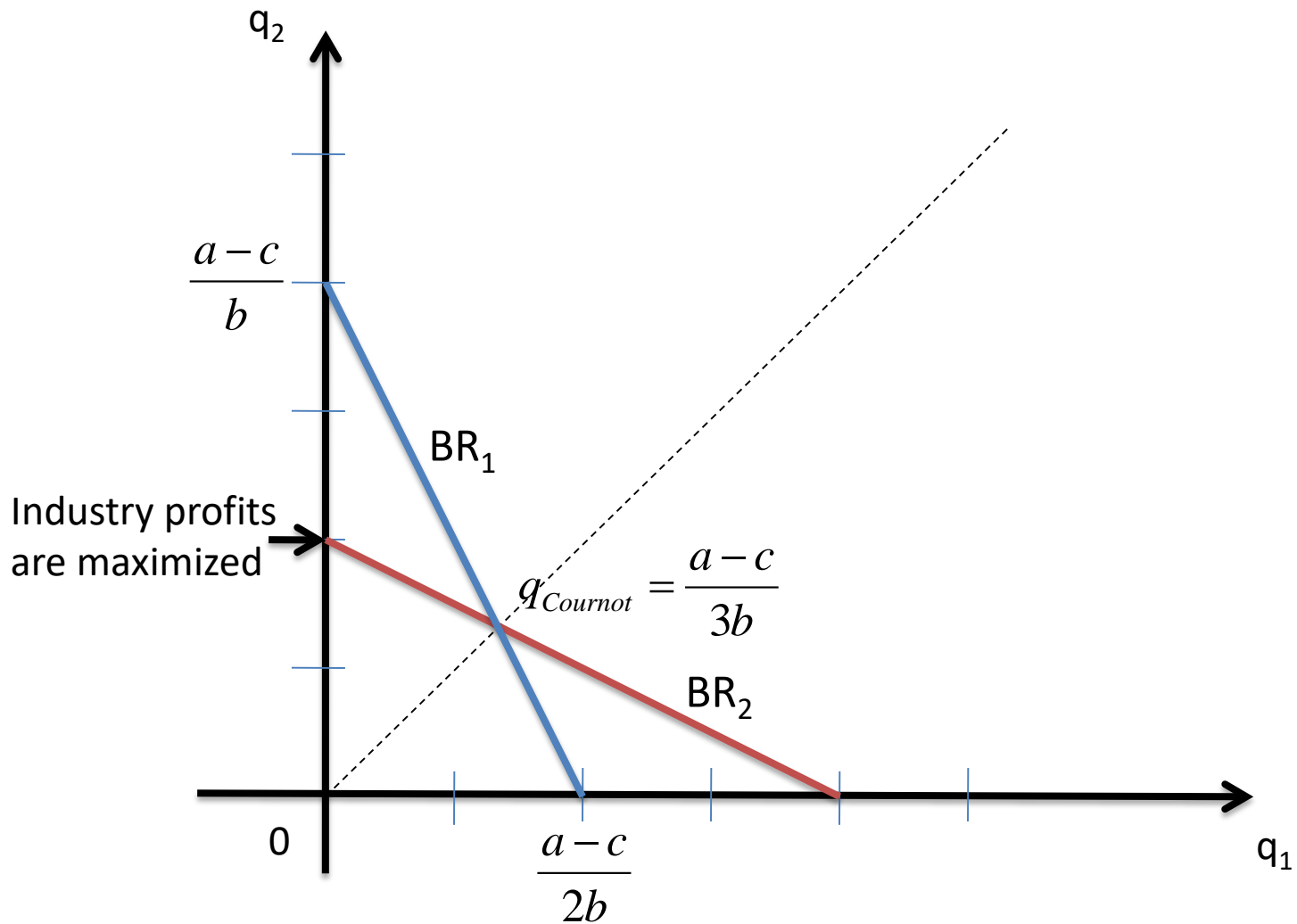
Where else are industry profits maximized?

q_2

$\dfrac{a-c}{b}$

BR$_1$

Both firms
produce half
of the monopoly
quantity

$q_{Cournot} = \dfrac{a-c}{3b}$

BR$_2$

0

$\dfrac{a-c}{2b}$

q_1

Cartels, Agreements …

- How could Firm 1 and Firm 2 set an agreement so as to profit more from the market?

- E.g.: they could decide both to produce half of the monopoly quantity and they would earn more

  – Can you see this on the previous plots?

Cartels, Agreements …

- What is wrong with this agreement?

- What is the BR for a player? Can you see on the graph where such an agreement would end up?

- Is there anything else wrong in this reasoning?
  - What happened to the production quantities?
  - The market is not fully exploited
  - So?

Cournot Duopoly: Final Word

- How do quantities and prices we've encountered so far compare?

| Perfect Competition | Cournot Quantity | Monopoly | PRICES |

$$\frac{a-c}{b} \qquad \frac{2(a-c)}{3b} \qquad \frac{a-c}{2b}$$

| Perfect Competition | Cournot Quantity | Monopoly | QUANTITIES |