Lecture 7

Repeated Games

1

Outline of Lecture: I

- Description and analysis of finitely repeated games.
- Example of a finitely repeated game with a unique equilibrium
- A general theorem on finitely repeated games.

2

Outline of Lecture: II

- A formula for computing discounted payoffs in repeated games.
- A description of infinitely repeated games.
- Examples of strategies in infinitely repeated games.
- How to support “better” equilibria in infinitely repeated games.
- Application to pricing games.

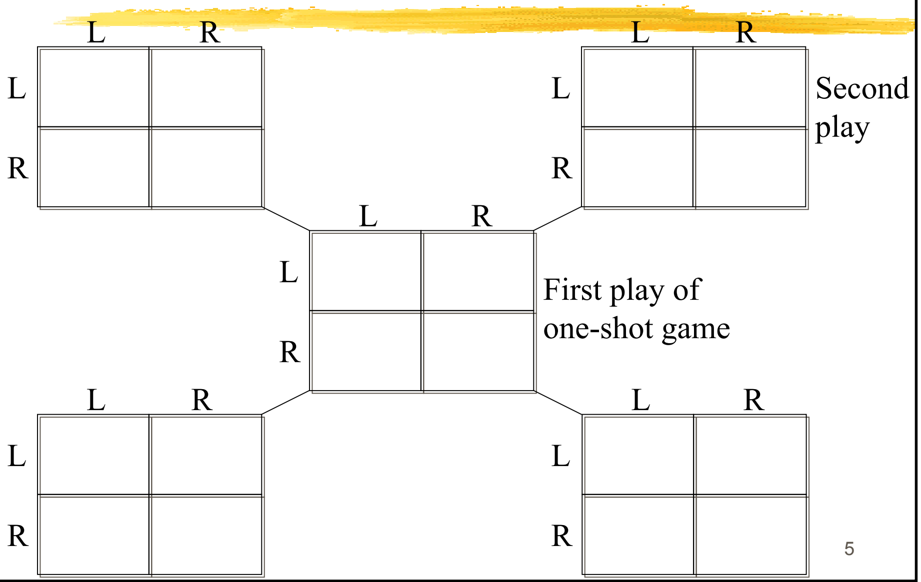
3

Strategies and payoffs for games played twice

- Finitely repeated games
- Discounted utility
- Complete plans of play for 2×2 games played twice
- Trigger strategies

4

A 2×2 game played twice



5

Strategies for playing a 2×2 game twice

Strategy No.	Round 1	After (L, L)	After (L, R)	After (R, L)	After (R, R)
1	L	L	L	L	L
2	L	L	L	L	R
3	L	L	L	R	L
4	L	L	L	R	R
…	…	…	…	…	…
…	…	…	…	…	…
17	R	L	L	L	L
18	R	L	L	L	R
…	…	…	…	…	…
…	…	…	…	…	…
31	R	R	R	R	L
32	R	R	R	R	R

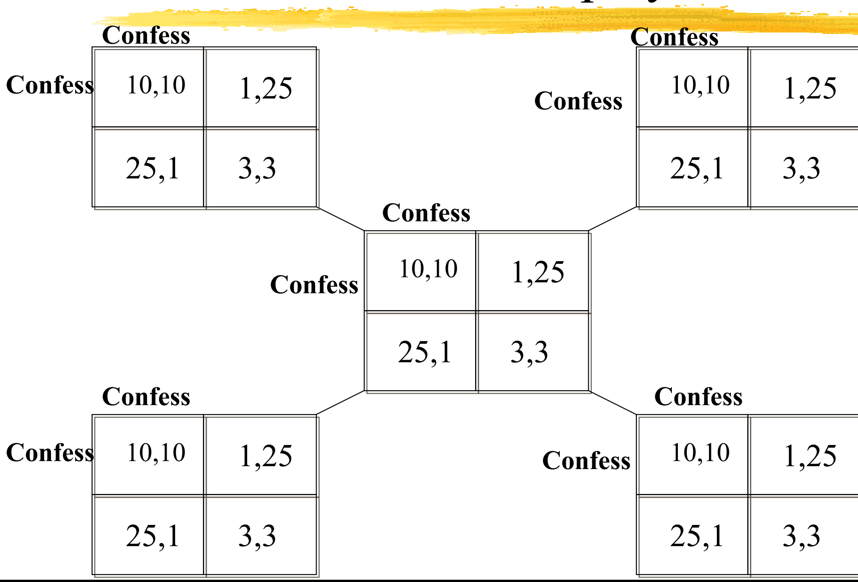
6

Repeated games with a single equilibrium in the “stage game”.

- A repeated game is just an extensive form game.
- Selten’s theorem on unique subgame perfect equilibria
- Repetition by itself does not solve a credibility problem

7

Prisoner’s Dilemma, played twice



8

Prisoner’s Dilemma, backward induction (second play): Pay-off matrix

Player 1 \ Player 2	Confess	Deny
Confess	10,10	1,25
Deny	25,1	3,3

9

Prisoner’s Dilemma, backward induction (second play): Player 1’s strategy

Player 1 \ Player 2	Confess	Deny
Confess	<u>10</u>,10	<u>1</u>,25
Deny	**25,1**	**3,3**

10

Prisoner's Dilemma, backward induction (second play): Player 2's strategy

Player 1 \ Player 2	Confess	Deny
Confess	10,10	1,25
Deny	25,1	3,3

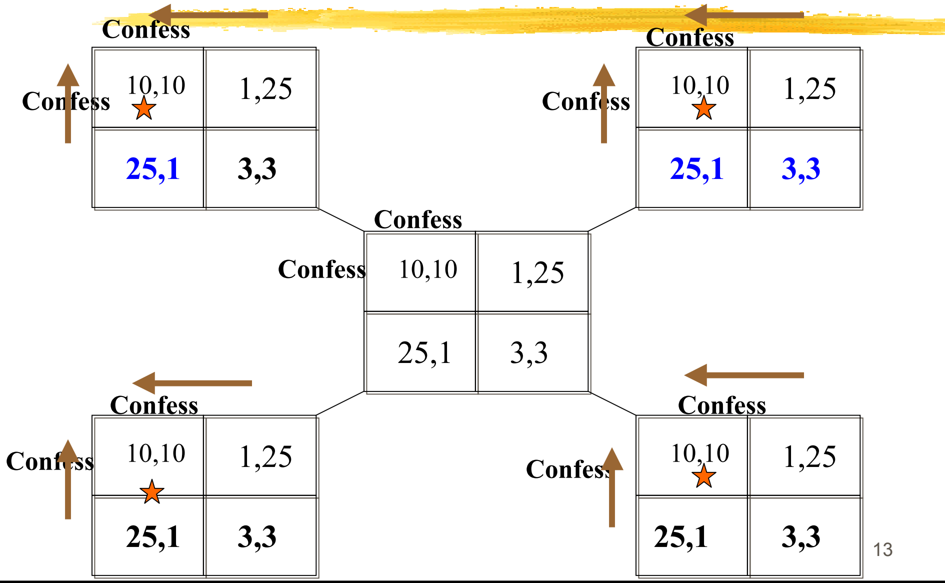
11

Prisoner's Dilemma, second play, by backward induction

Player 1 \ Player 2	Confess	Deny
Confess	10,10	1,25
Deny	**25,1**	**3,3**

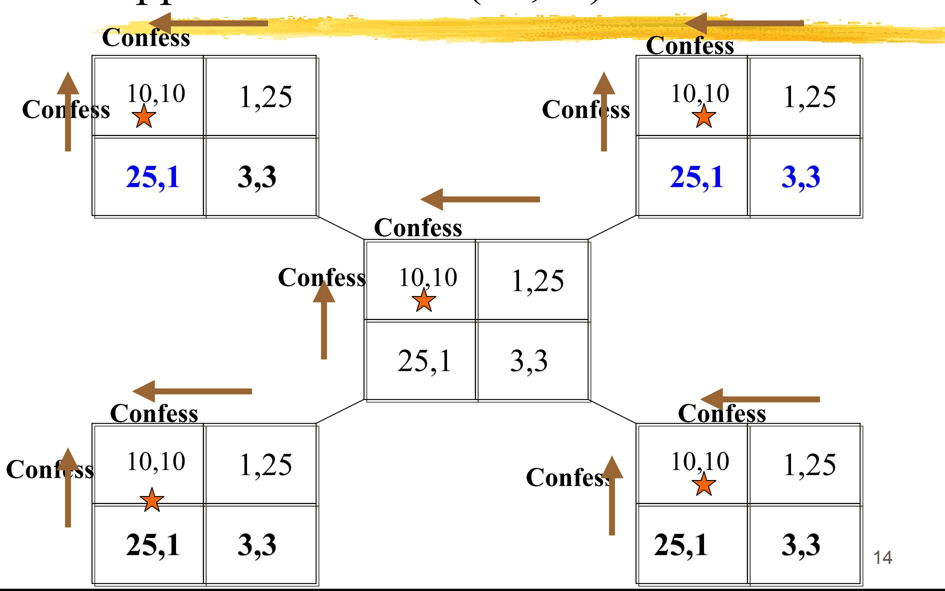
12

Prisoner’s Dilemma, first play: Get (10,10) in second play regardless of first



13

Prisoner’s Dilemma, played twice: Can’t support better than (10,10) in first



14

Selten's Theorem

If a game with a unique equilibrium is played finitely many times, its solution is that equilibrium played each and every time

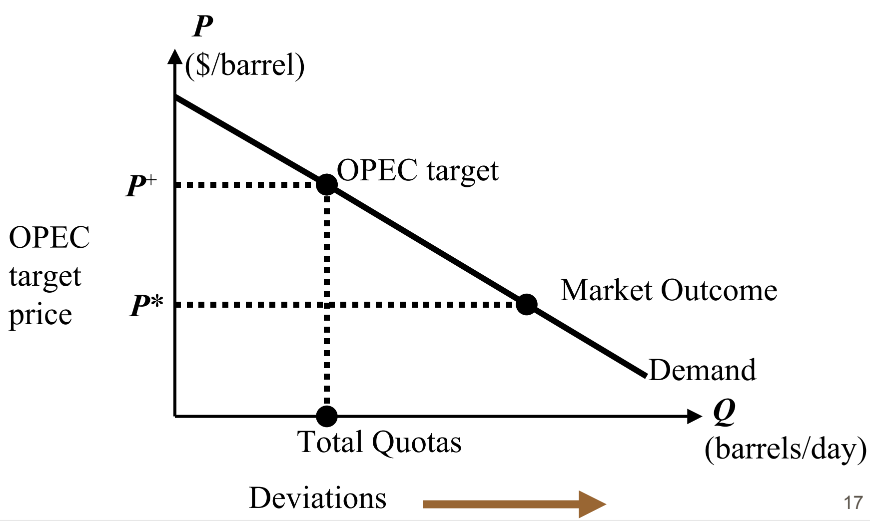
15

OPEC drops Quotas

- OPEC's quota system, 1973-1993
- The attempt to improve upon a one-shot Cournot equilibrium
- Finiteness of a resource and finiteness of a game

16

OPEC quotas



17

Achieving Higher Payoffs in Infinitely Repeated Games.

- Selten's theorem suggests that if the single period game has a unique equilibrium, then the repetition of that equilibrium is all that will occur in repeated games.
- This conclusion is only true for games that are repeated "finitely".

18

Infinitely repeated games: strategies and payoffs

- The "as if" interpretation of infinite repetition.
- Complete plans for infinite play.
- Discounting infinite series of payoffs.

19

Evaluating payoffs for an infinitely repeated game

Total payoff for player 1, $u_1 = \sum d^{\,t}\, u_1(t)$
t goes from 0 to ∞; d is the discount factor where $d < 1$

Note, if u is profits, $d=1/(1+r)$ where r is the interest rate or the internal rate of return for the firm.

RULE: When $u_1(t) = 1$ for all t, $u_1 = 1 + d + d^2 + d^3 + \ldots$

For $0 < d < 1$, the series sums to

$$u_1 = 1/(1 - d) = 1 + d + d^2 + d^3 + \ldots$$

When $u_1(t) = k$ for all t, $u_1 = k/(1 - d)$

20

Evaluating payoffs for an infinitely repeated game: Extension

An important modification: Suppose that in every period, there is a probability, 1-*p,* that the game ends.

Now, total payoff for player 1, $u_1 = \Sigma(dp)^t u_1(t)$
t goes from 0 to ∞; d is still the discount factor where $d < 1$ and $p<1$ so $dp<1$ as well.

If we call R the "effective rate of return", then

$1/(1+R)=dp$, or $R=(1-dp)/dp$.

See Dixit and Skeath, Chapter 8 and Appendix to Chapter 8.

21

Different Types of Strategies for repeated games.

- The repeated "One-shot" strategy:
  - if a profile of strategies form an equilibrium in the one-shot game, then any repetition of these strategies form an equilibrium in the repeated game.
- Thus, in an infinitely repeated Prisoner's Dilemma, Confess (or Do not Collude) in every period is an equilibrium of the repeated game.

22

Different Types of Strategies.

- The "Grim Trigger Strategy":
  - in the prisoner's dilemma game, a promise to Deny forever as long as the rival Denies supported by a threat to Confess forever if the rival Confesses can sometimes be an equilibrium.

23

Different Types of Strategies.

- The "Tit for Tat Strategy":
  - in the prisoner's dilemma game, a promise to Deny as long as the rival Denies supported by a threat to Confess for a period if the rival Confesses and then revert to Deny (as long as the deviant rival Denied in the punishment period) can sometimes be an equilibrium.
- Dixit and Skeath focus on this strategy, I look at the Grim Trigger Strategy.

24

Duopolists Play the Prisoner's Dilemma

- Consider the game where two firms choose high (cooperative) prices or low (deviant) prices in each period.
- If both choose High, they get profits of *3* each. If one chooses High and the other low, the High gets *0* and the low gets *4.*
- If both choose Low, they get *2* each.
- They play this game infinitely and *dp=.25.*

25

Prisoner's Dilemma: Duopoly Version

Firm 1 \ Firm 2	Cooperate	Deviate
Cooperate	3, 3	0, **4**
Deviate	4, 0	2, **2**

26

Duopolists Play the Prisoner's Dilemma

- We know that Deviate (Low) forever is an equilibrium. This pays each firm
  - $\Pi_1 = \sum (dp)^t u_1(t)$
  - $= \sum (.25)^t 2 = 2*(1/(1-.25)) = 8/3$

27

Duopolists Play the Prisoner's Dilemma

- If they play the grim trigger strategy,
  - $\Pi_1 = \sum (.25)^t 3 = 3*(1/(1-.25)) = 4.$
- But won't one Firm deviate?
- If I deviate, I get *4* right away, and then I go to the deviate equilibrium from next period onward which gives me 8/3.
- This strategy would give me
  - $4+(.25)*(8/3)=4+2/3$

28

Duopolists Play the Prisoner's Dilemma

- Since this is bigger than what I get from Cooperate forever, I should deviate.
- But what if *dp=.75?*
- Deviate forever gives me *2*(1/(1-.75))=8*
- Cooperate forever gives me *3*(1/(1-.75))=12*
- Now if I deviate, I get *4* plus a payoff of *8* from then on, but *4+.75*8=10<12.*
- I do not want to "Trigger" a price war!

Infinitely repeated Cournot market games

- A one-shot Cournot equilibrium, repeated infinitely often, is a subgame perfect equilibrium path
- Better paying equilibria than one-shot Cournot
- Monopoly-like equilibria when firms attach enough importance to the future
- A Folk Theorem for infinitely repeated games
- Infinitely repeated Bertrand market games

When Can Collusion Occur in Infinitely Repeated Games?

- Consider the General Version of the Prisoner's Dilemma Game.

31

Prisoner's Dilemma: Duopoly Version

Firm 1 \ Firm 2	Cooperate	Deviate
Cooperate	C,C	H,L
Deviate	L,H	D,D

32

Collusion In Prisoner's Dilemma Games

- Deviate forever gives $(1/(1-dp))D$.
- Cooperate forever gives $(1/(1-dp))C$.
- But a single period deviation from Cooperate forever gives $H+(dp/(1-dp))D$.
- We need to make sure that a firm does not want to deviate, or that $H+(dp/(1-dp))D< (1/(1-dp))C$.

33

Collusion In Prisoner's Dilemma Games

- $H+(dp/(1-dp))D< (1/(1-dp))C$.
- If and only if $(1-dp)*H+dp*D<C$. **
- Presumably $H>C$ and $D<C$ (or else this is not an interesting problem).
- Therefore, (**) is false for $dp=0$ and true for $dp=1$.

34

Collusion In Prisoner's Dilemma Games:Conclusions

- If the future matters a lot, (*dp* is close to one), then collusion is easier to support.
- If interest rate is very high (*d* close to zero) or the probability of end game very high (*p* close to zero), collusion is hard.
- If *D* is high, collusion is hard to attain.
- If *H* is high, collusion is hard to attain.
- If C is low, collusion is hard to attain.

35

Collusion In Prisoner's Dilemma Games:Conclusions

- Final Remarks: Observe why this suggests that it is easier to have collusion in Bertrand pricing games than in Cournot quantity games.
- In Bertrand pricing games, $D=0$. In Cournot games, $D>0$.
- Ironic conclusion: Collusion is easier to support in repeated games which have MORE competitive stage games!

36