بسم الله الرحمن الرحیم

Algorithmic Game Theory

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دانشگاه کاشان

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The Investment Game

- **The players:** you
- **The strategies:** each of you chose between investing nothing in a class project ($0) or invest ($10)
- **Payoffs:**
  - If you don't invest your payoff is $0
  - If you invest you're going to make a net profit of $5. This however requires more than 90% of the class to invest. Otherwise, you loose $10

- No communication!

The Investment Game

- What did you do?
  - Who invested?
  - Who did not invest?

- What is the NE in this game?

The Investment Game

- There are **2 NE** in this game
  - All invest
  - None invest
- Let's check:
  - If everyone invests, none would have **regrets**, and indeed the BR would be to invest
  - If nobody invests, then the BR would be to not invest

The Investment Game

- How did we find the NE?
    1. We could have checked rigorously what everyone's best response would be in each case
    2. We can just guess and check!
- Actually, **checking is easy**, **guessing is hard**
    - What does this remind you? Can you tell anything about the complexity of finding a NE?
- Note: checking is easy when you have many players but few strategies

The Investment Game

- What did you do in this game?
- Players: you
- Strategies: Not Invest ($0) or Invest $10
- Payoffs:
  - If no invest → $0
  - If invest $10 → $\begin{cases} \text{\$5 net profit if} \geq 90\% \text{ invest} \\ \text{-\$10 net profit if} < 90\% \text{ invest} \end{cases}$

The Investment Game

- I want you to play the game again, no communication please!!
- What did you do?
  - Who did invest?
  - Who did not invest?
- I want you to play again…

- Where are we going to?

The Investment Game

- We are **heading toward an equilibrium**

➔There are certain cases in which playing converges in a natural sense to an equilibrium

- But we're going towards only one of the two equilibria!
- Is any of these two NE better than the other?

The Investment Game

- Clearly, everyone investing is **a better NE**

- Nevertheless we were converging very rapidly to a bad equilibrium, where no one gets anything, in which all money is left on the table!

- How can that be?

The Investment Game

- Formally, we say that one NE ***pareto dominates*** the other

- Why did we end up going to a bad equilibrium?

The Investment Game

- Remember when we started playing?
  - We were more or less 50 % investing
- The starting point was already bad for the people who invested for them to lose confidence
- Then we just tumbled down

- What would have happened if we started with 95% of the class investing?

The Investment Game

- Note also the process of converging towards the “bad” equilibrium
- It coincides with the **idea of a self-fulfilling** prediction

➔ Provided you think other people are not going to invest, you are not going to invest

The Investment Game

- Does this game belong to the Prisoners' Dilemma family?
- Was there any strictly dominated strategy?

<u>Coordination Game</u>

The Investment Game

- Why is this a coordination game?
- We'd like everyone to coordinate their actions and <u>invest</u>

- There are a lot of **coordination problems in real life**
  - **Examples?**

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Coordination Games

A Trusted Third Party (TTP) could drive the crowd to a better equilibrium!

Coordination Games

- Let's try to compare this to the Prisoners' Dilemma
- In that case, even the presence of a TTP would not help, because the strategy β would be still dominated and people would chose α no matter!

- So why a TTP works in coordination games?

Happy New Year!

Coordination Games

- In coordination games ***communication helps***!
- Indeed, a TTP is not going to impose players to adopt a strictly dominated strategy, but is just leading the crowd towards a better NE point
- In the **PD game**, you **need to change the payoff** of the game to move people's actions

Coordination Game

		Player 2	
		l	r
Player 1	U	1,1	0,0
	D	0,0	1,1

- Clearly in this game what matters is coordination
- If you played this game, it is quite likely you would end-up being uncoordinated
- A little bit of leadership would make sure you coordinate

Coordination Game

Strategic Complements

- **Investment game:** the more people invest the more likely you are to invest
- **Partnership game:** the more the other person does, the more likely for me to do more

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The Battle of the Sexes

Going to the Movies

		Player 2: M	N	P
Player 1	M	2,1	0,0	0,-1
	N	0,0	1,2	0,-1
	P	-1,0	-1,0	-2,-2

- The "Going to the Movies" game
- A pair is meeting at the movies and have to decide which movies to watch
- How would you play this game?

Going to the Movies

		Player 2		
		M	N	P
Player 1	M	2,1	0,0	0,-1
	N	0,0	1,2	0,-1
	P	-1,0	-1,0	-2,-2

- Are there any dominated strategies?
- If so, how is the game transformed?

Going to the Movies

		Player 2	
		M	N
Player 1	M	2,1	0,0
	N	0,0	1,2

- How do we play this game?
- Let's try it out: form a pair, write down what you would do, **without showing**!!

Going to the Movies

		Player 2	
		M	**N**
Player 1	**M**	2,1	0,0
	N	0,0	1,2

- Which kind of game is this?
- Does ***communication*** help here?

- Let's find the Nash Equilibrium of this game

Going to the Movies

		Player 2	
		M	**N**
Player 1	**M**	2,1	0,0
	N	0,0	1,2

		Player 2	
		l	r
Player 1	U	1,1	0,0
	D	0,0	1,1

- NE: (M,M) *<u>and</u>* (N,N)
- So it looks like a standard coordination game, with two NE
- What is the trick here?

Coordination Games

- **Pure coordination games:** there is no conflict whether one NE is better than the other
  - E.g.: in the investment game, we all agreed that the NE with everyone investing was a "better" NE

- **General coordination games:** there is a source of conflict as players would agree to coordinate, but one NE is "better" for a player and not for the other
  - E.g.: The Battle of the Sexes

➔ Communication might fail in this case

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Two-person Zero-sum Games

- One of the first games studied
  - most well understood type of game
- Players interest are strictly opposed
  - what one player gains the other loses
  - game matrix has single entry (**i.e., gain to player 1**)
- Intuitive solution concept
  - players maximize gains

Analyzing the Game

- Player 1 maximizes matrix entry, while player 2 minimizes

Player 2

Player 1

	A	B	C	D
A	12	-1	1	0
B	3	1	3	-18
C	5	2	4	3
D	-16	1	2	-1

Strictly dominated strategy (dominated by C)

Strictly dominated strategy (dominated by B)

Solving the Game

- Iterated removal of strictly dominated strategies

Player 2

Player 1

	L	**M**	**R**
T	2	-1	1
B	3	2	3

  - Player 1 cannot remove any strategy (neither T or B dominates the other)
  - Player 2 can remove strategy R (dominated by M)
  - Player 1 can remove strategy T (dominated by B)
  - Player 2 can remove strategy L (dominated by M)
  - Solution: (B, M)
    - payoff of 2

Solving the Game

- Removal of strictly dominated strategies does not always work
- Consider the game

Player 2

Player 1

	A	**B**	**D**
A	12	-1	0
C	5	2	2
D	-16	0	5

- Strictly dominated strategy cannot help!
- Requires another solution concept

Analyzing the Game

Player 2

Player 1

	A	**B**	**D**
A	12	-1	0
C	5	2	2
D	-16	0	5

Outcome (C, B) seems "stable"

m saddle point of game

Saddle Points

- An outcome is a *saddle point* if it is both less than or equal to any value in its row and greater than or equal to any value in its column

- Saddle Point Principle
  - Players should choose outcomes that are saddle points of the game

- Value of the game
  - value of saddle point outcome if it exists

Why Play Saddle Points?

Player 2

Player 1

	A	**B**	**D**
A	12	-1	0
C	5	2	2
D	-16	0	5

- If player 1 believes player 2 will play B
  - player 1 should play best response to B (which is C)
- If player 2 believes player 1 will play C
  - player 2 should play best response to C (which is B)

Solving the Game (min-max algorithm)

	Player 2				
	A	**B**	**C**	**D**	
Player 1 A	4	3	2	5	2
B	-10	2	0	-1	-10
C	7	5	1	3	1
D	0	8	-4	-5	-5
	7	8	2	5	

- choose maximum entry in each column
- choose the minimum among these
- this is the minimax value

- choose minimum entry in each row
- choose the maximum among these
- this is maximin value

if minimax == maximin, then this is the saddle point of game

Multiple Saddle Points

- In general, game can have multiple saddle points

		Player 2				
		A	B	C	D	
Player 1	A	3	2	2	5	2
	B	2	-10	0	-1	-10
	C	5	2	2	3	2
	D	8	0	-4	-5	-5
		8	2	2	5	

- Same payoff in *every* saddle point
  - unique value of the game
- Strategies are interchangeable
  - Example: strategies (A, B) and (C, C) are saddle points
  - Then (A, C) and (C, B) are also saddle points

Games With no Saddle Points

		Player 2		
		A	**B**	**C**
Player 1	**A**	2	0	-1
	B	-5	3	1

- What should players do?
  - resort to randomness to select strategies

Wait we will get back to this!

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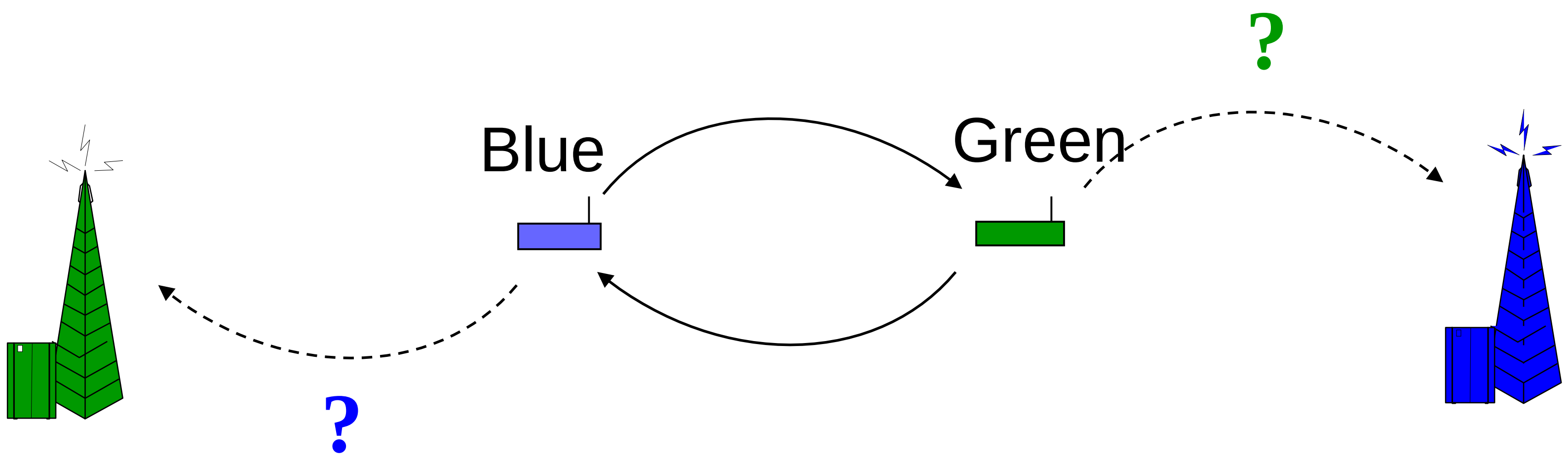
The Forwarder's Dilemma



Forwarder Game

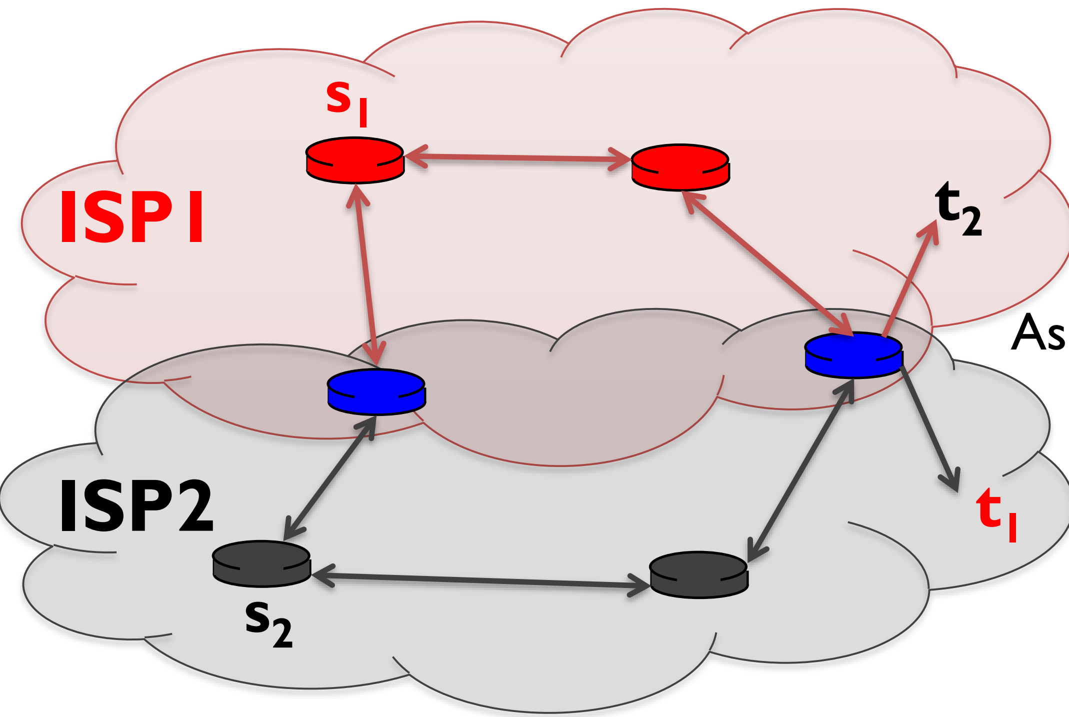
- users controlling the devices are ***rational*** = try to maximize their benefit

Blue \ Green	Forward	Drop
Forward	(1-c, 1-c)	(-c, 1)
Drop	(1, -c)	(0, 0)

- Reward for packet reaching the destination: 1
- Cost of packet forwarding: c ($0 < c << 1$)

(Drop , Drop) is NE

ISP Routing Games

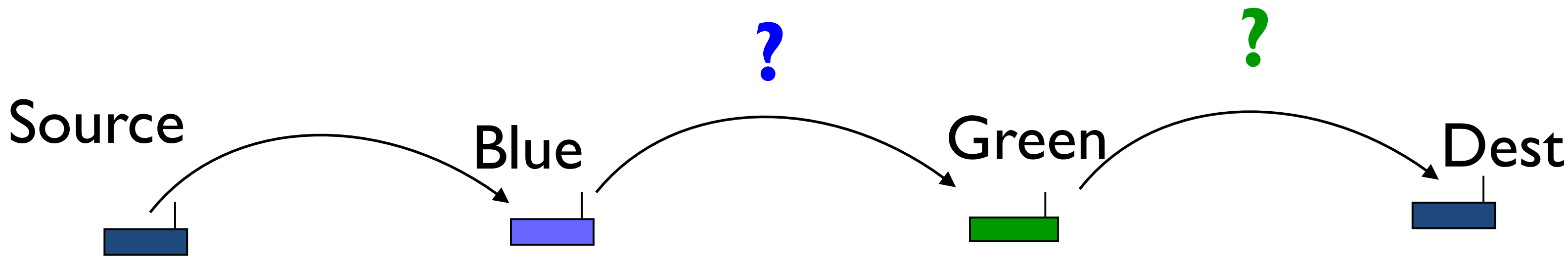


Assume that the unit cost along a link is 1

ISP1 \ ISP2	Hot Potato	Cooperate
Hot Potato	$\boxed{(-5, -5)}$	(-2,-6)
Cooperate	(-6, -2)	(-3, -3)

(Hot Potato, Hot Potato) is NE

The Joint Packet Forwarding Game

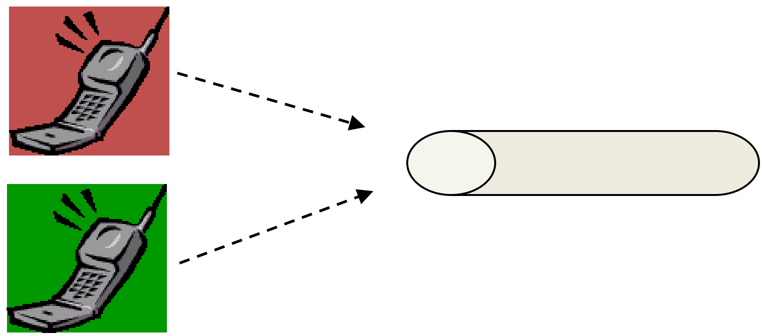


- Reward for packet reaching the destination: 1
- Cost of packet forwarding: c (0 < c << 1)

Blue \ Green	Forward	Drop
Forward	(1-c, 1-c)	(-c, 0)
Drop	(0, 0)	(0, 0)

(Forward , Forward) and (Drop , Drop) are NE

The Multiple Access game



Time-division channel

Reward for successful transmission: 1

Cost of transmission: c
($0 < c << 1$)

Blue \ Green	Quiet	Transmit
Quiet	(0, 0)	$\boxed{(0, 1-c)}$
Transmit	$\boxed{(1-c, 0)}$	(-c, -c)

There is no strictly dominating strategy

(Transmit , Quiet) and (Quiet , Transmit) are NE