Homework Assignment 1: Game Theory and Strategy

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Due Date: August 4, 08
Full Score: 100 pts

Late submissions will not be accepted. Homework must be typed. Mathematical expressions, graphs and tables may be written by hand. Show all your work and make sure all your diagrams are fully labeled. Please remember to write your name on your homework.

Q1. (60 points) Find all (pure strategy) Nash equilibria in the following games. If there is no pure strategy Nash equilibrium, state that. In each Nash equilibrium, how much payoff does each player obtain? Specify the numbers.

(1)

	Quiet	Fink
Quiet	(5, 5)	(8, 0)
Fink	(0, 8)	(2, 2)

(2)

	Quiet	Fink
Quiet	(-5, -5)	(8, 0)
Fink	(0, 8)	(2, 2)

(3)

	Quiet	Fink
Quiet	(5, 5)	(0, 8)
Fink	(8, 0)	(2, 2)

(4)

	Bach	Chopin
Bach	(3, 2)	(0, 0)
Chopin	(0, 0)	(2, 3)

(5)

	Bach	Chopin
Bach	(3, 2)	(0, 0)
Chopin	(0, 0)	(0, 0)

(6)

	Head	Tail
Head	(2, -2)	(-2, 2)
Tail	(-2, 2)	(2, -2)

(7)

	L	C	R
T	(9, 9)	(2, 2)	(4, 1)
M	(2, 2)	(5, 5)	(3, 6)
B	(1, 4)	(6, 3)	(0, 0)

(8)

	L	C	R
T	(1, 1)	(2, 2)	(4, 1)
M	(2, 2)	(5, 5)	(3, 6)
B	(1, 4)	(6, 3)	(0, 0)

(9)

	L	C	R
T	(1, -1)	(-1, -1)	(-1, 1)
M	(-1, -1)	(-1, 1)	(1, -1)
B	(-1, -1)	(1, -1)	(-1, 1)

(10)

	L	C	R
T	(1, -1)	(-1, -1)	(-1, 1)
M	(-1, -1)	(-1, 1)	(1, -1)
B	(0, 0)	(0, 0)	(0, 0)

Q2. (10 points) [Exercise 27.1 P. 27]
Each of two players has two possible actions, *Quiet* and *Fink*. Each action pair results in the players' receiving amounts of money equal to the numbers corresponding to that action pair in the following figure:

	Quiet	Fink
Quiet	(2, 2)	(0, 3)
Fink	(3, 0)	(1, 1)

The players are not selfish; rather, the preferences of each player i are represented by the payoff function $m_i(a) + \alpha\, m_j(a)$, where $m_i(a)$ is the amount of money received by player i when the action profile is a, j is the other player, and α is a given nonnegative number. Player 1's payoff to the action pair (Quiet, Quiet), for example, is $2 + 2\alpha$.

a. Formulate a strategic game that models this situation in the case $\alpha = 1$. Is this game the Prisoner's Dilemma?
b. Find the range of values of α for which the resulting game is the Prisoner's Dilemma. For values of α for which the game is not the Prisoner's Dilemma, find the Nash equilibria.

Q3. (10 points) [Exercise 37.1 p. 37]
a. Find the players' best response functions in the following games and verify the Nash equilibria of each game.

a-1.

	Quiet	Fink
Quiet	(2, 2)	(0, 3)
Fink	(3, 0)	(1, 1)

a-2.

	Bach	Chopin
Bach	(2, 1)	(0, 0)
Chopin	(0, 0)	(1, 2)

a-3.

	Head	Tail
Head	(1, -1)	(-1, 1)
Tail	(-1, 1)	(1, -1)

b. Find the Nash equilibria of the game in the following game by finding the players' best response functions.

	L	C	R
T	(2, 2)	(1, 3)	(0, 1)
M	(3, 1)	(0, 0)	(0, 0)
B	(1, 0)	(0, 0)	(0, 0)

Q4. (10 points) [Exercise 42.1 P. 42]
Find the Nash equilibria of the two-player strategic game in which each player's set of actions is the set of nonnegative numbers and the players' payoff functions are:

$$u_1(a_1, a_2) = a_1(a_2 - a_1);$$

and

$$u_2(a_1, a_2) = a_2(1 - a_2 - a_1).$$

Answer **one** of the following **three** problems Q5 – Q7.
Please make sure that you state the number of the question that you choose.

Q5. (10 points) [Exercise 44.1 p. 44]
Consider the model of contributing to a public good (talked about in the second lecture), which is described as follows: (again I explain here, and the description of the model is found in p. 42 — p. 44.)

- Denote person *i*'s wealth by w_i and the amount she contributes to the public good by c_i $(0 \leq c_i \leq w_i)$. She spends her remaining wealth $w_i - c_i$ on private goods. The amount of the public good is equal o the sum of the contributions.
- Suppose that payoff $u_i(c_1, c_2)$ is the sum of three parts: the amount $c_1 + c_2$ of the public good provided, the amount $w_i - c_i$ person *i* spends on private goods, and a term $(w_i - c_i)(c_1 + c_2)$ that reflects an interaction between the amount of the public

good and her private consumption – the greater the amount of the public good, the more she values her private consumption.

In summary, suppose that person i's payoff is $c_1 + c_2 + w_j - c_i + (w_i - c_i)(c_1 + c_2)$, or

$$w_j - c_j + (w_i - c_i)(c_1 + c_2),$$

where j is the other person. Assume that $w_1 = w_2 = w$ and that each player i's contribution c_i may be any number (positive or negative, possibly larger than w). Find the Nash equilibrium of the game that models this situation.

Q6. (10 points) [Exercise 34.2 P. 34]
Two candidates, A and B, compete in an election. Of the n citizens, k support candidate A and m $(= n-k)$ support candidate B. Each citizen decides whether to vote, at a cost, for the candidate she supports, or to abstain. A citizen who abstains receives the payoff of 2 if the candidate she supports wins, 1 if this candidate ties for first place, and 0 if this candidate loses. A citizen who votes receives the payoffs $2 - c$, $1 - c$, and $- c$ in these three cases, where $0 < c < 1$.

a. When $k = m = 1$, is the game the same (except for the names of the actions) as any considered so far in this class? State the name of the game.
b. For $k = m$, find the set of Nash equilibria.
c. What is the set of Nash equilibria for $k < m$?

Q7. (10 points) [Exercise 42.2 P. 42]
Two people are engaged in a joint project. If each person i puts in the effort x_i, a nonnegative number equal to at most 1, which costs her $c(x_i)$, the outcome of the project is worth $f(x_1, x_2)$. The worth of the project is split equally between the two people, regardless of their effort levels. Formulate this situation as a strategic game. Find the Nash equilibria of the game when

(a) $f(x_1, x_2) = 3x_1x_2$ and $c(x_i) = x_i^2$ for $i = 1, 2$;

and

(b) $f(x_1, x_2) = 4x_1x_2$ and $c(x_i) = x_i$ for $i = 1, 2$.

In each case, is there a pair of effort levels that yields higher payoffs for both players than do the Nash equilibrium effort levels?