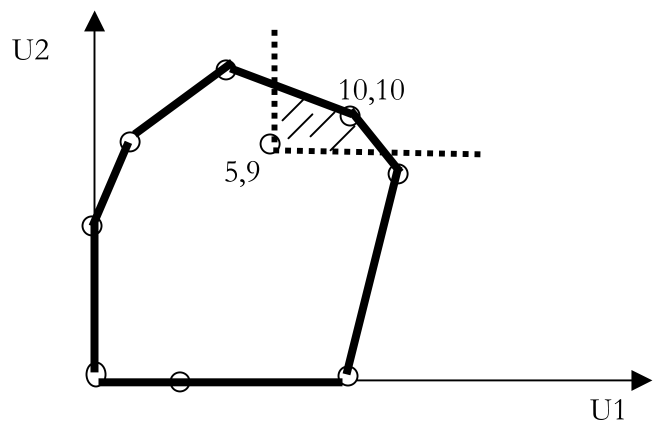
Spring 2007 Final – SOLUTIONS

1. ***(35%) Simultaneous and Repeated Games.***
   a. All strategies survive IESDS. No pure strategy of either player is strictly better than another for all actions of their opponent. Note, though it was not required, we could dominate T with a mixture of M and B. Then L is dominated by C and R. Finally M and C are eliminated and we're left with only B and R.
   b. (B,R) is the unique NE by the underlining method.
   c. The Grim-trigger strategy for player 1 (player 2) is as follows: Play M (C) in the first period and in all periods where no deviation has occurred. Play B (R) otherwise.
   d. Folk region:



   e. Note that cooperation yields (for both players)

$$U_i^c = 10(1+\delta_i+\delta_i^2+\ldots) = 10*(1/(1-\delta_i))$$

If player 1 deviates, he gets:

$$U_1^d = 11+5(\delta_1+\delta_1^2+\ldots) = 11 + 5*\delta_1/(1-\delta_1)$$

If player 2 deviates, he gets:

$$U_2^d = 11+9(\delta_2+\delta_2^2+\ldots) = 11 + 9*\delta_2/(1-\delta_2)$$

So if cooperation is better than deviating:

$$10(1/(1-\delta_1)) >= 11 + 5\delta_1/(1-\delta_1) \rightarrow \delta^*_1>=1/6$$

and,

$$10(1/(1-\delta_2)) >= 11 + 9\delta_2/(1-\delta_2) \rightarrow \delta^*_2>=1/2$$

   f. Note that both players get the same equilibrium path per period payoff and the same one period deviation. However, following a deviation, player 2 does much better compared to player 1 (9>5). Thus, player 2 has more of an incentive to deviate compared to player 1 and hence we require player 2 to be more patient if he's willing to pass up the short run gains today from deviating. In other words, we require player 2 to care more about the future than player 1 does.

2. ***(15%) Extensive game of Imperfect Information.***

   a. The game has 2 subgames: the subgame following player 1's choice of D, and the whole game.

   b. Player 1 has four strategies (A,B,C,D) and player 2 also has four strategies: (QX,QY,RX,RY). The strategic form is as follows:

	QX	**QY**	**RX**	**RY**
A	**0,0**	**0,0**	**0,1**	**0,1**
B	**0,1**	**0,1**	**0,0**	**0,0**
C	**0,4**	**0,4**	**0,0**	**0,0**
D	**0,1**	**0,3**	**0,1**	**0,3**

So the game has 8 Nash Equilibria:

NE={(A,RX); (A,RY); (B,QX); (B,QY);
(C,QX); (C,QY); (D,QY); (D,RY)}

   c. Note the only subgame besides the game itself starts following player 1's choice of D. So player 2 must act optimally from that subgame – ie, player 2 must choose Y following D. Thus the set of Subgame Perfect NE is:

SPNE={ (A,RY); (B,QY); (C,QY); (D,QY); (D,RY) }

(Just eliminate all those equilibria that involve player 2 choosing X).

3. ***(25%) Signaling.***

a. Note that since the receiver's information set following L is "off the equilibrium path," his beliefs, (p,1-p) do not depend on α. However, following R, we require:

$$E_{Rec}[D|R] >= E_{Rec}[U|R]$$
$$2(1-\alpha) >= \alpha$$
$$3\alpha <= 2$$
$$\alpha <= 2/3$$

b. Given the sender plays (L,L). Then p = 0.1. So following L, the receiver compares $E_{Rec}[U|L] = 2*0.1 = 0.2$ to $E_{Rec}[D|L] = 1(1-0.1) = 0.9$. So following L, the receiver plays D.

Next, consider the senders payoff from playing L. A strong and weak sender gets 2 and 3 respectively. A weak sender cannot profitably deviate since he is already attaining his highest potential payoff. However, a strong sender could deviate and get either 0 or 3. So a strong sender requires that a receiver play U following the R signal. So we need:

$$E_{Rec}[U|R] >= E_{Rec}[D|R]$$
$$q >= 2(1-q)$$
$$3q >= 2$$
$$q >= 2/3$$

Thus our PBE is:

PBE = { (L,L), (D,U), (p,1-p), (q,1-q) | p = 0.1, q >= 2/3 }

c. Both the sender and receiver each have 2 information sets. The sender's follow nature's choice of strong or weak. The receiver's follow the L and R signal.

4. ***(25%) Static Bertrand.***

a. Since firms 2 and 3 have equal and higher marginal cost compared to firm 1, any NE will involve firm 1 taking the whole market, while firms 2 and 3 will get nothing. There are two types of equilibria with discrete prices:

$$\{P_1 = \$1.99, [P_2 >= \$2, P_3 >= \$2; \text{at least one with equality}]\}$$
$$\{P_1 = \$2.00, [P_2 >= \$2.01, P_3 >= \$2.01; \text{at least one with equality}]\}$$

Note that we need either firm 2 or firm 3 to price right at \$2.00 (or \$2.01) in order to keep firm 1 from having a profitable deviation (by raising his price). Also note that the demand specification is included in this problem solely because you needed to know that choke price (\$20) was greater than the highest marginal cost of any firm (\$2).

b. With continuous prices, there is NO Nash Equilibrium. Firm 1 wants to undercut firms 2 and 3 by a very small amount but there is always a better deviation.

c. When firms have costs $\$1.00 = c_1 = c_2 < c_3 = \$2.00$, the low-cost firms (1 and 2) will compete away almost all of the profits. With discrete prices, we again have two types of NE:

$$\{P_1 = \$1.00, P_2 = \$1.00, P_3 >= \$1.01\}$$
$$\{P_1 = \$1.01, P_2 = \$1.01, P_3 >= \$1.02\}$$

Note in the second type of equilibria, firms 1 and 2 earn positive profits (on half the market). By lowering their price by one cent, they gain the whole market but make a zero margin on those customers. You might consider a NE where firms 1 and 2 set their price equal to \$1.02. See homework 1 for the proof that either firm HAS a profitable deviation from this price – namely lowering by 1 cent.

Finally, when prices are continuous, while the Nash Existence Theorem does not guarantee an equilibrium, we do have one (actually an infinite number). They are:

$$\{P_1 = \$1.00, P_2 = \$1.00, P_3 > \$1.00\}$$

Again, firm 3 just has to set a price above the others to make sure he doesn't win any of the market (and make a loss).