

Economics 644 – Final

Please answer ALL questions on this examination. Be sure to explain any non-standard notation that you use and JUSTIFY your answers. Each question's weight is shown in parentheses. Good Luck!

1. *Bayesian Game (35%)*. Consider the following game:

| | | Emily | |
|------|------|--------|--------|
| | | Left | Right |
| John | Up | (2, 2) | (0, 2) |
| | Down | (2, 0) | (1, 1) |

State 1
Probability = α

| | | Emily | |
|------|------|----------|--------|
| | | Left | Right |
| John | Up | (-2, -2) | (2, 0) |
| | Down | (0, 2) | (2, 2) |

State 2
Probability = $1 - \alpha$

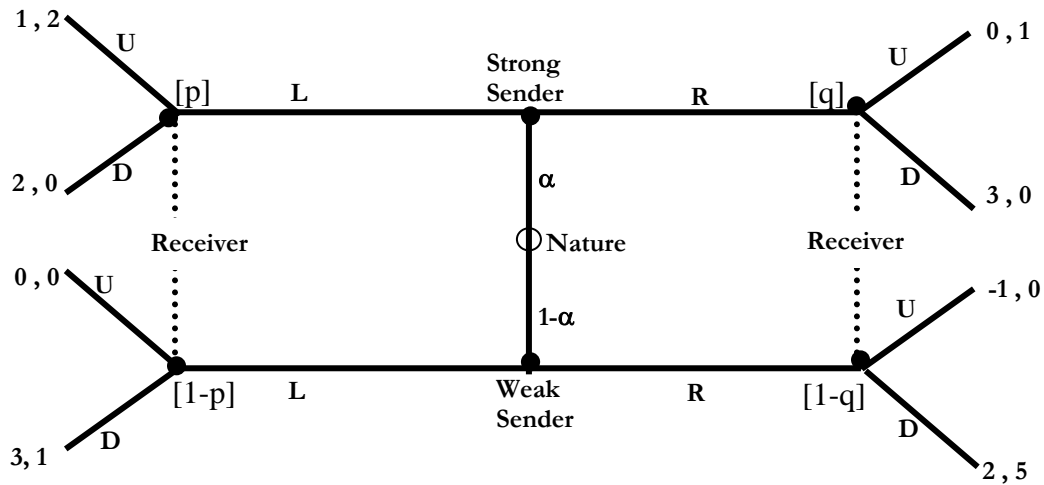
For part a, suppose $\alpha = 1$.

- Find all Nash Equilibria of the game (i.e., ignoring the game in state 2). Plot the best response correspondences on a graph.

For parts b and c, suppose $\alpha = \frac{1}{2}$. Assume both players are uncertain about the state of the world.

- How many strategies does John have?
- Solve for the Bayesian Nash Equilibria of the game.

2. Signaling (30%).



- Find the range of α such that there exists a Perfect Bayesian Equilibrium (PBE) involving the strategies (R,R) and (U,D) for the sender and receiver respectively.
- Now suppose $\alpha = 0.5$. Solve for a pooling PBE involving both types of sender playing L.
- How many information sets does each player have?

3. *Repeated Games (35%)*. Consider the following stage game, G:

| | | Player 2 | |
|----------|------|-----------|-----------|
| Player 1 | | Left | Right |
| | Up | (Y , B) | (0 , A) |
| | Down | (X , 0) | (Z , C) |

Assume $X > Y > Z > 0$ and $A > B > C > 0$. Note the payoff for player 2 if (Down, Left) is played is zero and the payoff for player 1 if (Up, Right) is played is zero.

- Find the Nash Equilibrium of the simultaneous static game.
- Write down Grim-Trigger strategies for each player to sustain (Y,B) as the average per-period payoff of $G(\infty, \delta)$, where $\delta = (\delta_1, \delta_2)$. I.e., players 1 and 2 may have different discount factors.
- Solve for the critical discount factors, $\delta^* = (\delta_1^*, \delta_2^*)$, such that cooperation [playing (Up, Left) in all periods] is optimal for both players.
- Solve for the critical discount factors (δ_1^*, δ_2^*) , assuming the following values:

| | |
|---------------|---------------|
| A = 30 | X = 40 |
| B = 25 | Y = 20 |
| C = 5 | Z = 10 |

Which player is required to be more patient in order for no deviations to occur? Explain the intuition behind this result given the values for each variable.