Stat 155 Homework # 5 Due March 3

Problems:

Q 1 Ferguson Chapter II Section 2.6 Q 6 (a)

Solution

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

The first and seventh rows are dominated by the second and sixth respectively.

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$

Columns 3 and 5 are dominated by columns 1 and 7 respectively.

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Row 3 is dominated by row 1.

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Column 3 is dominated by column 2.

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The remaining matrix is diagonal. Using the formula for diagonal matrices we have that the value is $\frac{1}{4}$ and the optimal strategies are $x = y = (\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4})$. In the original game this translates to strategies of

$$x = (0, \frac{1}{4}, \frac{1}{4}, 0, \frac{1}{4}, \frac{1}{4}, 0), \qquad y = (\frac{1}{4}, \frac{1}{4}, 0, 0, 0, \frac{1}{4}, \frac{1}{4})$$

Q 2 Ferguson Chapter II Section 2.6 Q 9

The payoff matrix is

$$\begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix}$$

The second row is dominated by the average of the first and third rows.

$$\begin{pmatrix} 0 & 1 & 2 \\ 2 & 1 & 0 \end{pmatrix}$$

The second column is dominated by the average of the first and third rows

$$\begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$$

By symmetry between the two states the optimal strategies are $x = y = (\frac{1}{2}, \frac{1}{2})$ and the value is 1. The optimal strategies in the original game are $x = y = (\frac{1}{2}, 0, \frac{1}{2})$.

Q 3 Ferguson Chapter II Section 3.7 Q 2 (a) The diagonal term is a saddle point so the value is 0.

(b) If $d_i > 0$ and $d_j < 0$ then (i, j) is a saddle point so the value is 0.

(c) If all the diagonal entries are negative then the value is $(d_1^{-1} + \ldots + d_n^{-1})^{-1}$.

Q 4 Ferguson Chapter II Section 3.7 Q 4

The payoff matrix is

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{2} & 1 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 1 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 1 \end{pmatrix}$$

We get the system of linear equations

$$\begin{aligned} x_1 + \frac{1}{2}(x_2 + x_3 + x_4) &= v \\ x_2 + \frac{1}{2}(x_3 + x_4) &= v \\ x_3 + \frac{1}{2}x_4 &= v \\ x_4 &= v \\ x_1 + x_2 + x_3 + x_4 &= 1 \end{aligned}$$

Solving these we get $x_4 = v$, $x_3 = v - \frac{1}{2}x_4 = v/2$, $x_2 = v - \frac{1}{2}(x_3 + x_4) = v/4$ and $x_1 = v - \frac{1}{2}(x_2 + x_3 + x_4) = v/8$. Then $1 = x_1 + x_2 + x_3 + x_4 = \frac{15}{8}v$ so $v = \frac{8}{15}$ and $x = (\frac{1}{15}, \frac{2}{15}, \frac{4}{15}, \frac{8}{15})$. Solving similarly for y we have that $y = (\frac{8}{15}, \frac{4}{15}, \frac{2}{15}, \frac{1}{15})$.