GEOS 639 – INSAR AND ITS APPLICATIONS

GEODETIC IMAGING AND ITS APPLICATIONS IN THE GEOSCIENCES

Lecturer:

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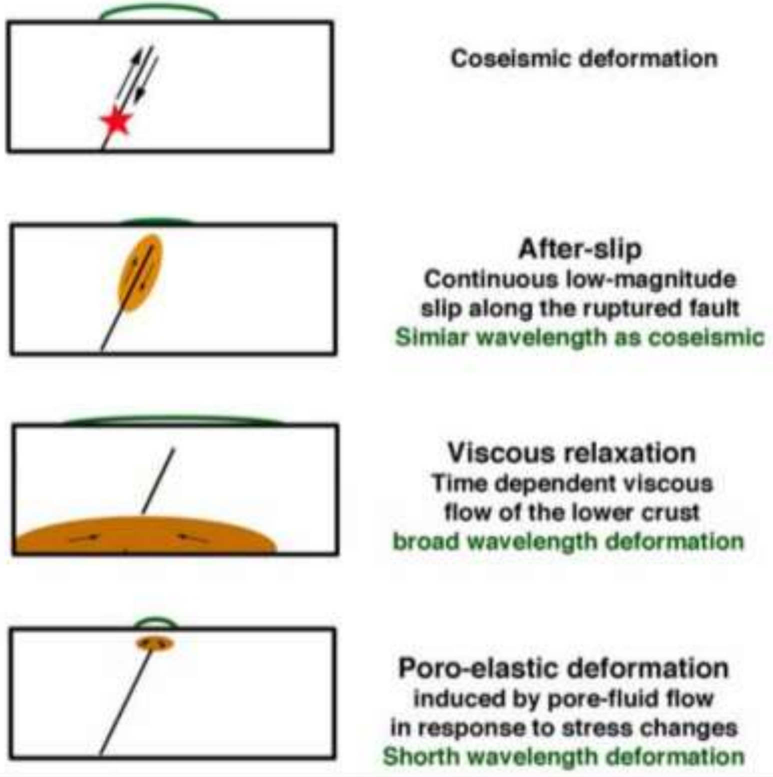
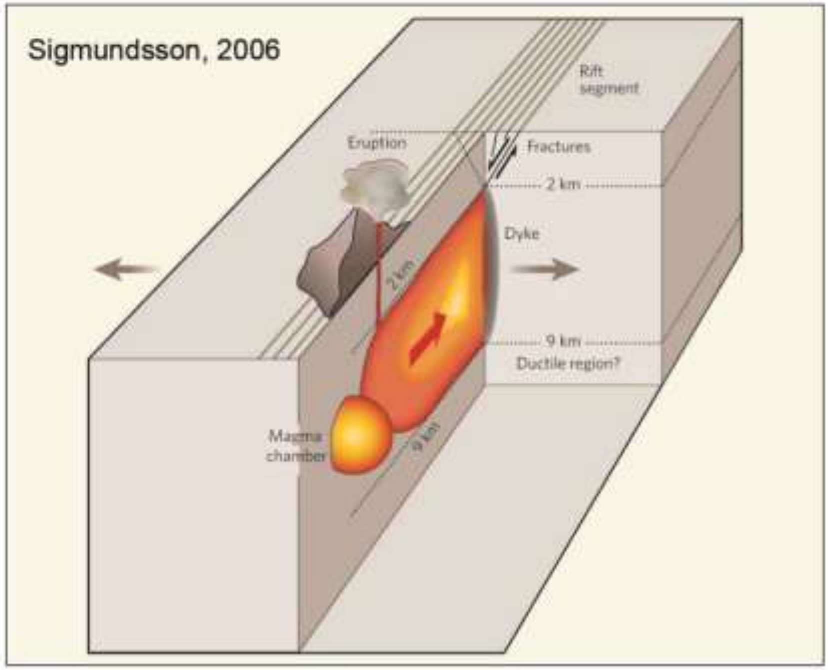
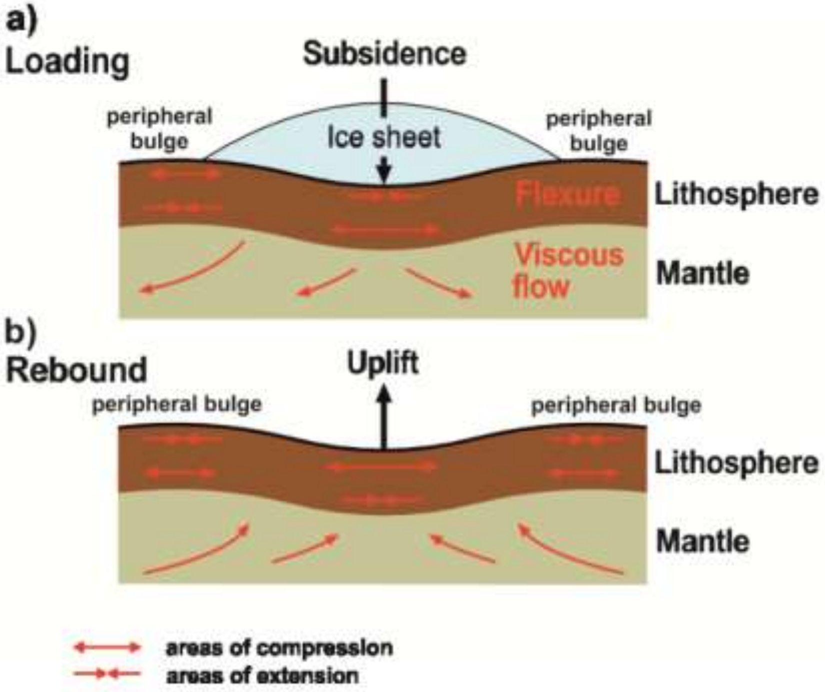
Lecture 10: On the Use of InSAR in Geophysics

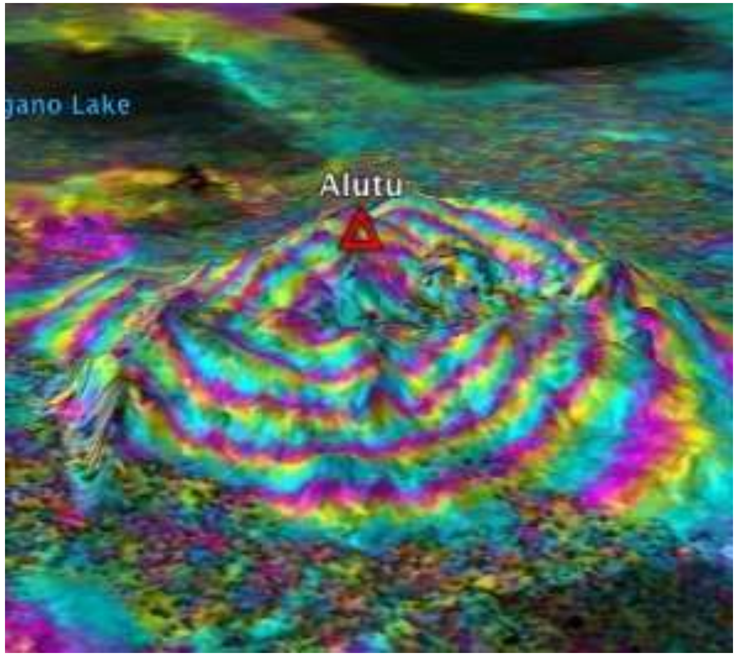
Geophysical Modeling from Geodetic Data

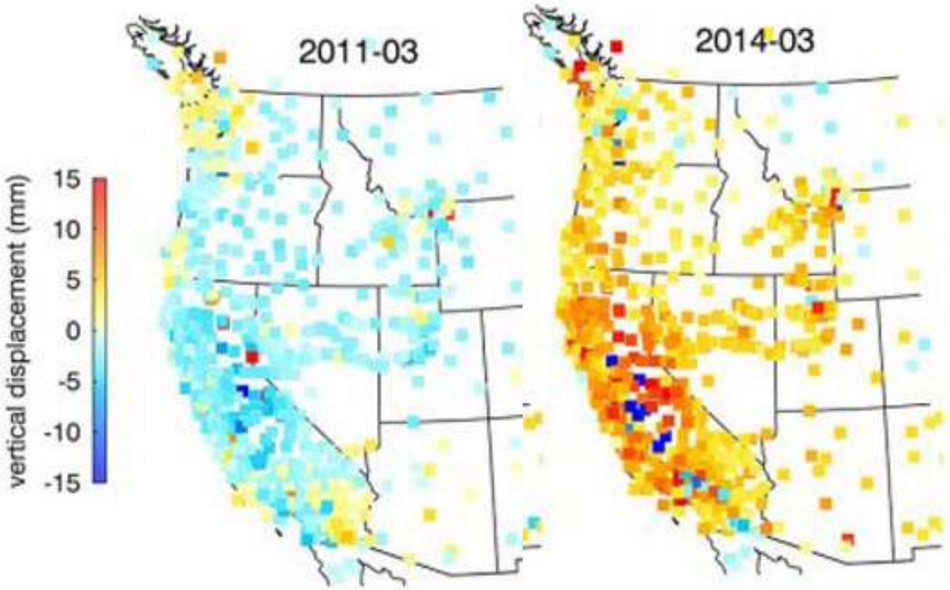
By geophysical **modeling**, we mean using idealized representations of the Earth to gain insight into its properties and processes

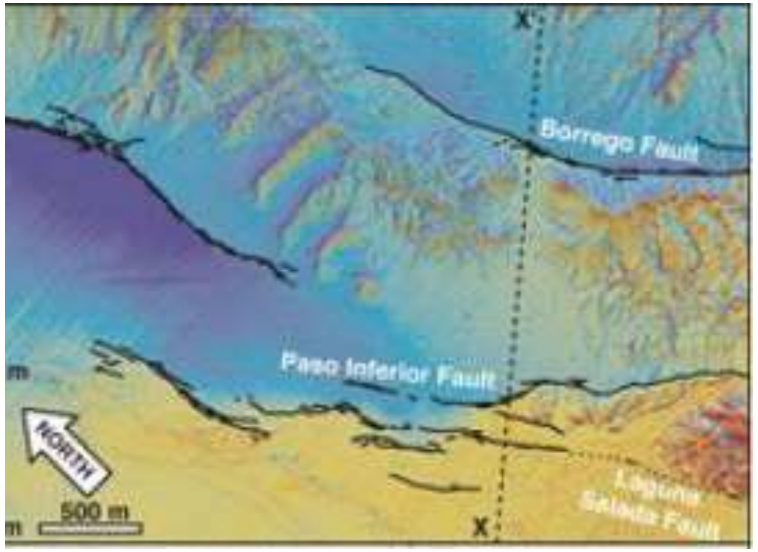












By geodetic **data**, we mean data that measure deformation (changes in shape) of the Earth's surface – e.g. InSAR, GPS/GNSS, differential lidar, optical image correlation…

InSAR Results needed for Geophysical Modeling

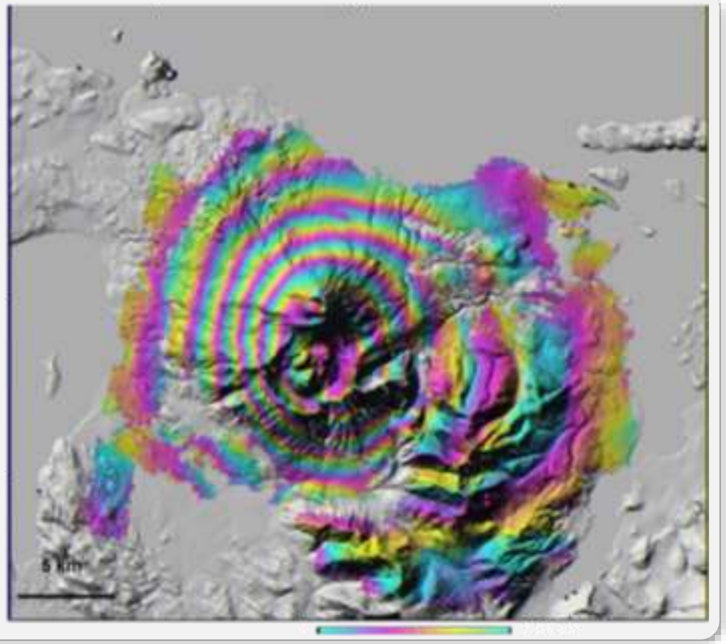
InSAR Inputs for Geophysical Modeling

InSAR Deformation Rate Information

- **Two Example Approaches to Arrive at a Deformation Rate Map**

Single Co-Seismic / Co-Eruptive InSAR Pair

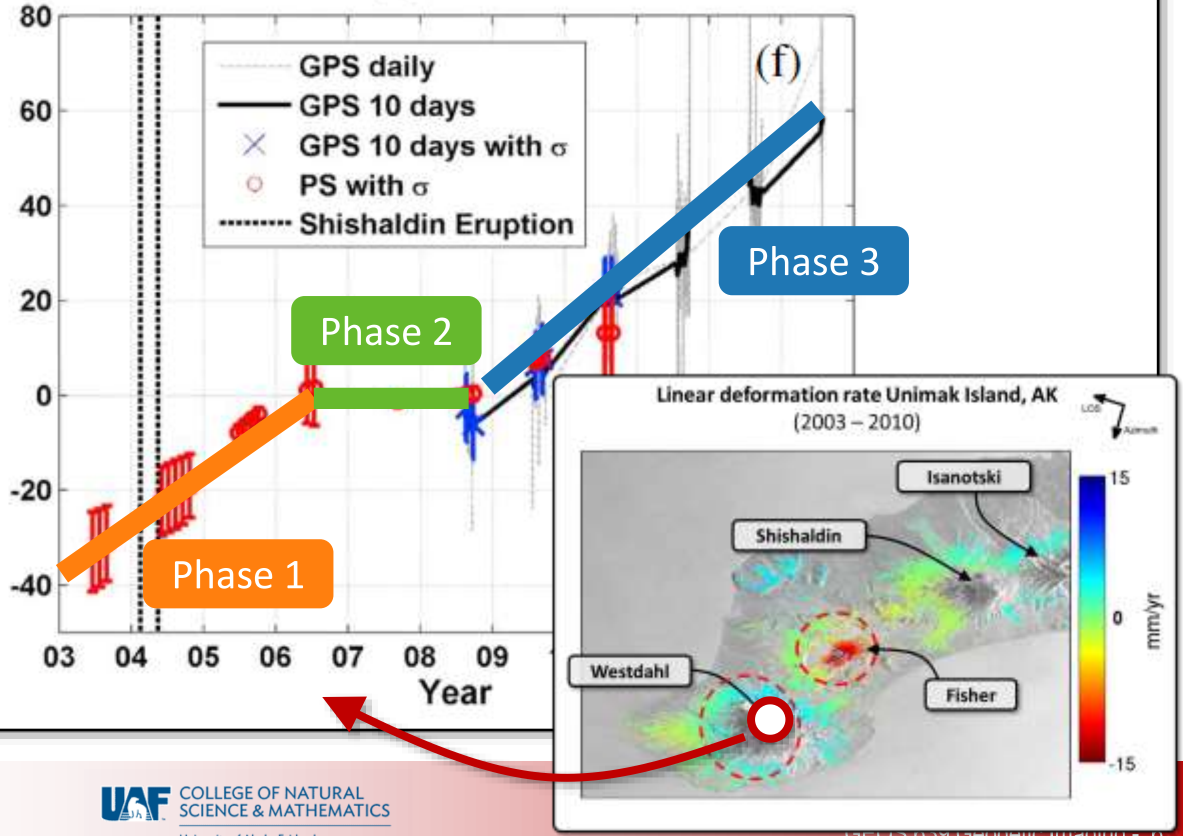
- Standard InSAR pair processing and phase unwrapping
- Assumption: Signal is large compared to noise from atmosphere, decorrelation, residual topography
- Coherence sufficient



InSAR pair of Peulik volcano showing ~17 cm of uplift centered on the volcano's southwest flank from October 1996 to October 1997

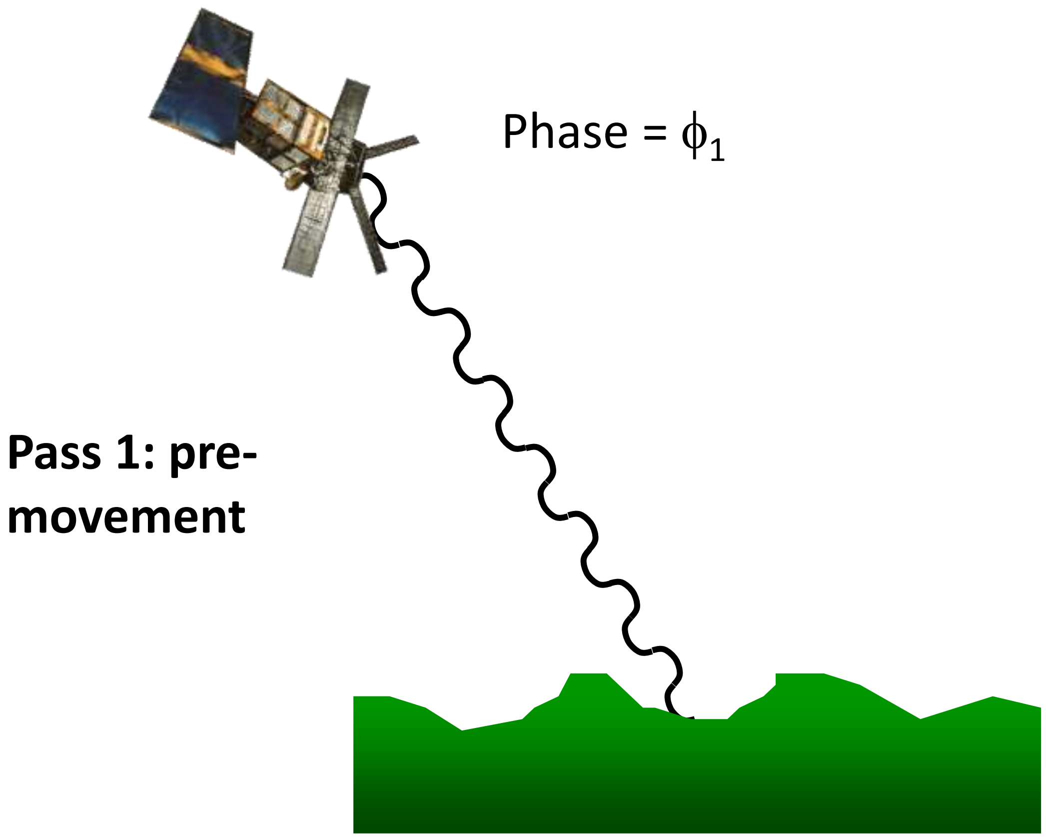
Time Series Solution

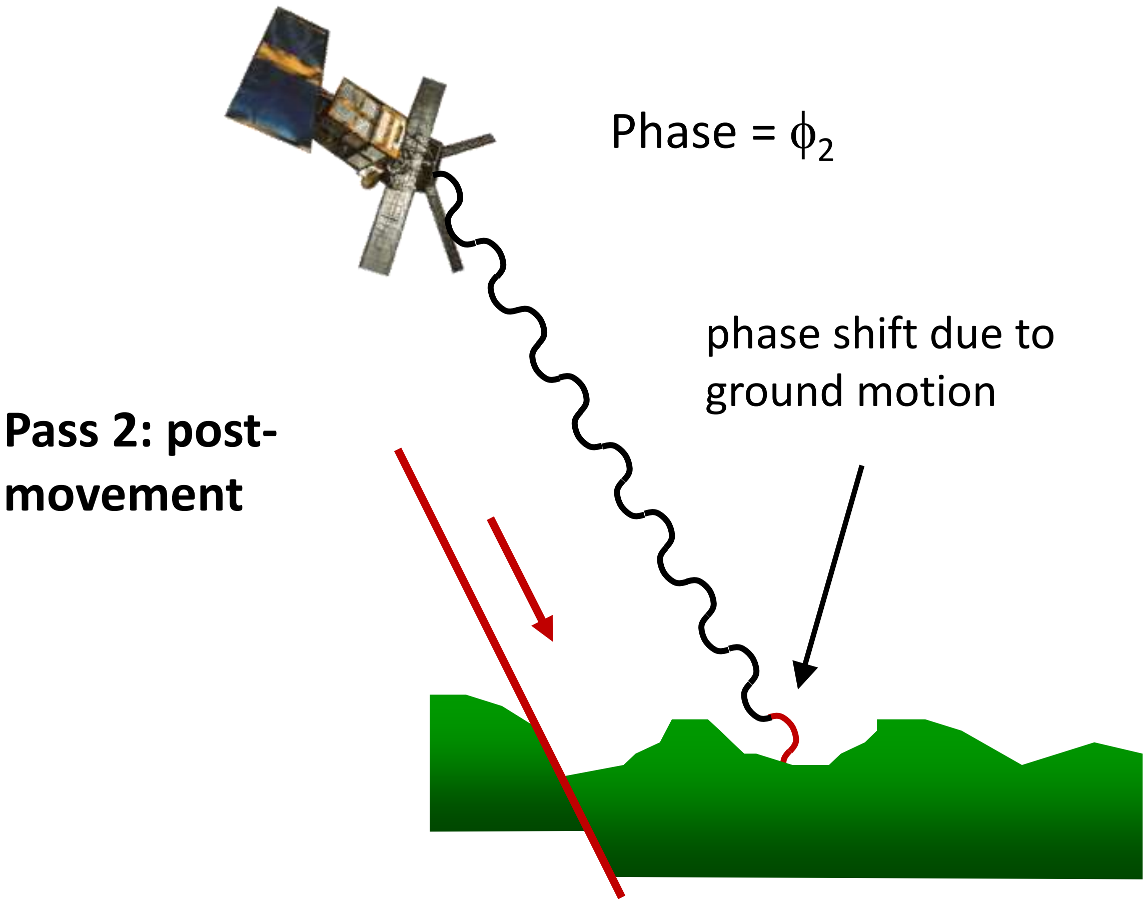
- Perform SBAS time series inversion
- Segment deformation time series into phases of consistent behavior → model each phase with linear rate
- Solve for geophysical parameters for each phase separately



Remember: InSAR Is Only Sensitive to the Line-Of-Sight Component of the 3-D Motion Vector

- **An individual SAR interferogram measures deformation in one dimension, in the radar line-of-sight**





Remember: InSAR Is Only Sensitive to the Line-Of-Sight Component of the 3-D Motion Vector

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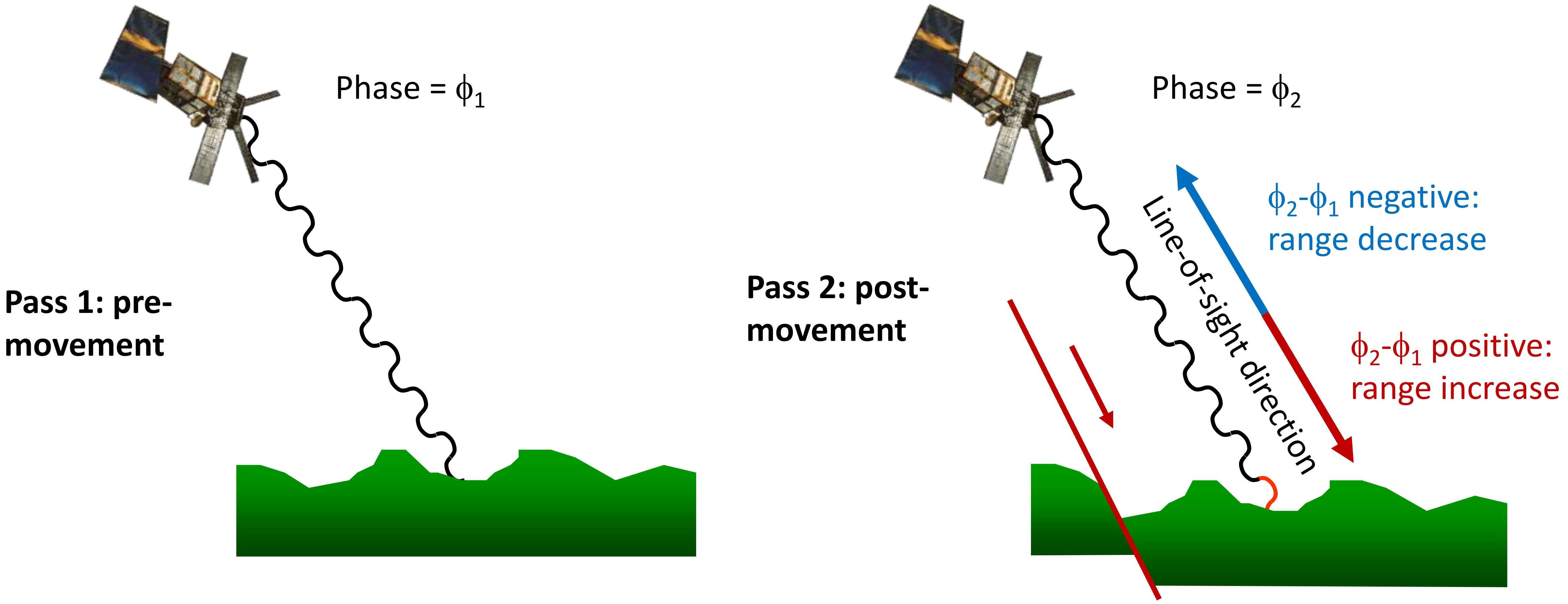
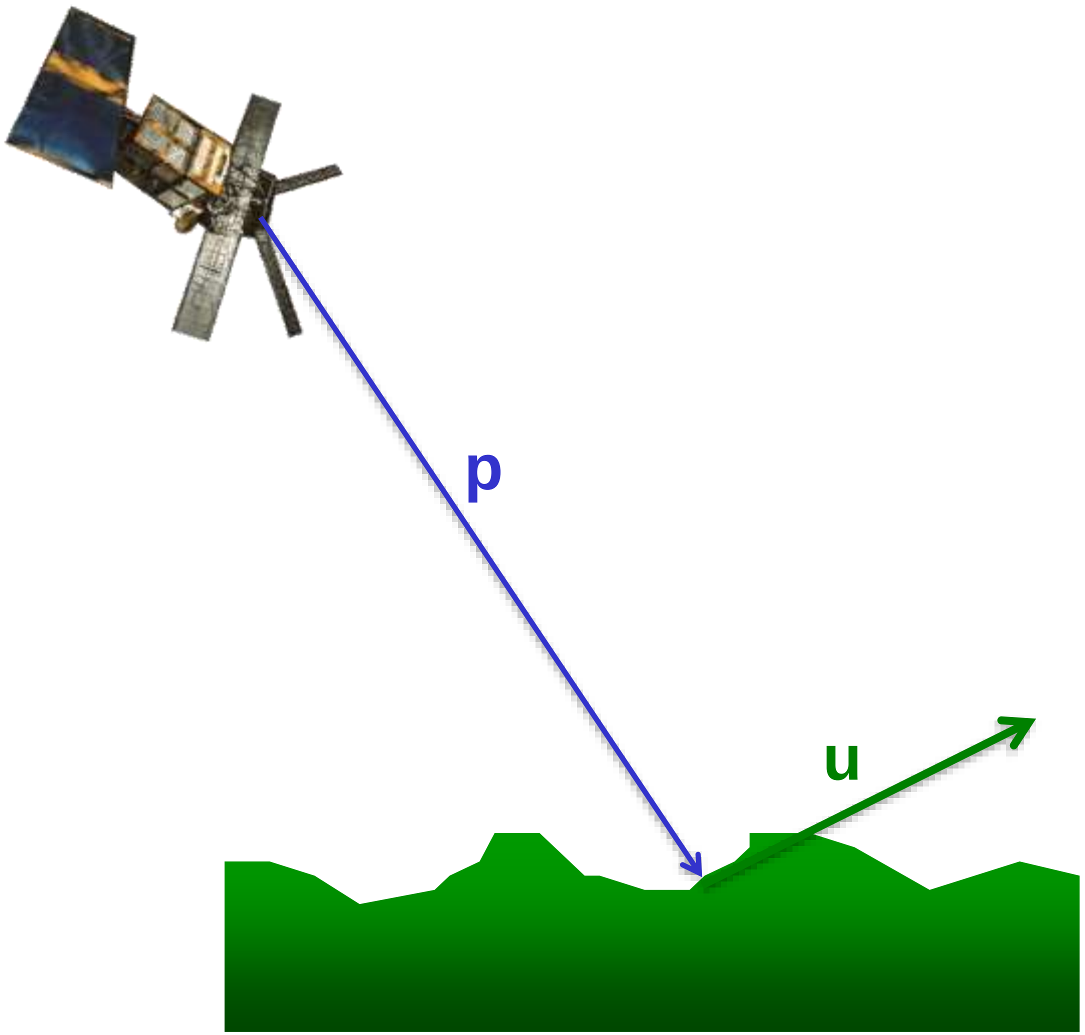


The Unit Pointing Vector

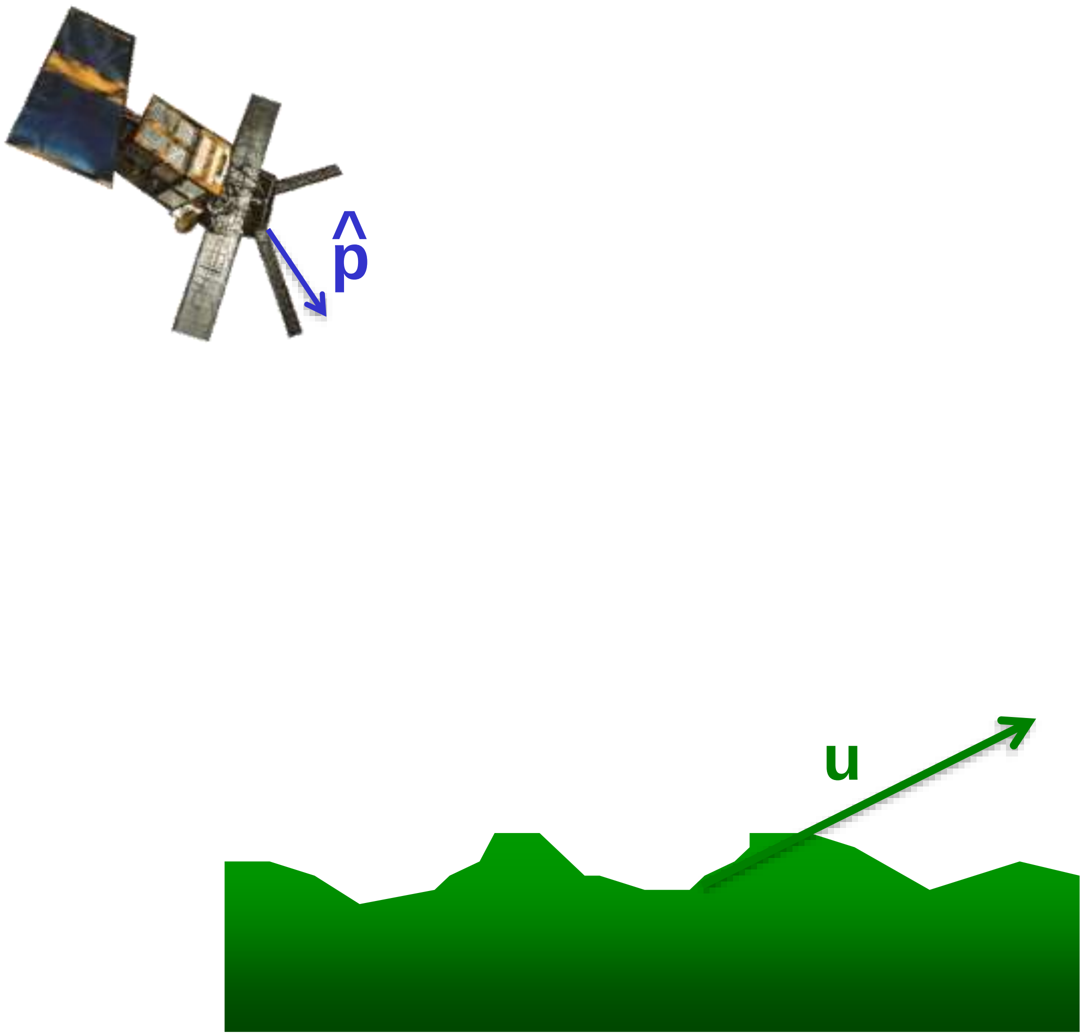


$\boldsymbol{u}$ = ground displacement vector

$\boldsymbol{p}$ = pointing vector (from satellite to ground target)

$\boldsymbol{p}$ is controlled by the satellite trajectory, beam mode (incidence angle) and position of the pixel within the swath

The Unit Pointing Vector

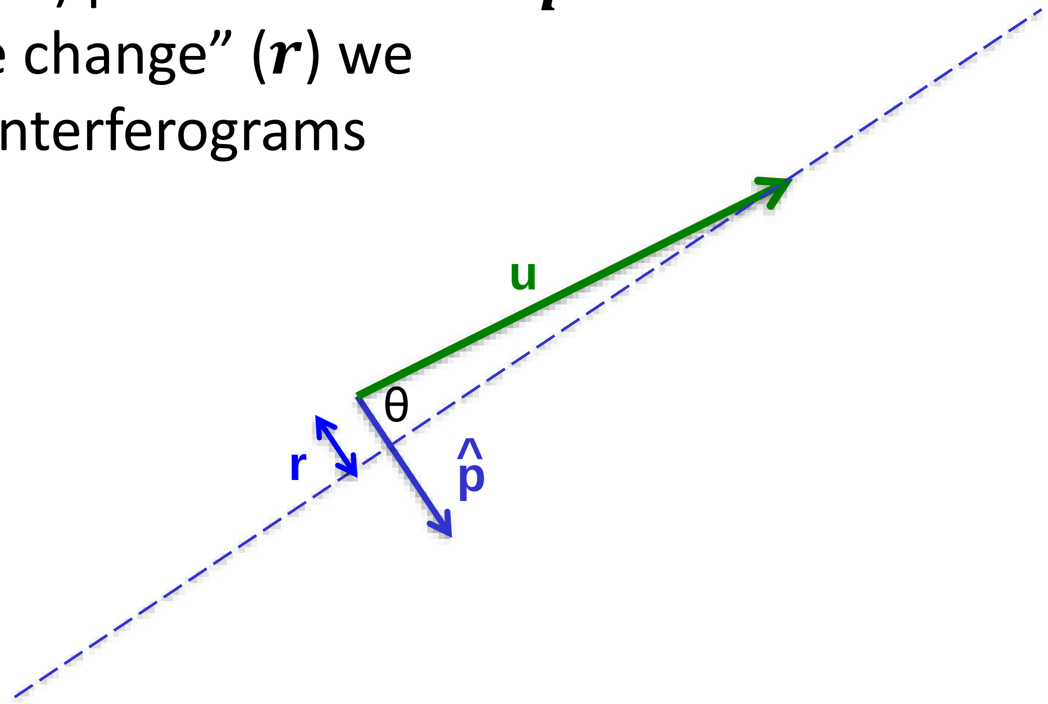


$\boldsymbol{u}$ = ground displacement vector

$\hat{\boldsymbol{p}}$ = <u>unit</u> pointing vector (from satellite to ground target)

Range Change

the scalar (dot) product of $\boldsymbol{u}$ and $\hat{\boldsymbol{p}}$ is the “range change” ($\boldsymbol{r}$) we measure in interferograms



cross-section view

$$r = \mathbf{u} \times \hat{\mathbf{p}}$$
$$= |\mathbf{u}||\hat{\mathbf{p}}|\cos q$$
$$= |\mathbf{u}|\cos q$$

therefore, the key to modeling InSAR data is having a code that can simulate the displacements $\boldsymbol{u}$

An Example of the Use of InSAR in Geophysics

Volcanic Deformation

The Deformation Modeling Problem

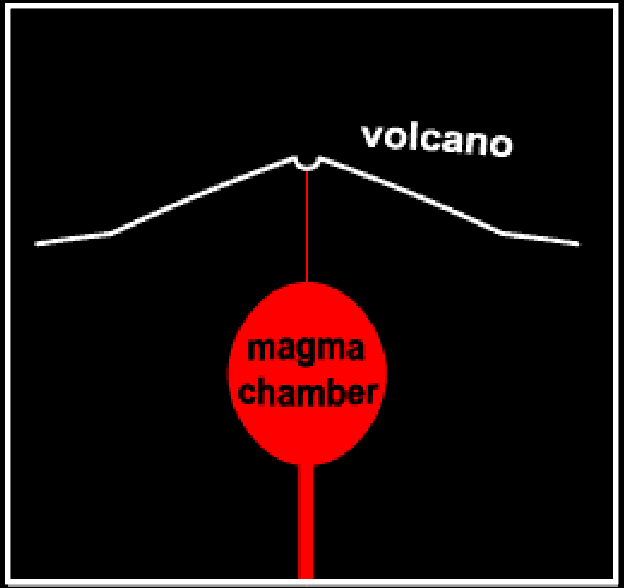
deformation:

what we see (InSAR)

?

magma dynamics:

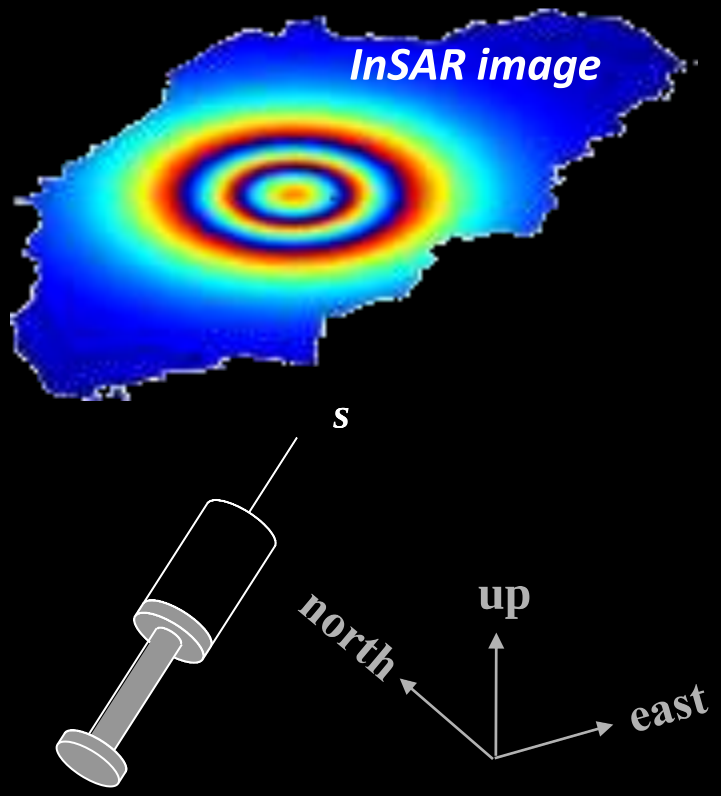
what we want to know



Magma intrusion

The Deformation Modeling Problem

Estimate source characteristics from InSAR deformation data



forward model

design matrix

$$G\, s = d$$

source parameters

displacement (vector)

inverse model

$$s = G^{inv}\, d$$

Solving for Model Parameters using Model Inversion

$$G \cdot x = b$$

- If the covariance matrix for errors in the observation (b) is Σ_b, then the weighted least-squares (maximum likelihood) solution for x is

$$\hat{x} = \left[G^T \cdot \Sigma_b^{-1} \cdot G\right]^{-1} \cdot \left[G^T \cdot \Sigma_b^{-1} \cdot b\right]$$

  and the covariance matrix for the estimated vector components is

$$\Sigma_x = \left[G^T \cdot \Sigma_b^{-1} \cdot G\right]^{-1}$$

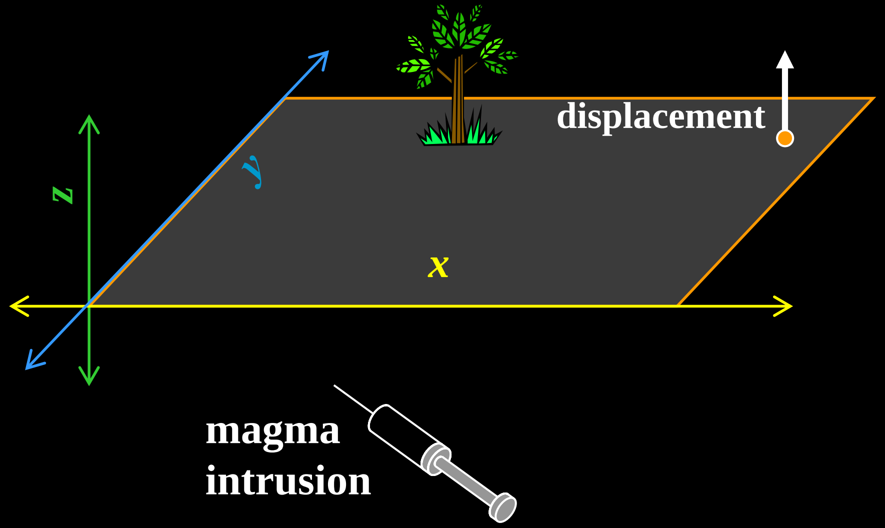
- In the case where we assume that observation errors are independent and have equal standard deviations, σ, we get

$$\Sigma_x = \sigma^2\left[G^T \cdot G\right]^{-1}$$

  – The square roots of the diagonal terms of Σ_x are the standard errors of the estimated parameters

What is the Forward Model in Volcano Deformation?

Predicts deformation (*u*) caused by magma intrusion (relates magma intrusion to deformation)



elasto-static behavior

$$\mu \nabla^2 u_i + \frac{\mu}{(1-2\upsilon)} \left[\frac{\partial^2 u_k}{\partial x_i \partial y_k} \right] = -F_i$$

$$\boxed{\underline{\mathbf{u}} = f(\textit{model parameters})}$$

What Is the Forward Model?

Simple Model: Inflating Point Source Model

- A component of deformation vector (u_i) and the displacement at the free surface ($x_3 = 0$) takes the form

$$u_i\left(x_1 - \boldsymbol{x'_1}, x_2 - \boldsymbol{x'_2}, -\boldsymbol{x'_3}\right) = \boldsymbol{C}\frac{x_i - \boldsymbol{x'_i}}{|R^3|}$$

  - $\boldsymbol{x'_i}$ is a source location, $\boldsymbol{C}$ is a combination of material properties and source strength, and R is the distance from the source to the surface location

- $\boldsymbol{C}$ is defined as follows:

$$\boldsymbol{C} = \Delta P(1-\nu)\frac{r_s^3}{G} = \boldsymbol{\Delta V}\frac{(1-\nu)}{\pi}$$

Unknown (target) parameters marked in **red**

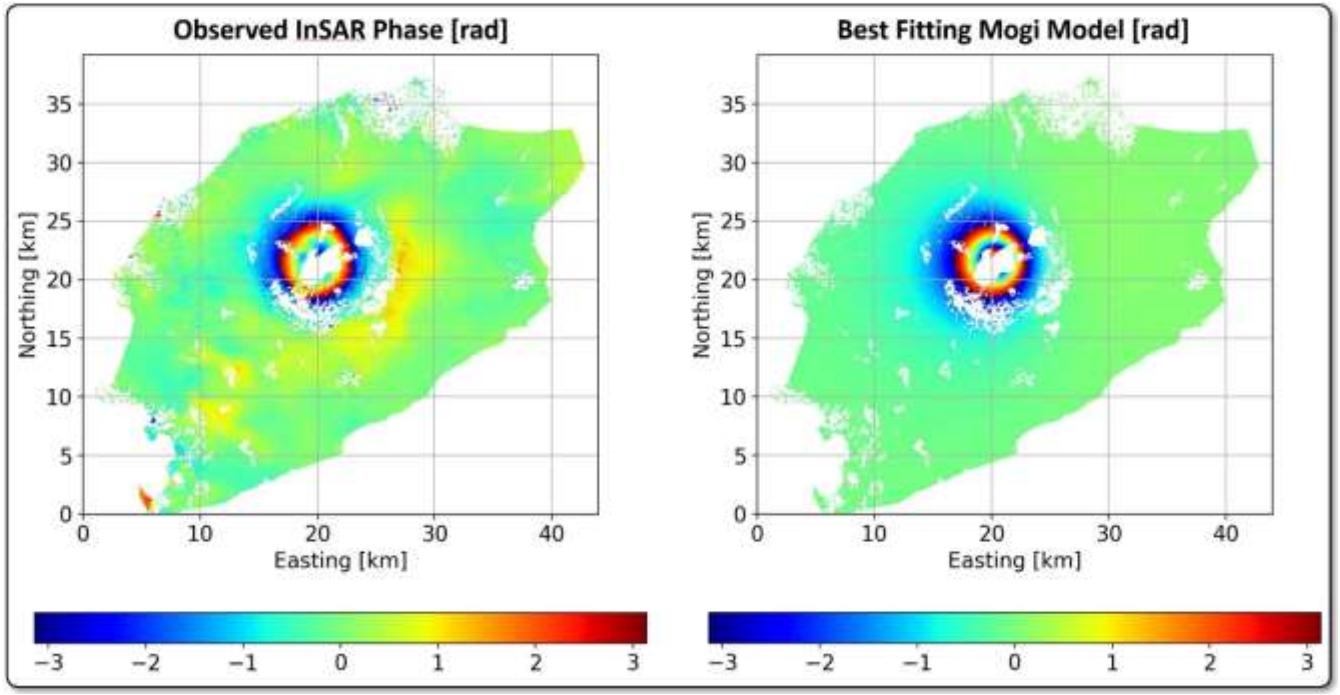
  - ΔP - change in pressure of magma chamber
  - ΔV - change in volume of magma chamber
  - ν - Poisson's ratio (material property)
  - r_s - radius of the sphere
  - G - shear modulus of country rock (material property)

Think – Pair – Share:

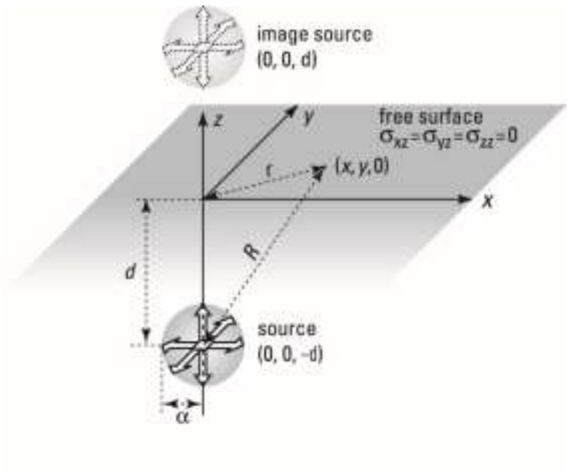
- **Limitations of Mogi Models**
  - Let's look at the Mogi model equations one more time

$$u_i(x_1 - \boldsymbol{x'_1}, x_2 - \boldsymbol{x'_2}, -\boldsymbol{x'_3}) = \boldsymbol{C}\frac{x_i - \boldsymbol{x'_i}}{|R^3|} \quad \text{with} \quad \boldsymbol{C} = \Delta P(1-\nu)\frac{r_s^3}{G} = \boldsymbol{\Delta V}\frac{(1-\nu)}{\pi}$$

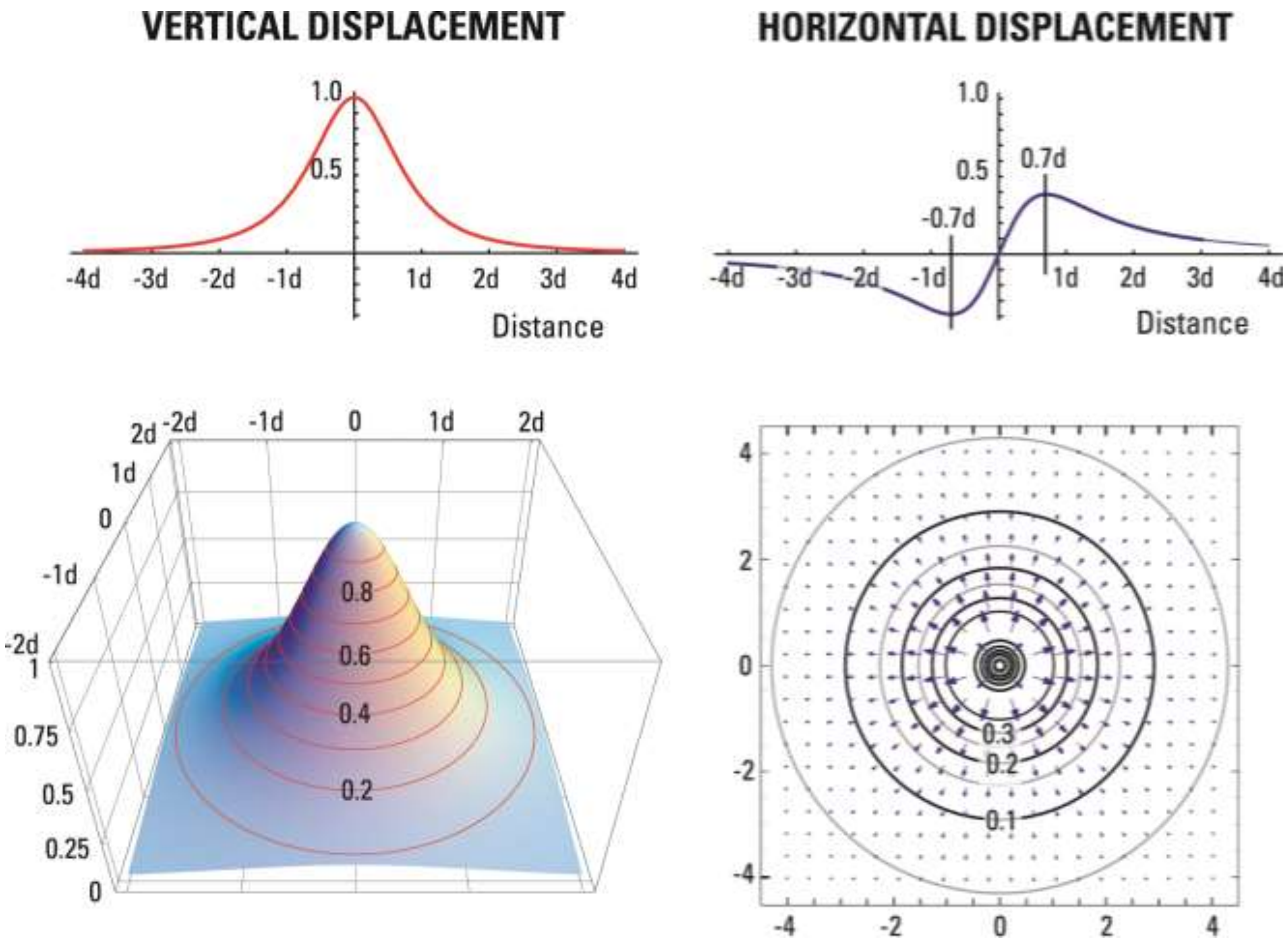
  - **Activity 1**: Discuss the limitations that may be brought on by how the variables ν and G are used in these equations.
  - **Activity 2**: Discuss the limitations that may be brought on by how the source geometry is captured in the equations.



Forward Model: Inflating Point Source

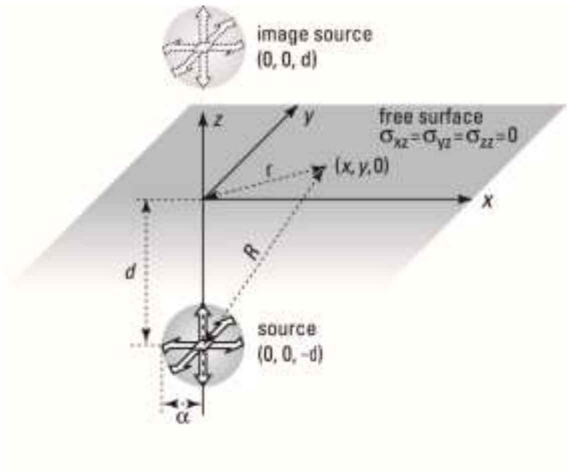


α << d



D. Dzurisin, 2007
Courtesy of M. Lisowski

Forward Model: Inflating Point Source



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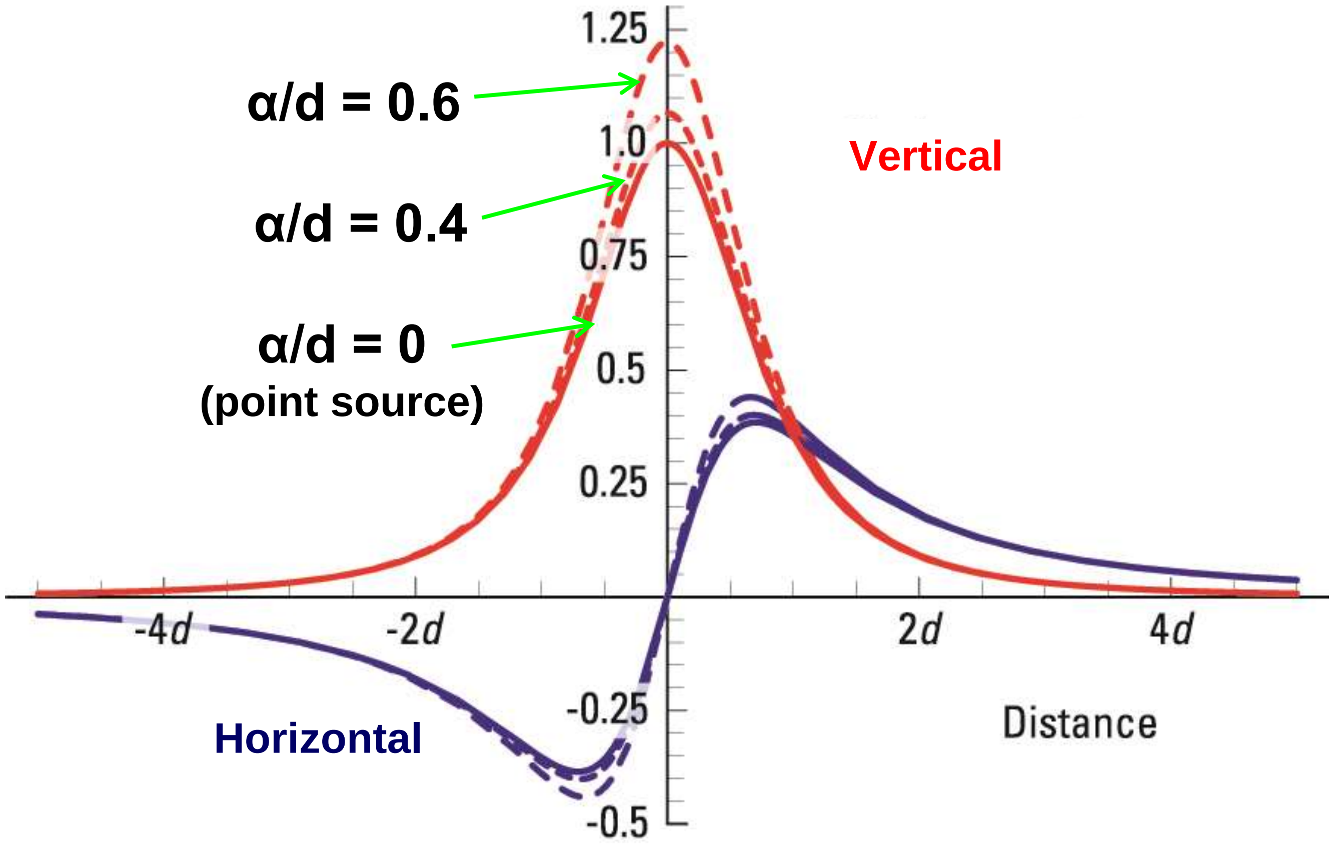

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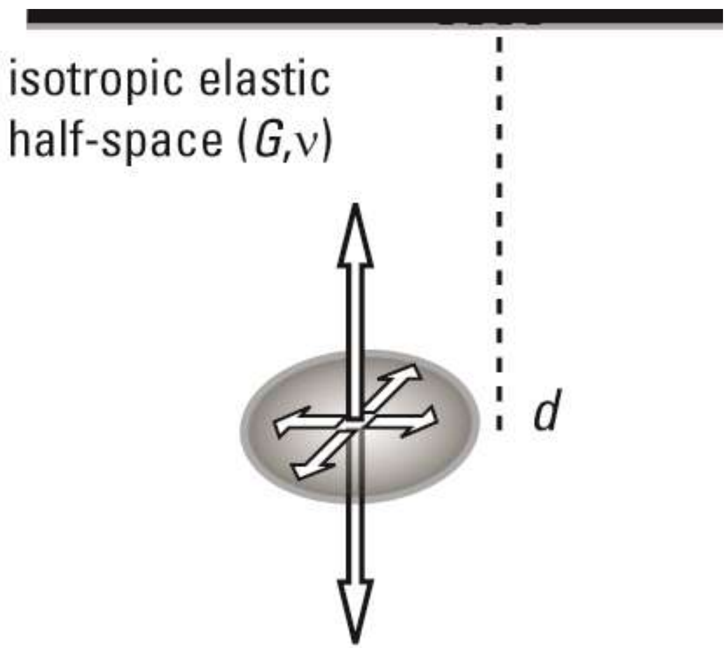
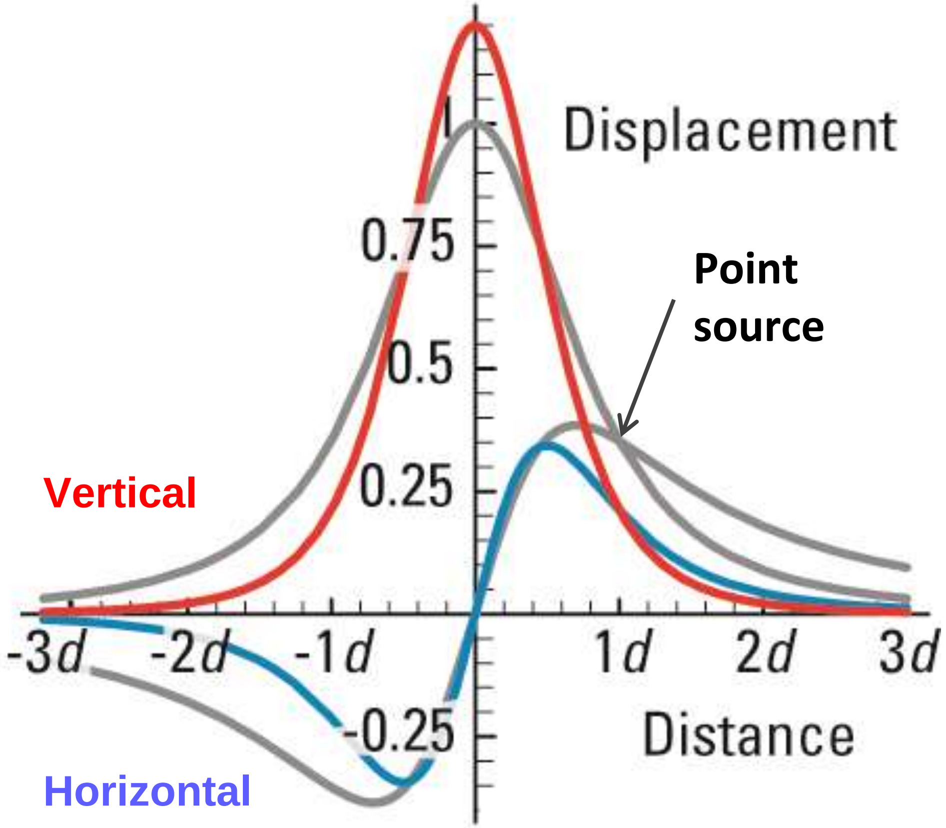
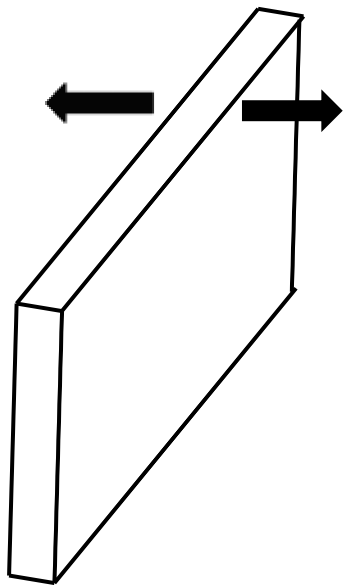
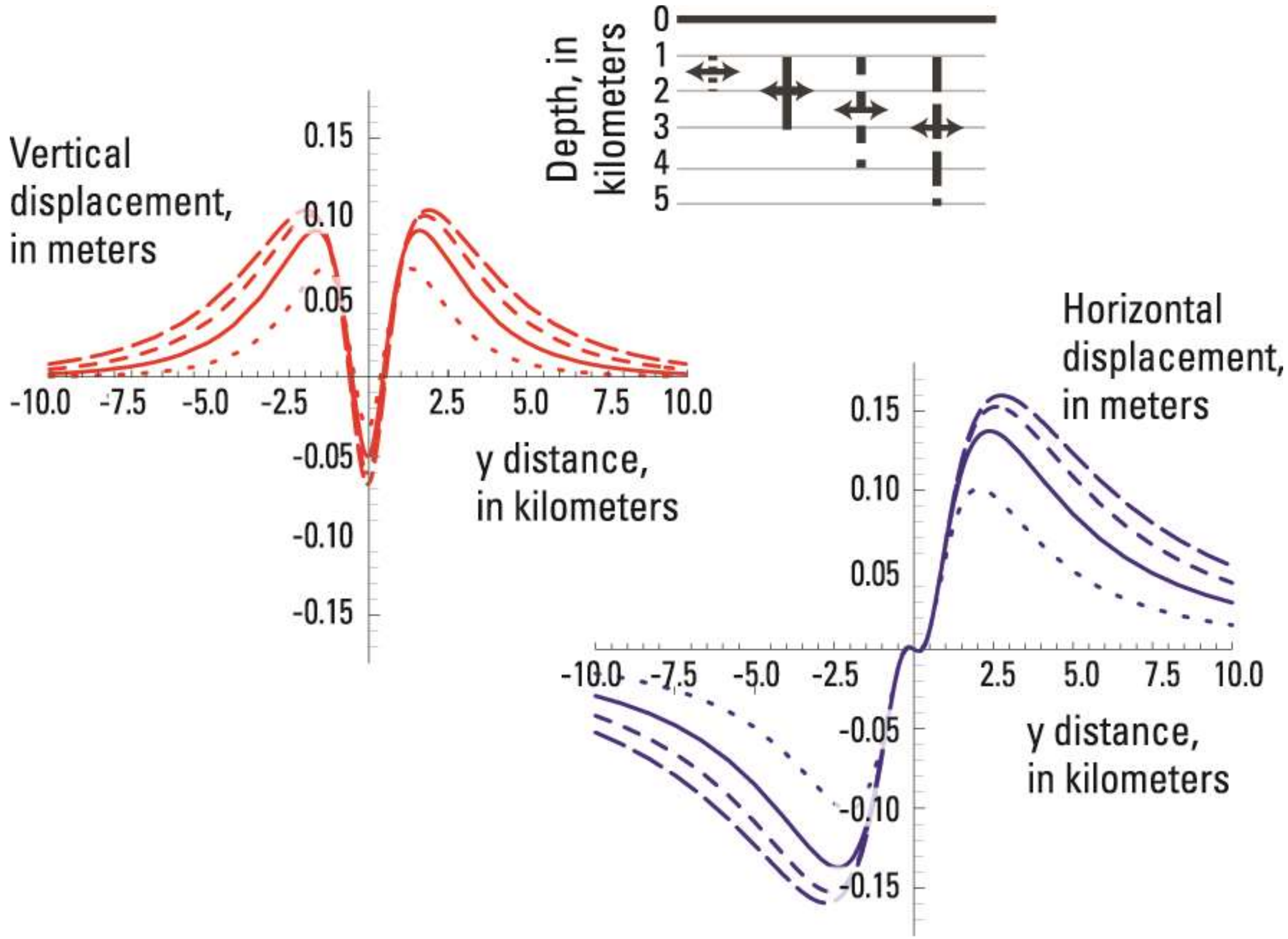
Forward Model: Sill Model





Forward Model: Dike Model





Ultimate Goal of Deformation Modeling:

Minimize

$$\sum [u_i(x,y) \bullet los_i(x,y) - obs_i(x,y)]^2$$

u_i is a theoretical calculation of ground surface deformation vector (i=1, 2, 3)
los_i is the InSAR line-of-sight vector
obs_i is the observed deformation (InSAR image)
(x, y) is the image coordinate

Non-linear inversion!!!!

Find the best-fitting Model Parameters

Grid Search: A Simple Approach

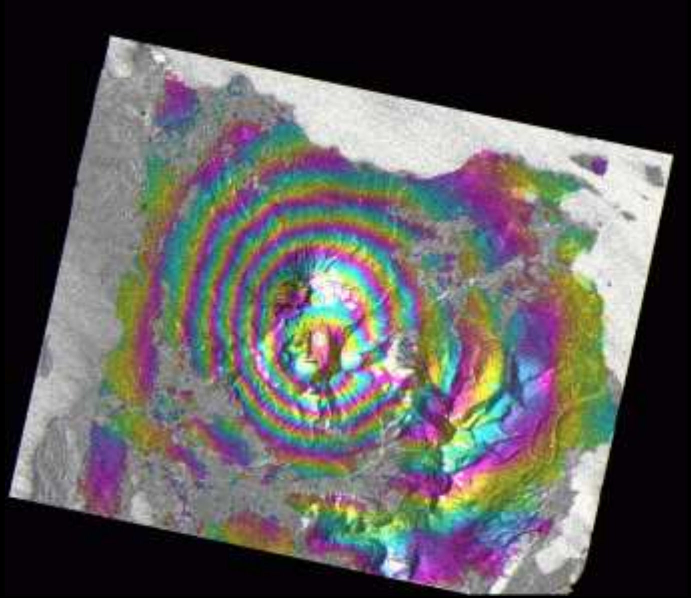
1. Loop through model parameters
2. calculate the residual (observed – modeled) for each set of model parameters
3. Find the set of model parameters that renders the smallest residual

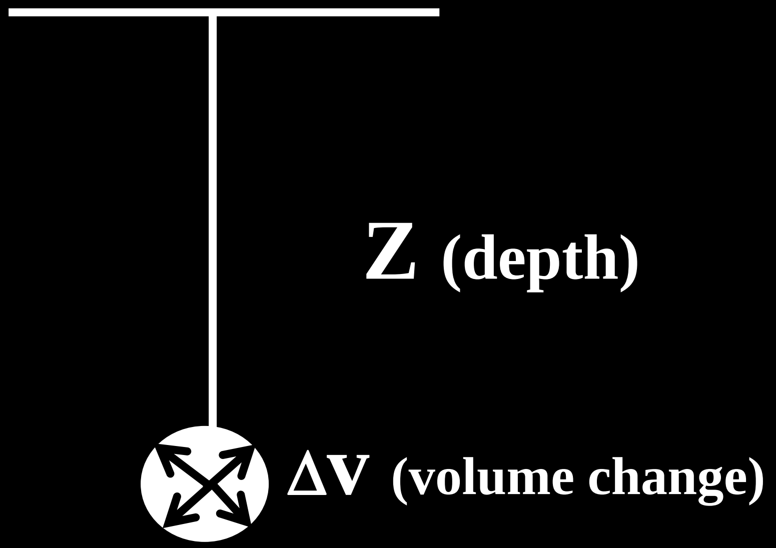
→ **best-fitting model parameters**

Next Week: A Jupyter Notebook Lab for Estimating Source Parameters

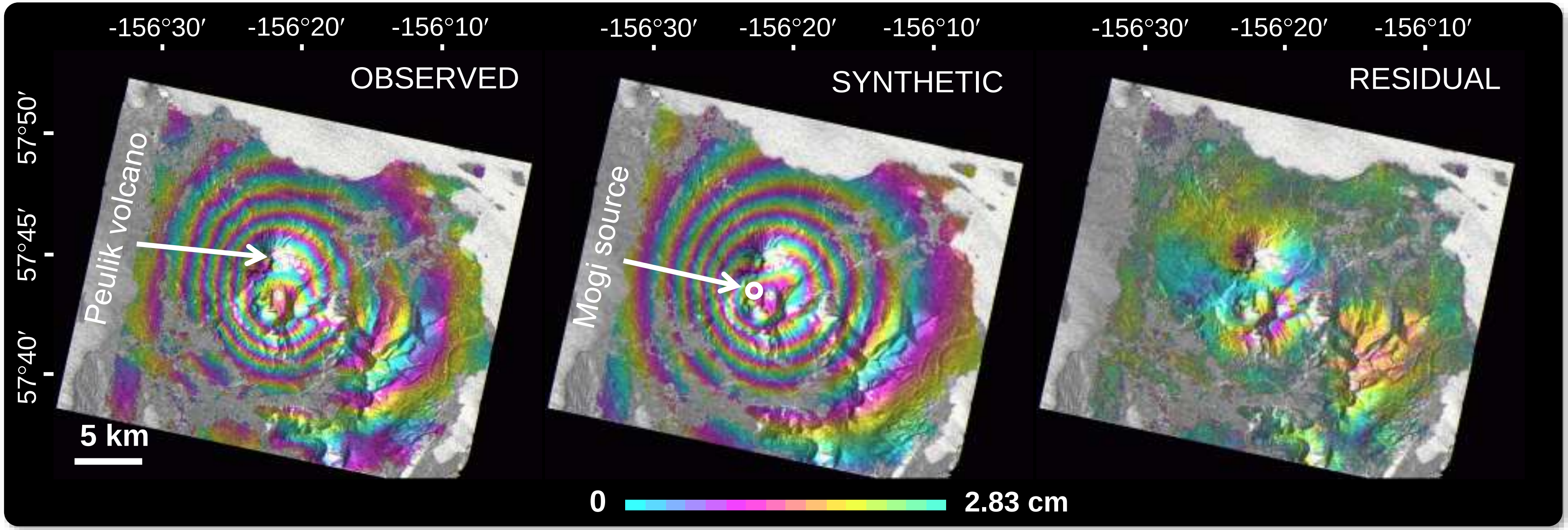
<u>What we will do in the lab:</u>

- We will define a search space for source model location
- We will assume that source depth and magma volume change are known and fixed
- **For each set of x and y coordinate parameters:**
  - We will run a forward model to produce *predicted surface deformation* results
  - Calculate difference (residuals) between predicted and measured deformation
- **Best fitting model parameters are those that minimize residuals between observations & model prediction**





Mt. Peulik Example

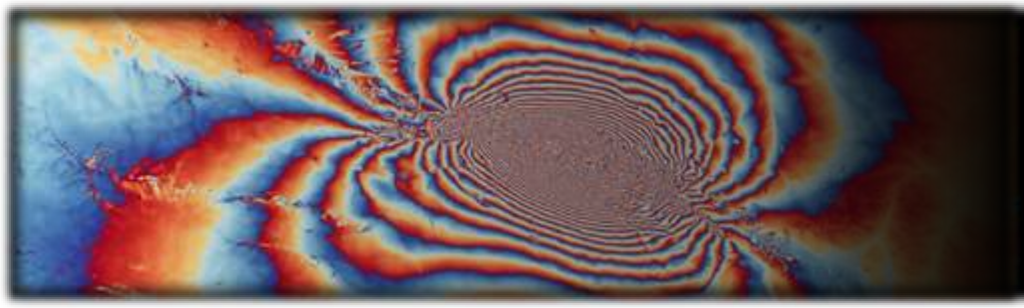


- Spherical Point Source Model (Mogi Source)

$$u_i\left(x_1 - \boldsymbol{x'_1}, x_2 - \boldsymbol{x'_2}, -\boldsymbol{x'_3}\right) = \boldsymbol{C}\frac{x_i - \boldsymbol{x'_i}}{|R^3|}$$

Where $\boldsymbol{x'_i}$ is source location, $\boldsymbol{C}$ is a combination of material properties and source strength, and R is the distance from the source to the surface location

- Best fit Source parameters:
  - Depth: $6.5 \pm 0.2km$; Volume change: $0.043 \pm 0.002km^3$



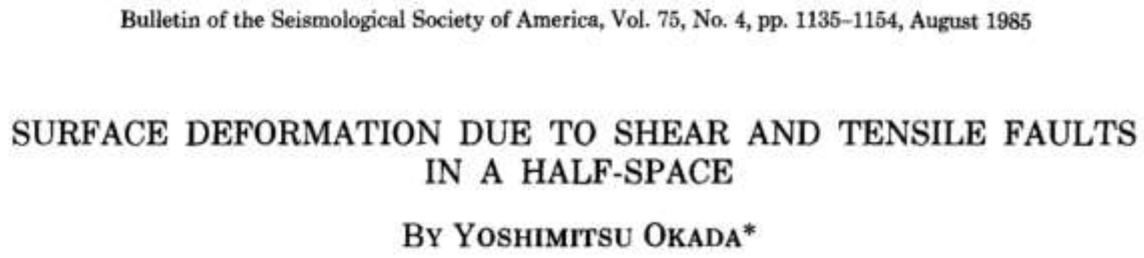
Other More General Displacement Models

The Okada Model

- **General solution for rectangular (1985) and point (1992) sources in an elastic half space**

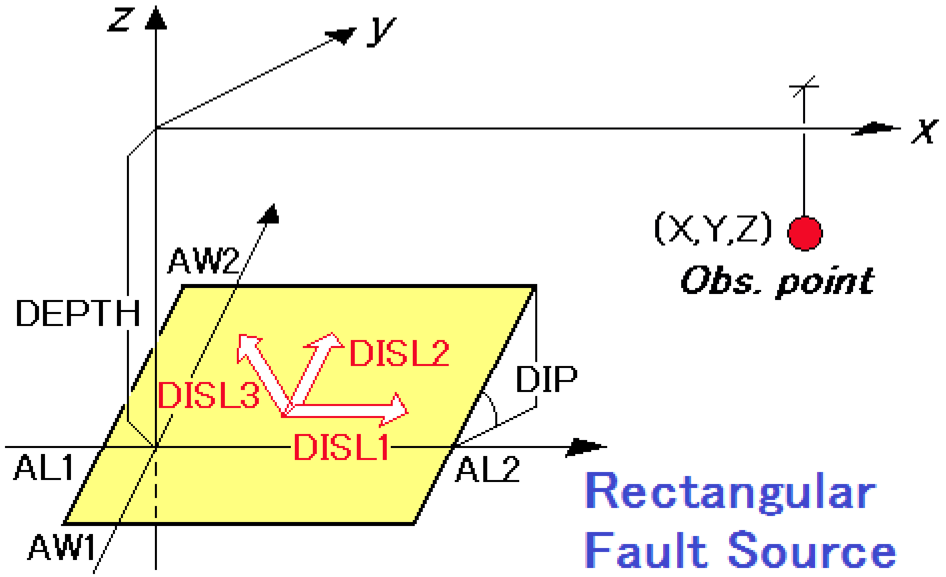
- **Pros**
  - Analytical solution, fast to compute
  - Can model shear (fault slip) and opening (dike or sill intrusion/collapse)

- **Cons**
  - Again, simplifying assumptions are not necessarily realistic
  - Cannot tesselate into complex surfaces


Bulletin of the Seismological Society of America, Vol. 75, No. 4, pp. 1135–1154, August 1985

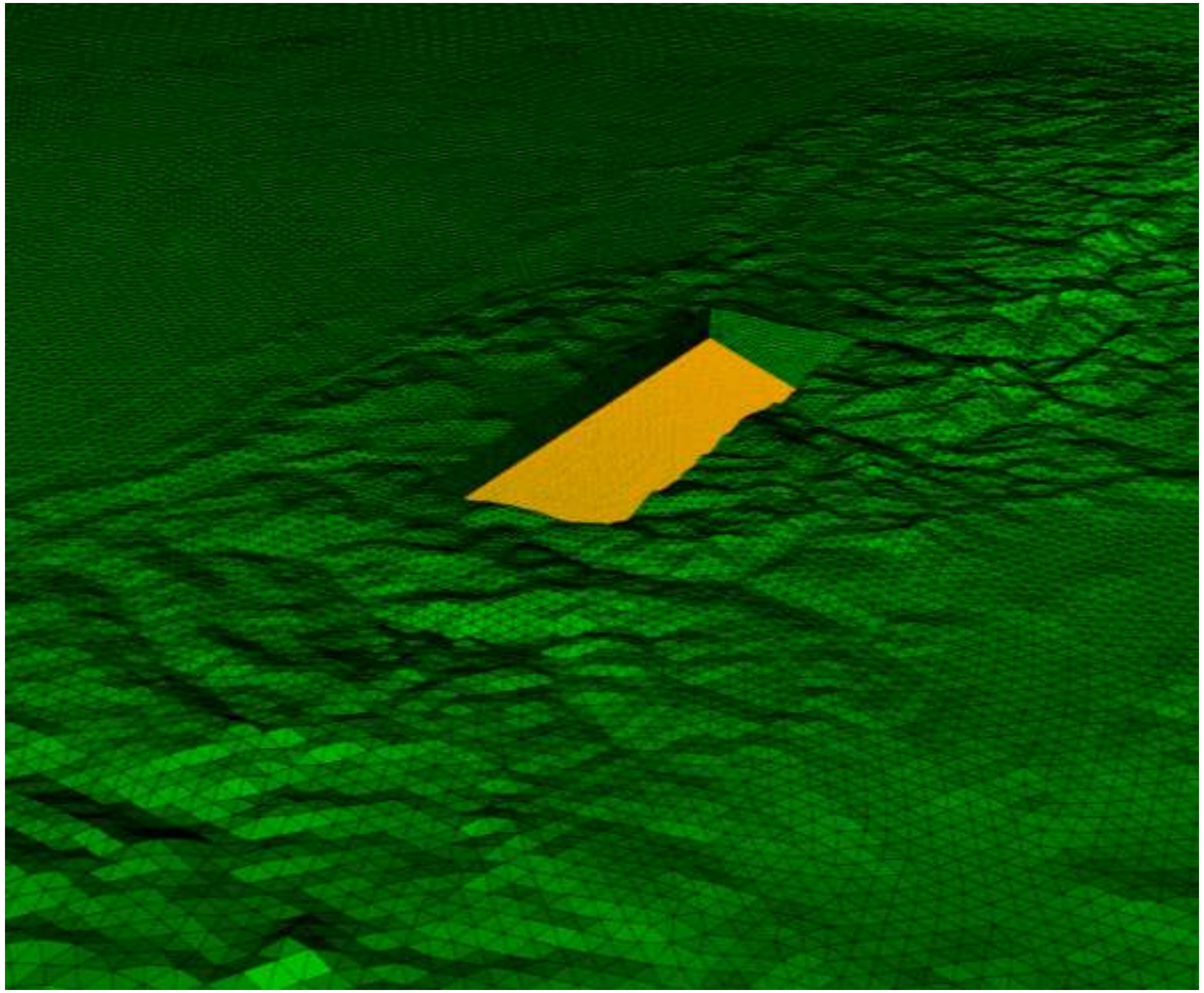
SURFACE DEFORMATION DUE TO SHEAR AND TENSILE FAULTS IN A HALF-SPACE

BY YOSHIMITSU OKADA*



Finite Element Models (FEMs)

- **Can compute displacements and stresses for generalized solids and sources**
- **Good**
  - Can incorporate heterogeneous material properties, nonplanar geometries, realistic topography
  - Can incorporate more complex rheologies (e.g. viscoelasticity)
- **Less good**
  - Making meshes is complicated and slow
  - Computing displacements is expensive (minutes to hours)



Gorkha, Nepal earthquake source region (7 million tetrahedral elements!)

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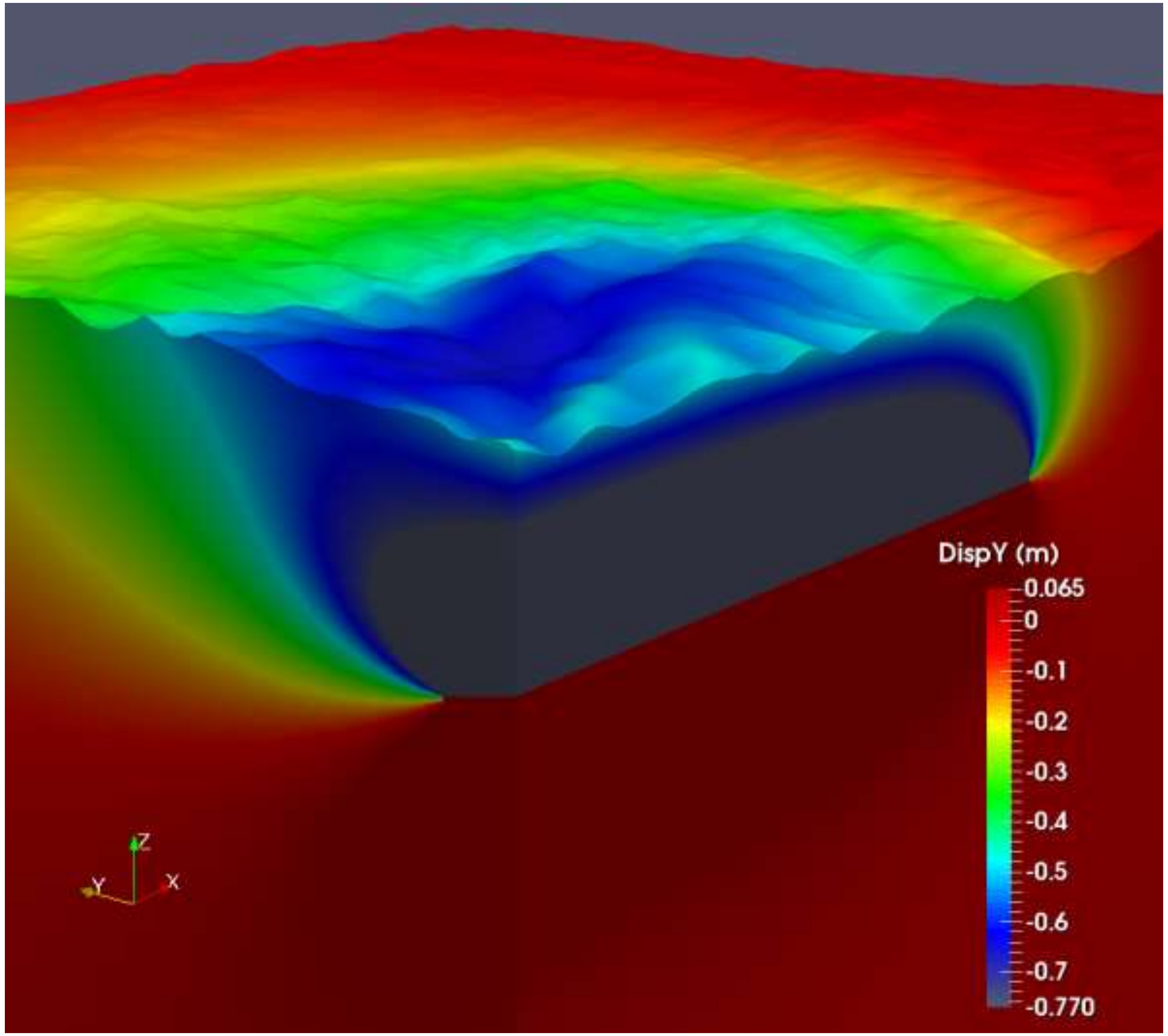
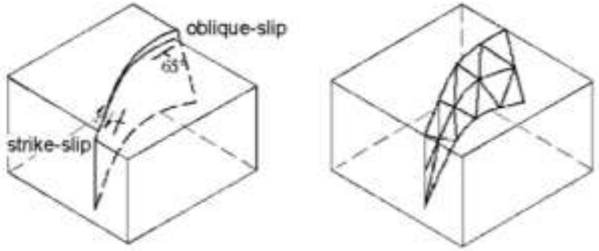


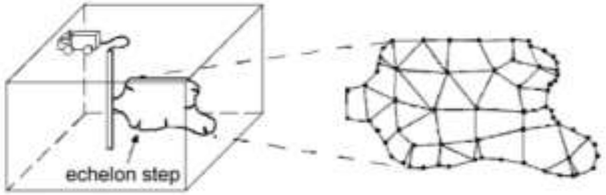
Boundary Element Models

- **Numerical method in which quantities are computed on surfaces rather than in volumes**
- **Yay**
  - Faster than FEMs
  - Polygonal elements can allow complex source geometries
  - Can compute stresses, use driving stresses
- **Nay**
  - Does not allow heterogeneous material properties
  - Slower than analytical codes

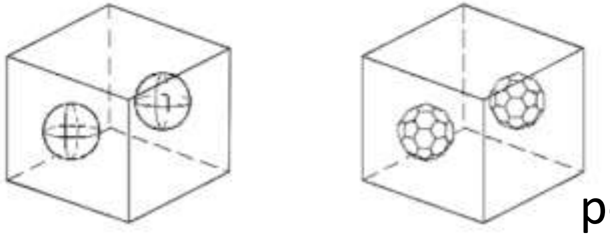
Fault surfaces which change in both strike and dip can be meshed without creating gaps.



Polygonal elements easily replicate the irregular boundary of a hydraulic fracture.



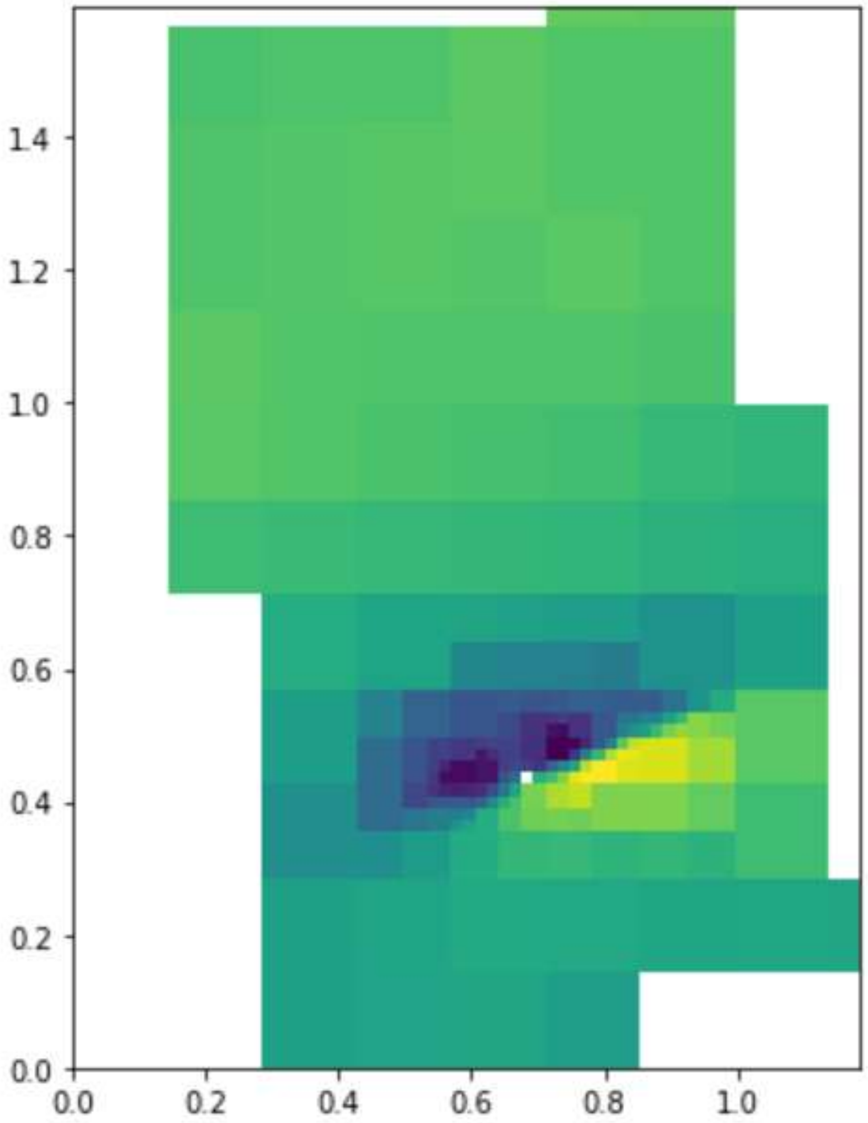
A spherical void can be modeled by assembling hexagonal and pentagonal elements in the manner of a soccer ball.



poly3d manual

Data Down Sampling

- InSAR data are highly spatially correlated – you do not need every pixel to capture information on a process
- Typically more data points → longer computation time → down sampling can make modeling more efficient
- For example, **quadtree decomposition** (right) can divide an unwrapped interferogram into a set of regions with similar variance



Linear vs Non-Linear Inverse Modeling

Linear Inverse Modeling

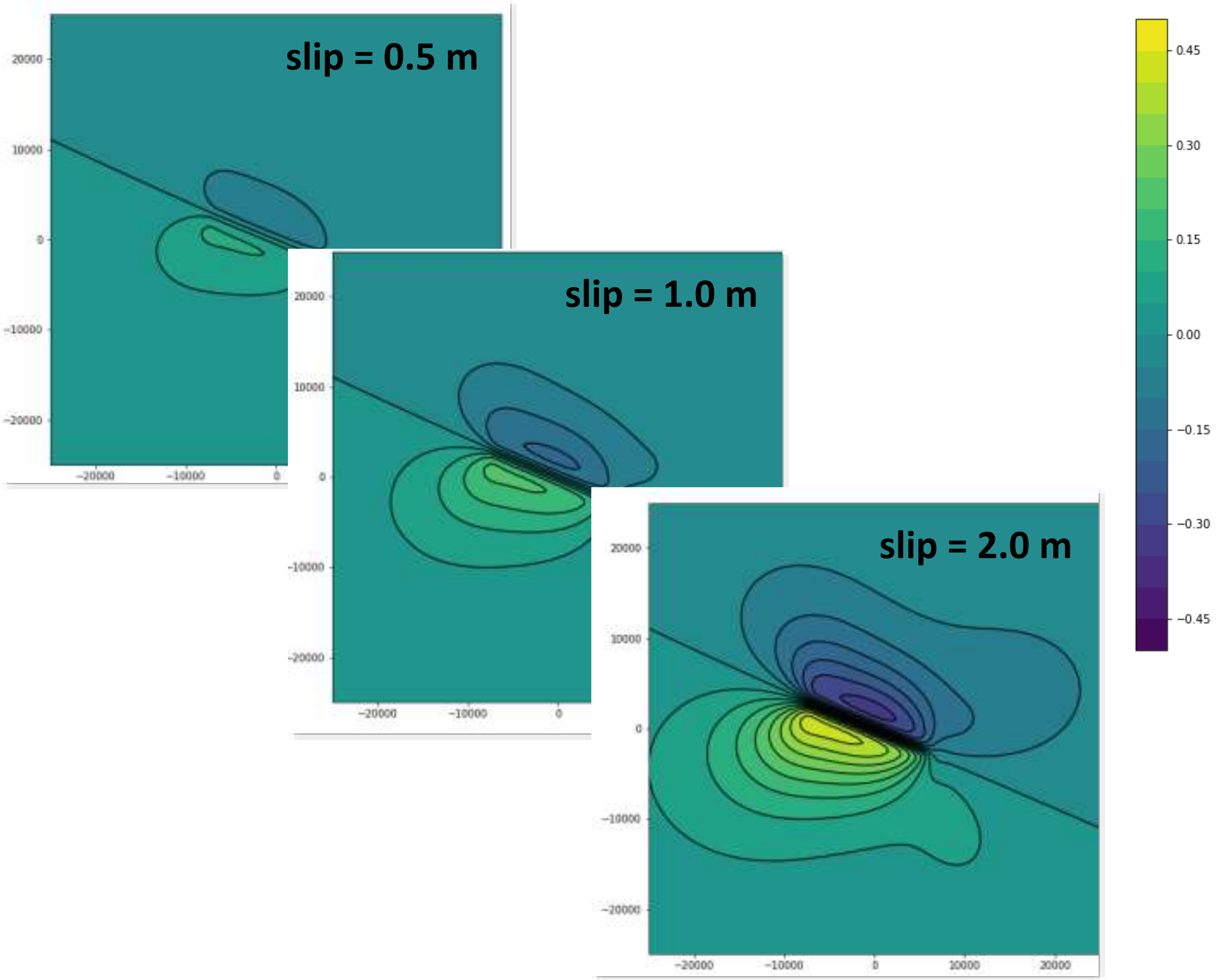
- If you have
  - Fixed model geometry
  - Linear relationship between model parameters and surface displacements

- Things are fairly straightforward!
  - Model simplifies to a matrix inversion problem:

$$\boldsymbol{d} = \boldsymbol{G}\boldsymbol{m}$$

$$\boldsymbol{m} = \left(\boldsymbol{G}^T\boldsymbol{G}\right)^{-1}\boldsymbol{G}^T\boldsymbol{d}$$

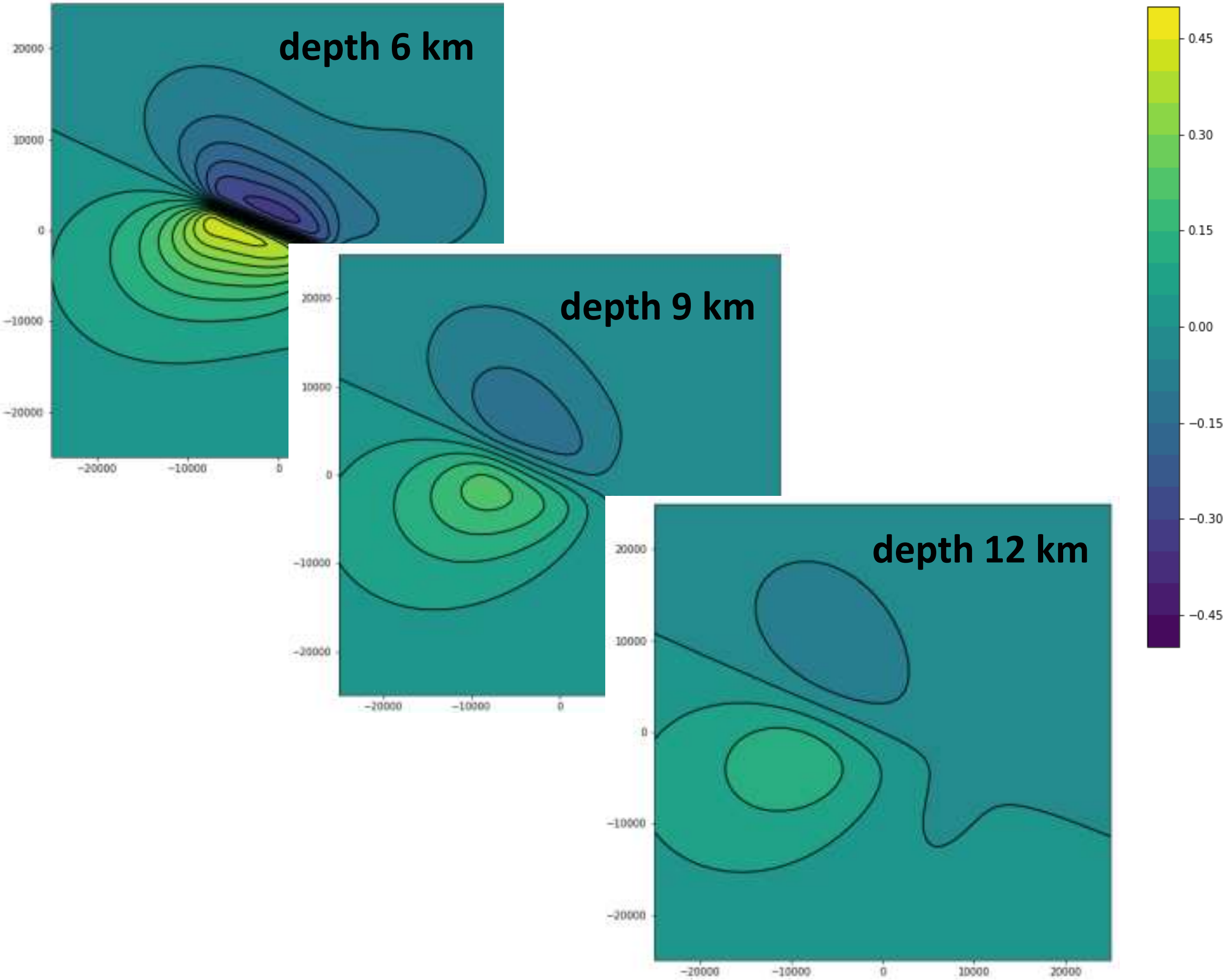
($\boldsymbol{G}$ is a matrix of Green's functions)



Linear vs Non-Linear Inverse Modeling

Non-Linear Inverse Modeling

- Unfortunately, not everything has a linear relationship with displacement!
  - changes in position/depth
  - changes in dimensions
  - changes in orientation
- In these cases, we may use an optimization approach:
  - forward model displacements using guessed model parameters
  - calculate the fit of the forward model to the data
  - vary the model parameters until a good fit is obtained (e.g. by using an algorithm!)



What's Next?

- **This is what awaits next:**
  - **Next week Tuesday**: Lab on Mogi source inversion from InSAR