The Emergence and Persistence of Oligarchy:
A Dynamic Model of Endogenous Political Power

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October 6, 2016

Endogenous Political Power

- The bargaining outcome **today** determines
  the players' bargaining power **tomorrow**.

Endogenous Political Power

- The bargaining outcome **today** determines
  the players' bargaining power **tomorrow**.

- Those who receive a greater share of the resources today
  have greater proposal and voting power tomorrow.

Main Questions

1. What are the distributions of resources and power in the long run?
2. What conditions lead to different long-run outcomes?

Existing Models

	Policy	Proposal power	Voting Power	Key Feature
Rubinstein (1982)	Once-for-all	Fixed	Fixed	Proposer's advantage
Baron and Ferejohn (1989)	Once-for-all	Fixed	Fixed	Proposer's advantage, MWC
Yildirim (2007, 2010)	Once-for-all	Endogenous	Fixed	Recognition rule competition
Penn (2009)	Continuing	Exogenous	Fixed	Farsighted players' preferences
Kalandrakis (2004, 2009)	Continuing	Fixed	Fixed	Proposer's dictatorial power
Diermeier and Fong (2012)	Continuing	Fixed	Fixed	Proposer's limited power
Nunnari (2011)	Continuing	Fixed	Fixed	Vetoer's dictatorial power
Duggan and Kalandrakis (2012)	Continuing	Endogenous	Endogenous	General existence result
Current Model	**Continuing**	**Endogenous**	**Endogenous**	**Tyranny or oligarchy**

Model: Setup

Multilateral Bargaining over a fixed amount of surplus

- $I = \{1, \ldots, n\}$
- $X = \{\boldsymbol{x} \in \mathbb{R}^n | \sum_{i \in I} x_i = 1, x_i \geq 0, \forall i \in I\}$
- Short-term utility $u_i(\boldsymbol{x}^t) = x_i^t$
- Long-term utility from $\{\boldsymbol{x}^1, \boldsymbol{x}^2, \ldots\}$: $\frac{\sum_{t=1}^{T} u_i(x^t)}{T}$
- State $\boldsymbol{s}^t = (\boldsymbol{x}^{t-1}, \boldsymbol{p}^t, \boldsymbol{w}^t)$
  - $\boldsymbol{x}^{t-1}$: status quo division
  - $\boldsymbol{p}^t$: recognition probability
  - $\boldsymbol{w}^t$: voting weights
- Evolution of proposal and voting power: $\boldsymbol{p}^{t+1} = \boldsymbol{w}^{t+1} = \boldsymbol{x}^t$

Model: Timing

Period t in state $\boldsymbol{s}^t = (\boldsymbol{x}^{t-1}, \boldsymbol{p}^t, \boldsymbol{w}^t)$,

1. Random selection of a proposer according to $\boldsymbol{p}^t$
2. Proposer makes a proposal $\boldsymbol{y} \in X$.
3. All players simultaneously vote *Yes* or *No* with $\boldsymbol{w}^t$.
4. $\boldsymbol{x}^t = \boldsymbol{y}$ if voting weights of $yes > no$; otherwise $\boldsymbol{x}^t = \boldsymbol{x}^{t-1}$
5. Short-term utility $u_i(\boldsymbol{x}^t) = x_i^t$ at the end of t

Period $t+1$ in state $\boldsymbol{s}^{t+1} = (\boldsymbol{x}^t, \boldsymbol{p}^{t+1}, \boldsymbol{w}^{t+1})$

Model: Strategies

- Each player's strategy=(proposal strategy, voting strategy)
- Markov strategies: same state and same player $\Rightarrow$ same action
- Player i's Markov strategy $\sigma_i = (\mu_i, A_i)$
  - $\mu_i : S \to \Delta X$,
    where ΔX is the set of probability distributions over X
  - $A_i : S \twoheadrightarrow X$

Model: Preferences

- Given $\boldsymbol{\sigma}$, player i's T-period expected utility from $\boldsymbol{x}$:

$$U_i^{\sigma,T}(\boldsymbol{x}) = \frac{1}{T}\left[u_i(\boldsymbol{x})+(T-1)v_i^{\sigma,T-1}(\boldsymbol{x})\right]$$

$$v_i^{\sigma,T-1}(x) \equiv \sum_{j\in I} x_j \Big[\sum_{y:\mu_j(y|x)>0} \mu_j(y|x)\Big[\frac{u_i(y)+(T-2)v_i^{\sigma,T-2}(y)}{T-1}\Big]I_{A(x)}(y) + \sum_{y:\mu_j(y|x)>0} \mu_j(y|x)\Big[\frac{u_i(x)+(T-2)v_i^{\sigma,T-2}(x)}{T-1}\Big]I_{X\setminus A(x)}(y)\Big]$$

- Continuation value $v_i^{\sigma,T-1}(x)$: player i's *ex ante* expected value from state x before the identity of the proposer is known.

Model: Preferences

- For $T \to \infty$, players' long-term preferences by the overtaking criterion
- For $x, y \in X$, i prefers x to y $[x \succeq_i y]$ in σ if and only if

$$\liminf_{T \to \infty} T[U_i^{\sigma,T}(x) - U_i^{\sigma,T}(y)] \geq 0.$$

- $x \succ_i y \quad$ if $\lim_{T\to\infty} U_i^{\sigma,T}(x) > \lim_{T\to\infty} U_i^{\sigma,T}(y)$
- If $\lim_{T\to\infty} U_i^{\sigma,T}(x) = \lim_{T\to\infty} U_i^{\sigma,T}(y)$, the overtaking criterion takes account of the payoff differences in the finite number of initial periods.

Model: Equilibrium Notion

Symmetric Markov perfect Nash equilibria
in stage-undominated voting strategies

- Symmetry: Players' names do not matter.
- Stage-undominated voting: $y \in A_i^*(s) \Leftrightarrow y \succeq_i s$
- Optimal proposal:
  $\mu_i^*(y|s) > 0 \Rightarrow y \in \{x \in A^*(s) : x \succeq_i x', \forall x' \in A^*(s)\}$
- Status quo bias: $s \sim_i y, \forall y \in A^*(s) \ \Rightarrow \mu_i^*(s) = s$

Analysis

- Think backwardly
  - from the states with the most concentrated power structure
- It simplifies the complexity involving the players' expected utility assessments on different resource allocation plans.

Partitions of States

Partitions of States: preliminary

- Decisive coalition: the sum of its members' voting weights $> \frac{1}{2}$
- A player has a veto if he can prevent the passing of the proposals that he disapproves.
- If a player has $\frac{1}{2}$ or more voting power, then he is a veto player.

Partitions of States: Tyrannical

- A state is **tyrannical** if there is a player who possesses the entire proposal and voting power.

Partitions of States: Dictatorial

- A state is **dictatorial** if there is a player who alone constitutes a decisive coalition.
- There is a player who has a majority of voting power.

Partitions of States: Oligarchic

- A state is **oligarchic** if there is a decisive coalition such that every member of which has a veto.
- Two veto players

Partitions of States: Collegial

- A state is **collegial** if there is at least one single player in every decisive coalition.
- A single veto player

Partitions of States: Noncollegial

- A state is **noncollegial** if no single player is in every decisive coalition.
- No veto player

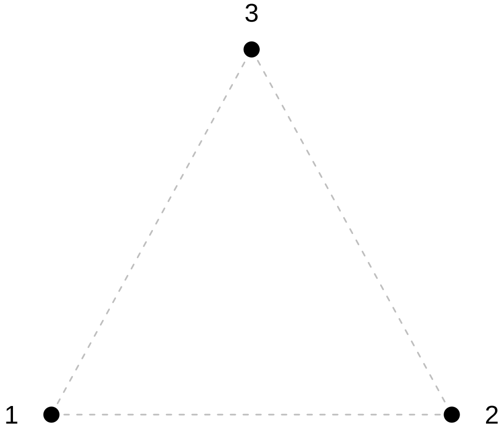
Partitions of States

A state is,

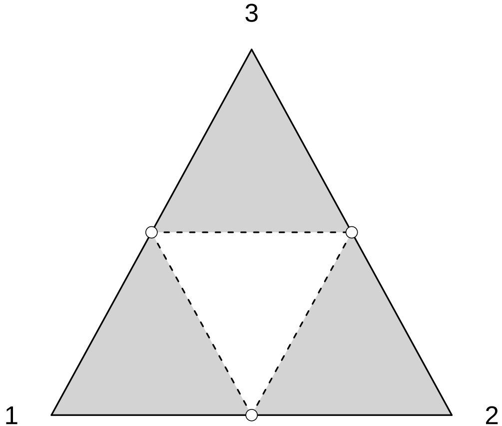
- noncollegial if no player is in every decisive coalition;
- collegial if there is at least one player in every decisive coalition;
- oligarchic if there is a decisive coalition such that every member of which has a veto;
- dictatorial if there is a player who alone constitutes a decisive coalition;
- tyrannical if there is a player who monopolizes proposal and voting power.

Partitions of states
Three-player

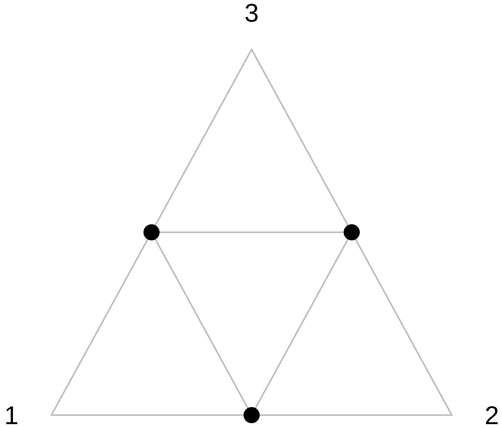
Classes of States: Tyrannical



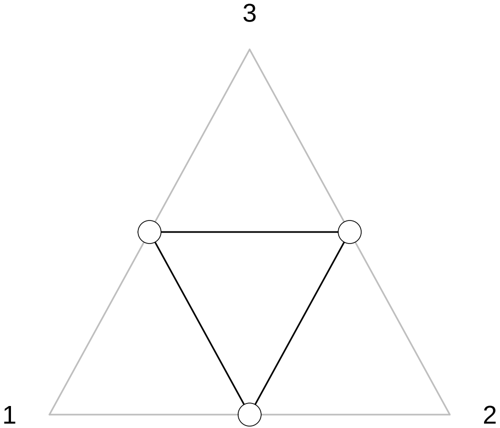
Classes of States: Dictatorial



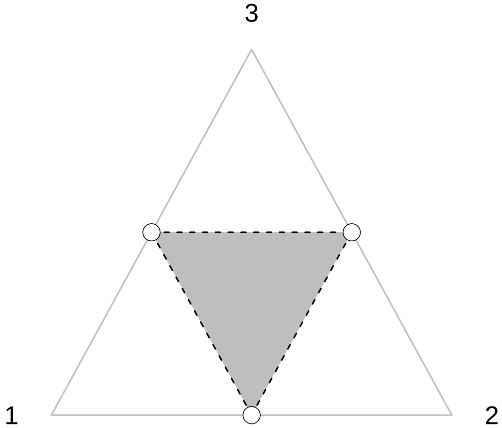
Classes of States: Oligarchic (Two veto players)



Classes of States: Collegial (One veto player)

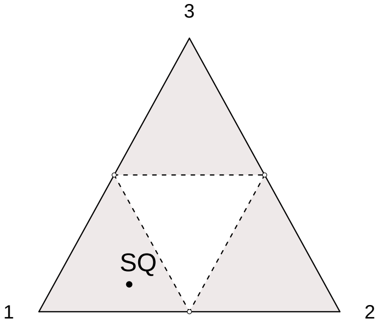


Classes of States: Noncollegial (No veto player)



Analysis

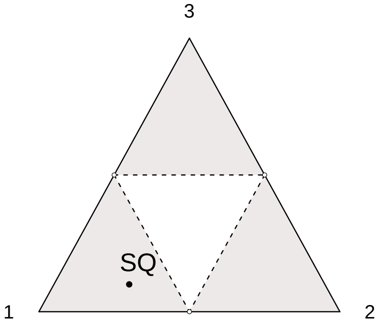
Dictatorial States

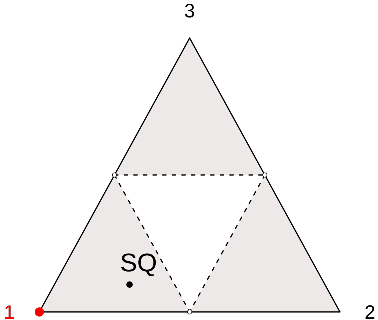


$\boldsymbol{x} = (x_1, x_2, x_3)$ with $x_1 \geq \frac{1}{2}$

Player 1 is a dictator.

Dictatorial States





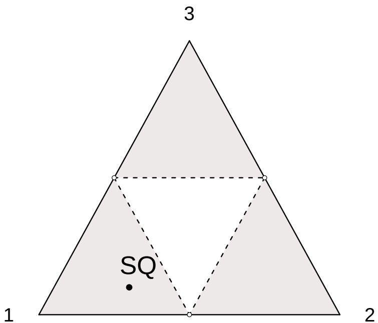
$\boldsymbol{x} = (x_1, x_2, x_3)$ with $x_1 \geq \frac{1}{2}$

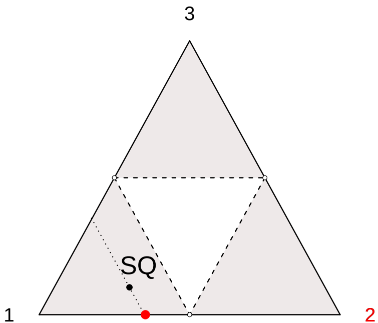
Player 1 is a dictator.

Player 1's proposal: $(1, 0, 0)$

Tyrannical State

Dictatorial States





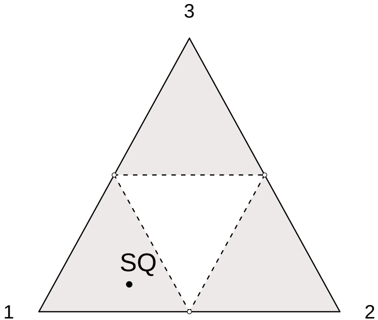
$\boldsymbol{x} = (x_1, x_2, x_3)$ with $x_1 \geq \frac{1}{2}$

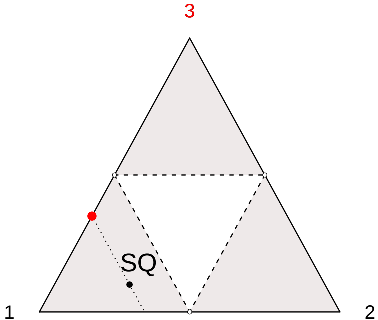
Player 1 is a dictator.

Player 2's proposal: $(x_1, 1 - x_1, 0)$

for any discount factor

Dictatorial States





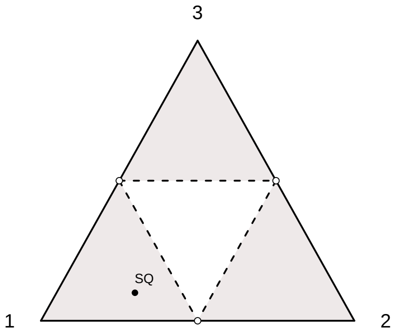
$\boldsymbol{x} = (x_1, x_2, x_3)$ with $x_1 \geq \frac{1}{2}$

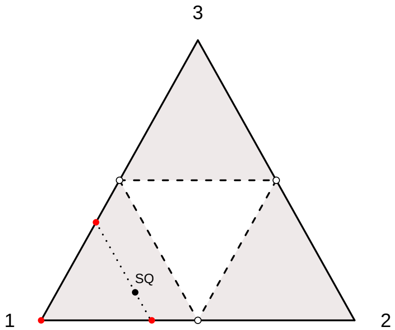
Player 1 is a dictator.

Player 3's proposal: $(x_1, 0, 1 - x_1)$

for any discount factor

Dictatorial States





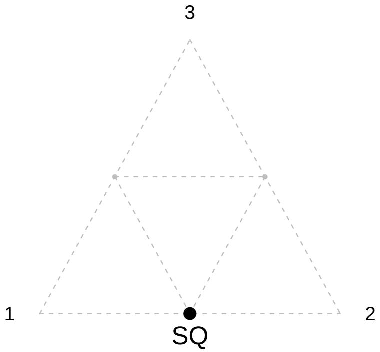
$x = (x_1, x_2, x_3)$ with $x_1 \geq \frac{1}{2}$

Player 1 is a dictator.

$x_1' = 1$ (tyrannical state)

or $x_1' = x_1$

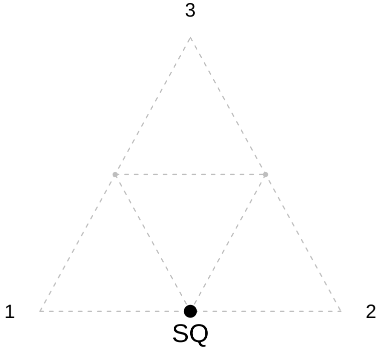
Oligarchic States

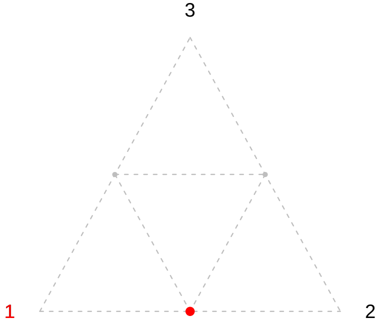


$$x = (\tfrac{1}{2}, \tfrac{1}{2}, 0)$$

Oligarchy of players 1 and 2

Oligarchic States





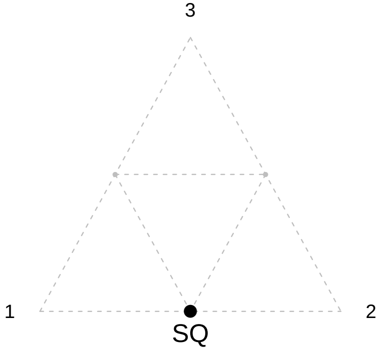
$x = (\frac{1}{2}, \frac{1}{2}, 0)$

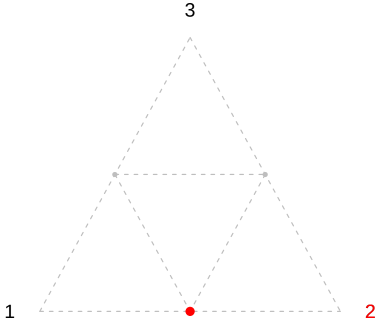
Oligarchy of players 1 and 2

Player 1's proposal is x

for any discount factor

Oligarchic States





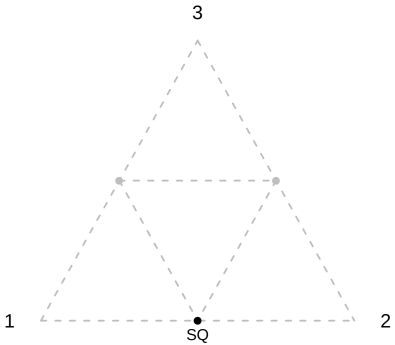
$x = (\frac{1}{2}, \frac{1}{2}, 0)$

Oligarchy of players 1 and 2

Player 2's proposal is x

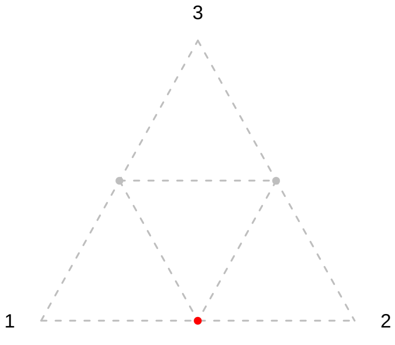
for any discount factor

Oligarchic States



$$\boldsymbol{x} = \left(\tfrac{1}{2}, \tfrac{1}{2}, 0\right)$$

Oligarchy of players 1 and 2

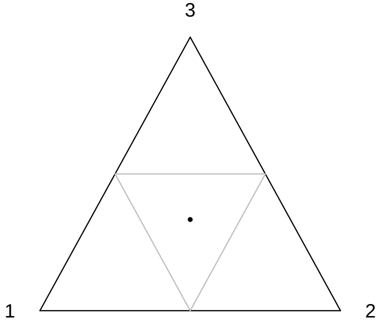


$$\boldsymbol{x}' = \boldsymbol{x}$$

The state perpetuates.

Collegial State

The equilibrium proposals in collegial states:
- Collegium player: status quo
- Other players: oligarchic division in which the collegium player and



self share the pie.

No Tyranny but Oligarchy

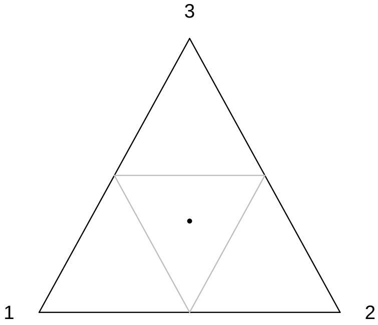
- There is no transition from noncollegial states to dictatorial or tyrannical state in equilibrium.
- If the initial state is noncollegial, the long-run outcome is a permanent oligarchy of two players in equilibrium.

Farsighted Players

- Non-dictator in a dictatorial state
- Any player in a non-dictatorial state

Myopic Players

Initial state $s(\boldsymbol{x})$ with $\boldsymbol{x} = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ and $\delta = 0$



Myopic Players

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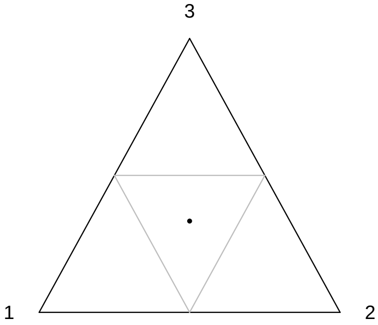
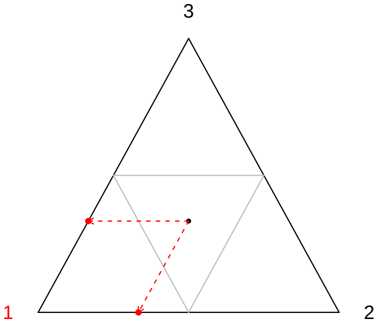
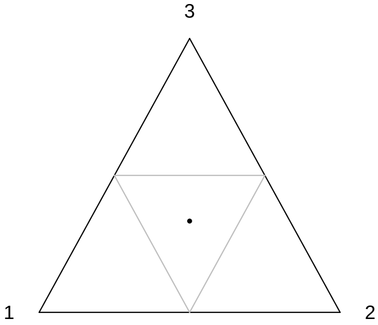
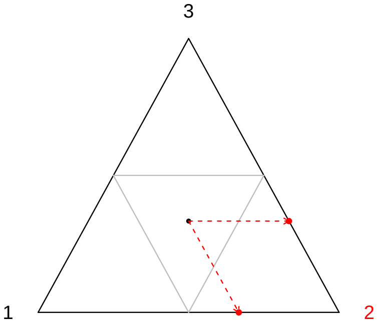




Myopic Players

Initial state $s(\boldsymbol{x})$ with $\boldsymbol{x} = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ and perfect myopia



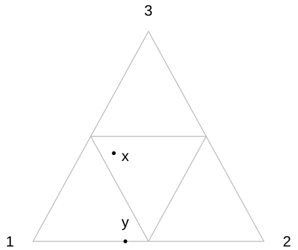


Myopic Players

- Permanent tyranny of a player in the long-run
- From almost all the initial states

Farsighted Players: No Dictatorship

If the players are farsighted, there is no transition from non-dictatorial states to any dictatorial states.



$$\boldsymbol{x} = (\tfrac{3}{7}, \tfrac{1}{7}, \tfrac{3}{7})$$

$$\boldsymbol{y} = (\tfrac{4}{7}, \tfrac{3}{7}, 0)$$

- Player 2's share: $\boldsymbol{y} > \boldsymbol{x}$
- Player 2's prefers $\boldsymbol{x}$ to $\boldsymbol{y}$
- Farsighted players bear short-term costs for long-term gains.
- Farsighted players care about other players' shares.

Main Result

There exists an equilibrium. In all equilibria, the following statements are true:

(i) If the initial state is dictatorial or tyrannical, the long-run outcome is a permanent tyranny of a single player. The dictator or tyrant in the initial state turns into a permanent tyrant.

(ii) If the initial state is not dictatorial or tyrannical, the long-run outcome is a permanent oligarchy of two players who equally share the entire wealth and power. If the initial state is oligarchic, the oligarchy perpetuates. If the initial state is collegial, the first proposer is always in the permanent oligarchy. If the initial state is noncollegial, the second proposer is always in the permanent oligarchy.

Conclusion

Regardless of the number of players,

- The long-run distribution of resources and power
  - Tyranny of one player
  - Oligarchy of two players

- Initial condition and farsightedness of players
  - Dictatorial initial state or myopic players $\Rightarrow$ Tyranny
  - Nondictatorial state and farsighted players $\Rightarrow$ Oligarchy