MATH 61A Problem Set 1

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Problem 1

Problem 1

Draw the truth table for $P \implies R$, and $P \implies Q$ and $Q \implies R$. Explain in words, referring to the truth tables, why the latter statement implies the former.

Truth table

P	Q	R	$P \implies Q$	$Q \implies R$	$P \implies R$	Explanation	$(P \implies Q$ and $Q \implies R) \implies (P \implies R)$
T	T	T	T	T	T	1	T
T	T	F	T	F	F	2	T
T	F	T	F	T	T	3	T
T	F	F	F	T	F	4	T
F	T	T	F	T	F	5	T
F	T	F	T	F	T	6	T
F	F	T	T	T	T	7	T
F	F	F	T	T	T	8	T

1. When P, Q, and R are all true, $P \implies Q$ and $Q \implies R$ are both true statements. $P \implies Q \wedge Q \implies R$ is true and true, which is true. Since $P \implies R$ is also true, we now have a true implies true statement, which is true.

2. In this case, $P \implies Q$ is true because true $\implies$ true is true. $Q \implies R$ and $P \implies R$ are false because true $\implies$ false is false. $P \implies Q \wedge Q \implies R$ is false because true and false is false. $P \implies R$ is false for the same reason. Now we have the statement false implies false, which is true.

3. $P \implies Q$ is false because true implies false is false. $Q \implies R$ is true because false implies true is true. $P \implies R$ is true because true implies true is true. $P \implies Q \wedge Q \implies R$ is true and false, which is false. Now we have the statement false implies true, which is true.

4. $P \implies Q$ is false because false implies true is false. $Q \implies R$ is true because false implies false is true. $P \implies R$ is false because true implies false is false. $P \implies Q \wedge Q \implies R$ is false and true, which is false. Now we have the statement false implies false, which is true.

5. $P \implies Q$ is false because false implies true is false. $Q \implies R$ is true because true implies true is true. $P \implies R$ is true because false implies true is true. $P \implies Q \wedge Q \implies R$ is false and true, which is false. Now we have the statement false implies false, which is true.

6. $P \implies Q$ is false because false implies true is true. $Q \implies R$ is false because true implies false is false. $P \implies R$ is true because false implies false is true. $P \implies Q \wedge Q \implies R$ is true and false, which is false. Now we have the statement false implies true, which is true.

7. $P \implies Q$ is true because false implies false is true. $Q \implies R$ is true because false implies true is true. $P \implies R$ is true because false implies true is true. $P \implies Q \wedge Q \implies R$ is true and true, which is true. Now we have the statement true implies true, which is true.

8. $P \implies Q$ is true because false implies false is true. $Q \implies R$ is true because false implies false is true. $P \implies R$ is true because false implies false is true. $P \implies Q \wedge Q \implies R$ is true and true, which is true. Now we have the statement true implies true, which is true.

1 Exercise 1.5.1

1. $\neg P$

P	$\neg P$
T	F
F	T

This is equivalent to $P|P$

P	P	$P|P$
T	T	F
F	F	T

2. $P \land Q$

P	Q	$P \land Q$
T	T	T
T	F	F
F	T	F
F	F	F

This is equivalent to $(P|Q)|(P|Q)$

$P|Q$	$P|Q$	$(P|Q)|(P|Q)$
F	F	T
T	T	F
T	T	F
T	T	F

3. $P \lor Q$

P	Q	$P \lor Q$
T	T	T
T	F	F
F	T	F
F	F	F

We can rewrite this as $\neg(\neg P \land \neg Q)$. From the first part of the problem, we can know that $\neg P = P|P$ and $\neg Q = Q|Q$. From the second problem, we know that $P \land Q = (P|Q)|(P|Q)$. Therefore, $\neg(\neg P \land \neg Q) = ((P|P)|(P|P))|((Q|Q)|(Q|Q))$.

2 1.5.7

Find the cardinalities of the following sets

1. $\{1, 2\} \cup \mathcal{P}(\{1, 2\})$

$$\mathcal{P}(\{1, 2\}) = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$$

We could have also calculated the cardinality of this set by using the general formular:

$$|\mathcal{P}(A)| = 2^{|A|}$$

$$|\mathcal{P}(\{1, 2\})| = 2^2 = 4$$

$$|\{1, 2\}| = 2$$

$$\{1, 2\} \cup \mathcal{P}(\{1, 2\}) = \{1, 2, \emptyset, \{1\}, \{2\}, \{1, 2\}\} = 2 + 4$$

There are 6 unique elements in this set, meaning the cardinality of this set is **6**

2. $\{\{1\}, \{2\}\} \cup \mathcal{P}(\{1, 2\})$

$$\{\{1\}, \{2\}\} \cup \mathcal{P}(\{1, 2\}) = \{\{1\}, \{2\}, \emptyset, \{1\}, \{2\}\} = \{\emptyset, \{1\}, \{2\}\}$$

Since the union of these sets only has 3 unique elements, the cardinality of this set is **3**.

3. $\{0, 1, 2, 3\} \times \{\emptyset, \{0, 1, 2, 3\}$

The cardinality of one set times another is the product of the cardinalities of the two sets.

$$|\{0, 1, 2, 3\}| = 4$$

$$|\{\emptyset, \{0, 1, 2, 3\}| = 2$$

$$|\{0, 1, 2, 3\} \times \{\emptyset, \{0, 1, 2, 3\}| = 4 \times 2 = 8$$

The cardinality of this set is **8**.