MATH 61A Problem Set 1

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Problem 1

Draw the truth table for $P \to R$, and $P \to Q$ and $Q \to R$. Explain in words, referring to the truth tables, why the former statement implies the latter.

Truth tables for $P \to R, P \to Q$, and $Q \to R$

P	R	$P \rightarrow R$		P	Q	P o Q	Q	R	$Q \to R$
Т	Т	Т		Т	Т	Т	Τ	Т	Τ
T	F	F	İ	Τ	F	F	Τ	F	\mathbf{F}
F	Т	Т		F	Т	Т	F	Т	${ m T}$
F	F	T		\mathbf{F}	F	Т	\mathbf{F}	F	${ m T}$

Truth table for $P \to Q$ and $Q \to R$

$P \rightarrow Q$	$Q \to R$	$P \to Q \text{ and } Q \to R$
T	Т	Т
T	F	F
F	Τ	F
F	F	F

From these truth tables, we can observe that $P \to R$ is false when P is true and R is false. There are two cases when P is true, and Q is either true or false, providing us with two cases, either $P \to Q$ is true or false. In the case where the statement is true, Q is true, and vice versa. Now, let us consider when R is false. When Q is false but R is true, $Q \to R$ is false. When Q and R are false, $Q \to R$ is true.

Using these observations, I will explain how the former statement implies the latter.