# MATH 32A Problem Set 7

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## 1 Question 1

Suppose the tangent plane to a function $g(x, y)$ at a point $P$ has a normal vector $\langle 0, -1, 1\rangle$. If you increase the value of $y$ by a small amount, do you expect that the function will increase in value, decrease in value, or stay the same? Explain your answer.

Let's take into consideration the tangent plane. Let $P = (x_0, y_0)$ The equation of the tangent plane is:

$$0(x - x_0) - 1(y - y_0) + (g(x, y) - g(x_0, y_0)) = 0$$

$$g(x, y) = y - y_0 + g(x_0, y_0)$$

From this simplified form, we can see that if we increase the value of $y$, the function $g(x, y)$ will increase as well.

## 2 Question 2

Let $I = W/H^2$ denote body mass index, where $W$ is body weight and $H$ is the body height. Suppose that $(W, H) = (30, 1.2)$. Use linearization and/or differentials to estimate the change in height that would cause BMI to change by 3, if weight is held constant.

$$I_H = -2\frac{W}{H^3}$$

$$-2\frac{W}{H^3} \cdot \Delta H = 3$$

$$-2\frac{30}{1.2^3} \cdot \Delta H = 3$$

$$\Delta H = -0.0864$$

# 3 Question 3

Let $g(x, y) = 3\sin^2(x)y^4$. Compute the directional derivative of $g$ in the direction of the vector $\langle 1, 1\rangle$ at the point $(0, 2)$

$$g_x = 6\sin(x)\cos(x)y^4 \qquad 12\sin^2(x)y^4$$

$$\nabla g(0, 2) = \langle 6\sin(x)\cos(x)y^4, 12\sin^2(x)y^3\rangle = \langle 0, 12\sin^2(x)y^3\rangle = \langle 0, 0\rangle$$

$$D_{\vec{u}}\nabla g = \nabla g \cdot \vec{u} = \langle 0, 0\rangle \cdot \frac{1}{\sqrt{2}}\langle 1, 1\rangle = 0 + 0 = 0$$

The directional derivative is simply 0

# 4 Question 4

Let $f(x, y, z) = x^2\sqrt{y} - 7z$. Determine the direction from the point $(1, 4, 1)$ in which $f$ is increasing the fastest, and calculate the rate of change in that direction

$$\nabla f = \langle 2x\sqrt{y}, \frac{x^2}{2\sqrt{y}}, -7\rangle$$

$$\nabla f(1, 4, 1) = \langle 4, \frac{1}{4}, -7\rangle$$

$$\text{rate} = \sqrt{4^2 + (\frac{1}{4})^2 + (-7)^2} = \sqrt{65 + \frac{1}{16}} = \sqrt{\frac{1041}{16}}$$

$$\text{The direction at which } f \text{ is increasing the fastest is } \sqrt{\frac{16}{1041}} \cdot \langle 4, \frac{1}{4}, -7\rangle$$

## 5 Question 5

Plot level curves for the function $f(x, y) = xy$ at heights -1, 0, and 1. Plot $\nabla(f)$ at the points $(0, 1), and(1, -1)$

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