Math 170E Lecture 1

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Basic Notions of Probability Theory

Consider an experiment with unknown probabilistic outcome, set of all possible outcomes S is called the *sample space*

Ex) For the experiment where a coin is flipped three times,

$$S = \{\text{TTT, TTH, THH, THT, HTT, HTH, HHT, HHH}\}$$

Elements $s \in S$ are outcomes, while $A \subset S$ are events

We consider a function $\mathbb{P} : \{A \subset S\} \to [0, 1]$, which heuristically assigns to each event A the probability $\mathbb{P}(A)$ that it occurs. P also satisfies some natural properties like $\mathbb{P}(A \sqcup B)$

$$A \sqcup B = A \cup B \text{ in the case that } A \cap B = \emptyset$$

Conditional Probability

$$\mathbb{P} := \frac{\mathbb{P} = (A \cap B)}{\mathbb{P}(B)}$$

This equation represents the conditional probability of event A occurring given that event B has already occurred. In other words, it's the probability that A happens under the condition that B is true.

Example Problem

Given that you flip a coin 3 times, what are the chances that

1. You get exactly 2 heads?
2. You get exactly 2 heads if the first flip is heads
3. The first flip was heads if you know there were exactly two heads

Problem 1

The total number of combinations is 2 * 2 * 2 = 8

Writing out the sample space, we know that the possible outcomes are:

$$S = \{\text{TTT, TTH, THH, THT, HTT, HTH, HHT, HHH}\}$$

There are 3 cases where there are 2 heads, so the answer is $\boxed{\frac{3}{8}}$

Problem 2

If we look at the cases where the first flip was heads, we are left with {HTT, HHT, HTH, HHH}. Then out of these 4 choices, we only have 2 cases where there are 2 heads, so the solution is $\boxed{\frac{1}{2}}$

Problem 2

The subset which includes all elements with 2 heads is $\{THH, HTH, HHT\}$. From this, we can observe that there are two elements where the combination begins with heads, so the solution is $\boxed{\frac{2}{3}}$