

MATH 32A Midterm 2 Review

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1 14.5 Question 33

Find the coefficient a_T and a_N as a function of t

$$r(t) = \langle t, \cos t, \sin t \rangle$$

When considering the decomposition of $\mathbf{a} = a_T T + a_N N$, we have

$$a_T = \mathbf{a} \cdot T = \frac{\mathbf{a} \cdot v}{\|v\|} \quad a_N = \mathbf{a} \cdot N = \sqrt{\|\mathbf{a}\|^2 - \|a_T\|^2}$$

$$v(t) = \langle 1, -\sin t, \cos t \rangle$$

$$a(t) = \langle 0, -\cos t, -\sin t \rangle$$

$$\mathbf{a} \cdot \mathbf{T} = \frac{\mathbf{a} \cdot \mathbf{v}}{\|\mathbf{v}\|} = \frac{\langle 1, -\sin t, \cos t \rangle \cdot \langle 0, -\cos t, -\sin t \rangle}{\sqrt{1^2 + (-\sin)^2 + \cos^2}} = \frac{0}{\sqrt{2}} = 0$$

$$a \cdot N = \sqrt{1 + 0^2} = 1$$

2 14.5 Question 43

Find the decomposition of $\mathbf{a}(t)$ into tangential and normal components at the points indicated

$$r(t) = \langle t, \cos t, t \sin t \rangle, t = \frac{\pi}{2}$$

$$a_T T = \frac{a \cdot v}{v \cdot v} v \quad a = \frac{a \cdot v}{v \cdot v} v$$

$$v(t) = \langle 1, -\sin t, \sin t + t \cos t \rangle$$

$$v\left(\frac{\pi}{2}\right) = \langle 1, -1, 1 \rangle$$

$$a(t) = \langle 0, -\cos t, \cos t + \cos t + t \sin t \rangle$$

$$a\left(\frac{\pi}{2}\right) = \langle 0, 0, \frac{\pi}{2} \rangle$$

$$a_T T = \frac{\frac{\pi}{2}}{3} \langle 1, -1, 1 \rangle = \langle \frac{\pi}{6}, -\frac{\pi}{6}, \frac{\pi}{6} \rangle$$

$$a_N N = \langle 0, 0, \frac{\pi}{2} \rangle - \langle \frac{\pi}{6}, -\frac{\pi}{6}, \frac{\pi}{6} \rangle = \langle -\frac{\pi}{6}, \frac{\pi}{6}, \frac{\pi}{3} \rangle$$