- ME408 Project Part 3 -

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29450

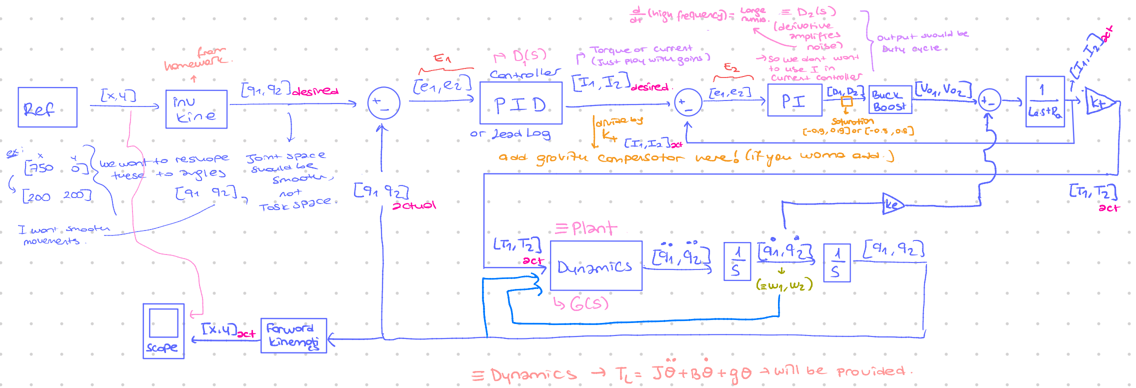
Project Report: Planar Elbow Manipulator Control Design

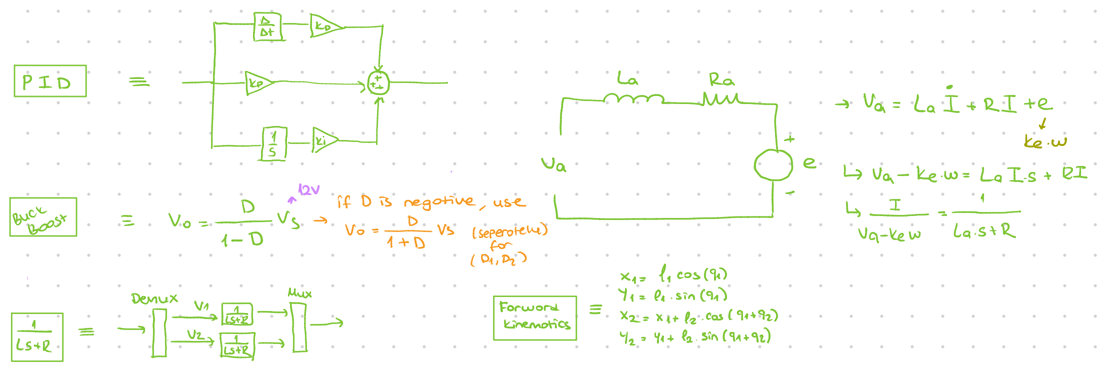
1. Introduction

The objective of this project was to design and implement control systems for a planar elbow manipulator. The control design involves two main loops: an inner loop for the electrical system and an outer loop for the manipulator dynamics. This report details the implementation process, including the tuning of controllers and troubleshooting steps taken to achieve the desired system performance.

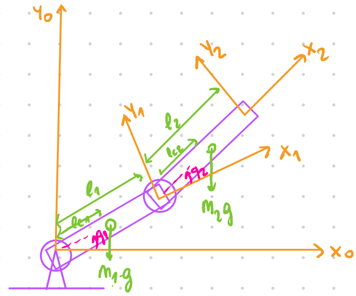
The following figure shows the complete model design and dynamics modelling.

Complete model design:





Simpified dynamics model:



$$\begin{bmatrix} x_{c1} \\ y_{c1} \\ z_{c1} \end{bmatrix} = \begin{bmatrix} l_{c1} \cdot \cos(q_1) \\ l_{c1} \cdot \sin(q_1) \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x_{c2} \\ y_{c2} \\ z_{c2} \end{bmatrix} = \begin{bmatrix} l_1 \cdot \cos(q_1) + l_{c2} \cdot \cos(q_1+q_2) \\ l_1 \cdot \sin(q_1) + l_{c2} \cdot \sin(q_1+q_2) \\ 0 \end{bmatrix}$$

(slender bar)

$$I_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{m \cdot l_1^2}{12} & 0 \\ 0 & 0 & \frac{m \cdot l_1^2}{12} \end{bmatrix} ; \quad I_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{m \cdot l_2^2}{12} & 0 \\ 0 & 0 & \frac{m \cdot l_2^2}{12} \end{bmatrix}$$

around z-axis

$${}^0R^1 = \begin{bmatrix} \cos(q_1) & -\sin(q_1) & 0 \\ \sin(q_1) & \cos(q_1) & 0 \\ 0 & 0 & 1 \end{bmatrix} ; \quad {}^0R^2 = \begin{bmatrix} -\cos(q_1+q_2) & -\sin(q_1+q_2) & 0 \\ \sin(q_1+q_2) & \cos(q_1+q_2) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\underset{(3\times1)}{\omega_1} = \begin{bmatrix} 0 \\ 0 \\ \dot{q}_1 \end{bmatrix} = \underset{(3\times2)}{J_{\omega_1}} \cdot \underset{(2\times1)}{\vec{\dot{q}}} = \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}}_{J_{\omega_1}} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} ; \quad \underset{(3\times1)}{\omega_2} = \begin{bmatrix} 0 \\ 0 \\ \dot{q}_1+\dot{q}_2 \end{bmatrix} = \underset{(3\times2)}{J_{\omega_2}} \cdot \underset{(2\times1)}{\vec{\dot{q}}} = \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}}_{J_{\omega_2}} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

$\frac{d}{dt}$

$$V|_{c_1} = \begin{bmatrix} -l_{c1} \cdot \sin(q_1) \cdot \dot{q}_1 \\ l_{c1} \cdot \cos(q_1) \cdot \dot{q}_1 \\ 0 \end{bmatrix} = \underbrace{\begin{bmatrix} -l_{c1} \cdot \cos(q_1) & 0 \\ l_{c1} \cdot \sin(q_1) & 0 \\ 0 & 0 \end{bmatrix}}_{J_{v_1}} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} =$$

$$V|_{c_2} = \begin{bmatrix} -l_1 \sin(q_1) \dot{q}_1 - l_{c2} \sin(q_1+q_2) \cdot \dot{q}_1 - l_{c2} \sin(q_1+q_2) \cdot \dot{q}_2 \\ l_1 \cdot \cos(q_1) \cdot \dot{q}_1 + l_{c2} \cdot \cos(q_1+q_2) \dot{q}_1 + l_{c2} \cos(q_1+q_2) \dot{q}_2 \\ 0 \end{bmatrix} = \underbrace{\begin{bmatrix} -l_1 \sin(q_1) - l_{c2} \cdot \sin(q_1+q_2) & -l_{c2} \cdot \sin(q_1+q_2) \\ l_1 \cdot \cos(q_1) + l_{c2} \cdot \sin(q_1+q_2) & l_{c2} \cdot \cos(q_1+q_2) \\ 0 & 0 \end{bmatrix}}_{J_{v_2}} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

$$D = \overbrace{m_1 \cdot J_{v_1}^T \cdot J_{v_1}}^{1} + \overbrace{J_{\omega_1}^T \cdot R_1 \, I_1 \, R_1^T \cdot J_{\omega_1}}^{2} + \overbrace{m_2 \, J_{v_2}^T \, J_{v_2}}^{3} + \overbrace{J_{\omega_2}^T \, R_2 \, I_2 \cdot R_2^T \cdot J_{\omega_2}}^{4}$$

$$1 = m_1 \cdot \begin{bmatrix} -l_{c1} \cdot \cos(q_1) & l_{c1} \cdot \sin(q_1) & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -l_{c1} \cdot \cos(q_1) & 0 \\ l_{c1} \cdot \sin(q_1) & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} m_1 \cdot l_{c1}^2 & 0 \\ 0 & 0 \end{bmatrix}$$

$$2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \cos(q_1) & -\sin(q_1) & 0 \\ \sin(q_1) & \cos(q_1) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{m \cdot l_1^2}{12} & 0 \\ 0 & 0 & \frac{m \cdot l_1^2}{12} \end{bmatrix} \begin{bmatrix} \cos(q_1) & \sin(q_1) & 0 \\ -\sin(q_1) & \cos(q_1) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} \frac{m_1 \cdot l_1^2}{12} & 0 \\ 0 & 0 \end{bmatrix}$$

$$③ = m_2 \begin{bmatrix} -l_1 \sin(q_1) - l_{c2} \sin(q_1+q_2) & l_1 \cos(q_1) + l_{c2} \sin(q_1+q_2) & 0 \\ -l_{c2} \sin(q_1+q_2) & l_{c2} \cos(q_1+q_2) & 0 \end{bmatrix} \begin{bmatrix} -l_1 \sin(q_1) - l_{c2} \sin(q_1+q_2) & -l_{c2} \sin(q_1+q_2) \\ l_1 \cos(q_1) + l_{c2} \sin(q_1+q_2) & l_{c2} \cos(q_1+q_2) \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} m_2(l_1^2 + l_{c2}^2 + l_1 \cdot l_2 \cdot \cos(q_2)) & m_2(l_{c2}^2 + l_1 \cdot l_{c2} \cdot \cos(q_2)) \\ m_2(l_{c2}^2 + l_1 \cdot l_{c2} \cdot \cos(q_2)) & m_2 \cdot l_{c2}^2 \end{bmatrix}$$

$$④ = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -\cos(q_1+q_2) & -\sin(q_1+q_2) & 0 \\ \sin(q_1+q_2) & \cos(q_1+q_2) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{m_2 l_2^2}{12} & 0 \\ 0 & 0 & \frac{m_2 l_2^2}{12} \end{bmatrix} \begin{bmatrix} -\cos(q_1+q_2) & \sin(q_1+q_2) & 0 \\ -\sin(q_1+q_2) & \cos(q_1+q_2) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{m_2 \cdot l_2^2}{12} & \frac{m_2 \cdot l_2^2}{12} \\ \frac{m_2 \cdot l_2^2}{12} & \frac{m_2 \cdot l_2^2}{12} \end{bmatrix}$$

$$D(q) = \begin{bmatrix} \overbrace{\frac{m_1 \cdot l_1^2}{3} + m_2\left(l_1^2 + \frac{l_2^2}{3} + l_1 \cdot l_2 \cdot \cos(q_2)\right)}^{d_{11}} & \overbrace{m_2\left(\frac{l_2^2}{3} + l_1 \cdot l_{c2} \cdot \cos(q_2)\right)}^{d_{12}} \\ m_2\left(\frac{l_2^2}{3} + l_1 \cdot l_{c2} \cdot \cos(q_2)\right) & \underbrace{m_2 \cdot \frac{l_2^2}{3}}_{d_{22}} \end{bmatrix}$$

$$K = \frac{1}{2} [\dot q_1 \ \dot q_2] \ D(q) \begin{bmatrix} \dot q_1 \\ \dot q_2 \end{bmatrix} = \frac{1}{2} \left[d_{11} \cdot \dot q_1^2 + 2 d_{12} \cdot \dot q_1 \dot q_2 + d_{22} \cdot \dot q_2^2 \right]$$

$$L = K - P = \frac{1}{2} \sum_{i,j} d_{ij}(q) \dot q_i \dot q_j - P(q)$$

$$\hookrightarrow \frac{\partial L}{\partial \dot q_k} = \sum_j d_{kj} \cdot \dot q_j \Rightarrow \frac{d}{dt} \frac{\partial L}{\partial \dot q_k} = \sum_j d_{kj} \cdot \ddot q_j + \sum_j \frac{d}{dt} d_{kj} \cdot \dot q_j = \sum_j d_{kj} \cdot \ddot q_j + \sum_{i,j} \frac{\partial d_{kj}}{\partial q_i} \cdot \dot q_i \cdot \dot q_j$$

$$\hookrightarrow \frac{\partial L}{\partial q_k} = \frac{1}{2} \sum_{i,j} \frac{\partial d_{ij}}{\partial q_k} \cdot \dot q_i \cdot \dot q_j - \frac{\partial P}{\partial q_k}$$

$$\underbrace{\sum_j d_{kj} \cdot \ddot q_j}_{①} + \underbrace{\sum_{i,j} \left\{ \frac{\partial d_{kj}}{\partial q_i} - \frac{1}{2} \frac{\partial d_{ij}}{\partial q_k} \right\} \dot q_i \cdot \dot q_j}_{②} - \underbrace{\frac{\partial P}{\partial q_k}}_{③} = \tau_k$$

$$* \sum_{i,j} \left\{ \frac{\partial d_{kj}}{\partial q_i} \right\} \dot q_i \cdot \dot q_j = \frac{1}{2} \sum_{i,j} \left\{ \frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} \right\} \dot q_i \cdot \dot q_j \Rightarrow c_{ijk} = \sum_{i,j} \frac{1}{2} \left\{ \frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k} \right\} \dot q_i \cdot \dot q_j$$

$$\hookrightarrow k=1 \rightarrow \sum_{i=1}^{2} \left(\sum_{j=1}^{2} c_{ij1} \cdot \dot q_j \right) \dot q_i$$

$$k=2 \rightarrow \sum_{i=1}^{2} \left(\sum_{j=1}^{2} c_{ij2} \cdot \dot q_j \right) \dot q_i$$

$$\begin{bmatrix} \sum_{j=1}^{2} c_{ij1} \cdot \dot q_j \\ \sum_{j=1}^{2} c_{ij2} \cdot \dot q_j \end{bmatrix} = \left[\sum_{j=1}^{2} c_{ijk} \cdot \dot q_j \right] \quad k = 1, 2$$

$$\hookrightarrow c = [\vec c_1 \ \ \vec c_2] \rightarrow c \cdot \vec{\dot q}_i = \dot q_1 \cdot \vec c_1 + \dot q_2 \cdot \vec c_2 = \sum_{i=1}^{2} \dot q_i \vec c_i$$

$$\vec c_1 = \begin{bmatrix} \sum_{j=1}^{2} c_{1j1} \cdot \dot q_j \\ \sum_{j=1}^{2} c_{1j2} \cdot \dot q_j \end{bmatrix} = \begin{bmatrix} c_{111} \cdot \dot q_1 + c_{121} \dot q_2 \\ c_{112} \cdot \dot q_1 + c_{122} \cdot \dot q_2 \end{bmatrix} \quad \vec c_2 = \begin{bmatrix} c_{211} \cdot \dot q_1 + c_{221} \cdot \dot q_2 \\ c_{212} \cdot \dot q_1 + c_{222} \cdot \dot q_2 \end{bmatrix}$$

$$\vec C = \begin{bmatrix} c_{111} \dot q_1 + c_{121} \dot q_2 & c_{211} \cdot \dot q_1 + c_{221} \dot q_2 \\ c_{112} \dot q_1 + c_{122} \dot q_2 & c_{212} \cdot \dot q_1 + c_{222} \cdot \dot q_2 \end{bmatrix}$$

$$i=1,2 \quad j=1,2 \quad k=1,2 \qquad C_{ijk} = \frac{1}{2}\left\{\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right\}$$

indep of q_1

$$C_{111}=\frac{1}{2}\left\{\cancel{\frac{\partial d_{11}}{\partial q_1}}^{0} + \cancel{\frac{\partial d_{11}}{\partial q_1}} - \cancel{\frac{\partial d_{11}}{\partial q_1}}\right\}=0 \; ; \; C_{112}=\frac{1}{2}\left\{\frac{\partial d_{21}}{\partial q_1}+\frac{\partial d_{21}}{\partial q_1}-\frac{\partial d_{11}}{\partial q_2}\right\}=\frac{m_2\cdot l_1\cdot l_2 \sin(q_2)}{2}$$

indep of q_1

$$C_{121}=C_{211}=\frac{1}{2}\left\{\cancel{\frac{\partial d_{12}}{\partial q_1}}+\frac{\partial d_{11}}{\partial q_2}-\cancel{\frac{\partial d_{12}}{\partial q_1}}\right\}=\frac{-m_2 l_1\cdot l_2 \sin(q_2)}{2} \; ; \; C_{122}=C_{212}=\frac{1}{2}\left\{\cancel{\frac{\partial d_{22}}{\partial q_1}}^{0}+\cancel{\frac{\partial d_{21}}{\partial q_2}}-\cancel{\frac{\partial d_{12}}{\partial q_2}}\right\}=0$$

indep. of q_2

$$C_{221}=\frac{1}{2}\left\{\frac{\partial d_{12}}{\partial q_2}+\frac{\partial d_{12}}{\partial q_2}-\frac{\partial d_{22}}{\partial q_1}\right\}=\frac{-m_2}{2} l_1\cdot l_2\cdot\sin(q_2) \; ; \; C_{222}=\frac{1}{2}\left\{\cancel{\frac{\partial d_{22}}{\partial q_2}}^{0}+\cancel{\frac{\partial d_{22}}{\partial q_2}}-\cancel{\frac{\partial d_{22}}{\partial q_2}}\right\}=0$$

2

$$\vec{C}(q,\dot q)=\begin{bmatrix} C_{111}\dot q_1+C_{121}\dot q_2 & C_{211}\dot q_1+C_{221}\dot q_2 \\ C_{112}\dot q_1+C_{122}\dot q_2 & C_{212}\dot q_1+C_{222}\dot q_2 \end{bmatrix}=\begin{bmatrix} -m_2\cdot l_1\cdot l_2\cdot\sin(q_2)\cdot\dot q_2 & \frac{-m_2\cdot l_1\cdot l_2\cdot\sin(q_2)}{2}\cdot\dot q_2 \\ \frac{m_2\cdot l_1\cdot l_2\cdot\sin(q_2)}{2}\dot q_1 & 0 \end{bmatrix}$$

$$* \; P=P_1+P_2 = m_1\cdot l_{c1}\cdot g\sin(q_1)+m_2\left(l_1\cdot\sin(q_1)+l_{c2}\cdot\sin(q_1+q_2)\right)g$$

3

$$G(q)=\begin{bmatrix}\frac{\partial P}{\partial q_1}\\ \frac{\partial P}{\partial q_2}\end{bmatrix}=\begin{bmatrix} m_1\cdot l_{c1}\, g\cdot\cos(q_1)+m_2\cdot l_1\cdot\cos(q_1)\cdot g + m_2\cdot l_{c2}\cdot g\cos(q_1+q_2) \\ m_2\cdot l_{c2}\cdot g\cdot\cos(q_1+q_2)\end{bmatrix}$$

$$\sum_j d_{kj}\cdot\ddot q_j + \sum_{i,j}\left\{\frac{\partial d_{kj}}{\partial q_i}-\frac{1}{2}\frac{\partial d_{ij}}{\partial q_k}\right\}\dot q_i\cdot\dot q_j - \frac{\partial P}{\partial q_k}=\tau_k$$

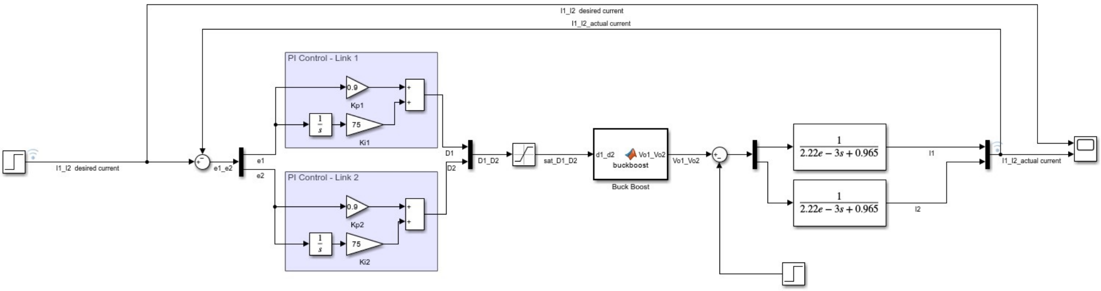
$$\tau_1=\left[\frac{m_1\cdot l_1^2}{3}+m_2\left(l_1^2+\frac{l_2^2}{3}+l_1\cdot l_2\cos(q_2)\right)\right]\ddot q_1+\left[m_2\left(\frac{l_2^2}{3}+l_1\cdot l_{c2}\cdot\cos(q_2)\right)\right]\ddot q_2$$

$$-m_2\cdot l_1\cdot l_2\cdot\sin(q_2)\dot q_1\dot q_2-\frac{m_2}{2}l_1\cdot l_2\cdot\sin(q_2)\cdot(\dot q_2)^2$$

$$+\left[m_1\cdot l_{c1}\cdot\cos(q_1)+m_2\cdot l_1\cdot\cos(q_1)+m_2\cdot l_{c2}\cdot\cos(q_1+q_2)\right]g$$

$$\tau_2=\left[m_2\left(\frac{l_2^2}{3}+l_1\cdot l_{c2}\cdot\cos(q_2)\right)\right]\ddot q_1+\left[m_2\frac{l_2^2}{3}\right]\ddot q_2+\frac{m_2\, l_1\cdot l_2\cdot\sin(q_2)}{2}(\dot q_1)^2+\left[m_2\cdot l_{c2}\cdot\cos(q_1+q_2)\right]g$$

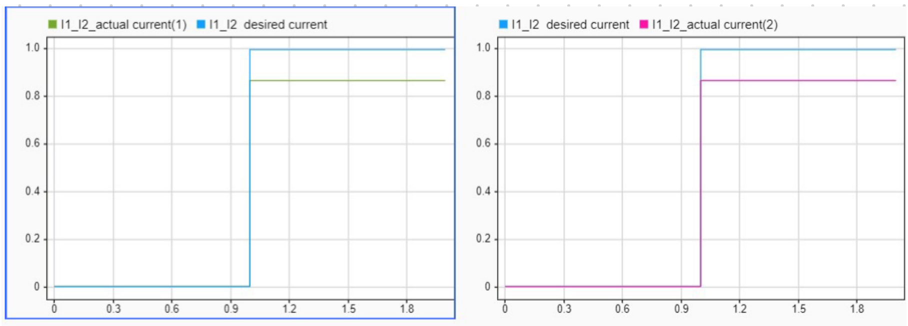
2. Inner Loop Design and Implementation



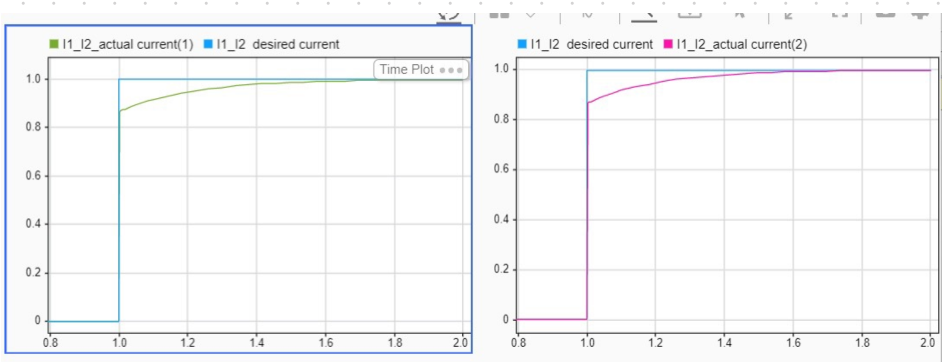
- Initial Tuning Attempts:

The inner loop involves controlling the electrical system using a PI controller. The initial attempt to tune the controller started with Kp=1 and Ki=0. However, the steady-state value was lower than the desired one. To address this, I introduced an integral gain Ki and set it to 5, which improved the results.

* kp=1 ; ki= 0 ;



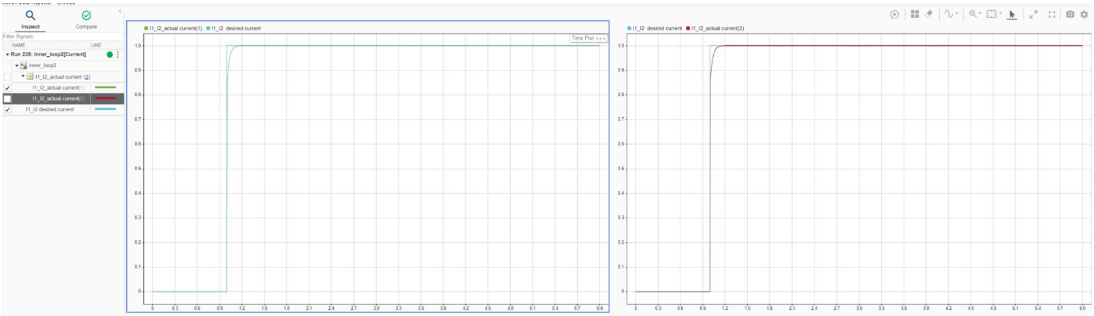
* kp=1 ; ki = 5



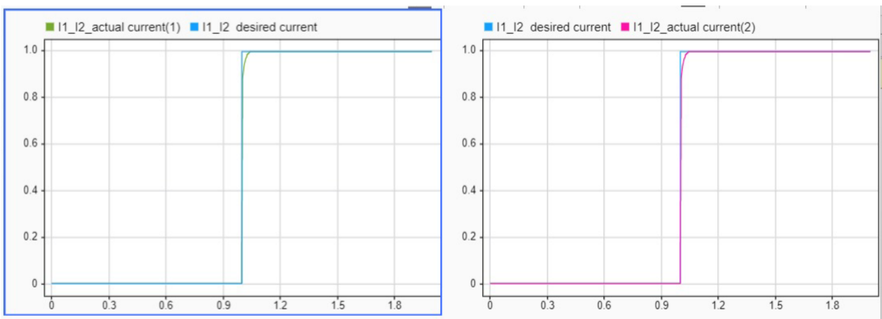
- Incremental Tuning

To further refine the performance, I gradually increased the
K i value through a series of steps: 30, 40, 50, 60, and finally 75. After iterative testing, I determined that the optimal results were achieved with Kp=0.9 and K i=75.

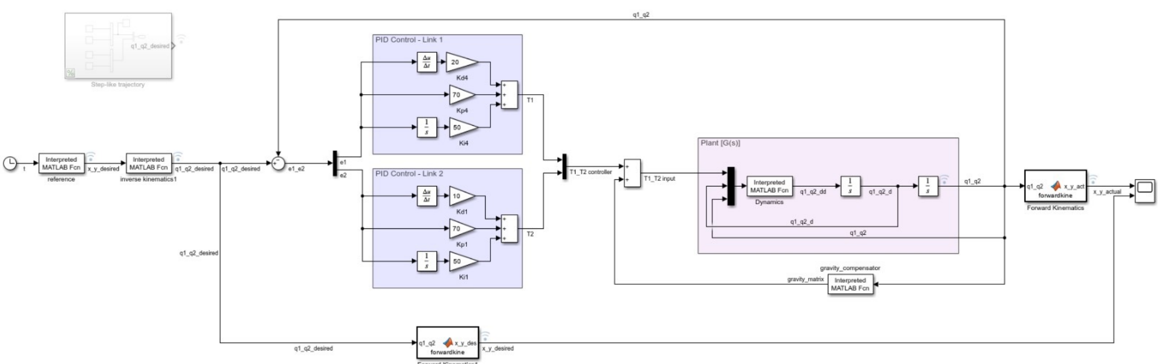
*Kp=1 ; Ki=30



* Kp=0.9 ; Ki=75



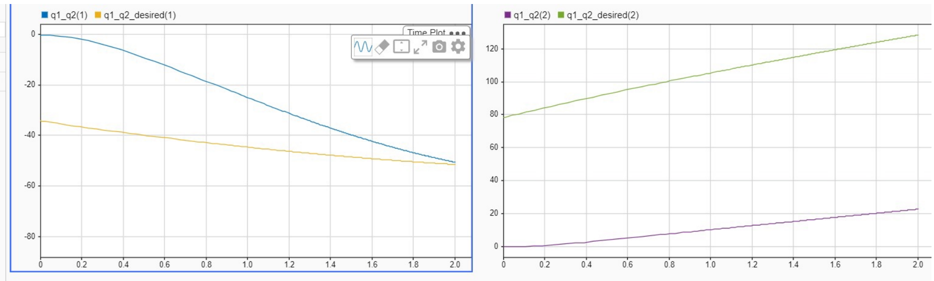
3. Outer Loop Design and Implementation



- Initial PID Tuning Challenges

The outer loop's initial performance was poor, and tuning the PID controller did not yield satisfactory results. Despite numerous adjustments to the Kp, Ki, and Kd gains, the system failed to meet the project requirements of a rise time of ≤2 seconds, overshoot of ≤10%, and steady-state error of ≤0.01 for a unit step input. An example of a failure is in the following figure.

* Kp=20; Kd=20

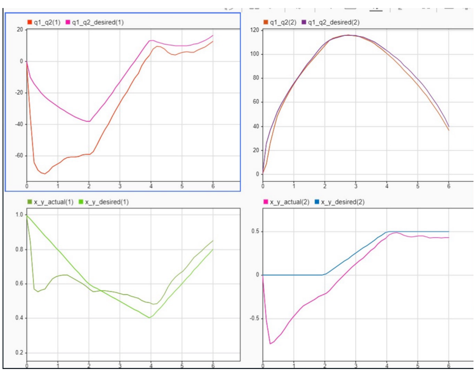


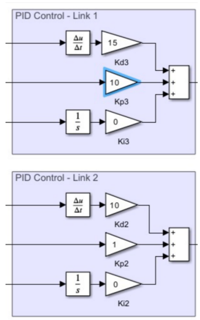
- Gravity Compensation

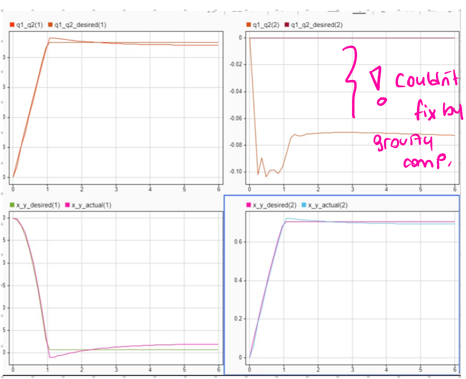
To improve the outer loop performance, I introduced a gravity compensator. This adjustment led to significant improvements, but the results were still not within acceptable limits.

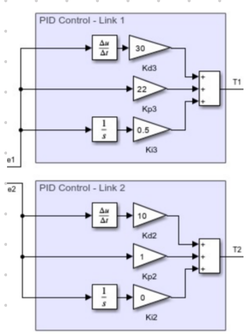
- Angle Conversion Issue

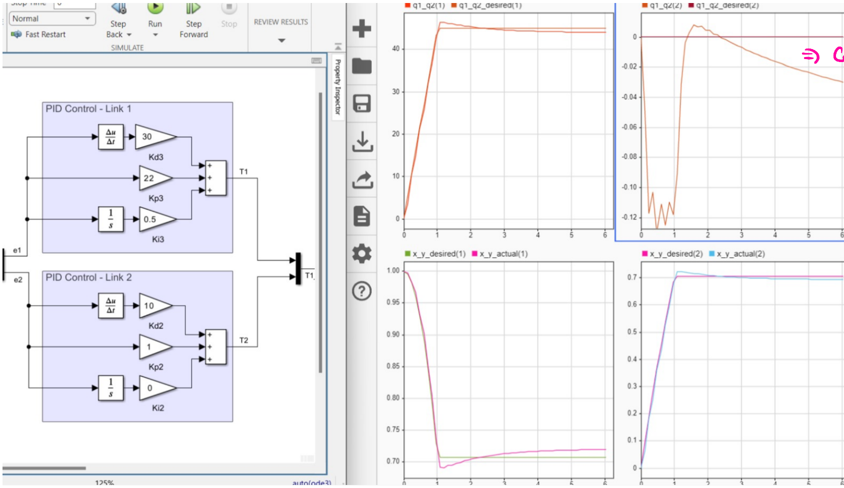
Upon further investigation, I discovered that the trigonometric functions (cosine and sine) were incorrectly using radians instead of degrees. Correcting this issue resulted in improved performance, though still not satisfactory.











⇒ Couldn't fix by Playing with the gains and the PID controller.

- Final Adjustments

After ensuring all parameter units were defined correct, I continued to struggle with achieving the desired response. Increasing Kp and Ki to address the steady-state error led to poor initial responses, and adjusting Kd resulted in unmanageable ripple effects.

- Gear Reduction Adjustment

As a last resort, I adjusted the gear reduction ratio r from 4 to 1/4. This change significantly improved the system's response, allowing it to meet the project requirements.

Note: we were allowed to chance r according to the project document.

J = motor inertia = 118.2e-3, B = motor friction = 129.6e-3

r = gear reduction ≈ 4. You can play around with this number to see its effects on the system dynamics.