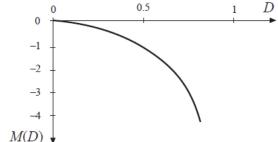
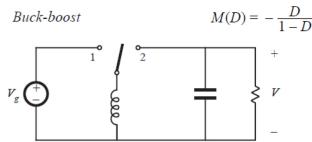
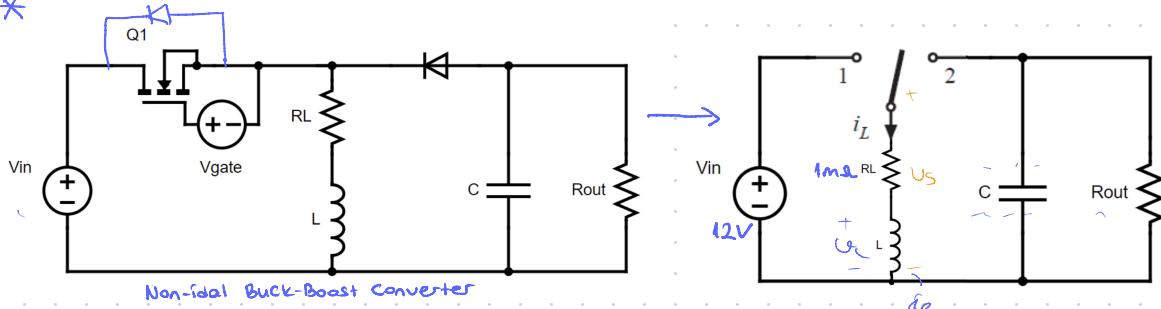


$$* \text{Duty cycle } (D) = \frac{t_{on}}{t_{on}+t_{off}} = \frac{t_{on}}{T_s}$$

$$* V_o = M(D) \cdot V_{in} \Rightarrow V_o = \frac{-D}{1-D} V_{in}$$

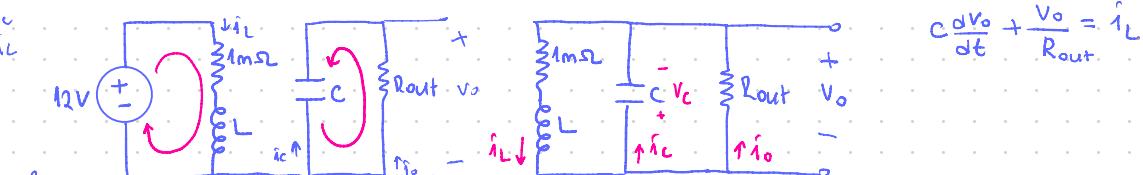


*



Switch at position 1:

Switch at position 2:



$$\Rightarrow L \frac{di_L}{dt} + R_L i_L = V_{in}$$

$$\therefore V_L = V_{in} - I \cdot R_L$$

$$\Rightarrow i_C = -i_O \Rightarrow C \frac{dV_C}{dt} = \frac{V_O}{R_{out}}$$

$$\therefore i_C = \frac{V}{R_{out}}$$

$$\text{Lap: } \left(\frac{1}{C} \int (i_L - i_O) dt \right) = -V_o$$

$$\frac{1}{CS} I_L(s) + \frac{1}{CS \cdot R_{out}} V_o(s) = -V_o(s)$$

$$\frac{1}{CS} \frac{V_o(s)}{I_L(s)} = -V_o(s) \left(\frac{CS \cdot R_{out} + 1}{CS \cdot R_{out}} \right) \Rightarrow \frac{V_o}{V_S} = -\frac{CS \cdot R_{out}}{CS \cdot (L + R_L) + CS \cdot R_{out} + 1} \Rightarrow H_T(s) = \frac{-R_{out}}{s^2(LC \cdot R_{out}) + s(L + C \cdot R_{out}) + 1}$$

$$V_S = R_L i_L + L \frac{di_L}{dt} \Rightarrow V_S(s) = L s I_L(s) + R_L I_L(s)$$

$$\therefore V_S(s) = I_L (Ls + R_L) \Rightarrow I_L(s) = \frac{V_S(s)}{(Ls + R_L)}$$

* You pick the switching frequency ($f > 1\text{kHz}$). You can add additional circuit elements to the above design (some will be necessary).

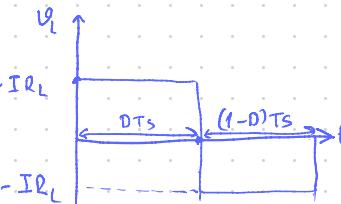
* Switching frequency:

$$f_s > 1\text{kHz} \rightarrow T_S < 0.001\text{ s} \quad 500\text{ } \mu\text{s}$$

I first tried $f_s = 20\text{kHz} \rightarrow T_S = 0.05\text{ms}$

$$\hookrightarrow f_{\text{cutoff}} = \frac{20000}{100} = 200\text{Hz}$$

(2 decades away from f_s)



$$(V_{in} - IR_L)DT_S + (1-D)T_S(V - IR_L) = 0$$

$$\hookrightarrow V_{in} \cdot D - IR_L \cdot D + V - IR_L - V \cdot D + IR_L \cdot D = 0$$

$$V_{in} \cdot D = -V + IR_L + VD$$

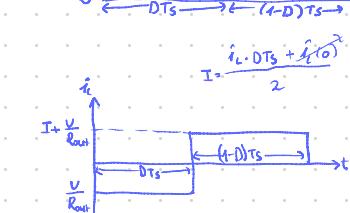
$$V_{in} = \frac{-V(1-D)}{D} + \frac{IR_L}{D} \Rightarrow IR_L = (V_{in} \cdot D) + V(1-D)$$

$$\hookrightarrow V = \frac{IR_L - V_{in} \cdot D}{(1-D)}$$

$$\hookrightarrow V_L(t) = L \cdot \frac{di_L}{dt}$$

$$(\text{switch @1}) \rightarrow \frac{2\Delta i_L}{\Delta t} = \frac{V_{in} - IR_L}{L} \Rightarrow 2\Delta i_L = \frac{(V_{in} - IR_L) \cdot DT_S}{L} \Rightarrow L = \frac{(V_{in} - IR_L)DT_S}{2\Delta i_L}$$

$$(\text{switch @2}) \rightarrow \frac{2\Delta i_L}{\Delta t} = \frac{V - IR_L}{L} \Rightarrow 2\Delta i_L = \frac{(V - IR_L)(1-D)T_S}{L}$$



$$\hookrightarrow C \cdot \frac{dV_C}{dt} = i_C = I + \frac{V}{R_{out}} \quad (@ \text{ switch 1})$$

$$\hookrightarrow (\frac{V}{R_{out}})DT_S + (I + \frac{V}{R_{out}})(1-D)T_S = 0$$

$$\rightarrow \frac{V \cdot D}{R_{out}} + I + \frac{V}{R_{out}} - ID - \frac{VD}{R_{out}} = 0$$

$$\rightarrow I(1-D) = \frac{V}{R_{out}} \rightarrow I = \frac{-V}{(1-D) \cdot R_{out}}$$

$$\rightarrow I = \frac{V_{in} \cdot D - IR_L}{R_{out} \cdot (1-D)^2}$$

$$\rightarrow R_{out} \cdot (1-D)^2 \cdot I + IR_L = V_{in} \cdot D$$

$$\hookrightarrow I(R_{out} \cdot (1-D)^2 + R_L) = V_{in} \cdot D$$

$$\hookrightarrow I = \frac{V_{in} \cdot D}{(R_{out} \cdot (1-D)^2 + R_L)}$$

$$C \cdot \frac{dV_C}{dt} = i_C = I + \frac{V}{R_{out}} \quad (@ \text{ switch 1})$$

$$C \cdot \frac{dV_C}{dt} = \frac{V_{in} \cdot D}{(R_{out} \cdot (1-D)^2 + R_L)} + \frac{IR_L - V_{in} \cdot D}{(1-D) \cdot R_{out}}$$

$$= \frac{V_{in} \cdot D}{(R_{out} \cdot (1-D)^2 + R_L)} + \frac{V_{in} \cdot D \cdot R_L}{((R_{out} \cdot (1-D)^2 + R_L)(1-D) \cdot R_{out})}$$

$$= \frac{V_{in} \cdot D (1-D) \cdot R_{out} + V_{in} \cdot D \cdot R_L + V_{in} \cdot D ((R_{out} \cdot (1-D)^2 + R_L))}{((R_{out} \cdot (1-D)^2 + R_L)(1-D) \cdot R_{out})}$$

$$= \frac{V_{in} \cdot R_{out} \cdot D (1-D) (2-D) + 2V_{in} \cdot D \cdot R_L}{((R_{out} \cdot (1-D)^2 + R_L) (1-D) \cdot R_{out})}$$

$$\hookrightarrow \Delta V_C = \left[\frac{V_{in} \cdot R_{out} \cdot D (1-D) (2-D) + 2V_{in} \cdot D \cdot R_L}{((R_{out} \cdot (1-D)^2 + R_L) (1-D) \cdot R_{out})} \right] \frac{DT_S}{C} \Rightarrow C = \left[\frac{V_{in} \cdot R_{out} \cdot D (1-D) (2-D) + 2V_{in} \cdot D \cdot R_L}{((R_{out} \cdot (1-D)^2 + R_L) (1-D) \cdot R_{out})} \right] \frac{DT_S}{\Delta V_C}$$

Matlab :

```
1 clc
2 clear all;
3
4 Vin = 12;
5 D = 0.7;
6 Ts = 50e-6;
7 Rout = 100;
8 R1 = 1e-3;
9
10 current_ripple_L = 0.05;
11 voltage_ripple_C = 0.2;
12
13 I = (Vin * D)/(Rout*(1-D)^2 + R1);
14
15 L = (Vin*D - I*R1) * D*Ts / (current_ripple_L)
16 C = (((Vin*D)/(Rout * (1-D)^2 + R1)) + ((I*R1 - Vin*D)/((1-D)*Rout))) * (D*Ts/voltage_ripple_C)
17
18
```

Command Window

L =
0.0059 = 5.9mH

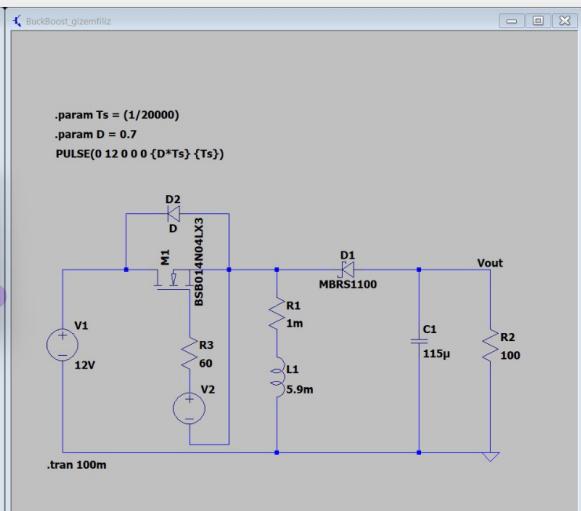
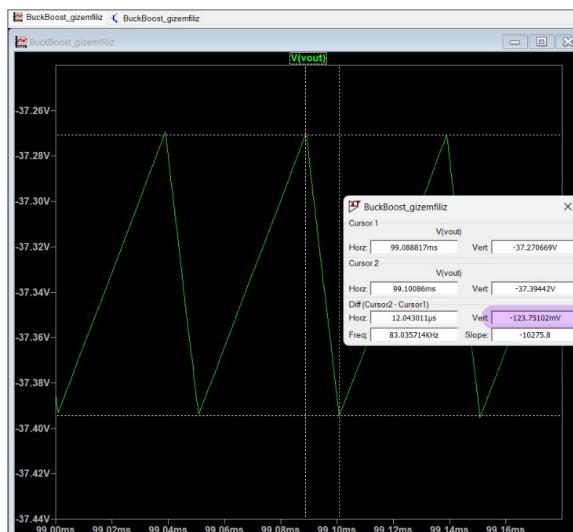
C =
1.1432e-04 = 114.32 μF

* We would like stable results especially around the $0.3 < D < 0.7$ band
(where D is the duty cycle). That's the main operation zone.

Current ripple is important for the motor and therefore the permissible inductor current ripple is 0.05A (peak to peak). $< 50\text{mA}$

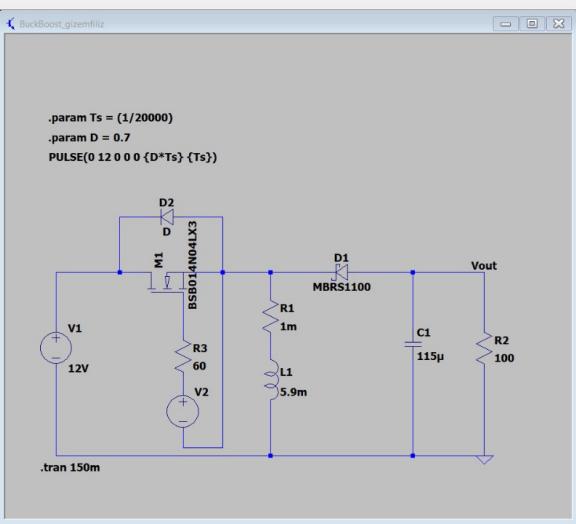
Permissible capacitor voltage is 0.2V (peak to peak). $< 200\text{mV}$

* $D=0.7$:



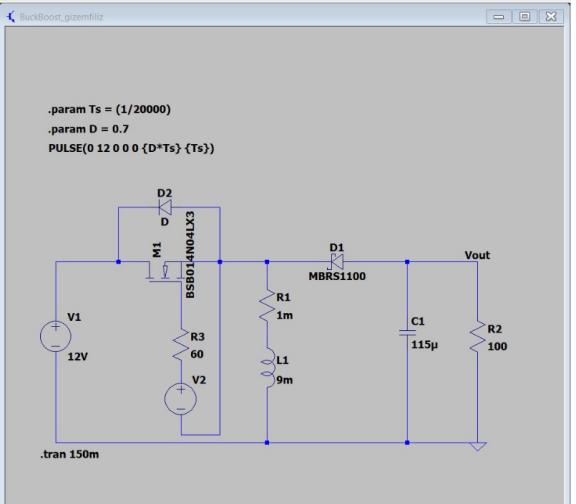
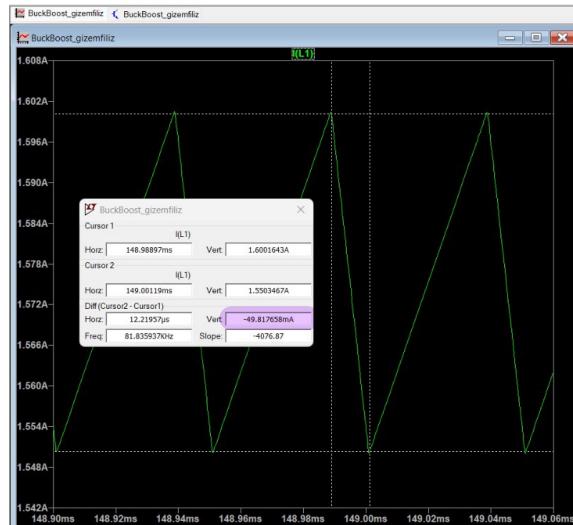
↳ i_L current ripple is inside the boundary.

However,



V_c voltage ripple exceed the boundary $76.26\text{mA} < 50\text{mA}$

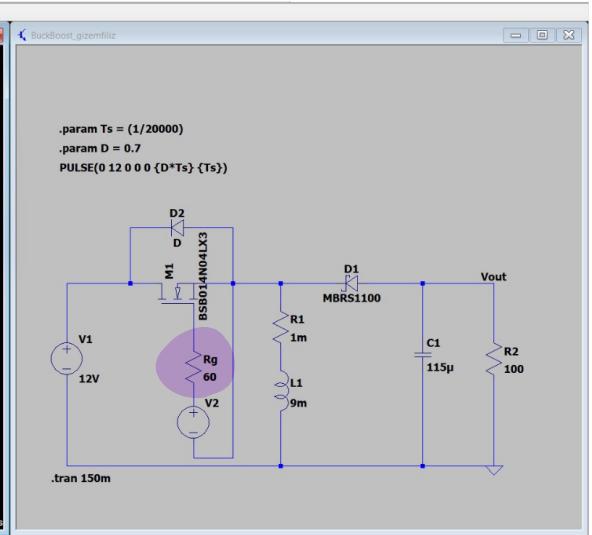
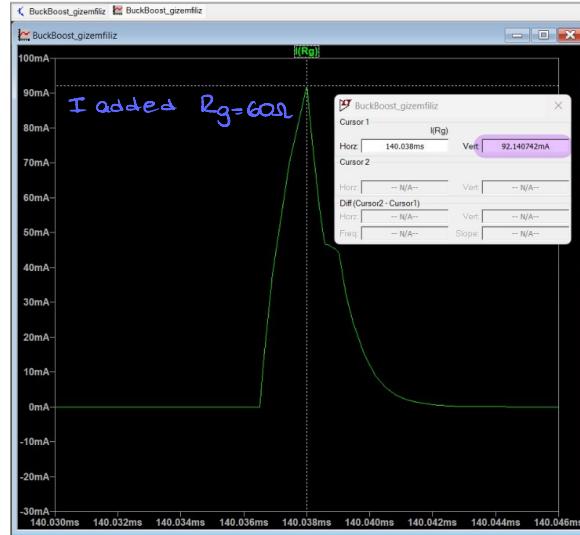
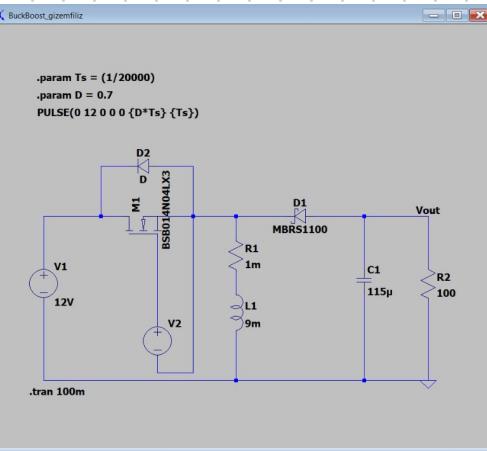
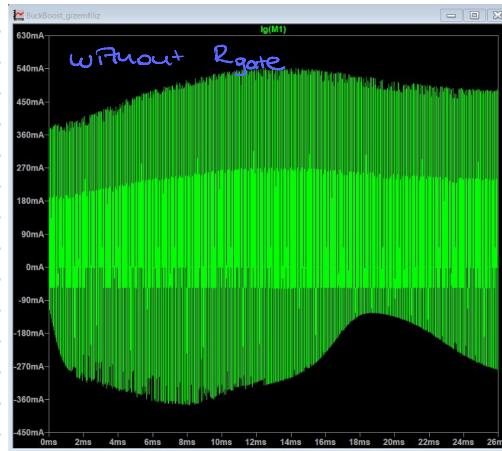
↳ So, I increased L1 inductance to 9mH.



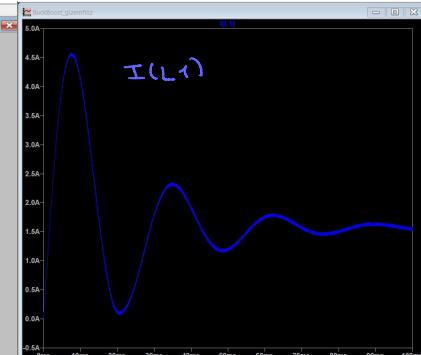
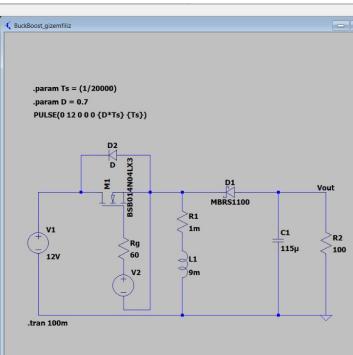
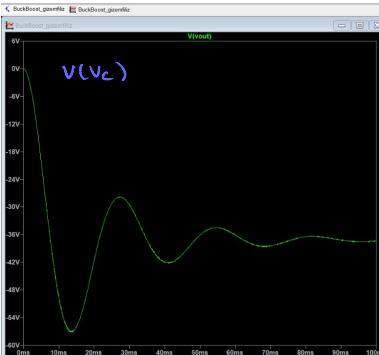
↳ i_L current ripple didn't change: $\approx 123\text{mV} < 200\text{mV}$ (inside the boundary.)

↳ V_c voltage ripple decreased to $49.81\text{mA} < 50\text{mA}$ (" " " ")

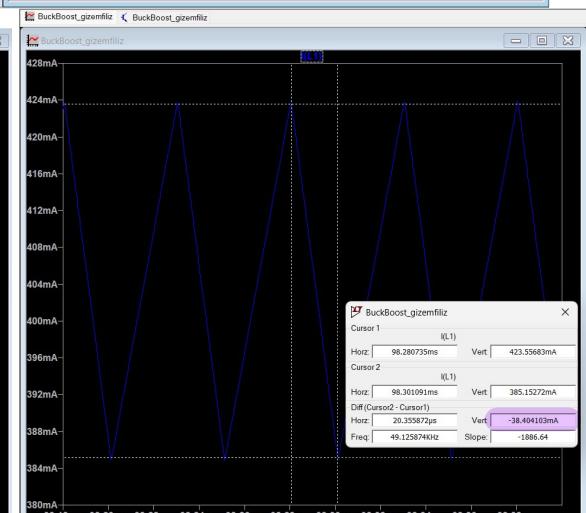
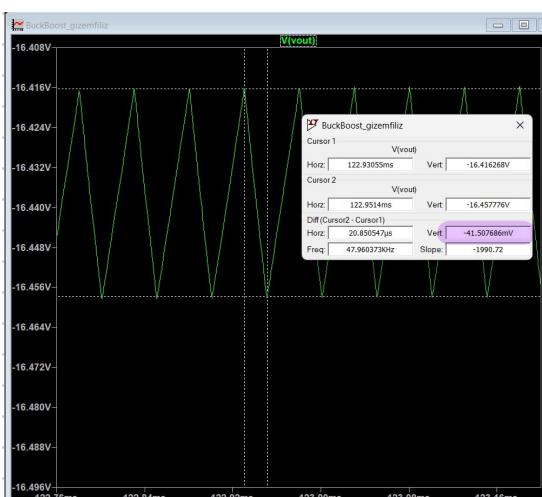
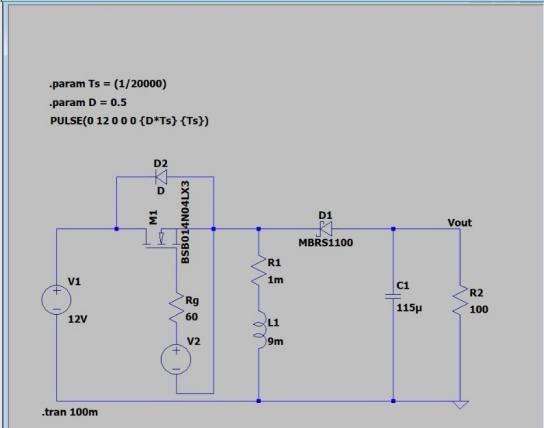
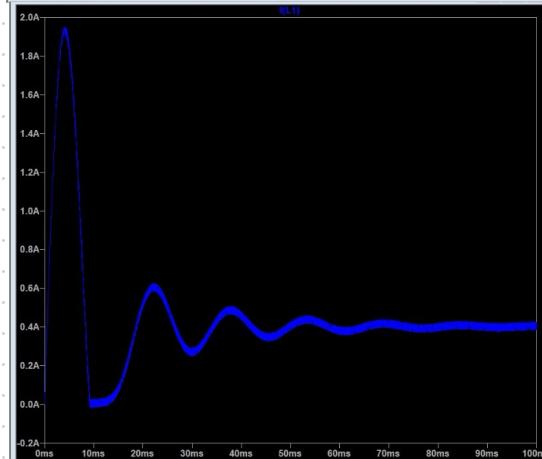
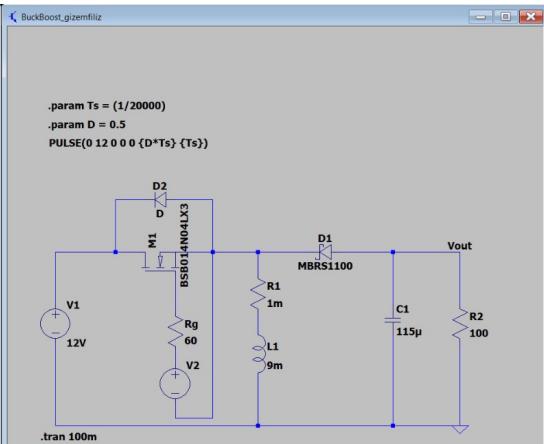
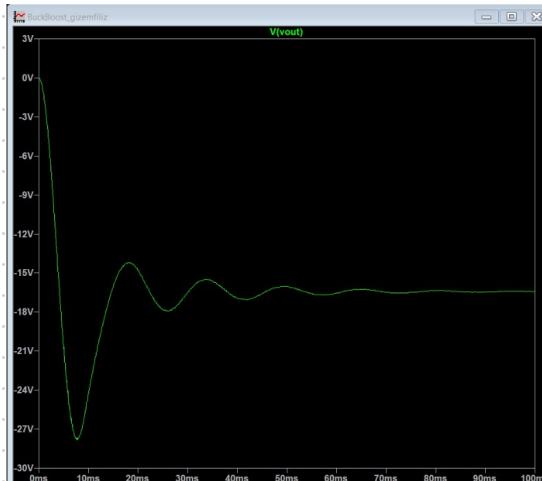
Peak value for i_g should not surpass 100mA. What can you change to reduce this current? Make the necessary changes. What is the tradeoff?



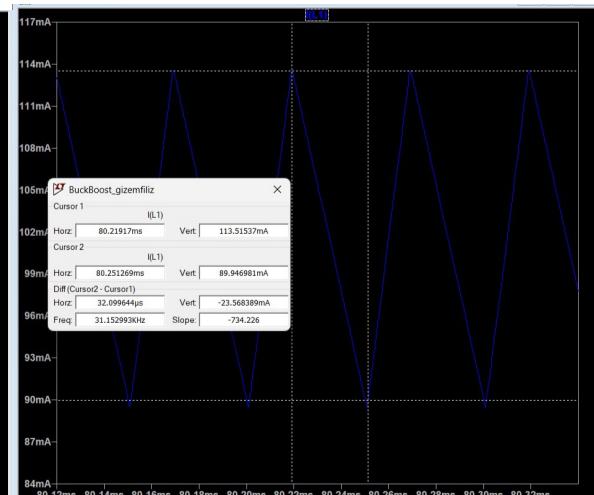
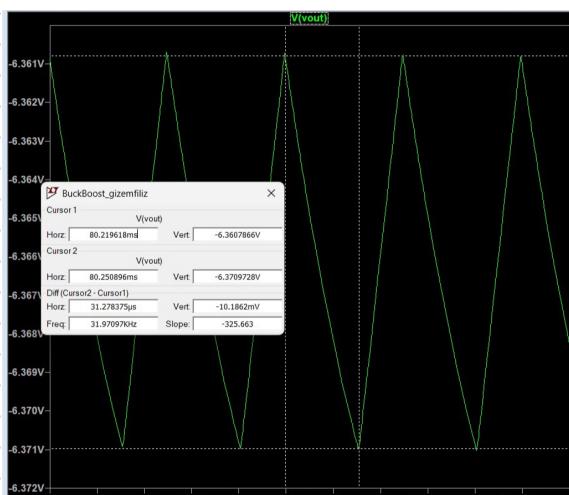
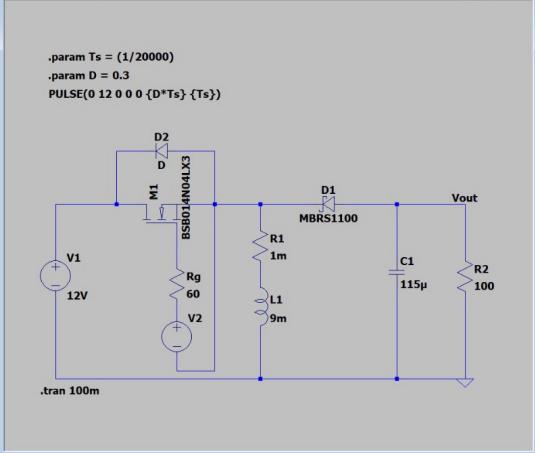
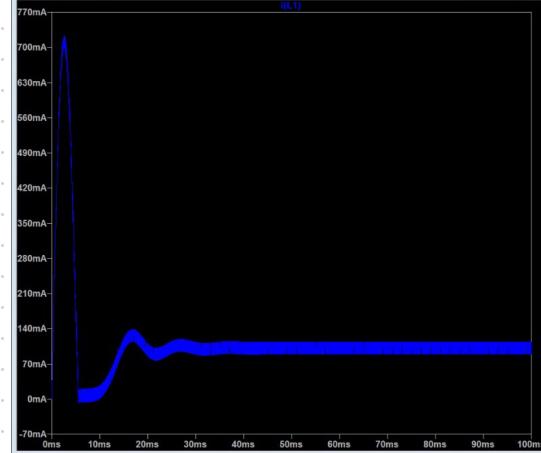
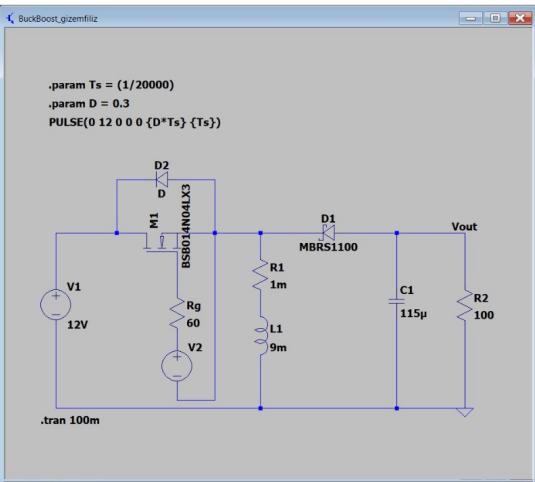
gate current $i_g = 92.140\text{mA}$ ($< 100\text{mA}$ (inside the boundary))



* D = 0.5



D=0.3



Q) @ Cutoff frequency $H_f(s) = \frac{1}{f_2}$

$$H_f(s) = \left| \frac{-R_{out}}{s^2(LC R_{out}) + s(L + C_{L,R_{out}}) H_f L} \right| = \frac{1}{f_2}$$

$$\text{Let } f_c = 200\text{Hz} \Rightarrow \omega_c = 2\pi 200 = 400\pi \text{ rad/s}$$

$$s = j\omega \Rightarrow s = 400\pi j$$

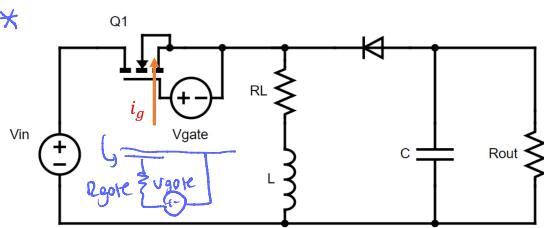
$$= \left| \frac{-100}{(400j)^2 (3 \times 10^{-3} \times 11.5 \times 10^{-6} \times 100) + (400j)(9 \times 10^{-3} + 11.5 \times 10^{-6} \times 10^3 \times 100) + 10^3} \right| =$$

$$= \left| \frac{-100}{-16,559 + 3,6046j} \right| = \left| \frac{1655,9 + 360,46j}{274,200481 + 12,99311116} \right| = 5,76579 ?$$

$$\left| \frac{-100}{(-\omega_c^2 \cdot (1.0238 \times 10^{-4}) + 0.001) - \omega_c j (1.15 \times 10^5)} \right| = \frac{1}{f_2}$$

$$\left| \frac{-100 \cdot (-\omega_c^2 \cdot (1.0238 \times 10^{-4}) + 0.001) - \omega_c j (1.15 \times 10^5)}{(-\omega_c^2 \cdot (1.0238 \times 10^{-4}) + 0.001)^2 + (\omega_c (1.15 \times 10^5))^2} \right| = \frac{1}{f_2}$$

→ constant value for ω_c

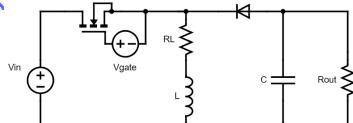


Peak value for i_g should not surpass 100mA. What can you change to reduce this current? Make the necessary changes. What is the tradeoff?

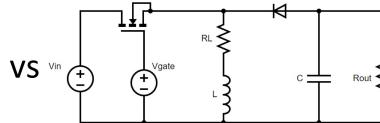
I added $R_{gate} = 60\Omega$ to my circuit
In order to decrease the i_g current
 $\hookrightarrow i_{gate} = \frac{V_{gate}}{R_{gate}}$

However this caused $R_{gate} \times C_{gate}$ to increase. Thus, τ_{gate} increased.
resulting in an increase in the rise and fall times of the MOSFET.
So the switching speed of the MOSFET dropped. This also causes power dissipation during each switching event.

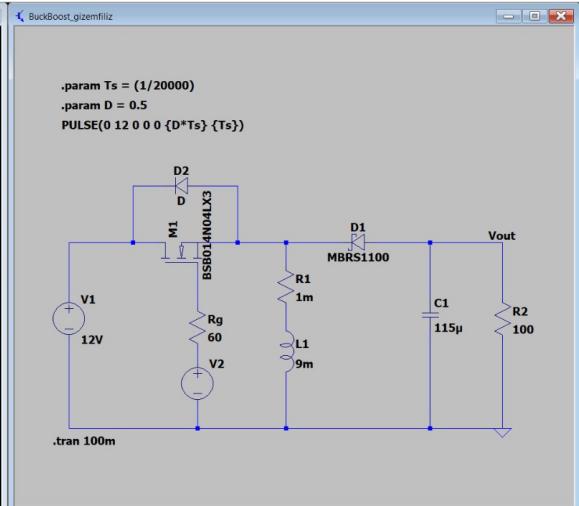
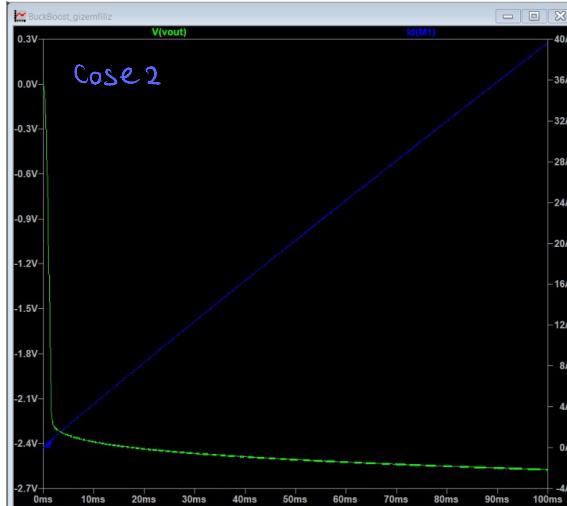
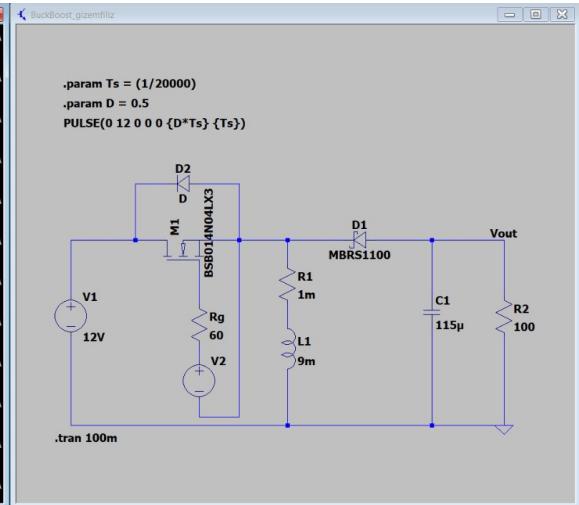
* Case 1:



Case 2:



For what reason is the left configuration was chosen for the gate pulse generator? Compare the current through the MOSFET drain source for both configurations. You can also check the same current for your buck converters (or the boost converter on Sucourse) to get a better idea.

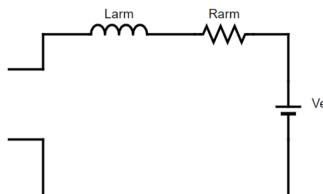


In case 2, we can see that Drain current grows unboundedly, which also results in lower output voltage (overdamped).

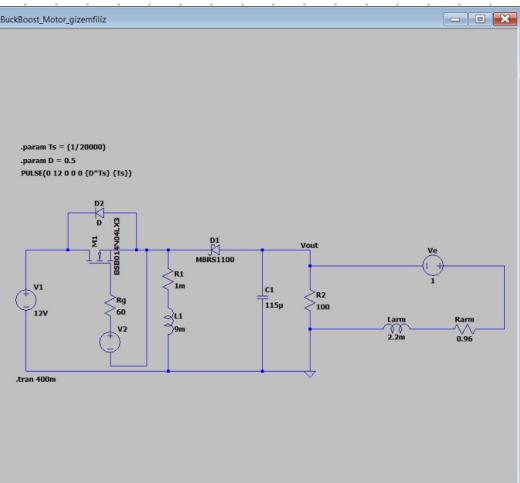
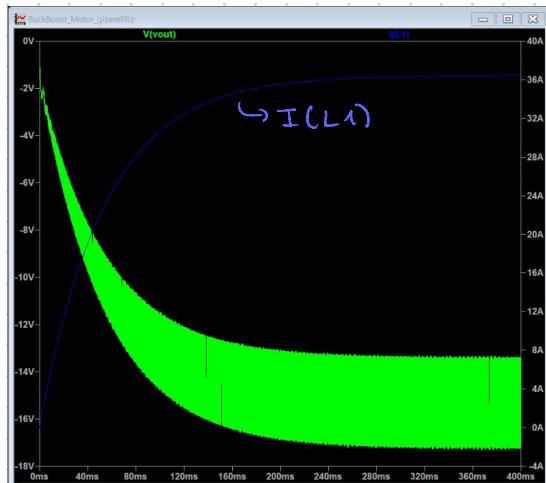
* Motor Armature:

The Dc motor is rated for 12-36Volts.

$$L_{arm} = 2.2\text{mH} \quad R_{arm} = 0.96\Omega$$

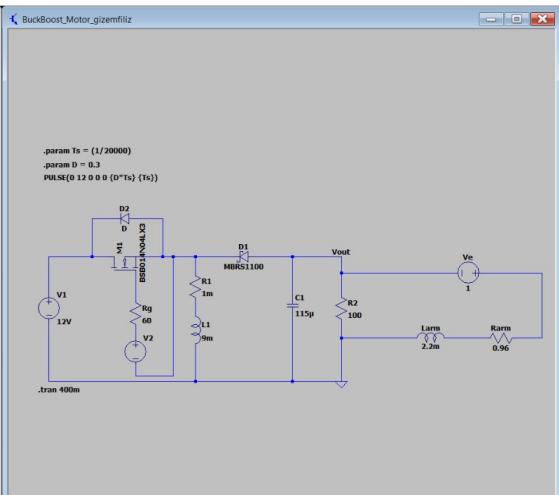
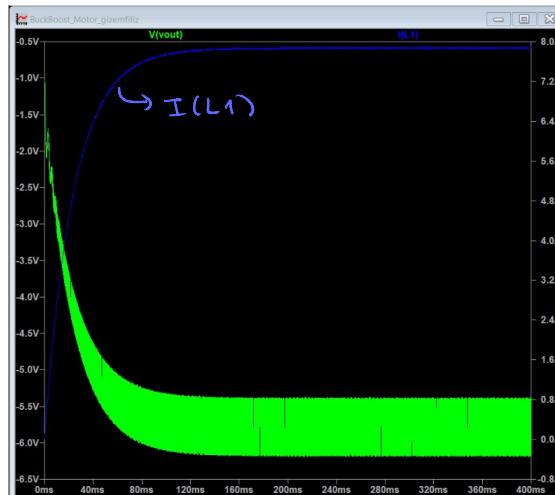


* D = 0.5

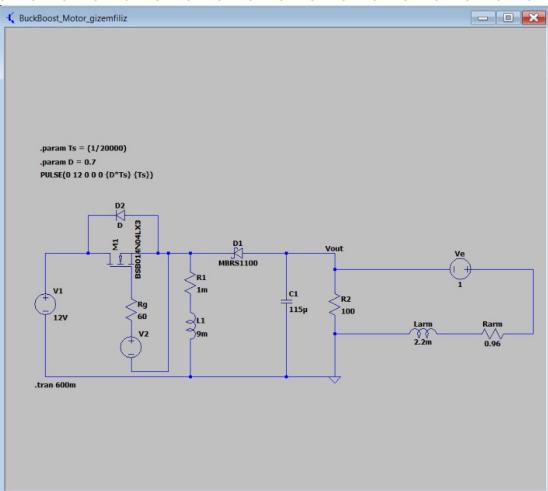
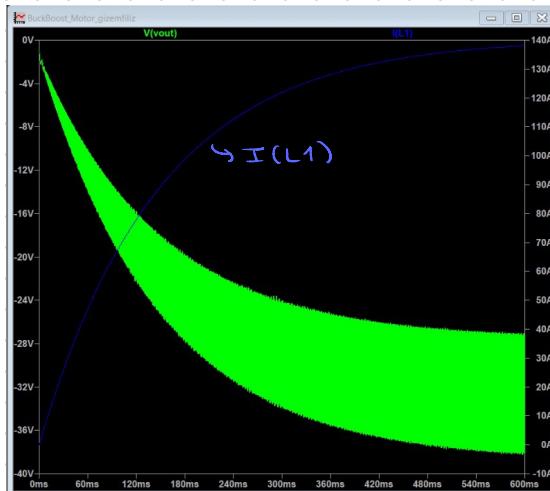


↳ $V_e = 1\text{V}$ resulted in an extreme increase of Voltage ripple, it became 3.7485V . This vibrations may cause hardware to break.
In order to overcome this issue, we may increase capacitor value or increase

* D = 0.3



* D = 0.7



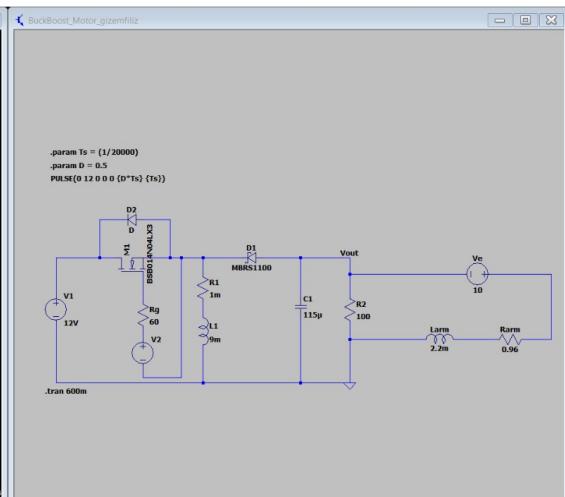
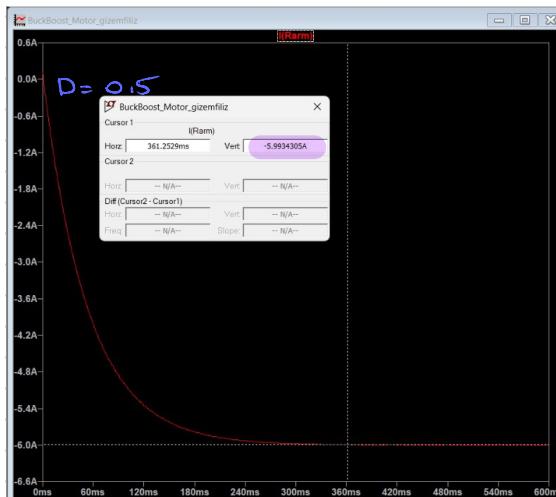
* Developed torque $T_e = k_m i_a$, Induced voltage $V_e = k_e \omega$

where $k_m = 0.5 \frac{N \cdot m}{A}$, $k_e = 0.5 \frac{V \cdot s}{rad}$, i_a is the armature current, and ω is the motor speed.

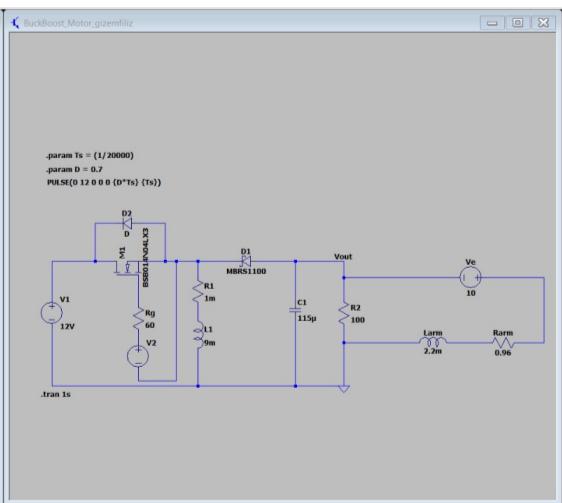
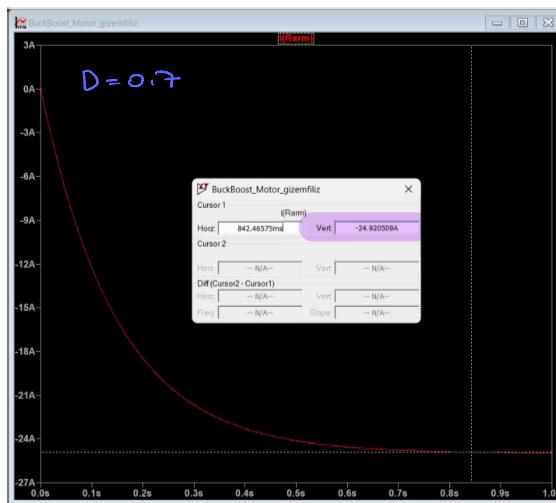
Given $\omega = 20 \text{ rad/s}$, find V_e . Measure the resulting armature current to calculate the torque.

Do the same for $\omega = 10 \text{ rad/s}$.

$$\omega = 20 \text{ rad/s} \Rightarrow V_e = 0.5 \frac{V \cdot s}{rad} \times 20 \frac{\text{rad}}{s} = 10 \text{ V}$$

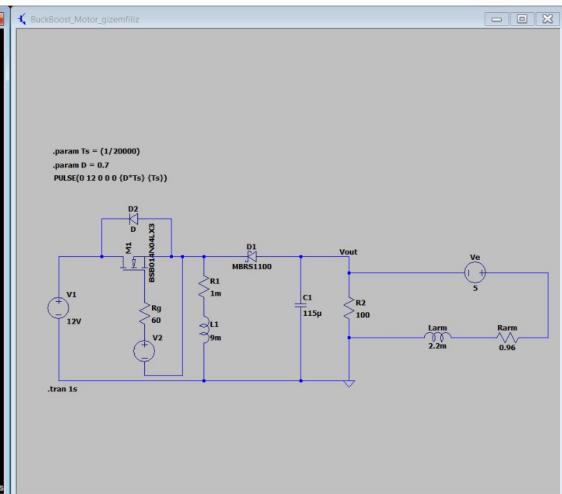
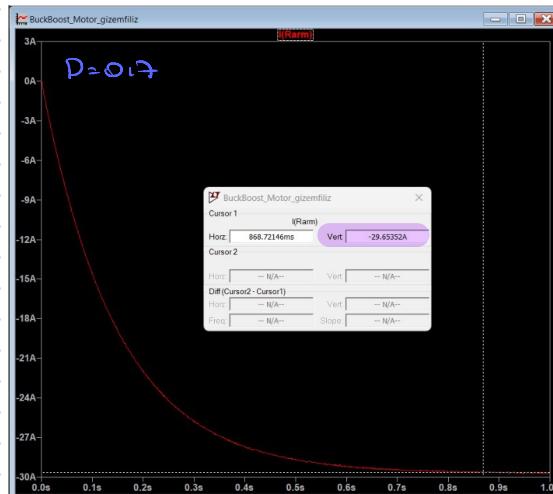


$$I_{\text{arm}} = 6 \text{ A} \Rightarrow T_e = 0.5 \frac{N \cdot m}{A} \times 6 \text{ A} = 3 \text{ N.m}$$



$$\hookrightarrow I_{\text{arm}} = 24.92 \text{ A} \Rightarrow \gamma_e = 0.15 \frac{\text{N}\cdot\text{m}}{\text{A}} \times 24.92 \text{ A} = 12.46 \text{ N.m}$$

$$\omega = 10 \text{ rad/s} \Rightarrow V_e = 0.15 \frac{\text{V.s}}{\text{rad}} \times 10 \frac{\text{rad}}{\text{s}} = 5 \text{ V}$$

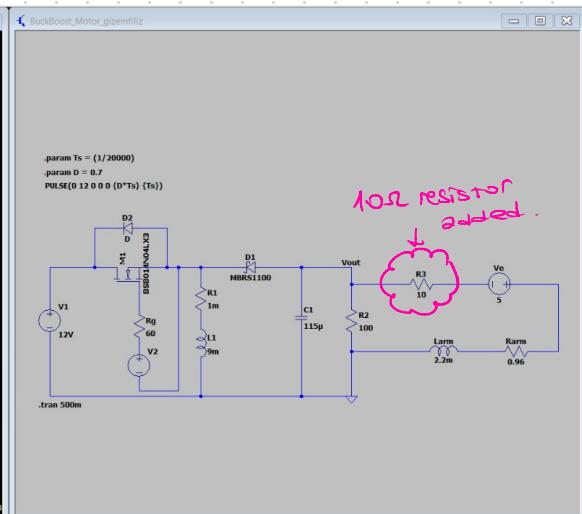
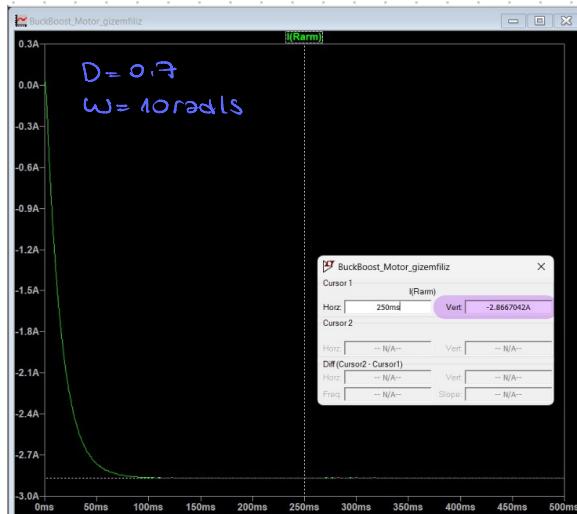


$$\hookrightarrow I_{\text{arm}} = 29.6535 \text{ A} \Rightarrow \gamma_e = 0.15 \frac{\text{N}\cdot\text{m}}{\text{A}} \times 29.6535 \text{ A} = 14.8267 \text{ N.m}$$

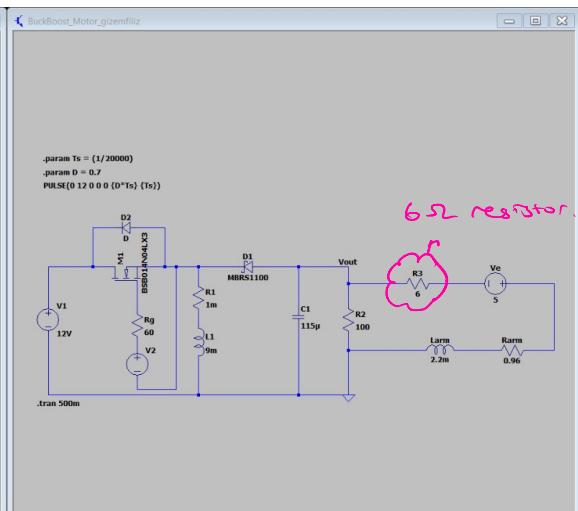
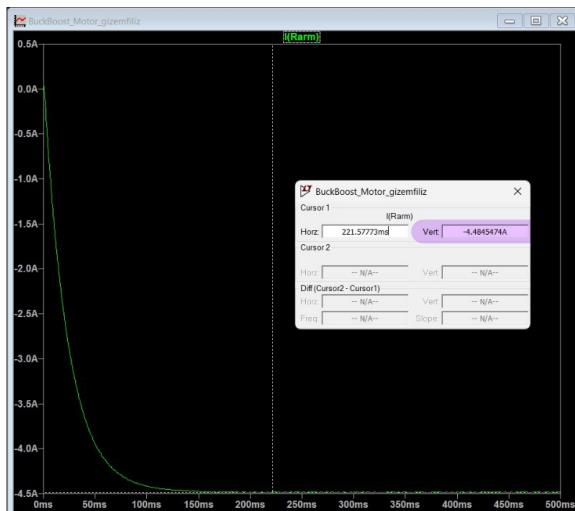
* The motor is rated for 5A current max. What changes can be made to protect the motor? What is the tradeoff?

If we limit the motor current to maximum of 5A, we will encounter a limit on the motor's torque output too.

↳ We can achieve this by adding a resistor to armature setup.



Armature current is reduced to 2.8667A by adding 10Ω resistor to armature setup.



With 6Ω resistor, armature current became 4.48A.

(without additional R)

$$\hookrightarrow I_{arm} = 29.6535A \Rightarrow \gamma_e = 0.15 \frac{N \cdot m}{A} \times 29.6535A = 14.8267 N \cdot m$$

(with R)

$$\hookrightarrow I_{arm} = 4.48A \Rightarrow \gamma_e = 0.15 \frac{N \cdot m}{A} \times 4.48A = 2.24 N \cdot m$$

\Rightarrow Adding R might protect the motor under high load conditions, however limiting armature current result in reduced torque and speed. Thus, limits the motor's operational capacity. We may prefer this strategy to reduce the risk of damaging circuit components and the motor.