Assignment 5

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1 Decision Version

Optimization Problem: c is a positive integer denoting the largest amount of effort that can be made on a subset of projects, how can we get the maximum profit from these projects, such that the total effort is at most c?

Decision Problem: A positive integer c denotes the maximum effort that can be put into a subset of projects, is it possible to get the highest amount of profit with at most total amount of c effort, by completing a subset of these projects?

Input: A set T of projects, where each project $t \in T$,
A positive integer e_t which denotes the amount of effort for a project t,
A positive integer p_t denoting the amount of profit gained by completing a project t,
A positive integer c which denotes the maximum amount of effort that can be spent on the projects.

Output:

YES: There is a way to get the highest amount of profit from a subset of projects $T' \subseteq T$, such that the total effort spent to complete the projects in T' is at most c.

NO: It is not possible to get the highest profit from any subset of projects $T' \subseteq T$, with at most total amount of effort c.

2 Proof

To prove that the decision version of this optimization problem is in NP, the problem should be non-deterministically polynomial. This means we should show that it is possible to verify the guessed solution in polynomial time. To verify the guessed solution, we need to verify two things happen in a polynomial time. Those are:

- The effort spent to complete a subset of projects $T' \subseteq T$ is at most c,
- The total profit gained from completing subset of projects $T' \subseteq T$ is greater or equal to p

Hence, given a guessed solution g, which is a subset of projects such that total amount of effort to complete the projects is at most c, and the profit gained from completing these projects is greater or equal to p, we can verify whether the effort is at most c, by summing the e_t's of every project and checking if it is smaller than c. This can be done in polynomial time.

Secondly, we can check if the profit gained from completing the project is the given threshold p, by adding up the values p_t, and checking if it is greater or equal to p. Then we can check other projects to see whether if we add that project to the subset, the value c is exceeding or not. If it is not exceeding, this means we can have a subset that makes more profit, so this proves that our guessed solution is incorrect. Otherwise, we can verify that it is correct. This computations can also be done in polynomial time.

To sum it up, we can say that it is possible to verify the decision problem in polynomial time, so it is in NP.