Calculation of Azimuth, Elevation and Polarization for non-horizontal aligned Antennas

Algorithm Description

Technical Document
TD-1205-a

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In Co-operation with

eutelsat COMMUNICATIONS

Change Log:

V1.3: Corrected error in the example: Values for latitude and longitude were inverted in the parameter table for the antenna's location. Correct values are now given as longitude=11° and latitude=50°

V1.4: Updated satellite table in chapter 2.3 Table of EUTELSAT Satellites
and updated formula $\vec{r}_{ESA\_ECEF}$ on page 12 and formula $\tau =$ on page 18.

V2.0: new release generated.

V2.1: Updated satellite table in chapter 2.3 Table of EUTELSAT Satellites

V2.2: error correction in text at page 8, numbering formulas
Updated satellite table in chapter 2.3 Table of EUTELSAT Satellites, 10.04.2018

Objective

Many SatCom applications are based on non-stationary terminals e.g. mounted on trucks or vessels. In the general case, the related antenna base is not horizontal whereas the offset angles are measured with adequate inclinometers. These offset angles have to be taken into account when commanding the azimuth, elevation and polarization drives of the non-horizontal terminal. This document presents a numeric algorithm that can calculate the pointing angles for a non-horizontal aligned antenna.

Decision Matrix: Which approach is most suitable for my application?

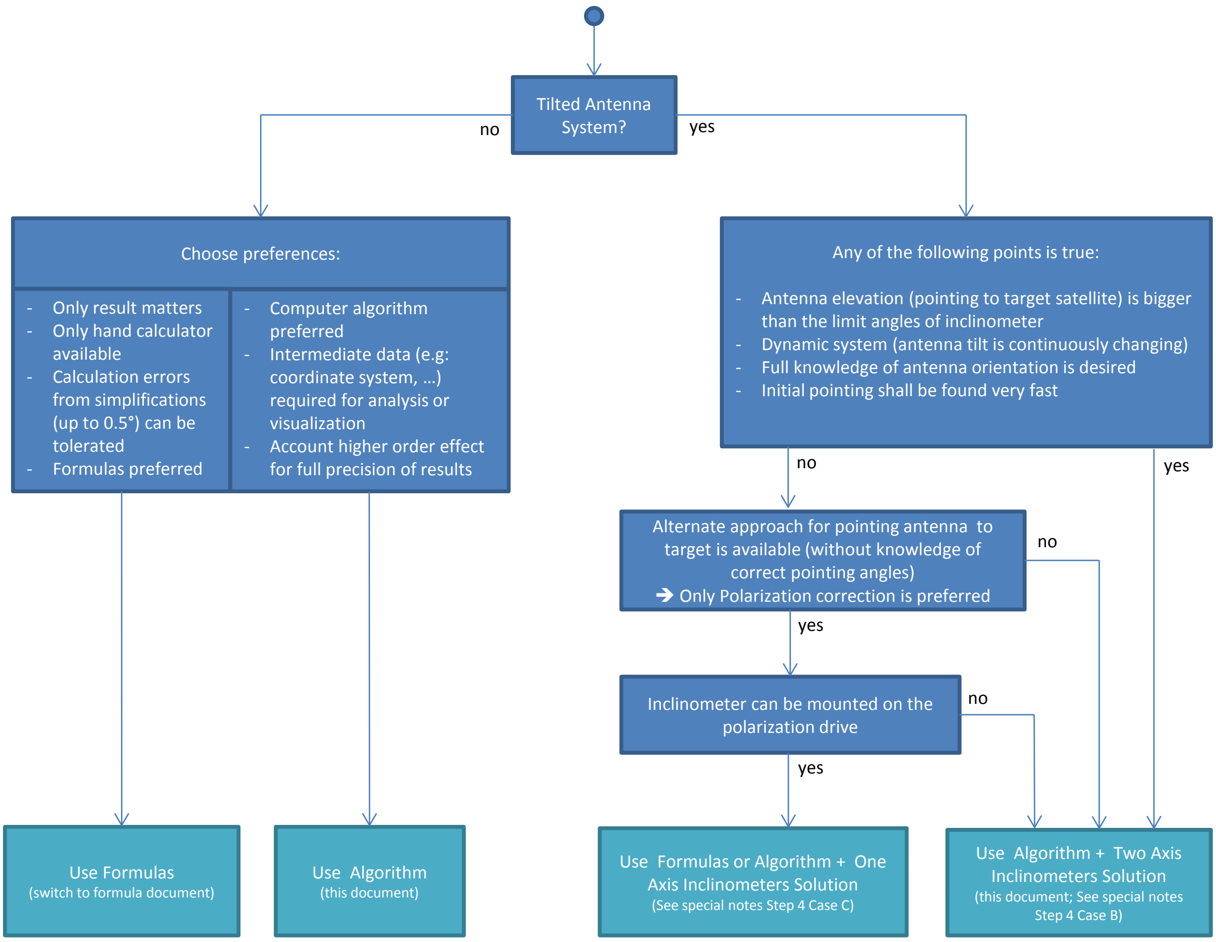

Table of Contents

1 Mathematical Background

The presented algorithm uses a lot of vector calculus. This chapter gives a short overview of the required basic operations. The reader should consult a mathematics book for a detailed description.

1.1 Matrix rules

$$A := \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \qquad B := \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \tag{1-1}$$

Matrix Transpose:

$$A^T = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix} \tag{1-2}$$

Matrix Multiplication:

$$\mathrm{C} := \mathrm{A} \cdot B$$

$$\begin{aligned}
C[1,1]; &= a_{11}\, b_{11} + a_{12}\, b_{21} + a_{13}\, b_{31} \\
C[2,1]; &= a_{21}\, b_{11} + a_{22}\, b_{21} + a_{23}\, b_{31} \\
C[3,1]; &= a_{31}\, b_{11} + a_{32}\, b_{21} + a_{33}\, b_{31} \\
C[1,2]; &= a_{11}\, b_{12} + a_{12}\, b_{22} + a_{13}\, b_{32} \\
C[2,2]; &= a_{21}\, b_{12} + a_{22}\, b_{22} + a_{23}\, b_{32} \\
C[3,2]; &= a_{31}\, b_{12} + a_{32}\, b_{22} + a_{33}\, b_{32} \\
C[1,3]; &= a_{11}\, b_{13} + a_{12}\, b_{23} + a_{13}\, b_{33} \\
C[2,3]; &= a_{21}\, b_{13} + a_{22}\, b_{23} + a_{23}\, b_{33} \\
C[3,3]; &= a_{31}\, b_{13} + a_{32}\, b_{23} + a_{33}\, b_{33}
\end{aligned} \tag{1-3}$$

Rotation Matrixes are defined as:

Information: cosd and sind in the following means that cos and sin are calculated using degrees.

$$\overline{R}_x(\alpha[°]) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cosd(\alpha) & -sind(\alpha) \\ 0 & sind(\alpha) & cosd(\alpha) \end{bmatrix} \quad \text{(Transformation around the x-Axis)} \tag{1-4}$$

$$\overline{R}_y(\alpha[°]) = \begin{bmatrix} cosd(\alpha) & 0 & sind(\alpha) \\ 0 & 1 & 0 \\ -sind(\alpha) & 0 & cosd(\alpha) \end{bmatrix} \quad \text{(Transformation around the y-Axis)} \tag{1-5}$$

$$\overline{R}_z(\alpha[°]) = \begin{bmatrix} cosd(\alpha) & -sind(\alpha) & 0 \\ sind(\alpha) & cosd(\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ (Transformation around the z-Axis) (1-6)
)

Coordinate System definitions:

Any Coordinate System consists of a transform and an origin vector.

Coordinate systems transforms are defined as:

$$\overline{T}_{SystemName\_BasisSystemName} = [\ \overrightarrow{e_x} \quad \overrightarrow{e_y} \quad \overrightarrow{e_z}\ ]$$
(1-7)

were $\overrightarrow{e_x}$ $\overrightarrow{e_y}$ $\overrightarrow{e_z}$ are the three perpendicular unit vectors of the coordinate system. The unit vectors are defined as column vectors. The document uses the following naming convention: A coordinate system transform is always depicted with the capital letter T. The subscript shows the name of the coordinate system as well as the coordinate system wherein it is defined.

In addition every coordinate system has its origin:

$$\vec{r}_{SystemName\_BasisSystemName} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$
(1-8)

The document uses the following naming convention: A coordinate system origin is always depicted with the letter r. The subscript naming rules are similar to the rules for coordinate system transforms.

1.2 Vector rules

Vectors are defined as column vectors.

$$x := \begin{bmatrix} a \\ b \\ c \end{bmatrix} \qquad y := \begin{bmatrix} d \\ e \\ f \end{bmatrix}$$
(1-9)

Cross Product rule:

$$x \times y = \begin{bmatrix} bf - ce \\ cd - af \\ ae - bd \end{bmatrix} = \begin{bmatrix} bf - ce \\ cd - af \\ ae - bd \end{bmatrix}$$
(1-10)

2 Numeric Algorithm Description

2.1 Algorithm Overview

The algorithm uses coordinate systems as well as matrix transformations to find the problem solution. The Earth-Centered, Earth-Fixed (ECEF) coordinate system is used as inertial frame. Its XY-Plane is the equatorial plane with X-Axis pointing in longitude 0° direction. The algorithm uses five stages to derive the pointing angles:

1. The GEO-Satellite coordinate system is defined in the ECEF coordinate system (multiple variants are feasible)
2. The Earth-Surface coordinate system at the antennas position is defined in the ECEF coordinate system (multiple variants are feasible, depending on the required accuracy)
3. The GEO-Satellite coordinate system is transformed to the Earth-Surface coordinate system. With this result the pointing coordinate system in the Earth-Surface coordinate system is derived.
4. The antenna coordinate system is defined in the Earth-Surface coordinate system
5. The pointing coordinate system is transformed to the antenna coordinate system and the Euler angles representing the antennas azimuth, elevation and polarization are calculated

2.2 Parameters

The user needs knowledge of the following parameters to calculate the antenna pointing angles:

Information	Name	Parameter	Unit	Value used in example calculation	Description
GEO Satellite	$Long_{sat}$	Satellite Position (Longitude)	[°]	7°	Measured positive in east direction. Value is in the range of 0° to 360°
	Lat_{sat}	Satellite Position (Latitude)	[°]	14°	Measured positive in north direction. Value is in the range of 90° to -90°
	Pol_{sat}	Polarization Angle (Is Skew Angle for $Lat_{sat} = 0$)	[°]	-22° (Note: For most Eutelsat Satellites the skew angle is 3.535°)	The value is the angle that aligns the Pol-Vector of the satellite antenna with the equatorial plane when rotating around the vector from the satellite center of mass to the earth center. If the Satellite is in the equatorial plane, implying $Lat_{sat} = 0$, this is the angle between the equatorial plane and the polarization direction, being the skew angle.
Antenna Position	$Long_{ant}$	Antenna Position Longitude	[°]	11°	Measured positive in east direction. Value is in the range of 0° to 360°
	Lat_{ant}	Antenna Position Latitude	[°]	50°	Measured positive in north direction. Value is in the range of 90° to -90°
	Alt_{ant} or r_{earth}	Antenna Altitude or Earth Radius	[m]	r_{earth} 6378000m	Altitude above mean sea level of the antenna or Earth radius Depends on the chosen earth model (See algorithm step 2 for details)
Antenna Orientation (See Chapter 2.4 Step 4)	$Roll_{ant}$	Antenna Roll (Prime Inclinometer)	[°]	14°	The prime inclinometer is placed in the rotating part of the station which generates the azimuth movement. It is aligned with the azimuth direction antenna main beam at elevation 0° (See Figure 5 Chapter 2.4) Measured Clockwise
	$Pitch_{ant}$	Antenna Pitch (Secondary Inclinometer)	[°]	8°	The secondary inclinometer is placed in the rotating part of the station which generates the azimuth movement. It is aligned with the elevation axis (See Figure 5 Chapter 2.4) Measured Clockwise
	$IncAz_{ant}$	In Plane (Horizon) azimuth angle of the prime Inclinometer	[°]	68°	Defines the angle of the prime inclinometer measured in the earth surface plane (See Figure 5 Chapter 2.4) Measured Clockwise
	$Az_{ant\,meas}$	Azimuth offset correction	[°]	52°	Optional when measuring parameters with an initial tracking cycle: Offset of the azimuth axis during measurement. Measured Clockwise (See Chapter 2.4 – Step 4B)

2.3 Table of EUTELSAT Satellites

EUTELSAT Name	Other Designations	EUTELSAT Code	International Designator	Orbital Location	Pol Skew Angle for LP (VO*-1)	Linnear / Circular Polarization
EUTELSAT 117 West A	SATMEX 8	**E117WA**	13012A	-116.8		LP
EUTELSAT 117 West B	SATMEX9	**E117WB**	16038B	-117.0		LP
EUTELSAT 115 West B	SATMEX 7	**E115WB**	15010B	-114.9		LP
EUTELSAT 113 West A	SATMEX 6	**E113WA**	06020A	-113.0		LP
	SATMEX 10			-113.0		LP (Future Satellite)
EUTELSAT 65 West A		**E65WA**	14006A	-65.2	0.000	C & Ku-Band: LP; Ka CP Q/V - LP (X-Horizontal)
EUTELSAT 36 West A	Atlantic Bird 1; E12WA	**E36WA**	02040A	-36.3	3.535	LP
Telstar 12 V	Telstar 12 Vantage	**TELSTAR-12V**	15068A	-15.0	0.000	LP; (CP for some non-EUTELSAT capacity)
EUTELSAT 12 West B	Atlantic Bird 2; AB2; New Bird; E8WA	**E12WB**	01042A	-12.5	3.535	LP
EUTELSAT 12 West C	QUANTUM; AnySat	**E12WC**		-12.5	3.535	*Future Satellite*
EUTELSAT 8 West B	AB2A	**E8WB**	15039B	-8.0	3.535	LP
EUTELSAT 7 West A	AB4R; AB7; AB4A	**E7WA**	11051A	-7.3	3.535	Ku-Band: LP; C-Band CP
EUTELSAT 5 West A	Atlantic Bird 3; Stellat 5	**E5WA**	02035A	-5.0	0.000	LP
EUTELSAT 5 West B		**E5WB**		-5.0		*Future Satellite*
EUTELSAT 3B	Newsat	**E3B**	14030A	3.1	3.535	LP
EUTELSAT 7A	W3A	**E7A**	04008A	7.0	3.535	LP
EUTELSAT 7B	E3D; W3D	**E7B**	13022A	7.0	3.535	LP
EUTELSAT 7C	E7X	**E7X**				*Future Satellite*
EUTELSAT 9B	ESA Data Relay; EDRS-A	**E9B**	16005A	9.0	3.535	LP
EUTELSAT KA-SAT 9A	KaSAT	**KA9A**	10069A	9.0	3.535	CP (LP for beacon)
EUTELSAT 10A	W2A	**E10A**	09016A	10.0	3.535	Ku-Band: LP; C-Band CP
BB4A	African Broadband Satellite (HTS)					*Future Satellite*
EUTELSAT HOT BIRD 13B	HB8	**HB13B**	06032A	13.0	3.535	LP
EUTELSAT HOT BIRD 13C	HB9	**HB13C**	08065A	13.0	3.535	LP
EUTELSAT HOT BIRD 13E	HB7A; Eurobird 9A; E9A	**HB13E**	06007B	13.0	3.535	LP

EUTELSAT 16A	W3C	**E16A**	11057A	16.0	3.535	LP
EUTELSAT 16C	SESAT 1	**E16C**	00019A	0.0	3.535	LP
EUTELSAT 21B	W6A	**E21B**	12062B	21.6	3.535	LP
EUTELSAT 25B	Es'hail; EB2A	**E25B**	13044A	25.5	3.535	Ku & DBS Band: LP Ka-Band: CP
EUTELSAT 28E	ASTRA 2E	**E28E**	13056A	28.5	0.000	
EUTELSAT 28F	ASTRA 2F	**E28F**	12051A	28.2	0.000	
EUTELSAT 28G	ASTRA 2G	**E28G**	14089A	28.2	0.000	
EUTELSAT 31A	e-Bird; EB3; E33A	**E31A**	03043A	30.9	3.535	LP
EUTELSAT 33C	W1R; E28A	**E33C**	01011A	33.1	3.535	LP
EUTELSAT 33E	HB10; AB4A; E3C; HB13D	**E33E**	09008B	33.1	3.535	LP
EUTELSAT 36B	W7	**E36B**	09065A	35.9	3.535	LP
EUTELSAT 36C	EXPRESS AMU1	**E36C**	15082A	36.1	3.535	LP
Y1B	Yahsat 1B	**Y1B**	12016A	47.6	0.000	
EUTELSAT 48D	E28B; W2M	**E48D**	08065B	48.1	3.535	LP
EUTELSAT 53A	EXPRESS-AM6; EXPRESS-AM22; SESAT 2	**E53A**	03060A	53.0	0.000	LP
YAMAL 402	2012-070	**Yamal402**	12070A	54.9		
EUTELSAT 56A	Express AT1	**EXPRESS AT1**	14010A	56.0		CP
EUTELSAT 70B	W5A	**E70B**	12069A	70.5	3.535	LP
EUTELSAT 140A	EXPRESS AT2	**EXPRESS AT2**	14010B	140.0		CP
EUTELSAT 172B		**E172B**	17029B	172.0	0.000	LP
EUTELSAT 174A	AMC23; GE-23;AMC 23, Worldsat 3, GE 2i,E172A	**E174A**	05052A	174.0	0.000	LP

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* **<u>Note about value of the skew of the Eutelsat satellites</u>** :The reference X-polarization (horizontal) is defined as that polarization whose plane makes an angle of 93.535° in an anti-clockwise direction, looking towards the earth, about a reference vector with respect to a plane containing this vector and the pitch axis. The reference vector is defined as the vector from the satellite in the direction 0.21° towards West and 6.07° towards north in satellite coordinates. The reference Y-polarization (vertical) is defined as that polarization whose plane is orthogonal to the X polarization plane and the reference vector defined above. In other words the polarization skew angle of the EUTELSAT satellites is +3.535°, clock-wise when looking at the satellite from the subsatellite point, while in the southern hemisphere the polarization skew angle of the EUTELSAT satellites is +183.535°, clock-wise.

2.4 Algorithm Details

Step 1: GEO-Satellite coordinate system

Goal: Define the position and orientation of the Satellite

The position (origin of the GEO-Satellite is calculated with spherical coordinates and is dependent on its radius, the longitude and latitude.

$$\vec{r}_{GEO\_ECEF} = \begin{bmatrix} r_{GEO} * \ cosd(Lat_{sat}) \ * cosd(Long_{sat}) \\ r_{GEO} * \ cosd(Lat_{sat}) \ * sind(Long_{sat}) \\ r_{GEO} * \ sind(Lat_{sat}) \end{bmatrix}$$ = GEO satellite origin in ECEF System

(2-1)

Example: $r_{GEO} = 42164200m \ \ Long_{sat} = 7° \ \ \ Lat_{sat} = 14°$

$$\vec{r}_{GEO\_ECEF} = \begin{bmatrix} 4.0606793 \ 10^7 \\ 4.9858871 \ 10^6 \\ 1.0200443 \ 10^7 \end{bmatrix}$$

The GEO satellite coordinate system transform is defined as following: X-Axis is parallel to the nadir (earth center) direction. The X-Axis points away from earth. The Y/Z-Axes are aligned with the polarization planes of the GEO satellite antenna. The matrix results from three rotations: Longitude, Latitude and Polarization:

$$\overline{T}_{GEO\_ECEF} = \overline{R}_Z(Long_{sat}) * \ \overline{R}_Y(-Lat_{sat}) * \overline{R}_x(Pol_{sat} \ )$$ = GEO transform in ECEF System

(2-2)

Example: $Long_{sat} = 7° \quad Lat_{sat} = 14° \quad Pol_{sat} = -22°$

$$\overline{T}_{GEO\_ECEF} = \begin{bmatrix} 0.993 & -0.122 & 0. \\ 0.122 & 0.993 & 0. \\ 0. & 0. & 1. \end{bmatrix} * \begin{bmatrix} 0.970 & 0. & -0.242 \\ 0. & 1. & 0. \\ 0.242 & 0. & 0.970 \end{bmatrix} * \begin{bmatrix} 1. & 0. & 0. \\ 0. & 0.928 & 0.374 \\ 0. & -0.374 & 0.928 \end{bmatrix}$$

$$\overline{T}_{GEO\_ECEF} = \begin{bmatrix} 0.962 & -0.122 & -0.240 \\ 0.118 & 0.992 & -0.0295 \\ 0.242 & 0. & 0.970 \end{bmatrix} * \begin{bmatrix} 1. & 0. & 0. \\ 0. & 0.928 & 0.374 \\ 0. & -0.374 & 0.928 \end{bmatrix}$$

$$\overline{T}_{GEO\_ECEF} = \begin{bmatrix} 0.962 & -0.0230 & -0.268 \\ 0.118 & 0.931 & 0.345 \\ 0.242 & -0.364 & 0.899 \end{bmatrix}$$

Output: The origin vector ($\vec{r}_{GEO\_ECEF}$) and transform ($\overline{T}_{GEO\_ECEF}$) of the Geo-Satellite in the ECEF-Frame

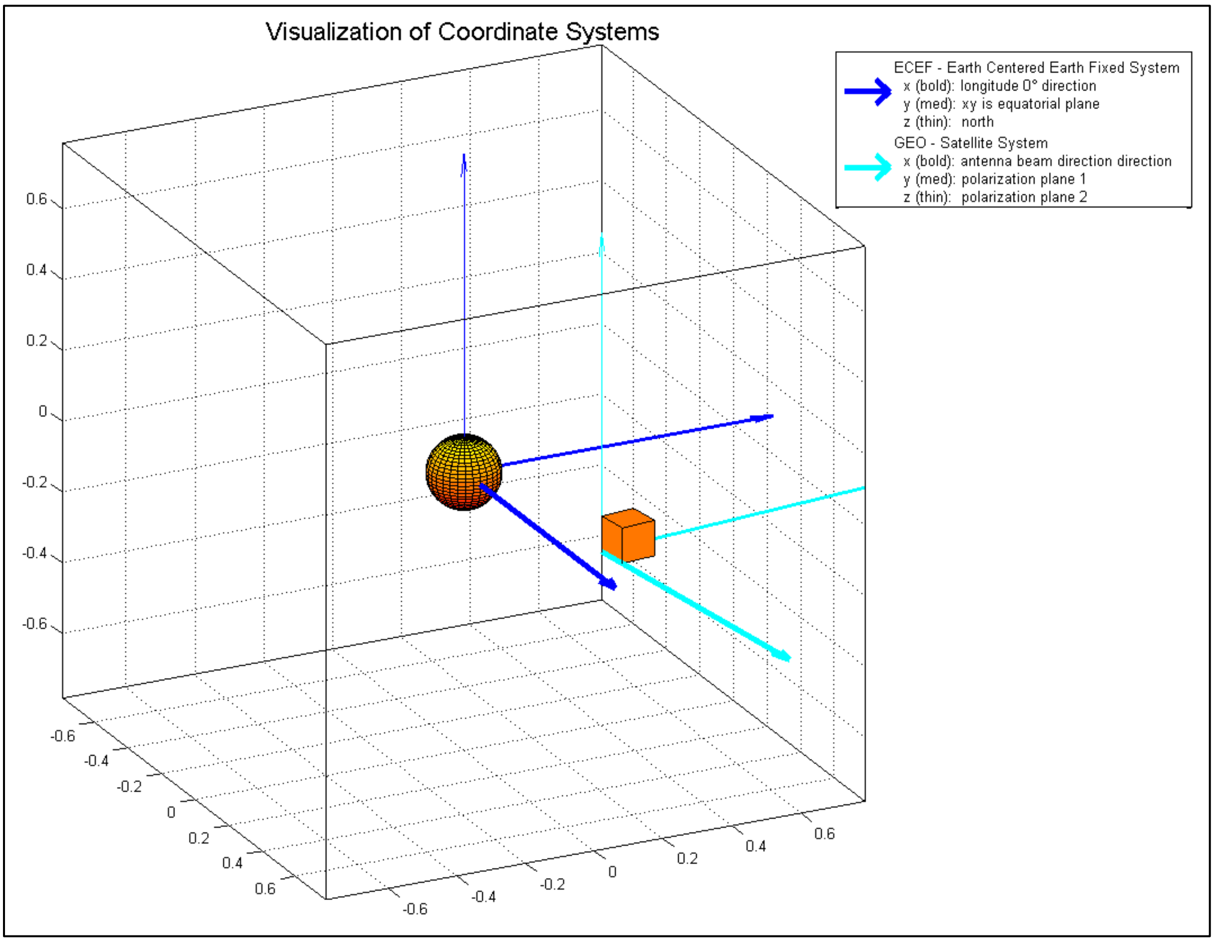

Figure 1: ECEF System and the Satellite System

Step 2: Earth-Surface coordinate system

Goal: Find the position and orientation of the earth surface coordinate system (at the antennas position)

Note: There are several earth models available that can be used for this purpose. Two alternatives are depicted here. First the usage of a spherical earth model that is very easy to handle but introduces pointing errors up to around 0.8°. Second the WGS84 model that handles the first order gravitational anomalies and is therefore much more accurate but involves more calculation work.

a) Spherical earth model

The antennas position being the earth surface coordinate system origin is calculated using longitude and latitude in spherical coordinates. With the earth radius the position is calculated as following:

$$\vec{r}_{ESA\_ECEF} = \begin{bmatrix} r_{Earth} * cosd(Lat_{ant}) * cosd(Long_{ant}) \\ r_{Earth} * cosd(Lat_{ant}) * sind(Long_{ant}) \\ r_{Earth} * sind(Lat_{ant}) \end{bmatrix} \quad \text{= Earth Surface origin in ECEF System}$$

(2-3)

Example: $r_{Earth} = 6378000 \quad Long_{ant} = 50° \quad Lat_{ant} = 11°$

$$\vec{r}_{ESA\_ECEF} = \begin{bmatrix} 4.024376\;10^{6} \\ 7.822598\;10^{5} \\ 4.885832\;10^{6} \end{bmatrix}$$

The earth surface coordinate system transform matrix is constructed from unit vectors. The XY-Plane is parallel to the ground. The Z-Axis points to nadir (earth center) direction. The X-Axis points in the north direction.

$$\overrightarrow{e_z}_{ESA\_ECEF} = -\frac{\vec{r}_{ESA\_ECEF}}{|\vec{r}_{ESA\_ECEF}|}$$ is the negative normalized origin vector (2-4)

$$\overrightarrow{e_y}_{ESA\_ECEF} = \frac{\overrightarrow{e_z}_{ESA\_ECEF} \times \overrightarrow{e_z}_{ECEF\_0}}{|\overrightarrow{e_z}_{ESA\_ECEF} \times \overrightarrow{e_z}_{ECEF\_0}|}$$ is perpendicular to the north direction (2-5)

$$\overrightarrow{e_x}_{ESA\_ECEF} = \frac{\overrightarrow{e_y}_{ESA\_ECEF} \times \overrightarrow{e_z}_{ESA\_ECEF}}{|\overrightarrow{e_y}_{ESA\_ECEF} \times \overrightarrow{e_z}_{ESA\_ECEF}|}$$ points to north and completes the orthogonal system (2-6)

$$\overline{T}_{ESA\_ECEF} = \left[\overrightarrow{e_x}_{ESA\_ECEF} \;\; \overrightarrow{e_y}_{ESA\_ECEF} \;\; \overrightarrow{e_z}_{ESA\_ECEF}\right]$$ = Earth Surface transform in ECEF System (2-7)

Example:

$$e_{z\;ESA\;ECEF} = -\frac{\begin{bmatrix} 4.02437610^{6} \\ 7.82259810^{5} \\ 4.88583210^{6} \end{bmatrix}}{\sqrt{\left(4.02437610^{6}\right)^{2} + \left(7.82259810^{5}\right)^{2} + \left(4.88583210^{6}\right)^{2}}} = \begin{bmatrix} -0.631 \\ -0.123 \\ -0.766 \end{bmatrix}$$

$$e_{y\;ESA\;ECEF} = \frac{\begin{bmatrix} -0.631 \\ -0.123 \\ -0.766 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}{\square \begin{bmatrix} -0.631 \\ -0.123 \\ -0.766 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \square} = \frac{\begin{bmatrix} -0.123 \\ 0.631 \\ 0. \end{bmatrix}}{\sqrt{(-0.123)^{2} + (0.631)^{2} + (0)^{2}}} = \begin{bmatrix} -0.1913 \\ 0.9815 \\ 0. \end{bmatrix}$$

$$e_{x\;ESA\;ECEF} = \frac{\begin{bmatrix} -0.1913 \\ 0.9815 \\ 0. \end{bmatrix} \times \begin{bmatrix} -0.631 \\ -0.123 \\ -0.766 \end{bmatrix}}{\square 1 \square} = \begin{bmatrix} -0.7518 \\ -0.1465 \\ 0.6428 \end{bmatrix}$$

$$\overline{T}_{ESA\_ECEF} = \begin{bmatrix} e_{x\,ESA\,ECEF} & e_{y\,ESA\,ECEF} & e_{z\,ESA\,ECEF} \end{bmatrix} = \begin{bmatrix} -0.7518 & -0.1913 & -0.631 \\ -0.1465 & 0.9815 & -0.123 \\ 0.6428 & 0. & -0.766 \end{bmatrix}$$

b) WGS84 earth model

The earth ellipsoid is used in the calculations to increase accuracy. The calculation requires the values of the major and minor semi-axes of the earth ellipsoid. These values are constants and defined as:

$a = 6378137.0 \quad b = 6356752.314$

Additional we require the following values:

$$e1 = \frac{\sqrt{a^2 - b^2}}{a} \qquad e2 = \frac{\sqrt{a^2 - b^2}}{b} \qquad n = \frac{a}{\sqrt{1 - e1^2 \sin^2(Lat_{ant})}} \tag{2-8}$$

With the antenna altitude (Height above mean sea level) the position is calculated as following.

$$\vec{r}_{ESA\_ECEF} = \begin{bmatrix} (n + Alt_{ant}) * cosd(Lat_{ant}) * cosd(Long_{ant}) \\ (n + Alt_{ant}) * cosd(Lat_{ant}) * sind(Long_{ant}) \\ (n * (1 - e1^2) + Alt_{ant}) * sind(Lat_{ant}) \end{bmatrix}$$ = Earth Surface origin in ECEF System

(2-9)

For the calculation of the surface system we require the following two additional values:

$$t0 = \text{atand}\left(\frac{a\, \vec{r}_{ESA\_ECEF\_z}}{b\, \vec{r}_{ESA\_ECEF\_x}}\right)$$

$$u = \text{atand}\left(\frac{\vec{r}_{ESA\_ECEF\_z} + e2^2 \,\text{sind}^3(t0)}{\vec{r}_{ESA\_ECEF\_x} - e2^2\, cosd^3(t0)}\right)$$

(2-10)

The orientation matrix is constructed from unit vectors. The XY-Plane is perpendicular to the gravitational vector. The Z-Axis points downwards. The X-Axis points in the north direction.

$$\overrightarrow{e_z}_{ESA\_ECEF} = -\begin{bmatrix} cosd(u) * cosd(Long_{ant}) \\ cosd(u) * sind(Long_{ant}) \\ sind(u) \end{bmatrix}$$ is parallel to the gravitation (2-11)

$$\overrightarrow{e_y}_{ESA\_ECEF} = \frac{\overrightarrow{e_z}_{ESA\_ECEF} \times \overrightarrow{e_z}_{ECEF\_0}}{\left|\overrightarrow{e_z}_{ESA\_ECEF} \times \overrightarrow{e_z}_{ECEF\_0}\right|}$$ is perpendicular to the north direction (2-12)

$$\overrightarrow{e_x}_{ESA\_ECEF} = \frac{\overrightarrow{e_y}_{ESA\_ECEF} \times \overrightarrow{e_z}_{ESA\_ECEF}}{\left|\overrightarrow{e_y}_{ESA\_ECEF} \times \overrightarrow{e_z}_{ESA\_ECEF}\right|}$$ points to north and completes the orthogonal system (2-13)

$$\overline{T}_{ESA\_ECEF} = \left[\overrightarrow{e_x}_{ESA\_ECEF} \;\; \overrightarrow{e_y}_{ESA\_ECEF} \;\; \overrightarrow{e_z}_{ESA\_ECEF}\right]$$ = Earth Surface transform in ECEF System (2-14)

Output: The origin vector ($\vec{r}_{ESA\_ECEF}$) and transform matrix ($\overline{T}_{ESA\_ECEF}$) of the Earth-Surface coordinate system in the ECEF-Frame

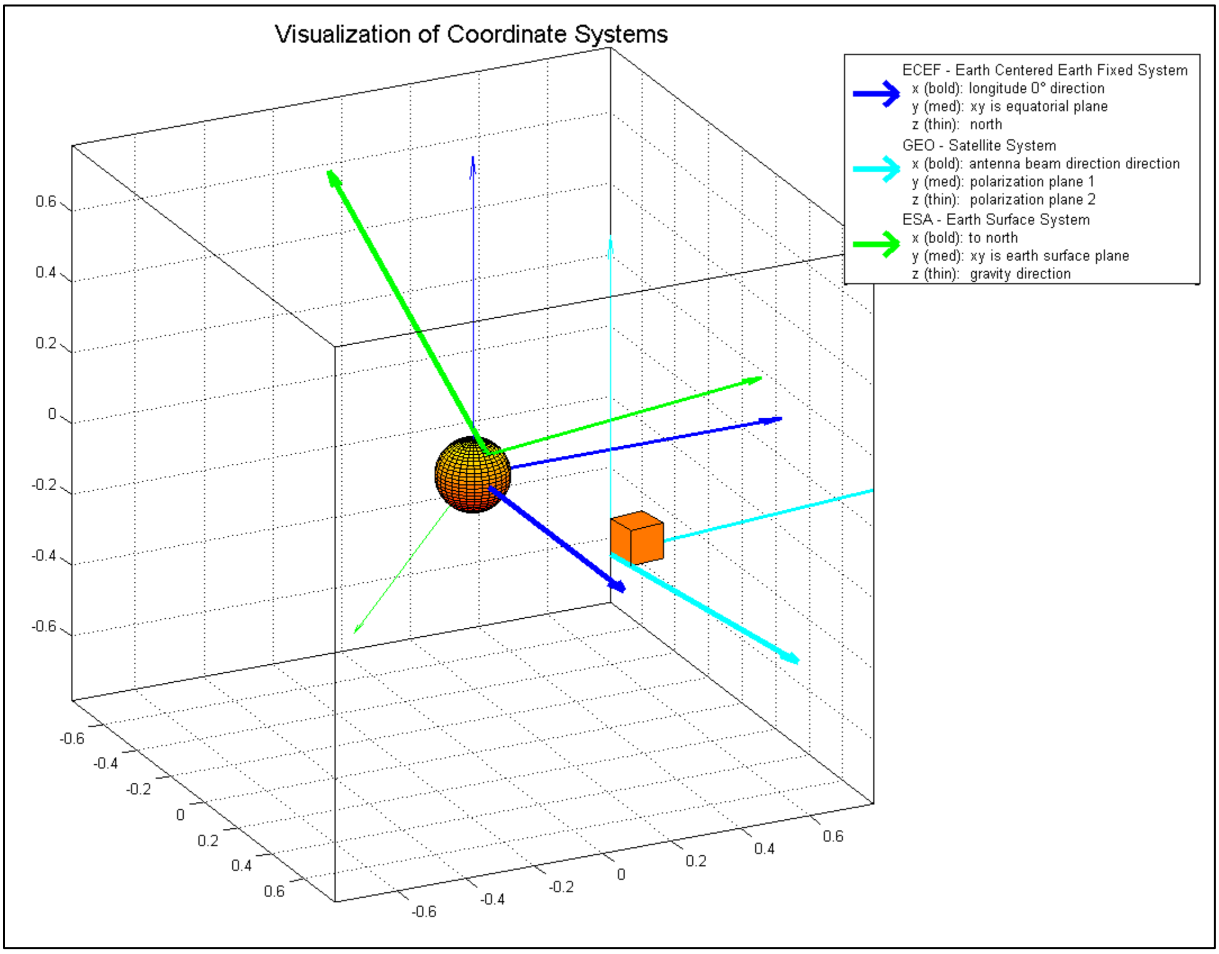

Figure 2: Added Earth Surface System

Step 3: Derive the untilted pointing system

Goal: Find the coordinate system, that describes the correct pointing for the untilted antenna

The GEO-Satellite position and orientation is defined in the ECEF frame (results from step 1). For the calculation of the pointing system we require the GEO-Satellite position and orientation as seen from the earth surface system (results from step 2). Therefore we apply a transformation. Note that all arguments on the right side of the equations have to be defined in the same base system (here ECEF) for the transformation to be valid.

$$\vec{r}_{GEO\_ESA} = \overline{T}_{ESA\_ECEF}^{\;T} * \left(\vec{r}_{GEO\_ECEF} - \vec{r}_{ESA\_ECEF}\right)$$ = GEO origin in Earth Surface System (2-15)

$$\overline{T}_{GEO\_ESA} = \overline{T}_{ESA\_ECEF}^{\;T} * \overline{T}_{GEO\_ECEF}$$ = GEO transform in Earth Surface System (2-16)

Example: With the systems from the steps above

$$\vec{r}_{GEO\_ESA} = \begin{bmatrix} -0.7518 & -0.1913 & -0.631 \\ -0.1465 & 0.9815 & -0.123 \\ 0.6428 & 0. & -0.766 \end{bmatrix}^T * \left(\begin{bmatrix} 4.0606793\ 10^7 \\ 4.9858876\ 10^6 \\ 1.0200443\ 10^7 \end{bmatrix} - \begin{bmatrix} 4.024376\ 10^6 \\ 7.822598\ 10^5 \\ 4.885832\ 10^6 \end{bmatrix} \right)$$

$$\vec{r}_{GEO\_ESA} = \begin{bmatrix} -0.7518 & -0.1465 & 0.6428 \\ -0.1913 & 0.9815 & 0. \\ -0.631 & -0.123 & -0.766 \end{bmatrix} * \begin{bmatrix} 3.6582417\,10^7 \\ 4.2036279\,10^6 \\ 5.314611\,10^6 \end{bmatrix} = \begin{bmatrix} -2.4702260\,10^7 \\ -2.8723555\,10^6 \\ -2.767154310^7 \end{bmatrix}$$

$$\overline{T}_{GEO\_ESA} = \begin{bmatrix} -0.7518 & -0.1913 & -0.631 \\ -0.1465 & 0.9815 & -0.123 \\ 0.6428 & 0. & -0.766 \end{bmatrix}^T * \begin{bmatrix} 0.962 & -0.0230 & -0.268 \\ 0.118 & 0.931 & 0.345 \\ 0.242 & -0.364 & 0.899 \end{bmatrix} = \begin{bmatrix} -0.5857 & -0.3525 & 0.7303 \\ -0.0684 & 0.9201 & 0.3890 \\ -0.8075 & 0.1779 & -0.5619 \end{bmatrix}$$

Now we have the vector from the station to the satellite. Thus we can calculate the range as following:

$$Range = |\vec{r}_{GEO\_ESA}| = \sqrt{(r_{x\ GEO\_ESA})^2 + (r_{y\ GEO\_ESA})^2 + (r_{z\ GEO\_ESA})^2} \tag{2-17}$$

Example:

$$Range = \sqrt{(-2.4702260\,10^7)^2 + (-2.8723555\,10^6)^2 + (-2.767154310^7)^2} = 3.720438639\,10^7$$

Next the required pointing coordinate system can be derived. The Antenna must point to the GEO satellite and be aligned with the polarization axes of the GEO satellite antenna. The pointing direction is easy to find as it is parallel to the calculated range vector. For the polarization directions: Remember that the Y/Z-Axes of the GEO transform had been chosen parallel to the polarization axes in step 1. Therefore we can use cross product to project the directions. The formulas are:

$$\overrightarrow{e_x}_{POINTING\_ESA} = \frac{\vec{r}_{GEO\_ESA}}{Range}$$ is the normalized vector to the GEO satellite (2-18)

$$\overrightarrow{e_y}_{POINTING\_ESA} = \frac{\overrightarrow{e_z}_{GEO\_ESA} \times \overrightarrow{e_x}_{POINTING\_ESA}}{|\overrightarrow{e_z}_{GEO\_ESA} \times \overrightarrow{e_x}_{POINTING\_ESA}|}$$ is aligned with the polarization plane (2-19)

$$\overrightarrow{e_z}_{POINTING\_ESA} = \frac{\overrightarrow{e_x}_{POINTING\_ESA} \times \overrightarrow{e_y}_{POINTING\_ESA}}{|\overrightarrow{e_x}_{POINTING\_ESA} \times \overrightarrow{e_y}_{POINTING\_ESA}|}$$ completes the orthogonal system (2-20)

$$\overline{T}_{POINTING\_ESA} = \left[\overrightarrow{e_x}_{POINTING\_ESA}\ \ \overrightarrow{e_y}_{POINTING\_ESA}\ \ \overrightarrow{e_z}_{POINTING\_ESA} \right] \tag{2-21}$$

= Pointing transform in Earth Surface System

Example:

$$e_{x\ POINTING\ ESA} = \frac{\begin{bmatrix} -2.4702260 10^{7} \\ -2.8723555 10^{6} \\ -2.7671543 10^{7} \end{bmatrix}}{3.720438639 10^{7}} = \begin{bmatrix} -0.6639 \\ -0.0772 \\ -0.7437 \end{bmatrix}$$

$$e_{y\ POINTING\ ESA} = \frac{\begin{bmatrix} 0.7303 \\ 0.3890 \\ -0.5619 \end{bmatrix} \times \begin{bmatrix} -0.6639 \\ -0.0772 \\ -0.7437 \end{bmatrix}}{\square \begin{bmatrix} 0.7303 \\ 0.3890 \\ -0.5619 \end{bmatrix} \times \begin{bmatrix} -0.6639 \\ -0.0772 \\ -0.7437 \end{bmatrix} \square} = \frac{\begin{bmatrix} -0.3326 \\ 0.9161 \\ 0.2018 \end{bmatrix}}{\sqrt{(-0.3326)^2 + (0.9161)^2 + (0.2018)^2}} = \begin{bmatrix} -0.3342 \\ 0.9205 \\ 0.2028 \end{bmatrix}$$

$$e_{z\ POINTING\ ESA} = \frac{\begin{bmatrix} -0.6639 \\ -0.0772 \\ -0.7437 \end{bmatrix} \times \begin{bmatrix} -0.3342 \\ 0.9205 \\ 0.2028 \end{bmatrix}}{1} = \begin{bmatrix} 0.6689 \\ 0.3831 \\ -0.6369 \end{bmatrix}$$

$$\overline{T}_{POINTING\_ESA} = \begin{bmatrix} e_{x\ POINTING\ ESA} & e_{y\ POINTING\ ESA} & e_{z\ POINTING\ ESA} \end{bmatrix} = \begin{bmatrix} -0.6639 & -0.3342 & 0.6689 \\ -0.0772 & 0.9205 & 0.3831 \\ -0.7437 & 0.2028 & -0.6369 \end{bmatrix}$$

Output: The pointing matrix ($\overline{T}_{POINTING\_ESA}$) in the Earth-Surface System and optional the Range if required

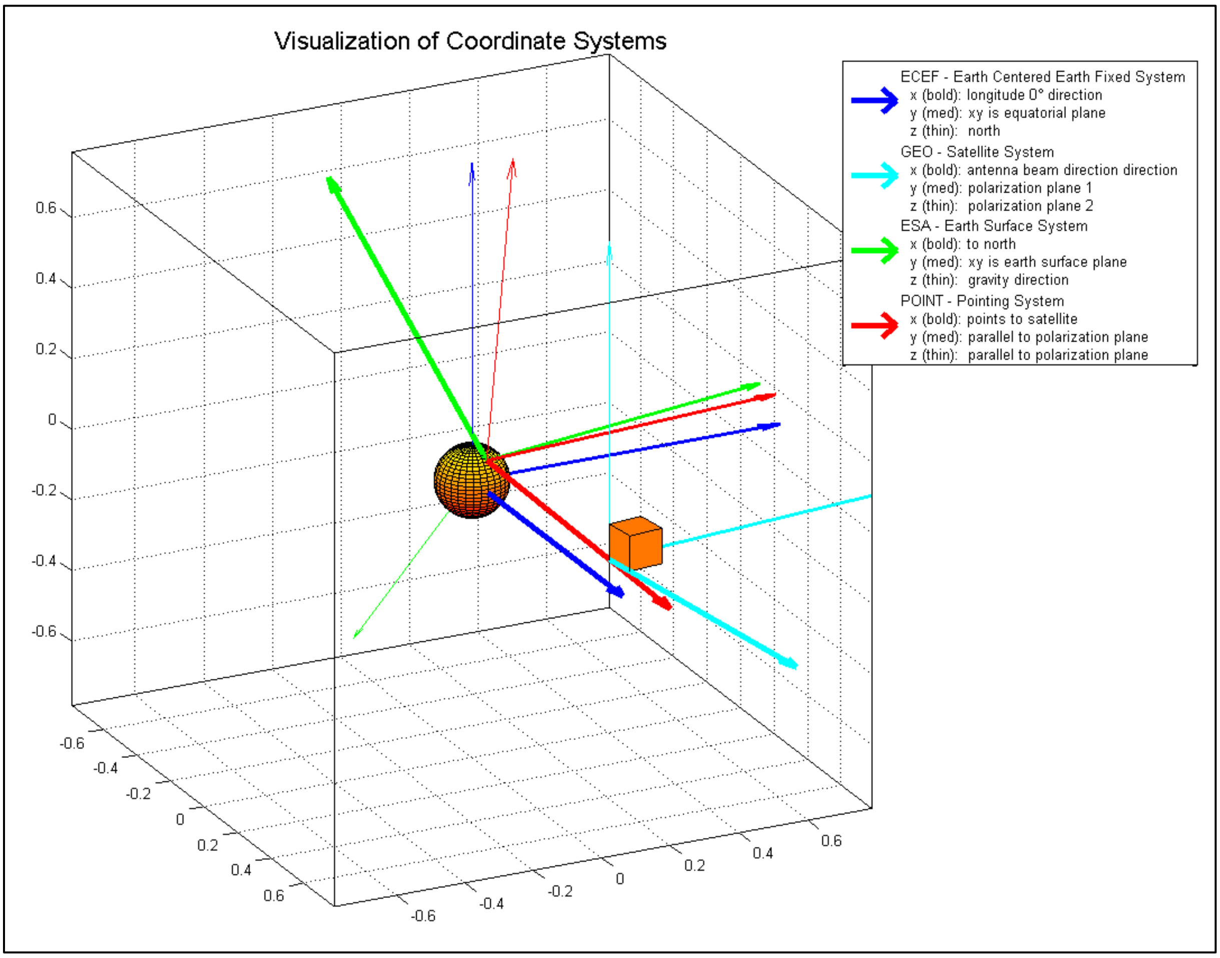

Figure 3: Added Pointing System

Step 4: Antenna coordinate system

Goal: Find the position and orientation of the antenna in the earth surface system

Note: The following chapter gives a detailed insight over the methods and backgrounds required to handle a tilted antenna system. It describes the mathematical backgrounds of a tilted antenna system as well as the details required for error handling and correction. It gives engineering tips that can be applied to improve the antenna system for tilted applications. It states contingency procedures that can be applied to measure or calculate parameters required for antenna calibration and setup. The reader should always bear in mind that step 4 has only one goal: Find the antenna orientation matrix in the earth surface system ($\overline{T}_{ANT\_ESA}$). Once found the algorithm can continue with step 5.

Case A: Untilted antenna system

For the trivial case where the antenna has no tilt and is aligned with the earth surface system (Azimuth=0° is north; Elevation=0° is horizon), the orientation matrix results in the identity matrix

$$\overline{T}_{ANT\_ESA} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2-22}$$

The algorithm can continue with step 5.

Case B: Tilted antenna system (Two axis inclinometer solution ➔ Get all pointing angles)

As the three-dimensional-space has three degrees of freedom for rotation, three parameters have to be known (measured) for a complete definition of the antennas orientation. We use two tilts against the horizon and one rotation around the normal vector of the horizontal plane.

Axis definitions:
The two tilt parameters are measured with two inclinometers. Figure 4 below shows the inclinometer system (xyz) located in the earth surface system (XYZ). Turning the inclinometer system around the roll inclinometer axis (x) with the angle $-Roll_{ant}$ aligns the pitch inclinometer axis (y) with the horizon plane (XY). This vector is called the horizon aligned pitch axis (y'). In the same way turning the system around the pitch inclinometer axis (y) with the angle $-Pitch_{ant}$ aligns the roll inclinometer axis (x) with the horizon plane (XY). This vector is called the horizon aligned roll axis (x'). The rotation around the normal vector of the horizontal plane (called the in plane azimuth of the prime inclinometer axis $IncAz_{ant}$) is defined as the angle from the north direction (X) to the horizon aligned roll axis (x').

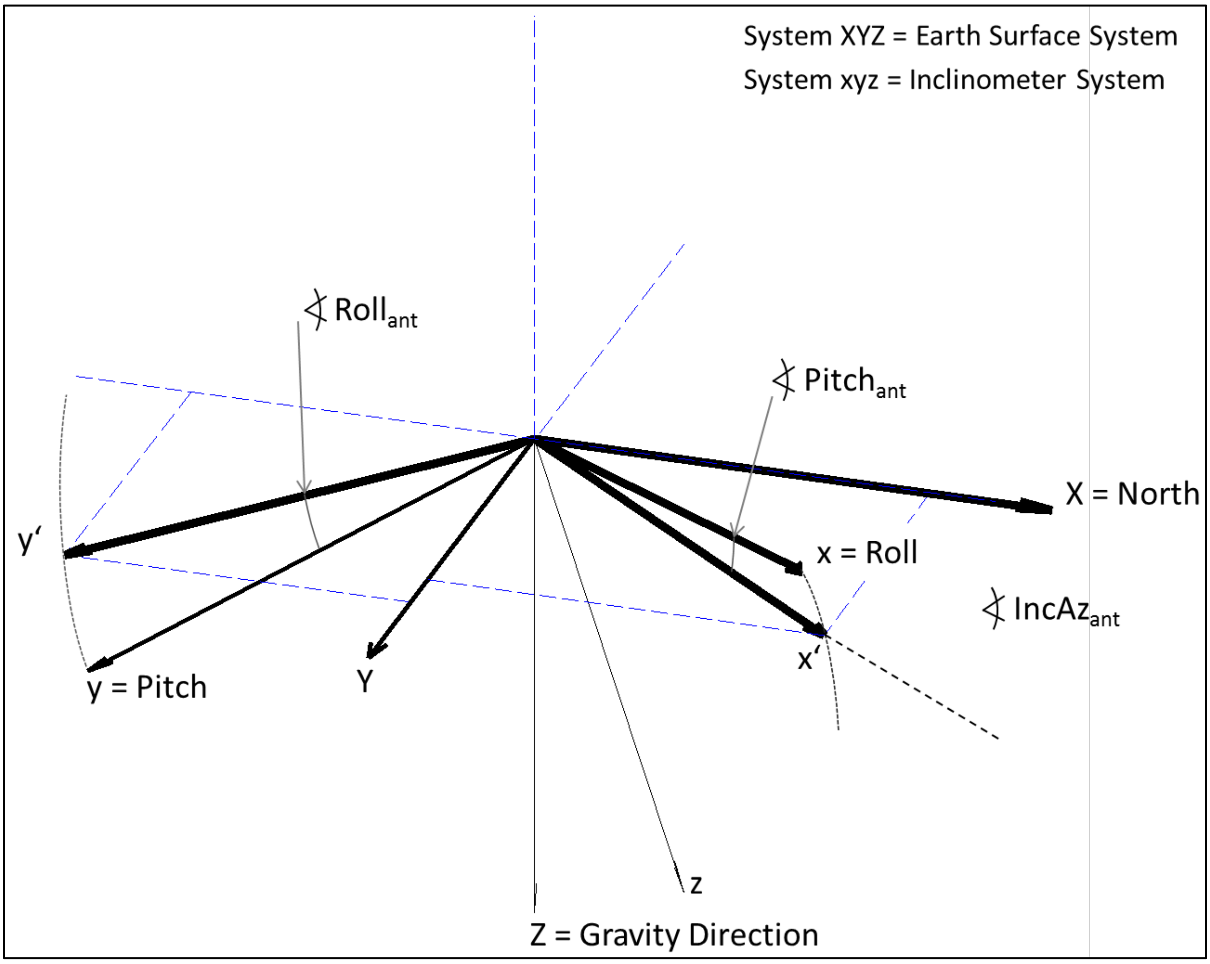

Figure 4: Earth Surface System and Inclinometer System

Choosing the right inclinometers for tilt measurement:
It is important that both inclinometer axes are perpendicular. The user should prefer the usage of a dual axis inclinometer in one package instead of using two separate inclinometers.

Choosing the correct inclinometer mounting:
The inclinometers are placed in the rotating part of the station which generates the azimuth movement (See Figure 5 below). The inclinometers should be perfectly aligned with the antenna axes. In this case the transformation from the inclinometer system to the antenna system is trivial. The primary inclinometer axis (measuring $Roll_{ant}$) is parallel to the polarization axis at elevation 0° and the secondary inclinometer axis (measuring $Pitch_{ant}$) is parallel to the elevation axis. Misalignment or alternate mounting concepts will require additional workload during antenna setup.

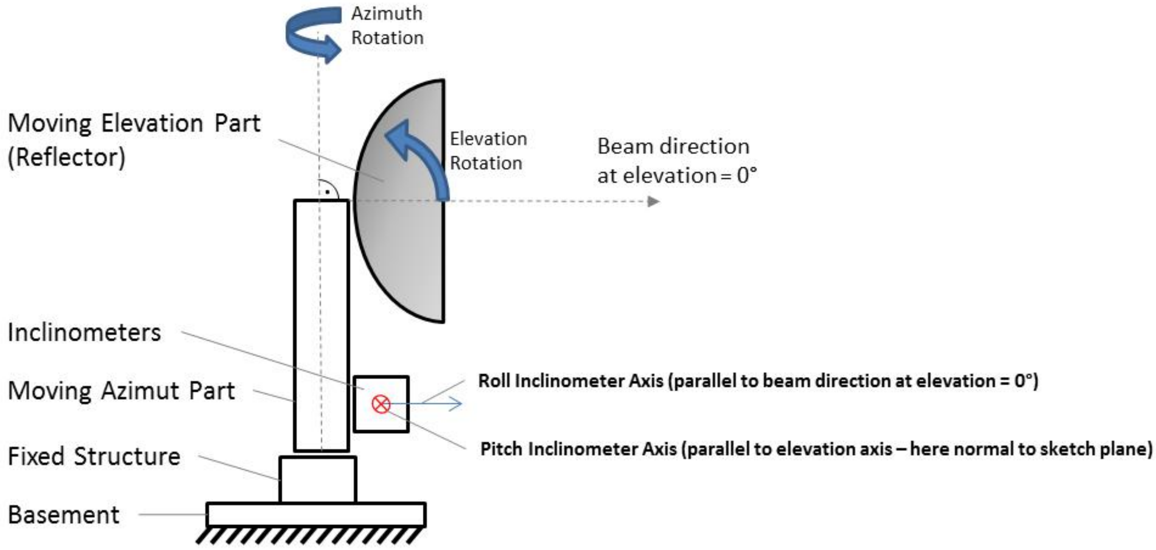

Figure 5: Two-Axis Inclinometer - Mounting on Antenna

Calculating the antenna system:
Keep in mind that the inclinometers are fixed to the moving azimuth part of the antenna. This implies that the inclinometer system is not fixed in the earth surface system. The three parameters that define the inclinometer system ($\mathrm{Roll_{ant}}$ $\mathrm{Pitch_{ant}}$ $\mathrm{IncAz_{ant}}$) change their values as the antennas azimuth axis is turned. If we measure a set of the three parameters to define the inclinometer system, we also have to remember the corresponding antennas azimuth that was set during the measurement. This parameter is called $\mathrm{Az_{ant\,meas}}$ in the following.

With the four parameters, the orientation matrix of the antenna can be calculated. This is done with four rotations. The first three rotations are used to receive the virtual inclinometer system. It is called virtual as it defines the orientation of the inclinometers bound to the antennas azimuth where the parameters were measured. The fourth rotation corrects the azimuth offset of the antenna, which was present during measurement of the parameters. This is possible as the inclinometer system is aligned with the antenna system. The formulas are:

$$\tau = atand\big(cosd(Pitch_{ant})tand(Roll_{ant})\big) \tag{2-23}$$

(Info: As the roll is changed during the pitch rotation it has to be adapted. The new adapted roll is called τ here. See the document analytical details for the derivation)

$$\overline{T}_{ANT\_ESA} = \underbrace{\overline{R}_z(\,IncAz_{ant}) * \overline{R}_x(\tau) * \overline{R}_y(Pitch_{ant}\,)}_{Inclinometer\ system\ (at\ Azimuth\ =\ Az_{ant\ meas})} \quad * \quad \underbrace{\overline{R}_z(\,-Az_{ant\ meas})}_{Azimuth\ offset\ correction} \tag{2-24}$$

= Antenna transform in Earth Surface System

Example: $Roll_{ant} = 14°$ $Pitch_{ant} = 8°$ $IncAz_{ant} = 68°$
$Az_{ant\,meas} = 52°$ *(Azimuth encoder value during parameter measurement)*

$\tau =$ *13.8690°* *(0.2421 rad)*

$$\overline{T}_{ANT\_ESA} = \begin{bmatrix} 0.372 & -0.928 & 0. \\ 0.928 & 0.372 & 0. \\ 0. & 0. & 1. \end{bmatrix} * \begin{bmatrix} 1. & 0. & 0. \\ 0. & 0.971 & -0.240 \\ 0. & 0.240 & 0.971 \end{bmatrix} * \begin{bmatrix} 0.990 & 0. & 0.139 \\ 0. & 1. & 0. \\ -0.139 & 0. & 0.990 \end{bmatrix} * \begin{bmatrix} 0.616 & 0.788 & 0. \\ -0.788 & 0.616 & 0. \\ 0. & 0. & 1. \end{bmatrix}$$

$$\overline{T}_{ANT\_ESA} = \begin{bmatrix} 0.372 & -0.901 & 0.223 \\ 0.928 & 0.361 & -0.0893 \\ 0. & 0.240 & 0.971 \end{bmatrix} * \begin{bmatrix} 0.990 & 0. & 0.139 \\ 0. & 1. & 0. \\ -0.139 & 0. & 0.990 \end{bmatrix} * \begin{bmatrix} 0.616 & 0.788 & 0. \\ -0.788 & 0.616 & 0. \\ 0. & 0. & 1. \end{bmatrix}$$

$$\overline{T}_{ANT\_ESA} = \begin{bmatrix} 0.337 & -0.901 & 0.273 \\ 0.931 & 0.361 & 0.0406 \\ -0.135 & 0.240 & 0.961 \end{bmatrix} * \begin{bmatrix} 0.616 & 0.788 & 0. \\ -0.788 & 0.616 & 0. \\ 0. & 0. & 1. \end{bmatrix}$$

$$\overline{T}_{ANT\_ESA} = \begin{bmatrix} 0.918 & -0.289 & 0.273 \\ 0.289 & 0.956 & 0.0406 \\ -0.272 & 0.0415 & 0.961 \end{bmatrix}$$

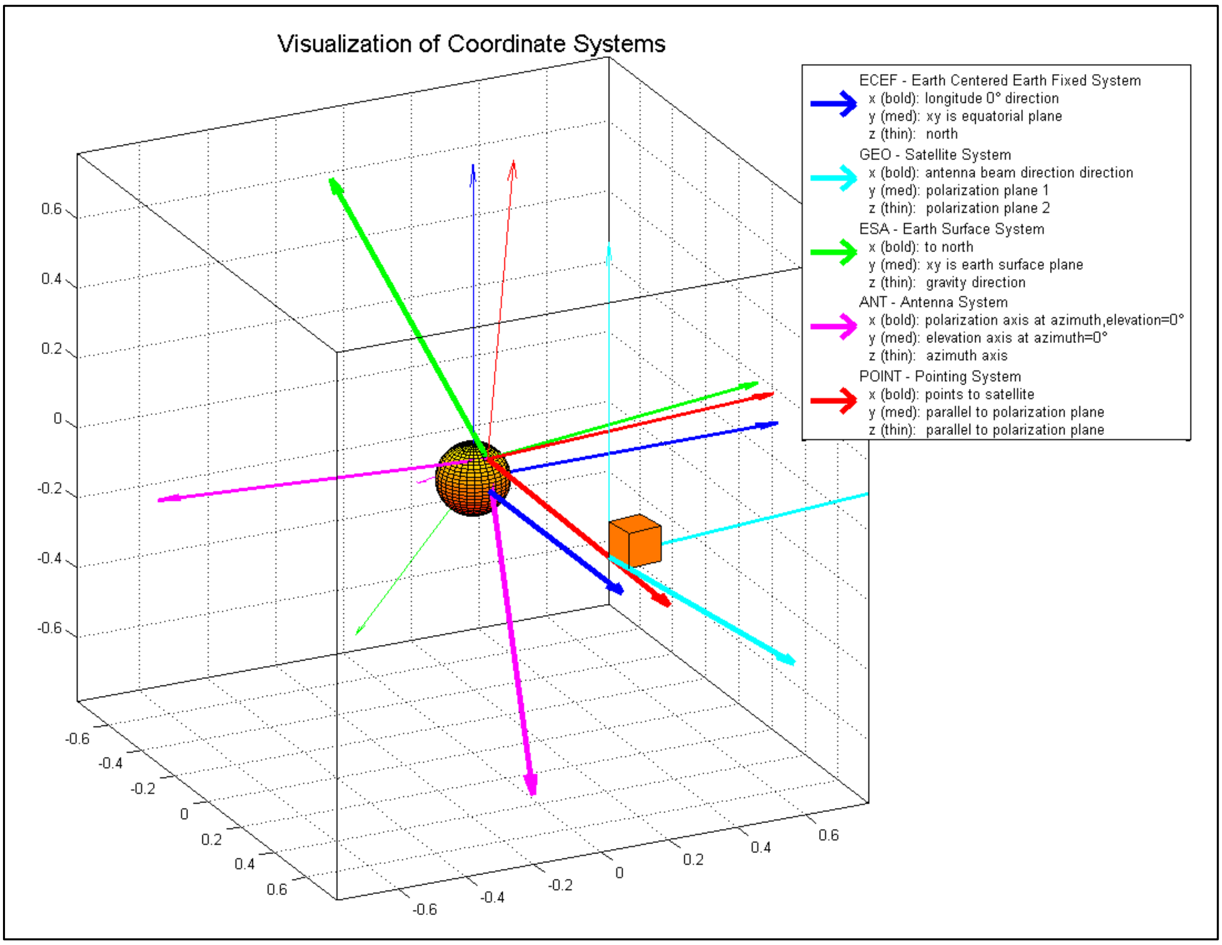

Figure 6: Added (tilted) Antenna System

How to receive a set of parameters using direct measurement (Low accuracy):

1. Get the current encoder value of the azimuth stage. This value is the parameter $\mathrm{Az}_{\mathrm{ant\,meas}}$. For the rest of the measurement the antenna is not allowed to move.
2. Get the measurement values of both inclinometers. These are the values $\mathrm{Pitch}_{\mathrm{ant}}$ and and $\mathrm{Roll}_{\mathrm{ant}}$
3. Measurement of the parameter $\mathrm{IncAz}_{\mathrm{ant}}$ can be accomplished with a compass. As this approach has a low accuracy it is only applicable for rough initial tracking. The measurement is performed as following: Put the compass on top of the inclinometer. Hold the compass as good as possible aligned with the horizon. For an analog compass turn the north position to the compass needle. Now read the angle between north and primary inclinometer axis. The angle is measured from north to east and is in the range of 0° to 360°. This is the value of $\mathrm{IncAz}_{\mathrm{ant}}$
4. Your parameter set is now complete.

How to receive a set of parameters using an initial tracing cycle (Preferred method):

1. Point the antenna to a known satellite using auto tracking. The antenna should now point to maximum beacon.
2. Measure the encoder value of the azimuth stage. This value is the parameter $\mathrm{Az}_{\mathrm{ant\,meas}}$. For the rest of the measurement the antenna is not allowed to move.
3. Get the measurement values of both inclinometers. These are the values $\mathrm{Pitch}_{\mathrm{ant}}$ and and $\mathrm{Roll}_{\mathrm{ant}}$
4. Calculate the antenna azimuth and elevation in case of an untilted antenna. The user could apply step 5 of the algorithm assuming an untitled antenna for this purpose (Using Case A of Step 4). The calculated untitled azimuth and elevation are called A and E here.
5. With the four values (A, E, Roll, Pitch) the antenna In-Plane Azimuth of the Prime Inclinometer ($IncAz_{ant}$) can be calculated. This approach is far more accurate as direct measurement. The value calculates to:

$$\tau = atand(cosd(Pitch_{ant})tand(Roll_{ant}))$$

$$IncAz_{ant} = 180 - acosd\left(\frac{-cosd(A)\sqrt{\frac{cosd^2(\tau) - 1 + cosd^2(E)}{cosd^2(\tau)}}cosd(\tau) + \ sind(\tau)sind(E)sind(A)}{cosd(E)cosd(\tau)}\right) \tag{2-25}$$

6. Optional: The angle to the horizon around the elevation axis can also be calculated. Comparing this angle to the measurement of the elevation encoder and the pitch measurement gives the elevation offset error of the antenna. The value can be used for additional error correction at the end of step 5.

$$El_{offset} = El_{Encoder} - asin\left(\frac{\sin(E)}{\cos(\tau)}\right) + Pitch_{ant} \tag{2-26}$$

7. Your parameter set is now complete.

Case C: Tilted antenna system (One axis inclinometer solution ➔ Get only polarization angle)

A single inclinometer mounted on the elevation axis of the antenna and having its measurement axes parallel to the antenna lobe can be used to get the polarization pointing angle for the tilted antenna system. However the correct azimuth and elevation pointing angles for the tilted system must be derived by alternate approaches in this case. (E.g. for small tilt values: Using the untitled pointing angles in combination with an initial tracing cycle)

Choosing the correct inclinometer mounting:
The inclinometer is placed in the moving elevation part of the station (See Figure 7 below). The inclinometer should be perfectly aligned with the polarization (antenna lobe) axis. The 0° direction on the inclinometer must be aligned with the 0° direction of the polarization drive, or the offset must be known for correction.

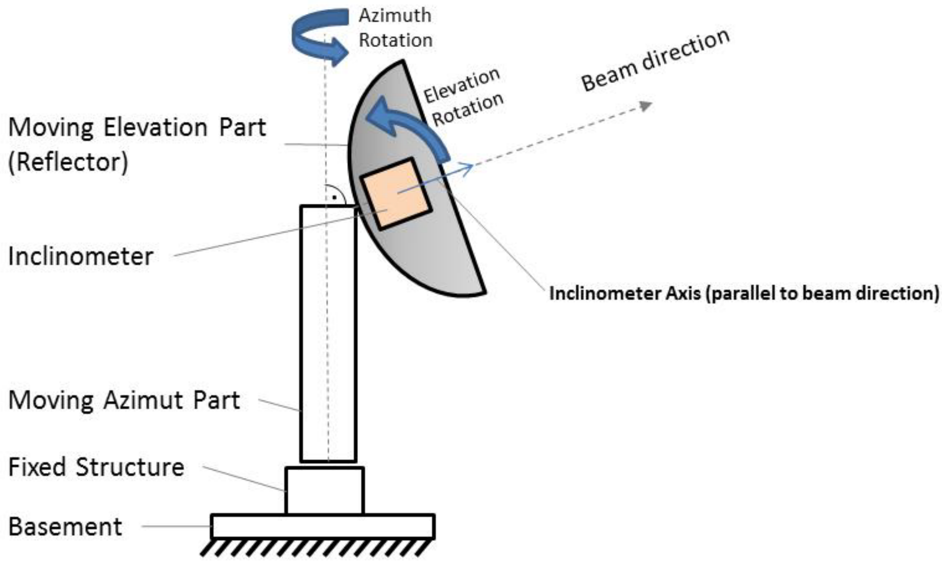

Figure 7: One-Axis Inclinometer - Mounting on Antenna

The calculated untitled polarization pointing angle can now be corrected by the measured value of the inclinometer when pointing to the target satellite.

Step 5: Pointing angles

Goal: Receive the pointing angles for the antenna

The pointing transform is defined in the earth surface system (results from step 3). For the calculation of the pointing angles we require the pointing transform as seen from the antenna system (results from step 4). Therefore we apply a transformation. Note that all arguments on the right side of the equation have to be defined in the same base system (here ESA) for the transformation to be valid.

$$\overline{T}_{POINTING\_ANT} = \overline{T}_{ANT\_ESA}^{\;T} * \overline{T}_{POINTING\_ESA} \qquad \text{= Pointing transform in Antenna System} \qquad (2\text{-}27)$$

Example:

For the untitled system:

$$\overline{T}_{POINTING\_ANT} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} -0.6639 & -0.3342 & 0.6689 \\ -0.0772 & 0.9205 & 0.3831 \\ -0.7437 & 0.2028 & -0.6369 \end{bmatrix} = \begin{bmatrix} -0.6639 & -0.3342 & 0.6689 \\ -0.0772 & 0.9205 & 0.3831 \\ -0.7437 & 0.2028 & -0.6369 \end{bmatrix}$$

For the tilted system:

$$\overline{T}_{POINTING\ ANT} = \begin{bmatrix} 0.918 & -0.289 & 0.273 \\ 0.289 & 0.956 & 0.0406 \\ -0.272 & 0.0415 & 0.961 \end{bmatrix}^T * \begin{bmatrix} -0.6639 & -0.3342 & 0.6689 \\ -0.0772 & 0.9205 & 0.3831 \\ -0.7437 & 0.2028 & -0.6369 \end{bmatrix}$$

$$\overline{T}_{POINTING\ ANT} = \begin{bmatrix} -0.4297 & -0.0989 & 0.8975 \\ 0.0861 & 0.9850 & 0.1497 \\ -0.8988 & 0.1416 & -0.4147 \end{bmatrix}$$

Euler angles representing the antenna azimuth, elevation and polarization can be calculated form the pointing matrix:

Notes:

- $\overline{T}_{POINTIN\ ANT}(3{,}1)$ means the element in row 3 and column 1 of the pointing matrix (In this example the lower left one).
- The algorithm only works for Elevation < 90°. In the very unlikely case that the elevation is exactly 90° the solution will fail, as Azimuth and Polarization are aligned and the solution is singular. Choosing slightly different parameters in that case will solve this problem.

The two-quadrant atan is used (atan2). It is defined as:

$$atan2(y,x) = 2\ \mathrm{atan}\left(\frac{\sqrt{x^2+y^2}-x}{y}\right) \tag{2-28}$$

The angels in degree calculate to:

$$Elevation = -\,\mathrm{asin}\left(\overline{T}_{POINTING_{ANT}}(3{,}1)\right) * \frac{180}{\pi} \tag{2-29}$$

$$Azimut = \mathrm{atan2}\left(\frac{\overline{T}_{POINTING\_ANT}(2{,}1)}{\cos(Elevation)}\ ;\ \frac{\overline{T}_{POINTING\_ANT}(1{,}1)}{\cos(Elevation)}\right) * \frac{180}{\pi} \tag{2-30}$$

$$Polarization = \mathrm{atan2}\left(\frac{\overline{T}_{POINTING\_ANT}(3{,}2)}{\cos(Elevation)}\ ;\ \frac{\overline{T}_{POINTING\_ANT}(3{,}3)}{\cos(Elevation)}\right) * \frac{180}{\pi} \tag{2-31}$$

Example:

For the tilted system:

$$Elevation = -\operatorname{asin}(-0.8988) * \frac{180}{\pi} = 64.01°$$

$$Azimut = \operatorname{atan2}\left(\frac{0.0861}{\operatorname{cosd}(64.01)}\,;\,\frac{-0.4297}{\operatorname{cosd}(64.01)}\right) * \frac{180}{\pi} = 168.67°$$

$$Polarization = \operatorname{atan2}\left(\frac{0.1416}{\operatorname{cosd}(64.01)}\,;\,\frac{-0.4147}{\operatorname{cosd}(64.01)}\right) * \frac{180}{\pi} = 161.15°$$

For the untilted system:

$$Elevation = 48.05°$$

$$Azimut = -173.41°$$

$$Polarization = 162.27°$$

Finally the offsets of the encoders can be corrected:

$$Elevation = Elevation + El_{off}$$

$$Polarization = Polarization + Pol_{off}$$ (If Pol_{off} is known)

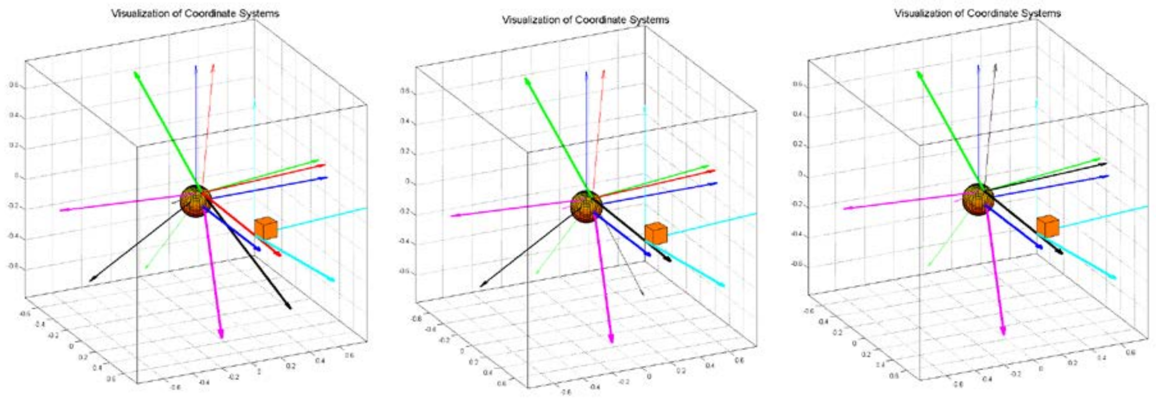

Figure 8: Visualization of the sequence when the tilted antenna is pointed to the satellite using the calculated values. The sequence shows how the magenta colored antenna system in stow position (azimuth, elevation, polarization = 0°) gets aligned with the red colored pointing system using the three rotations around azimuth, elevation, polarization with the calculated values. The black colored system shows from left to right: pointing after azimuth rotation (elevation, polarization = 0°); pointing after elevation rotation (polarization = 0°); pointing after polarization rotation = aligned with desired pointing system.

Calculation of Azimuth, Elevation and Polarization for non-horizontal aligned Antennas

Analytic Formulas

Technical Document
TD-1205-b

Version 2.2
04.03.2016
(with update in page 4/5 of 10.04.2018)

In Co-operation with

Objective

This document presents formulas that can be used to calculate the pointing angles of the antenna.

Table of Contents

Change Log:

V1.3: Updated satellite table in chapter 1.2 Table of EUTELSAT Satellites

V2.0: new release generated.

V2.1: Updated satellite table in chapter 1.2 Table of EUTELSAT Satellites

V2.2: Updated satellite table in chapter 1.2 Table of EUTELSAT Satellites, 10.04.2018

1.1 Parameters

The user needs knowledge of the following parameters to calculate the antenna pointing angles:

Information	Name	Parameter	Unit	Value used in example calculation	Description
GEO Satellite	$Long_{sat}$	Satellite Position (Longitude)	[°]	7°	Measured positive in east direction. Value is in the range of 0° to 360°
	Lat_{sat}	Satellite Position (Latitude)	[°]	14°	Measured positive in north direction. Value is in the range of 90° to -90°
	Pol_{sat}	Polarization Angle (Is Skew Angle for $Lat_{sat} = 0$)	[°]	-22° (Note: For most Eutelsat Satellites the skew angle is 3.535°)	The value is the angle that aligns the Pol-Vector of the satellite antenna with the equatorial plane when rotating around the vector from the satellite center of mass to the earth center. If the Satellite is in the equatorial plane, implying $Lat_{sat} = 0$, this is the angle between the equatorial plane and the polarization direction, being the skew angle.
Antenna Position	$Long_{ant}$	Antenna Position Longitude	[°]	50°	Measured positive in east direction. Value is in the range of 0° to 360°
	Lat_{ant}	Antenna Position Latitude	[°]	11°	Measured positive in north direction. Value is in the range of 90° to -90°
	Alt_{ant} or r_{earth}	Antenna Altitude or Earth Radius	[m]	r_{earth} 6378000m	Altitude above mean sea level of the antenna or Earth radius. Depends on the chosen earth model (See algorithm step 2 for details)

1.2 Table of EUTELSAT Satellites

EUTELSAT Name	Other Designations	EUTELSAT Code	International Designator	Orbital Location	Pol Skew Angle for LP (VO*-1)	Linnear / Circular Polarization
EUTELSAT 117 West A	SATMEX 8	**E117WA**	13012A	-116.8		LP
EUTELSAT 117 West B	SATMEX9	**E117WB**	16038B	-117.0		LP
EUTELSAT 115 West B	SATMEX 7	**E115WB**	15010B	-114.9		LP
EUTELSAT 113 West A	SATMEX 6	**E113WA**	06020A	-113.0		LP
	SATMEX 10			-113.0		LP (Future Satellite)
EUTELSAT 65 West A		**E65WA**	14006A	-65.2	0.000	C & Ku-Band: LP; Ka CP Q/V - LP (X-Horizontal)
EUTELSAT 36 West A	Atlantic Bird 1; E12WA	**E36WA**	02040A	-36.3	3.535	LP
Telstar 12 V	Telstar 12 Vantage	**TELSTAR-12V**	15068A	-15.0	0.000	LP; (CP for some non-EUTELSAT capacity)
EUTELSAT 12 West B	Atlantic Bird 2; AB2; New Bird; E8WA	**E12WB**	01042A	-12.5	3.535	LP
EUTELSAT 12 West C	QUANTUM; AnySat	**E12WC**		-12.5	3.535	*Future Satellite*
EUTELSAT 8 West B	AB2A	**E8WB**	15039B	-8.0	3.535	LP
EUTELSAT 7 West A	AB4R; AB7; AB4A	**E7WA**	11051A	-7.3	3.535	Ku-Band: LP; C-Band CP
EUTELSAT 5 West A	Atlantic Bird 3; Stellat 5	**E5WA**	02035A	-5.0	0.000	LP
EUTELSAT 5 West B		**E5WB**		-5.0		*Future Satellite*
EUTELSAT 3B	Newsat	**E3B**	14030A	3.1	3.535	LP
EUTELSAT 7A	W3A	**E7A**	04008A	7.0	3.535	LP
EUTELSAT 7B	E3D; W3D	**E7B**	13022A	7.0	3.535	LP
EUTELSAT 7C	E7X	**E7X**				*Future Satellite*
EUTELSAT 9B	ESA Data Relay; EDRS-A	**E9B**	16005A	9.0	3.535	LP
EUTELSAT KA-SAT 9A	KaSAT	**KA9A**	10069A	9.0	3.535	CP (LP for beacon)
EUTELSAT 10A	W2A	**E10A**	09016A	10.0	3.535	Ku-Band: LP; C-Band CP
BB4A	African Broadband Satellite (HTS)					*Future Satellite*
EUTELSAT HOT BIRD 13B	HB8	**HB13B**	06032A	13.0	3.535	LP
EUTELSAT HOT BIRD 13C	HB9	**HB13C**	08065A	13.0	3.535	LP

EUTELSAT HOT BIRD 13E	HB7A; Eurobird 9A; E9A	**HB13E**	06007B	13.0	3.535	LP
EUTELSAT 16A	W3C	**E16A**	11057A	16.0	3.535	LP
EUTELSAT 16C	SESAT 1	**E16C**	00019A	0.0	3.535	LP
EUTELSAT 21B	W6A	**E21B**	12062B	21.6	3.535	LP
EUTELSAT 25B	Es'hail; EB2A	**E25B**	13044A	25.5	3.535	Ku & DBS Band: LP Ka-Band: CP
EUTELSAT 28E	ASTRA 2E	**E28E**	13056A	28.5	0.000	
EUTELSAT 28F	ASTRA 2F	**E28F**	12051A	28.2	0.000	
EUTELSAT 28G	ASTRA 2G	**E28G**	14089A	28.2	0.000	
EUTELSAT 31A	e-Bird; EB3; E33A	**E31A**	03043A	30.9	3.535	LP
EUTELSAT 33C	W1R; E28A	**E33C**	01011A	33.1	3.535	LP
EUTELSAT 33E	HB10; AB4A; E3C; HB13D	**E33E**	09008B	33.1	3.535	LP
EUTELSAT 36B	W7	**E36B**	09065A	35.9	3.535	LP
EUTELSAT 36C	EXPRESS AMU1	**E36C**	15082A	36.1	3.535	LP
Y1B	Yahsat 1B	**Y1B**	12016A	47.6	0.000	
EUTELSAT 48D	E28B; W2M	**E48D**	08065B	48.1	3.535	LP
EUTELSAT 53A	EXPRESS-AM6; EXPRESS-AM22; SESAT 2	**E53A**	03060A	53.0	0.000	LP
YAMAL 402	2012-070	**Yamal402**	12070A	54.9		
EUTELSAT 56A	Express AT1	**EXPRESS AT1**	14010A	56.0		CP
EUTELSAT 70B	W5A	**E70B**	12069A	70.5	3.535	LP
EUTELSAT 140A	EXPRESS AT2	**EXPRESS AT2**	14010B	140.0		CP
EUTELSAT 172B		**E172B**	17029B	172.0	0.000	LP
EUTELSAT 174A	AMC23; GE-23;AMC 23, Worldsat 3, GE 2i,E172A	**E174A**	05052A	174.0	0.000	LP

Status: 10 April 2018

*** Notes:**

For precise orbital location refer to Eutelsat ephemeris; regarding value of the skew of the Eutelsat satellites: The reference X-polarization (horizontal) is defined as that polarization whose plane makes an angle of 93.535° in an anti-clockwise direction, looking towards the earth, about a reference vector with respect to a plane containing this vector and the pitch axis. The reference vector is defined as the vector from the satellite in the direction 0.21° towards West and 6.07° towards north in satellite coordinates. The reference Y-polarization (vertical) is defined as that polarization whose plane is orthogonal to the X polarization plane and the reference vector defined above. In other words the polarization skew angle of the EUTELSAT satellites is +3.535°, clock-wise when looking at the satellite from the earth, from anywhere on the meridian (in the northern hemisphere) corresponding to the orbital location of the satellite. In the southern hemisphere the polarization skew angle of the EUTELSAT satellites is +183.535°, clock-wise, from anywhere on the meridian corresponding to the orbital location of the satellite.

1.3 Analytical Formulas

1.3.1 Untitled System – General Case – Spherical Earth

This Chapter shows the formulas for calculation of azimuth, elevation and polarization for the general case ($Lat_{sat} \neq 0$). For the derivation of the formulas, see the document on the analytical details.

The two-quadrant atan is used for the calculation (atan2). It is defined as:

$$atan2(y, x) = 2 \operatorname{atan}\left(\frac{\sqrt{x^2 + y^2} - x}{y}\right)$$

or using cases:

$$\operatorname{atan2}(y, x) = \begin{cases} \arctan\left(\frac{y}{x}\right) & x > 0 \\ \arctan\left(\frac{y}{x}\right) + \pi & y \geq 0, x < 0 \\ \arctan\left(\frac{y}{x}\right) - \pi & y < 0, x < 0 \\ +\frac{\pi}{2} & y > 0, x = 0 \\ -\frac{\pi}{2} & y < 0, x = 0 \\ \text{undefined} & y = 0, x = 0 \end{cases}$$

To simplify the formulas we define:

$$w := Long_{ant} - Long_{sat}$$

Azimuth and Elevation can be calculated as following:

$$Range = \sqrt{\left(r_{geo} \cos(Lat_{sat}) \cos(w) - r_e \cos(Lat_{ant})\right)^2 + r_{geo}^2 \cos(Lat_{sat})^2 \sin(w)^2 + \left(r_{geo} \sin(Lat_{sat}) - r_e \sin(Lat_{ant})\right)^2}$$

$$Elevation = -\operatorname{asin}\left(-\frac{\cos(Lat_{ant}) r_{geo} \cos(Lat_{sat}) \cos(w) + r_{geo} \sin(Lat_{sat}) \sin(Lat_{ant}) - r_e}{Range}\right)$$

$$Azimuth = atan2\left(-\frac{r_{geo} \cos(Lat_{sat}) \sin(w)}{Range \ \cos(Elevation)}, -\frac{r_{geo}\left(\sin(Lat_{ant}) \cos(Lat_{sat}) \cos(w) - \cos(Lat_{ant}) \sin(Lat_{sat})\right)}{Range \ \cos(Elevation)}\right)$$

With the arguments x and y as below the polarization can be calculated:

$$
\begin{aligned}
y &= \cos(Azimuth)\sin(Elevation)\,\big(\sin(Lat_{ant})\sin(Lat_{sat})\\
&\quad \sin(Pol_{sat})\cos(w) - \sin(Lat_{ant})\cos(Pol_{sat})\sin(w)\\
&\quad + \cos(Lat_{ant})\cos(Lat_{sat})\sin(Pol_{sat})\big)\\
&\quad + \sin(Azimuth)\sin(Elevation)\,\big(\sin(Lat_{sat})\sin(Pol_{sat})\sin(w)\\
&\quad + \cos(Pol_{sat})\cos(w)\big)\\
&\quad + \cos(Elevation)\,\big(\cos(Lat_{ant})\sin(Lat_{sat})\sin(Pol_{sat})\cos(w)\\
&\quad - \cos(Lat_{ant})\cos(Pol_{sat})\sin(w)\\
&\quad - \sin(Lat_{ant})\cos(Lat_{sat})\sin(Pol_{sat})\big)
\end{aligned}
$$

$$
\begin{aligned}
x = -\sin(Azimuth)\,&\big(\sin(Lat_{ant})\sin(Lat_{sat})\sin(Pol_{sat})\cos(w)\\
&- \sin(Lat_{ant})\cos(Pol_{sat})\sin(w)\\
&+ \cos(Lat_{ant})\cos(Lat_{sat})\sin(Pol_{sat})\big)\\
&+ \cos(Azimuth)\,\big(\sin(Lat_{sat})\sin(Pol_{sat})\sin(w)\\
&+ \cos(Pol_{sat})\cos(w)\big)
\end{aligned}
$$

$$Polarization = atan2(y, x)$$

Example using the example values from the parameter table:

Elevation = 48.05°
Azimuth = -173.41°
Polarization = 162.44°

1.3.2 Untitled System – Spherical Earth – Satellite Latitude = 0

This Chapter shows the formulas for calculation of azimuth, elevation and polarization for a satellite at $Lat_{sat} = 0$. The formulas result from a simplification of the general case above. For the derivation of the formulas, see the document on the analytical details.

The two-quadrant atan is used for the calculation (atan2). It is defined as:

$$atan2(y, x) = 2\ \mathrm{atan}\left(\frac{\sqrt{x^2 + y^2} - x}{y}\right)$$

or using cases:

$$
\operatorname{atan2}(y,x) = \begin{cases}
\arctan\left(\frac{y}{x}\right) & x > 0\\
\arctan\left(\frac{y}{x}\right) + \pi & y \geq 0, x < 0\\
\arctan\left(\frac{y}{x}\right) - \pi & y < 0, x < 0\\
+\frac{\pi}{2} & y > 0, x = 0\\
-\frac{\pi}{2} & y < 0, x = 0\\
\text{undefined} & y = 0, x = 0
\end{cases}
$$

To simplify the formulas we define:

$w := Long_{ant} - Long_{sat}$

With $Lat_{sat} = 0$ the formulas from the general case simplify to:

$$Range = \sqrt{\left(r_{geo}\cos(w) - r_e\cos(Lat_{ant})\right)^2 + r_{geo}^2\sin(w)^2 + r_e^2\sin(Lat_{ant})^2}$$

$$Elevation = asin\left(\frac{r_{geo}\cos(w)\cos(Lat_{ant}) - r_e}{Range}\right)$$

$$Azimuth = atan2\left(-\frac{r_{geo}\sin(w)}{Range\cos(Elevation)}, -\frac{r_{geo}\sin(Lat_{ant})\cos(w)}{Range\cos(Elevation)}\right)$$

$$\begin{aligned} y = {} & \cos(Azimuth)\sin(Elevation)\left(-\sin(Lat_{ant})\cos(Pol_{sat})\sin(w)\right. \\ & \left.+ \cos(Lat_{ant})\sin(Pol_{sat})\right) \\ & + \sin(Azimuth)\sin(Elevation)\cos(Pol_{sat})\cos(w) \\ & + \cos(Elevation)\left(-\cos(Lat_{ant})\cos(Pol_{sat})\sin(w)\right. \\ & \left.- \sin(Lat_{ant})\sin(Pol_{sat})\right) \end{aligned}$$

$$\begin{aligned} x = {} & -\sin(Azimuth)\left(-\sin(Lat_{ant})\cos(Pol_{sat})\sin(w)\right. \\ & \left.+ \cos(Lat_{ant})\sin(Pol_{sat})\right) + \cos(Azimuth)\cos(Pol_{sat})\cos(w) \end{aligned}$$

$$Polarization = atan2(y, x)$$

1.3.3 Polarization Correction for tilted antenna

If the antenna is tilted the polarization offset can be calculated with the following formula.

Some values have to be measured or calculated prior to using the formula:

$Pitch_{ant}$ and $Roll_{ant}$ are measured with the antenna pointing to the satellite

$Pol_{Untilted}$ and $Elevation_{Untilted}$ can be calculated with the formulas for the untilted antenna

The polarization is calculated to:

$$\tau = -atand(cosd(Pitch_{ant})tand(Roll_{ant}))$$

$$Pol_{Tilted} = Pol_{Untilted} \underbrace{-acosd\left(\frac{\sqrt{\left(cosd(\tau)\right)^2 - \left(sind(Elevation_{Untilted})\right)^2}}{cosd(Elevation_{Untilted})}\right)}_{Elevaion\ Correction\ for\ tilted\ applications}$$

Calculation of Azimuth, Elevation and Polarization for non-horizontal aligned Antennas

Appendix: Analytic Details

Technical Document
TD-1205-c

Version 2.1
09.06.2015

In Co-operation with

Objective

This Appendix shows how the formulas used in the other documents can be derived.

Table of Contents

Change Log:

V2.0: new release generated.

V2.1: new release generated, following update of algorithm and formulas documents

1 Analytical Details

1.1 Adapted Roll

The pitched system shall be derived with rotations. As the roll is changed during the second rotation (pitch) it has to be adapted in the first place. The new adapted roll is called τ here. The following logic is applied to derive it:

After the three rotations:

$$\overline{R}_x(\tau) * \overline{R}_Y(Pitch_{ant}) * \overline{R}_x(-Roll_{ant})$$

the y-axis (Pitch direction) must be aligned with the XY plane.

This is easily solvable symbolically:

$$Rx(a) := \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(a) & -\sin(a) \\ 0 & \sin(a) & \cos(a) \end{bmatrix}$$

$$a \rightarrow Matrix(3, 3, \{(1,1) = 1, (1,2) = 0, (1,3) = 0, (2,1) = 0, (2,2) = \cos(a), (2,3) = -\sin(a), (3,1) = 0, (3,2) = \sin(a), (3,3) = \cos(a)\})$$

$$Ry(a) := \begin{bmatrix} \cos(a) & 0 & \sin(a) \\ 0 & 1 & 0 \\ -\sin(a) & 0 & \cos(a) \end{bmatrix}$$

$$a \rightarrow Matrix(3, 3, \{(1,1) = \cos(a), (1,2) = 0, (1,3) = \sin(a), (2,1) = 0, (2,2) = 1, (2,3) = 0, (3,1) = -\sin(a), (3,2) = 0, (3,3) = \cos(a)\})$$

$$Rz(a) := \begin{bmatrix} \cos(a) & -\sin(a) & 0 \\ \sin(a) & \cos(a) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$a \rightarrow Matrix(3, 3, \{(1,1) = \cos(a), (1,2) = -\sin(a), (1,3) = 0, (2,1) = \sin(a), (2,2) = \cos(a), (2,3) = 0, (3,1) = 0, (3,2) = 0, (3,3) = 1\})$$

$$Rg := Rx(\tau) \cdot Ry(p) \cdot Rx(r)$$

$$[[\cos(p), \sin(p)\sin(r), \sin(p)\cos(r)], [\sin(\tau)\sin(p), \cos(\tau)\cos(r) - \sin(\tau)\cos(p)\sin(r), -\cos(\tau)\sin(r) - \sin(\tau)\cos(p)\cos(r)], [-\cos(\tau)\sin(p), \sin(\tau)\cos(r) + \cos(\tau)\cos(p)\sin(r), -\sin(\tau)\sin(r) + \cos(\tau)\cos(p)\cos(r)]]$$

$$y := \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$res(ys) := \begin{bmatrix} xs \\ ys \\ 0 \end{bmatrix}$$

$$ys \rightarrow Vector(3, \{1 = xs, 2 = ys, 3 = 0\})$$

$$xs^2 + ys^2 = 1$$

$$xs^2 + ys^2 = 1$$

$$\xrightarrow{\text{solve for ys}}$$

$$\left[\left[ys = \sqrt{1 - xs^2}\right], \left[ys = -\sqrt{1 - xs^2}\right]\right]$$

$$R := Rg \cdot y - res\left(\sqrt{1 - xs^2}\right)$$

$$\begin{bmatrix} \sin(p)\sin(r) - xs \\ \cos(\tau)\cos(r) - \sin(\tau)\cos(p)\sin(r) - \sqrt{1 - xs^2} \\ \sin(\tau)\cos(r) + \cos(\tau)\cos(p)\sin(r) \end{bmatrix}$$

R[3]

$$\sin(\tau)\cos(r) + \cos(\tau)\cos(p)\sin(r)$$

$$\xrightarrow{\text{isolate for tau}}$$

$$\tau = -\arctan(\cos(p)\tan(r))$$

1.2 Prime Inclinometer In-Plane Azimuth form initial tracking

The problem includes the following 7 variables:

- Untitled Antenna Azimuth A; Untitled Antenna Elevation E
- Corrected Antenna Roll (See section above); Antenna Pitch; In-Plane (Earth Surface) Azimuth of the Prime Inclinometer
- Tilted Antenna Azimuth; Tilted Antenna Elevation

The following logic is applied for solving:

The rotations with the untitled A and E have to produce the same pointing vector as the rotations with the tilted A and E and the correct antenna orientation parameters (Roll, Pitch, Azimuth of the Prime Inclinometer).

In detail this means:

$$\overline{R}_z(A) * \overline{R}_Y(-E)$$

and

$$\overline{R}_z(IncAz_{ant}) * \overline{R}_x(\tau) * \overline{R}_y(Pitch_{ant}) * \overline{R}_y(-Tilted\ Elevation)$$

Have to produce the same pointing vector (x-Axis)

As the Roll and Pitch are depending on the Azimuth, the knowledge of 4 parameters is required. We choose: Untitled Antenna Azimuth A; Untitled Antenna Elevation E; Antenna Roll; Antenna Pitch

This is easily solvable symbolically:

$$Rx(a) := \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(a) & -\sin(a) \\ 0 & \sin(a) & \cos(a) \end{bmatrix}$$

$$a \to Matrix(3, 3, \{(1, 1) = 1, (1, 2) = 0, (1, 3) = 0, (2, 1) = 0, (2, 2) = \cos(a), (2, 3) = -\sin(a), (3, 1) = 0, (3, 2) = \sin(a), (3, 3) = \cos(a)\})$$

$$Ry(a) := \begin{bmatrix} \cos(a) & 0 & \sin(a) \\ 0 & 1 & 0 \\ -\sin(a) & 0 & \cos(a) \end{bmatrix}$$

$$a \to Matrix(3, 3, \{(1, 1) = \cos(a), (1, 2) = 0, (1, 3) = \sin(a), (2, 1) = 0, (2, 2) = 1, (2, 3) = 0, (3, 1) = -\sin(a), (3, 2) = 0, (3, 3) = \cos(a)\})$$

$$Rz(a) := \begin{bmatrix} \cos(a) & -\sin(a) & 0 \\ \sin(a) & \cos(a) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$a \to Matrix(3, 3, \{(1, 1) = \cos(a), (1, 2) = -\sin(a), (1, 3) = 0, (2, 1) = \sin(a), (2, 2) = \cos(a), (2, 3) = 0, (3, 1) = 0, (3, 2) = 0, (3, 3) = 1\})$$

$\tau = -\arctan(\cos(p)\tan(r))$ (See the section on the adapted roll)

$$Rq := Rz(\alpha) \cdot Rx(\tau) \cdot Ry(p - e) \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\alpha)\cos(e - p) + \sin(\alpha)\sin(\tau)\sin(e - p) \\ \sin(\alpha)\cos(e - p) - \cos(\alpha)\sin(\tau)\sin(e - p) \\ \cos(\tau)\sin(e - p) \end{bmatrix}$$

$$Rs := Rz(A) \cdot Ry(-E) \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \cos(A)\cos(E) \\ \sin(A)\cos(E) \\ \sin(E) \end{bmatrix}$$

$$R := Rq - Rs$$

$$\begin{bmatrix} \cos(\alpha)\cos(e-p) + \sin(\alpha)\sin(\tau)\sin(e-p) - \cos(A)\cos(E) \\ \sin(\alpha)\cos(e-p) - \cos(\alpha)\sin(\tau)\sin(e-p) - \sin(A)\cos(E) \\ \cos(\tau)\sin(e-p) - \sin(E) \end{bmatrix}$$

$R2 := isolate(R[2], \sin(\alpha))$

$$\sin(\alpha) = \frac{\cos(\alpha)\sin(\tau)\sin(e-p) + \sin(A)\cos(E)}{\cos(e-p)}$$

$R1 := subs(R2, R[1])$

$$\cos(\alpha)\cos(e-p) + \frac{1}{\cos(e-p)}\big((\cos(\alpha)\sin(\tau)\sin(e-p) + \sin(A)\cos(E))\sin(\tau)\sin(e-p)\big) - \cos(A)\cos(E)$$

$isolate(R1, \alpha)$

$$\alpha = \pi - \arccos\left(\frac{1}{\cos(e-p)^2 + \sin(\tau)^2\sin(e-p)^2}\big(\cos(E)\,(-\cos(A)\cos(e-p) + \sin(\tau)\sin(e-p)\sin(A))\big)\right)$$

$isolate(R[3], e)$

$$e = \arcsin\left(\frac{\sin(E)}{\cos(\tau)}\right) + p$$

$subs\left(e - p = \arcsin\left(\frac{\sin(E)}{\cos(\tau)}\right), De\right)$

$$\alpha = \pi - \arccos\left(\left(\cos(E)\left(-\cos(A)\cos\left(\arcsin\left(\frac{\sin(E)}{\cos(\tau)}\right)\right) + \sin(\tau)\sin\left(\arcsin\left(\frac{\sin(E)}{\cos(\tau)}\right)\right)\sin(A)\right)\right) \Big/ \left(\cos\left(\arcsin\left(\frac{\sin(E)}{\cos(\tau)}\right)\right)^2 + \sin(\tau)^2\sin\left(\arcsin\left(\frac{\sin(E)}{\cos(\tau)}\right)\right)^2\right)\right)$$

$\overset{\text{simplify}}{=}$

$$\alpha = \pi - \arccos\left(\frac{1}{\cos(E)\cos(\tau)}\left(-\cos(A)\sqrt{\frac{\cos(\tau)^2 - 1 + \cos(E)^2}{\cos(\tau)^2}}\cos(\tau) + \sin(\tau)\sin(E)\sin(A)\right)\right)$$

1.3 Untilted System – General Case – Sperical Earth

$$Rx(a) := \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(a) & -\sin(a) \\ 0 & \sin(a) & \cos(a) \end{bmatrix}$$

$$a \rightarrow Matrix(3, 3, \{(1, 1) = 1, (1, 2) = 0, (1, 3) = 0, (2, 1) = 0, (2, 2) = \cos(a), (2, 3) = -\sin(a), (3, 1) = 0, (3, 2) = \sin(a), (3, 3) = \cos(a)\})$$

$$Ry(a) := \begin{bmatrix} \cos(a) & 0 & \sin(a) \\ 0 & 1 & 0 \\ -\sin(a) & 0 & \cos(a) \end{bmatrix}$$

$$a \rightarrow Matrix(3, 3, \{(1, 1) = \cos(a), (1, 2) = 0, (1, 3) = \sin(a), (2, 1) = 0, (2, 2) = 1, (2, 3) = 0, (3, 1) = -\sin(a), (3, 2) = 0, (3, 3) = \cos(a)\}$$

$$Rz(a) := \begin{bmatrix} \cos(a) & -\sin(a) & 0 \\ \sin(a) & \cos(a) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$a \rightarrow Matrix(3, 3, \{(1, 1) = \cos(a), (1, 2) = -\sin(a), (1, 3) = 0, (2, 1) = \sin(a), (2, 2) = \cos(a), (2, 3) = 0, (3, 1) = 0, (3, 2) = 0, (3, 3) = 1\})$$

$$NormFunc(x) := \sqrt{x[1]^2 + x[2]^2 + x[3]^2}$$

$$x \rightarrow \sqrt{x_1^2 + x_2^2 + x_3^2}$$

$$arctan2(y, x) := 2 \arctan\left(\frac{\sqrt{x^2 + y^2} - x}{y}\right)$$

$$(y, x) \rightarrow 2 \arctan\left(\frac{\sqrt{x^2 + y^2} - x}{y}\right)$$

$$Rz(\alpha) \cdot Ry(\beta) \cdot Rx(\gamma)$$

$$[[\cos(\alpha)\cos(\beta), -\sin(\alpha)\cos(\gamma) + \cos(\alpha)\sin(\beta)\sin(\gamma), \sin(\alpha)\sin(\gamma) + \cos(\alpha)\sin(\beta)\cos(\gamma)], [\sin(\alpha)\cos(\beta), \cos(\alpha)\cos(\gamma) + \sin(\alpha)\sin(\beta)\sin(\gamma), -\cos(\alpha)\sin(\gamma) + \sin(\alpha)\sin(\beta)\cos(\gamma)], [-\sin(\beta), \cos(\beta)\sin(\gamma), \cos(\beta)\cos(\gamma)]]$$

$$Base := Rz(Long_{ant})$$

$$\begin{bmatrix} \cos(Long_{ant}) & -\sin(Long_{ant}) & 0 \\ \sin(Long_{ant}) & \cos(Long_{ant}) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$r_{geo\,ecef} := \begin{bmatrix} r_{geo}\cos(Lat_{sat})\cos(Long_{sat}) \\ r_{geo}\cos(Lat_{sat})\sin(Long_{sat}) \\ r_{geo}\sin(Lat_{sat}) \end{bmatrix}$$

$$\begin{bmatrix} r_{geo}\cos(Lat_{sat})\cos(Long_{sat}) \\ r_{geo}\cos(Lat_{sat})\sin(Long_{sat}) \\ r_{geo}\sin(Lat_{sat}) \end{bmatrix}$$

$$r_{geo\,base} := simplify(LinearAlgebra{:-}Transpose(Base) \cdot r_{geo\,ecef})$$

$$[[r_{geo}\cos(Lat_{sat})\,(\cos(Long_{ant})\cos(Long_{sat}) + \sin(Long_{ant})\sin(Long_{sat}))], [-r_{geo}\cos(Lat_{sat})\,(\sin(Long_{ant})\cos(Long_{sat}) - \cos(Long_{ant})\sin(Long_{sat}))], [r_{geo}\sin(Lat_{sat})]]$$

$$r_{geo\,base} := algsubs(\sin(Long_{ant})\cos(Long_{sat}) - \cos(Long_{ant})\sin(Long_{sat}) = \sin(Long_{ant} - Long_{sat}), r_{geo\,base})$$

$$[[r_{geo}\cos(Lat_{sat})\,(\cos(Long_{ant})\cos(Long_{sat}) + \sin(Long_{ant})\sin(Long_{sat}))], [-r_{geo}\cos(Lat_{sat})\sin(Long_{ant} - Long_{sat})], [r_{geo}\sin(Lat_{sat})]]$$

$$r_{geo\,base} := algsubs(\cos(Long_{ant})\cos(Long_{sat}) + \sin(Long_{ant})\sin(Long_{sat}) = \cos(Long_{ant} - Long_{sat}), r_{geo\,base})$$

$$\begin{bmatrix} r_{geo}\cos(Lat_{sat})\cos(Long_{ant} - Long_{sat}) \\ -r_{geo}\cos(Lat_{sat})\sin(Long_{ant} - Long_{sat}) \\ r_{geo}\sin(Lat_{sat}) \end{bmatrix}$$

$$r_{geo\,base} := algsubs(Long_{ant} - Long_{sat} = w, r_{geo\,base})$$

$$\begin{bmatrix} r_{geo}\cos(Lat_{sat})\cos(w) \\ -r_{geo}\cos(Lat_{sat})\sin(w) \\ r_{geo}\sin(Lat_{sat}) \end{bmatrix}$$

$$T_{geo\,ecef} := Rz(Long_{sat}) \cdot Ry(-Lat_{sat}) \cdot Rx(Pol_{sat})$$

$$\left[\left[\cos(Long_{sat})\cos(Lat_{sat}), -\sin(Long_{sat})\cos(Pol_{sat}) - \cos(Long_{sat})\sin(Lat_{sat})\sin(Pol_{sat}), \sin(Long_{sat})\sin(Pol_{sat}) - \cos(Long_{sat})\sin(Lat_{sat})\cos(Pol_{sat})\right],\right.$$
$$\left[\sin(Long_{sat})\cos(Lat_{sat}), \cos(Long_{sat})\cos(Pol_{sat}) - \sin(Long_{sat})\sin(Lat_{sat})\sin(Pol_{sat}), -\cos(Long_{sat})\sin(Pol_{sat}) - \sin(Long_{sat})\sin(Lat_{sat})\cos(Pol_{sat})\right],$$
$$\left.\left[\sin(Lat_{sat}), \cos(Lat_{sat})\sin(Pol_{sat}), \cos(Lat_{sat})\cos(Pol_{sat})\right]\right]$$

$$T_{geo\ base} := simplify(LinearAlgebra{:}\text{-}Transpose(Base) \cdot T_{geo\ ecef})$$

$$\left[\left[\cos(Lat_{sat})\left(\cos(Long_{ant})\cos(Long_{sat}) + \sin(Long_{ant})\sin(Long_{sat})\right),\right.\right.$$
$$-\cos(Long_{ant})\sin(Long_{sat})\cos(Pol_{sat}) - \cos(Long_{ant})\cos(Long_{sat})\sin(Lat_{sat})\sin(Pol_{sat}) + \sin(Long_{ant})\cos(Long_{sat})\cos(Pol_{sat}) - \sin(Long_{ant})\sin(Long_{sat})\sin(Lat_{sat})\sin(Pol_{sat}),$$
$$\cos(Long_{ant})\sin(Long_{sat})\sin(Pol_{sat}) - \cos(Long_{ant})\cos(Long_{sat})\sin(Lat_{sat})\cos(Pol_{sat}) - \sin(Long_{ant})\cos(Long_{sat})\sin(Pol_{sat}) - \sin(Long_{ant})\sin(Long_{sat})\sin(Lat_{sat})\cos(Pol_{sat})\Big],$$
$$\Big[-\cos(Lat_{sat})\left(\sin(Long_{ant})\cos(Long_{sat}) - \cos(Long_{ant})\sin(Long_{sat})\right),$$
$$\sin(Long_{ant})\sin(Long_{sat})\cos(Pol_{sat}) + \sin(Long_{ant})\cos(Long_{sat})\sin(Lat_{sat})\sin(Pol_{sat}) + \cos(Long_{ant})\cos(Long_{sat})\cos(Pol_{sat}) - \cos(Long_{ant})\sin(Long_{sat})\sin(Lat_{sat})\sin(Pol_{sat}),$$
$$-\sin(Long_{ant})\sin(Long_{sat})\sin(Pol_{sat}) + \sin(Long_{ant})\cos(Long_{sat})\sin(Lat_{sat})\cos(Pol_{sat}) - \cos(Long_{ant})\cos(Long_{sat})\sin(Pol_{sat}) - \cos(Long_{ant})\sin(Long_{sat})\sin(Lat_{sat})\cos(Pol_{sat})\Big],$$
$$\left.\left[\sin(Lat_{sat}), \cos(Lat_{sat})\sin(Pol_{sat}), \cos(Lat_{sat})\cos(Pol_{sat})\right]\right]$$

$$T_{geo\ base} := algsubs\left(\sin(Long_{ant})\cos(Long_{sat}) - \cos(Long_{ant})\sin(Long_{sat}) = \sin(Long_{ant} - Long_{sat}), T_{geo\ base}\right)$$

$$\Big[\big[\cos(Lat_{sat})\,(\cos(Long_{ant})\cos(Long_{sat}) + \sin(Long_{ant})\sin(Long_{sat})),$$
$$-\cos(Long_{ant})\cos(Long_{sat})\sin(Lat_{sat})\sin(Pol_{sat}) + \cos(Pol_{sat})\sin(Long_{ant}-Long_{sat}) - \sin(Long_{ant})\sin(Long_{sat})\sin(Lat_{sat})\sin(Pol_{sat}),$$
$$-\cos(Long_{ant})\cos(Long_{sat})\sin(Lat_{sat})\cos(Pol_{sat}) - \sin(Pol_{sat})\sin(Long_{ant}-Long_{sat}) - \sin(Long_{ant})\sin(Long_{sat})\sin(Lat_{sat})\cos(Pol_{sat})\big],$$
$$\big[-\cos(Lat_{sat})\sin(Long_{ant}-Long_{sat}),$$
$$\cos(Long_{ant})\cos(Long_{sat})\cos(Pol_{sat}) + \sin(Long_{ant})\sin(Long_{sat})\cos(Pol_{sat}) + \sin(Lat_{sat})\sin(Pol_{sat})\sin(Long_{ant}-Long_{sat}),$$
$$-\cos(Long_{ant})\cos(Long_{sat})\sin(Pol_{sat}) - \sin(Long_{ant})\sin(Long_{sat})\sin(Pol_{sat}) + \sin(Lat_{sat})\cos(Pol_{sat})\sin(Long_{ant}-Long_{sat})\big],$$
$$\big[\sin(Lat_{sat}),\cos(Lat_{sat})\sin(Pol_{sat}),\cos(Lat_{sat})\cos(Pol_{sat})\big]\Big]$$

$$T_{geo\ base} := algsubs(\cos(Long_{ant})\cos(Long_{sat}) + \sin(Long_{ant})\sin(Long_{sat}) = \cos(Long_{ant}-Long_{sat}), T_{geo\ base})$$

$$\Big[\big[\cos(Lat_{sat})\cos(Long_{ant}-Long_{sat}),$$
$$-\sin(Lat_{sat})\sin(Pol_{sat})\cos(Long_{ant}-Long_{sat}) + \cos(Pol_{sat})\sin(Long_{ant}-Long_{sat}),$$
$$-\sin(Lat_{sat})\cos(Pol_{sat})\cos(Long_{ant}-Long_{sat}) - \sin(Pol_{sat})\sin(Long_{ant}-Long_{sat})\big],$$
$$\big[-\cos(Lat_{sat})\sin(Long_{ant}-Long_{sat}),$$
$$\sin(Lat_{sat})\sin(Pol_{sat})\sin(Long_{ant}-Long_{sat}) + \cos(Pol_{sat})\cos(Long_{ant}-Long_{sat}),$$
$$\sin(Lat_{sat})\cos(Pol_{sat})\sin(Long_{ant}-Long_{sat}) - \sin(Pol_{sat})\cos(Long_{ant}-Long_{sat})\big],$$
$$\big[\sin(Lat_{sat}),\cos(Lat_{sat})\sin(Pol_{sat}),\cos(Lat_{sat})\cos(Pol_{sat})\big]\Big]$$

$$T_{geo\ base} := algsubs(Long_{ant} - Long_{sat} = w, T_{geo\ base})$$

$$[[\cos(Lat_{sat})\cos(w), -\sin(Lat_{sat})\sin(Pol_{sat})\cos(w) + \cos(Pol_{sat})\sin(w), -\sin(Lat_{sat})\cos(Pol_{sat})\cos(w) - \sin(Pol_{sat})\sin(w)],$$
$$[-\cos(Lat_{sat})\sin(w), \sin(Lat_{sat})\sin(Pol_{sat})\sin(w) + \cos(Pol_{sat})\cos(w), \sin(Lat_{sat})\cos(Pol_{sat})\sin(w) - \sin(Pol_{sat})\cos(w)],$$
$$[\sin(Lat_{sat}), \cos(Lat_{sat})\sin(Pol_{sat}), \cos(Lat_{sat})\cos(Pol_{sat})]]$$

$$r_{esa\,ecef} := \begin{bmatrix} r_e\cos(Lat_{ant})\cos(Long_{ant}) \\ r_e\cos(Lat_{ant})\sin(Long_{ant}) \\ r_e\sin(Lat_{ant}) \end{bmatrix}$$

$$\begin{bmatrix} r_e\cos(Lat_{ant})\cos(Long_{ant}) \\ r_e\cos(Lat_{ant})\sin(Long_{ant}) \\ r_e\sin(Lat_{ant}) \end{bmatrix}$$

$$r_{esa\,base} := simplify(LinearAlgebra\text{:-}Transpose(Base) \cdot r_{esa\,ecef})$$

$$\begin{bmatrix} r_e\cos(Lat_{ant}) \\ 0 \\ r_e\sin(Lat_{ant}) \end{bmatrix}$$

$$e_{z\,esa\,ecef} := -\frac{r_{esa\,ecef}}{r_e}$$

$$\begin{bmatrix} -\cos(Lat_{ant})\cos(Long_{ant}) \\ -\cos(Lat_{ant})\sin(Long_{ant}) \\ -\sin(Lat_{ant}) \end{bmatrix}$$

$$NormFunc(e_{z\,esa\,ecef})$$

$$\left(\cos(Lat_{ant})^2\cos(Long_{ant})^2 + \cos(Lat_{ant})^2\sin(Long_{ant})^2 + \sin(Lat_{ant})^2\right)^{1/2}$$

$$\xrightarrow{\text{simplify symbolic}}$$

$$1$$

$check : length\ e_x\ ok$

$$eytemp := \frac{\left(e_{z\,esa\,ecef} \times \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right)}{NormFunc\left(e_{z\,esa\,ecef} \times \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right)}$$

$$\begin{bmatrix} -\frac{\cos(Lat_{ant}) \sin(Long_{ant})}{\sqrt{\cos(Lat_{ant})^2 \sin(Long_{ant})^2 + \cos(Lat_{ant})^2 \cos(Long_{ant})^2}} \\ \frac{\cos(Lat_{ant}) \cos(Long_{ant})}{\sqrt{\cos(Lat_{ant})^2 \sin(Long_{ant})^2 + \cos(Lat_{ant})^2 \cos(Long_{ant})^2}} \\ 0 \end{bmatrix}$$

$e_{y\,esa\,ecef} := simplify(eytemp, symbolic)$

$$\begin{bmatrix} -\sin(Long_{ant}) \\ \cos(Long_{ant}) \\ 0 \end{bmatrix}$$

$NormFunc(e_{y\,esa\,ecef})$

$$\sqrt{\cos(Long_{ant})^2 + \sin(Long_{ant})^2}$$

$\xrightarrow{\text{simplify symbolic}}$

$$1$$

check : length e_y *ok*

$extemp := e_{y\,esa\,ecef} \times e_{z\,esa\,ecef}$

$$\begin{bmatrix} -\cos(Long_{ant}) \sin(Lat_{ant}) \\ -\sin(Long_{ant}) \sin(Lat_{ant}) \\ \sin(Long_{ant})^2 \cos(Lat_{ant}) + \cos(Long_{ant})^2 \cos(Lat_{ant}) \end{bmatrix}$$

$e_{x\,esa\,ecef} := simplify(extemp, symbolic)$

$$\begin{bmatrix} -\cos(Long_{ant}) \sin(Lat_{ant}) \\ -\sin(Long_{ant}) \sin(Lat_{ant}) \\ \cos(Lat_{ant}) \end{bmatrix}$$

$NormFunc(e_{x\,esa\,ecef})$

$$\left(\cos(Long_{ant})^2 \sin(Lat_{ant})^2 + \sin(Long_{ant})^2 \sin(Lat_{ant})^2 + \cos(Lat_{ant})^2\right)^{1/2}$$

$\xrightarrow{\text{simplify symbolic}} 1$

check : length e_z *ok*

$T_{esa\,ecef} := \langle e_{x\,esa\,ecef} | e_{y\,esa\,ecef} | e_{z\,esa\,ecef} \rangle$

$$[[-\cos(Long_{ant})\sin(Lat_{ant}), -\sin(Long_{ant}), -\cos(Lat_{ant})\cos(Long_{ant})], [-\sin(Long_{ant})\sin(Lat_{ant}), \cos(Long_{ant}), -\cos(Lat_{ant})\sin(Long_{ant})], [\cos(Lat_{ant}), 0, -\sin(Lat_{ant})]]$$

$$T_{esa\,base} := simplify(LinearAlgebra\text{:-}Transpose(Base) \cdot T_{esa\,ecef})$$

$$\begin{bmatrix} -\sin(Lat_{ant}) & 0 & -\cos(Lat_{ant}) \\ 0 & 1 & 0 \\ \cos(Lat_{ant}) & 0 & -\sin(Lat_{ant}) \end{bmatrix}$$

$$Range := NormFunc(r_{geo\,base} - r_{esa\,base})$$

$$\left(\left(r_{geo}\cos(Lat_{sat})\cos(w) - r_e\cos(Lat_{ant})\right)^2 + r_{geo}^2\cos(Lat_{sat})^2\sin(w)^2 + \left(r_{geo}\sin(Lat_{sat}) - r_e\sin(Lat_{ant})\right)^2 \right)^{1/2}$$

$$r_{geo\,esa} := simplify(LinearAlgebra\text{:-}Transpose(T_{esa\,base}) \cdot (r_{geo\,base} - r_{esa\,base}))$$

$$[[-r_{geo}\left(\sin(Lat_{ant})\cos(Lat_{sat})\cos(w) - \cos(Lat_{ant})\sin(Lat_{sat})\right)], [-r_{geo}\cos(Lat_{sat})\sin(w)], [-\cos(Lat_{ant})\,r_{geo}\cos(Lat_{sat})\cos(w) - r_{geo}\sin(Lat_{sat})\sin(Lat_{ant}) + r_e]]$$

$$T_{geo\,esa} := simplify(LinearAlgebra\text{:-}Transpose(T_{esa\,base}) \cdot T_{geo\,base})$$

$$[[-\sin(Lat_{ant})\cos(Lat_{sat})\cos(w)+\cos(Lat_{ant})\sin(Lat_{sat}),$$
$$\sin(Lat_{ant})\sin(Lat_{sat})\sin(Pol_{sat})\cos(w)$$
$$-\sin(Lat_{ant})\cos(Pol_{sat})\sin(w)$$
$$+\cos(Lat_{ant})\cos(Lat_{sat})\sin(Pol_{sat}),$$
$$\sin(Lat_{ant})\sin(Lat_{sat})\cos(Pol_{sat})\cos(w)$$
$$+\sin(Lat_{ant})\sin(Pol_{sat})\sin(w)$$
$$+\cos(Lat_{ant})\cos(Lat_{sat})\cos(Pol_{sat})],$$
$$[-\cos(Lat_{sat})\sin(w),\sin(Lat_{sat})\sin(Pol_{sat})\sin(w)$$
$$+\cos(Pol_{sat})\cos(w),\sin(Lat_{sat})\cos(Pol_{sat})\sin(w)$$
$$-\sin(Pol_{sat})\cos(w)],$$
$$[-\cos(Lat_{ant})\cos(Lat_{sat})\cos(w)-\sin(Lat_{ant})\sin(Lat_{sat}),$$
$$\cos(Lat_{ant})\sin(Lat_{sat})\sin(Pol_{sat})\cos(w)$$
$$-\cos(Lat_{ant})\cos(Pol_{sat})\sin(w)$$
$$-\sin(Lat_{ant})\cos(Lat_{sat})\sin(Pol_{sat}),$$
$$\cos(Lat_{ant})\sin(Lat_{sat})\cos(Pol_{sat})\cos(w)$$
$$+\cos(Lat_{ant})\sin(Pol_{sat})\sin(w)$$
$$-\sin(Lat_{ant})\cos(Lat_{sat})\cos(Pol_{sat})]]$$

$$T0_{geo\ esa} := subs(Lat_{ant}=0, T_{geo\ esa})$$
$$[[\sin(Lat_{sat}),\cos(Lat_{sat})\sin(Pol_{sat}),\cos(Lat_{sat})\cos(Pol_{sat})],$$
$$[-\cos(Lat_{sat})\sin(w),\sin(Lat_{sat})\sin(Pol_{sat})\sin(w)$$
$$+\cos(Pol_{sat})\cos(w),\sin(Lat_{sat})\cos(Pol_{sat})\sin(w)$$
$$-\sin(Pol_{sat})\cos(w)],$$
$$[-\cos(Lat_{sat})\cos(w),\sin(Lat_{sat})\sin(Pol_{sat})\cos(w)$$
$$-\cos(Pol_{sat})\sin(w),\sin(Lat_{sat})\cos(Pol_{sat})\cos(w)$$
$$+\sin(Pol_{sat})\sin(w)]]$$

$$e_{x\ point\ esa} := simplify\left(\frac{r_{geo\ esa}}{range}\right)$$

$$\left[\left[-\frac{r_{geo}\left(\sin\left(Lat_{ant}\right)\cos\left(Lat_{sat}\right)\cos(w)-\cos\left(Lat_{ant}\right)\sin\left(Lat_{sat}\right)\right)}{range}\right],\left[-\frac{r_{geo}\cos\left(Lat_{sat}\right)\sin(w)}{range}\right],\left[-\frac{1}{range}\left(\cos\left(Lat_{ant}\right)r_{geo}\cos\left(Lat_{sat}\right)\cos(w)+r_{geo}\sin\left(Lat_{sat}\right)\sin\left(Lat_{ant}\right)-r_{e}\right)\right]\right]$$

$$T_{polplane\,esa} := T_{esa\,base}\cdot Rz(a)\cdot Ry(e)$$

$$\left[\left[-\sin\left(Lat_{ant}\right)\cos(a)\cos(e)+\cos\left(Lat_{ant}\right)\sin(e),\sin\left(Lat_{ant}\right)\sin(a),-\sin\left(Lat_{ant}\right)\cos(a)\sin(e)-\cos\left(Lat_{ant}\right)\cos(e)\right],\left[\sin(a)\cos(e),\cos(a),\sin(a)\sin(e)\right],\left[\cos\left(Lat_{ant}\right)\cos(a)\cos(e)+\sin\left(Lat_{ant}\right)\sin(e),-\cos\left(Lat_{ant}\right)\sin(a),\cos\left(Lat_{ant}\right)\cos(a)\sin(e)-\sin\left(Lat_{ant}\right)\cos(e)\right]\right]$$

$$e_{y\,poldir\,polplane} := LinearAlgebra{:}\text{-}Transpose(Rz(Azimuth)\cdot Ry(Elevation))\cdot T_{geo\,esa}\cdot\begin{bmatrix}0\\1\\0\end{bmatrix}$$

$$\Big[\Big[\cos(Azimuth)\cos(Elevation)\,\big(\sin(Lat_{ant})\sin(Lat_{sat})\sin(Pol_{sat})\cos(w) - \sin(Lat_{ant})\cos(Pol_{sat})\sin(w) + \cos(Lat_{ant})\cos(Lat_{sat})\sin(Pol_{sat})\big) + \sin(Azimuth)\cos(Elevation)\,\big(\sin(Lat_{sat})\sin(Pol_{sat})\sin(w) + \cos(Pol_{sat})\cos(w)\big) - \sin(Elevation)\,\big(\cos(Lat_{ant})\sin(Lat_{sat})\sin(Pol_{sat})\cos(w) - \cos(Lat_{ant})\cos(Pol_{sat})\sin(w) - \sin(Lat_{ant})\cos(Lat_{sat})\sin(Pol_{sat})\big)\Big],$$

$$\Big[-\sin(Azimuth)\,\big(\sin(Lat_{ant})\sin(Lat_{sat})\sin(Pol_{sat})\cos(w) - \sin(Lat_{ant})\cos(Pol_{sat})\sin(w) + \cos(Lat_{ant})\cos(Lat_{sat})\sin(Pol_{sat})\big) + \cos(Azimuth)\,\big(\sin(Lat_{sat})\sin(Pol_{sat})\sin(w) + \cos(Pol_{sat})\cos(w)\big)\Big],$$

$$\Big[\cos(Azimuth)\sin(Elevation)\,\big(\sin(Lat_{ant})\sin(Lat_{sat})\sin(Pol_{sat})\cos(w) - \sin(Lat_{ant})\cos(Pol_{sat})\sin(w) + \cos(Lat_{ant})\cos(Lat_{sat})\sin(Pol_{sat})\big) + \sin(Azimuth)\sin(Elevation)\,\big(\sin(Lat_{sat})\sin(Pol_{sat})\sin(w) + \cos(Pol_{sat})\cos(w)\big) + \cos(Elevation)\,\big(\cos(Lat_{ant})\sin(Lat_{sat})\sin(Pol_{sat})\cos(w) - \cos(Lat_{ant})\cos(Pol_{sat})\sin(w) - \sin(Lat_{ant})\cos(Lat_{sat})\sin(Pol_{sat})\big)\Big]\Big]$$

$$\#Elevation := -arcsin\left(e_{x\,point\,esa}[3]\right)$$

$$\#Azimuth := arctan2\left(\frac{e_{x\,point\,esa}[2]}{\cos(Elevation)}, \frac{e_{x\,point\,esa}[1]}{\cos(Elevation)}\right)$$

$$e_{y\,poldir\,polplane}[3]$$

$$\cos(Azimuth)\sin(Elevation)\left(\sin\left(Lat_{ant}\right)\sin\left(Lat_{sat}\right)\sin\left(Pol_{sat}\right)\cos(w)-\sin\left(Lat_{ant}\right)\cos\left(Pol_{sat}\right)\sin(w)+\cos\left(Lat_{ant}\right)\cos\left(Lat_{sat}\right)\sin\left(Pol_{sat}\right)\right)+\sin(Azimuth)\sin(Elevation)\left(\sin\left(Lat_{sat}\right)\sin\left(Pol_{sat}\right)\sin(w)+\cos\left(Pol_{sat}\right)\cos(w)\right)+\cos(Elevation)\left(\cos\left(Lat_{ant}\right)\sin\left(Lat_{sat}\right)\sin\left(Pol_{sat}\right)\cos(w)-\cos\left(Lat_{ant}\right)\cos\left(Pol_{sat}\right)\sin(w)-\sin\left(Lat_{ant}\right)\cos\left(Lat_{sat}\right)\sin\left(Pol_{sat}\right)\right)$$

$e_{y\,poldir\,polplane}[2]$

$$-\sin(Azimuth)\left(\sin\left(Lat_{ant}\right)\sin\left(Lat_{sat}\right)\sin\left(Pol_{sat}\right)\cos(w)-\sin\left(Lat_{ant}\right)\cos\left(Pol_{sat}\right)\sin(w)+\cos\left(Lat_{ant}\right)\cos\left(Lat_{sat}\right)\sin\left(Pol_{sat}\right)\right)+\cos(Azimuth)\left(\sin\left(Lat_{sat}\right)\sin\left(Pol_{sat}\right)\sin(w)+\cos\left(Pol_{sat}\right)\cos(w)\right)$$

$\#Pol := acrtan2\left(e_{y\,poldir\,polplane}[3], e_{y\,poldir\,polplane}[2]\right)$