Problem 1

The statement that $A \subseteq B$ implies that every element in A is also contained within B. Additionally, the statement $B \subseteq A$ implies that every element in B is also contained within A. If A contains all of the elements in B, and B contains all of the elements in A, then it follows (as night follows day), that A and B must contain the same elements.

Problem 2

The empty set is not a proper subset of itself. The definition of proper subset requires that the second operand contain elements not contained within the first operand. This would imply that no set could be a proper subset of itself, because it cannot possibly meet the requirement for additional elements to be present. This property holds for the empty set as well.

Problem 3

Theorem 1 *If $A \subseteq B$, then $A \bigcup C \subseteq B \bigcup C$.*

Proof. The elements from C getting added to both A and B means that we know that the intersection of A and B contains, at the very least, the elements of C. Since we already know that $A \subseteq B$, then the addition of the elements from C to both of these sets will mean that B contains at least the elements in C and the elements in A. Thus, $A \bigcup C \subseteq B \bigcup C$.
■

Theorem 2 *If $A \subseteq B$, then $A \bigcap C \subseteq B \bigcap C$.*

Proof. If $A \subseteq B$, then we know that B contains all of the elements in A. The intersection $A \bigcap C$ must contain only elements elements of A. The intersection $B \bigcap C$ must contain any elements in B. If any elements are present in $A \bigcap C$, then those elements must also be in $B \bigcap C$ because B contains those elements from A since $A \subseteq B$. Thus, $A \bigcap C \subseteq B \bigcap C$.
■

Theorem 3 *If $A \subseteq B$, then $A - B \subseteq C - A$*

Proof. If $A \subseteq B$, then we know that B contains all of the elements in A. Thus, the operation $A - B$ will be $\emptyset$ since B contains everything in A. Since $A - B = \emptyset$, we know that $A - B$ is a subset of every set, since $\emptyset$ is a subset of every set. Thus, $A - B \subseteq C - A$.
■

Theorem 4 $A \bigcap (B - A) = \emptyset$

Proof. The operation $B - A$ will remove all of the elements of A from B. Since $B - A$ will contain none of the elements of A, then the intersection of that set with A will be $\emptyset$ since the intersection of disjoint sets is the emptyset. Thus, $A \bigcap (B - A) = \emptyset$. ■