Homework 7

Christopher Chapline

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1 Chapter 1.9

1.0.1 Problem 17

Let $T : R^4 \to R^4$

$$\begin{bmatrix} \mathrm{x}_1 \\ \mathrm{x}_2 \\ \mathrm{x}_3 \\ \mathrm{x}_4 \end{bmatrix} \to \begin{bmatrix} \mathrm{x}_1 + 2x_2 \\ 0 \\ 2\mathrm{x}_2 + x_4 \\ \mathrm{x}_2 - x_4 \end{bmatrix}$$

Let's determine the image of $\vec{e_1}, \vec{e_2}, \vec{e_3}$, and $\vec{e_4}$:

$$\vec{e_1} \to \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \vec{e_2} \to \begin{bmatrix} 2 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \vec{e_3} \to \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \vec{e_4} \to \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}$$

Given these images for e_1, e_2, e_3, and e_4, we can redefine T as such:
$T : R^4 \to R^4$

$$\begin{bmatrix} \mathrm{x}_1 \\ \mathrm{x}_2 \\ \mathrm{x}_3 \\ \mathrm{x}_4 \end{bmatrix} \to A\vec{x}, \text{ where } A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 1 & 0 & -1 \end{bmatrix}$$

1.0.2 Problem 19

Let $T : R^3 \to R^2$

$$\begin{bmatrix} \mathrm{x}_1 \\ \mathrm{x}_2 \\ \mathrm{x}_3 \end{bmatrix} \to \begin{bmatrix} \mathrm{x}_1 - 5x_2 + 4x_3 \\ \mathrm{x}_2 - 6x_3 \end{bmatrix}$$

Let's determine the image of $\vec{e_1}, \vec{e_2}$, and $\vec{e_3}$,

$\vec{e_1} \to \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\vec{e_2} \to \begin{bmatrix} -5 \\ 1 \end{bmatrix}$, $\vec{e_3} \to \begin{bmatrix} 4 \\ -6 \end{bmatrix}$

Given these images for e_1, e_2, and e_3,, we can redefine T as such:
$T : R^4 \to R^4$

$$\begin{bmatrix} \mathrm{x}_1 \\ \mathrm{x}_2 \\ \mathrm{x}_3 \end{bmatrix} \to A\vec{x}, \text{ where } A = \begin{bmatrix} 1 & -5 & 4 \\ 0 & 1 & -6 \end{bmatrix}$$

1.0.3 Problem 21

Let $T : R^2 \to R^2$

$$\begin{bmatrix} \mathrm{x}_1 \\ \mathrm{x}_2 \end{bmatrix} \to \begin{bmatrix} \mathrm{x}_1 + x_2 \\ 4\mathrm{x}_1 + 5x_2 \end{bmatrix}$$

First, we must find a matrix transformation equivalent to this linear transformation:

$\vec{e_1} \to \begin{bmatrix} 1 \\ 4 \end{bmatrix}$, $\vec{e_2} \to \begin{bmatrix} 1 \\ 5 \end{bmatrix}$, $T : \vec{x} \to A\vec{x}$ where $A = \begin{bmatrix} 1 & 1 \\ 4 & 5 \end{bmatrix}$

To find a vector $\vec{x}$ where $T\vec{x} = \begin{bmatrix} 3 \\ 8 \end{bmatrix}$, we must find the solution to the matrix A augmented by $\begin{bmatrix} 3 \\ 8 \end{bmatrix}$:

$$\begin{bmatrix} 1 & 1 & 3 \\ 4 & 5 & 8 \end{bmatrix} \backsim \begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & -4 \end{bmatrix} \mathrm{R}_2 - = 4R_1 \backsim \begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & -4 \end{bmatrix} \mathrm{R}_1 - = R_2$$

Therefore, $T\vec{x} = \begin{bmatrix} 3 \\ 8 \end{bmatrix}$ is consistent when $\vec{x} = \begin{bmatrix} 7 \\ -4 \end{bmatrix}$

2 Chapter 2.1

2.0.4 Problem 2

Let: $A = \begin{bmatrix} 2 & 0 & -1 \\ 4 & -5 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 7 & -5 & 1 \\ 1 & -4 & -3 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 3 & 5 \\ -1 & 4 \end{bmatrix}$,

and $E = \begin{bmatrix} -5 \\ 3 \end{bmatrix}$

2.0.5 Problem 4

2.0.6 Problem 8

2.0.7 Problem 12

2.0.8 Problem 15

2.0.9 Problem 16

2.0.10 Problem 22