1 Chapter 1.9

1.0.1 Problem 17

Let $T : R^4 \to R^4$

$$\begin{bmatrix} \mathrm{x}_1 \\ \mathrm{x}_2 \\ \mathrm{x}_3 \\ \mathrm{x}_4 \end{bmatrix} \to \begin{bmatrix} \mathrm{x}_1 + 2x_2 \\ 0 \\ 2\mathrm{x}_2 + x_4 \\ \mathrm{x}_2 - x_4 \end{bmatrix}$$

Let's determine the image of $\vec{e_1}, \vec{e_2}, \vec{e_3}$, and $\vec{e_4}$:

$$\vec{e_1} \to \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \vec{e_2} \to \begin{bmatrix} 2 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \vec{e_3} \to \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \vec{e_4} \to \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}$$

Given these images for e_1, e_2, e_3, and e_4, we can redefine T as such:
$T : R^4 \to R^4$

$$\begin{bmatrix} \mathrm{x}_1 \\ \mathrm{x}_2 \\ \mathrm{x}_3 \\ \mathrm{x}_4 \end{bmatrix} \to A\vec{x}, \text{ where } A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 1 & 0 & -1 \end{bmatrix}$$

1.0.2 Problem 19

Let $T : R^3 \to R^2$

$$\begin{bmatrix} \mathrm{x}_1 \\ \mathrm{x}_2 \\ \mathrm{x}_3 \end{bmatrix} \to \begin{bmatrix} \mathrm{x}_1 - 5x_2 + 4x_3 \\ \mathrm{x}_2 - 6x_3 \end{bmatrix}$$

Let's determine the image of $\vec{e_1}, \vec{e_2}$, and $\vec{e_3}$,

$$\vec{e_1} \to \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \vec{e_2} \to \begin{bmatrix} -5 \\ 1 \end{bmatrix}, \vec{e_3} \to \begin{bmatrix} 4 \\ -6 \end{bmatrix}$$

Given these images for e_1, e_2, and e_3,, we can redefine T as such:
$T : R^4 \to R^4$

$$\begin{bmatrix} \mathrm{x}_1 \\ \mathrm{x}_2 \\ \mathrm{x}_3 \end{bmatrix} \to A\vec{x}, \text{ where } A = \begin{bmatrix} 1 & -5 & 4 \\ 0 & 1 & -6 \end{bmatrix}$$

1.0.3 Problem 21

Let $T : R^2 \to R^2$

$$\begin{bmatrix} \mathrm{x}_1 \\ \mathrm{x}_2 \end{bmatrix} \to \begin{bmatrix} \mathrm{x}_1 + x_2 \\ 4\mathrm{x}_1 + 5x_2 \end{bmatrix}$$

First, we must find a matrix transformation equivalent to this linear transformation:

$\vec{e_1} \to \begin{bmatrix} 1 \\ 4 \end{bmatrix}$, $\vec{e_2} \to \begin{bmatrix} 1 \\ 5 \end{bmatrix}$, $T : \vec{x} \to A\vec{x}$ where $A = \begin{bmatrix} 1 & 1 \\ 4 & 5 \end{bmatrix}$

To find a vector $\vec{x}$ where $T\vec{x} = \begin{bmatrix} 3 \\ 8 \end{bmatrix}$, we must find the solution to the matrix A augmented by $\begin{bmatrix} 3 \\ 8 \end{bmatrix}$:

$$\left[\begin{array}{cc|c} 1 & 1 & 3 \\ 4 & 5 & 8 \end{array}\right] \backsim \left[\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & 1 & -4 \end{array}\right] \mathrm{R}_2 - = 4R_1 \backsim \left[\begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & -4 \end{array}\right] \mathrm{R}_1 - = R_2$$

Therefore, $T\vec{x} = \begin{bmatrix} 3 \\ 8 \end{bmatrix}$ is consistent when $\vec{x} = \begin{bmatrix} 7 \\ -4 \end{bmatrix}$.

2 Chapter 2.1

2.0.4 Problem 2

Let: $A = \begin{bmatrix} 2 & 0 & -1 \\ 4 & -5 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 7 & -5 & 1 \\ 1 & -4 & -3 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 3 & 5 \\ -1 & 4 \end{bmatrix}$, and $E = \begin{bmatrix} -5 \\ 3 \end{bmatrix}$

$$A + 3B = \begin{bmatrix} 2 & 0 & -1 \\ 4 & -5 & 2 \end{bmatrix} + \begin{bmatrix} 21 & -15 & 3 \\ 3 & -12 & -9 \end{bmatrix} = \begin{bmatrix} 23 & -15 & 2 \\ 7 & -17 & -7 \end{bmatrix}$$

$2C - 3E$ is not defined because C and E are different sizes.

$$DB = \begin{bmatrix} (21 + 5) & -15 - 20 & (3 - 15) \\ (-7 + 4) & 5 - 16 & (-1 - 12) \end{bmatrix} = \begin{bmatrix} 26 & -35 & -12 \\ -3 & -11 & -13 \end{bmatrix}$$

EC is not defined because E has 1 column and C has 2 rows.

2.0.5 Problem 4

Let $A = \begin{bmatrix} 5 & -1 & 3 \\ -4 & 3 & -6 \\ -3 & 1 & 2 \end{bmatrix}$.

$$A - 5I_3 = \begin{bmatrix} 5 & -1 & 3 \\ -4 & 3 & -6 \\ -3 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 3 \\ -4 & -2 & -6 \\ -3 & 1 & -3 \end{bmatrix}$$

$$(5I_3)A = 5(I_3A) = 5A = \begin{bmatrix} 25 & -5 & 15 \\ -20 & 15 & -30 \\ -15 & 5 & 10 \end{bmatrix}.$$

2.0.6 Problem 8

The matrix B would need to have 5 rows. The number of rows in the product is determined by the number of rows in the left operand of matrix multiplication, and since BC has 5 rows, so must B.

2.0.7 Problem 12

Let $A = \begin{bmatrix} 3 & -6 \\ -2 & 4 \end{bmatrix}$

To get AB to be equal to zero, we need to get the linear combinations that make up the multiplication to each result in the zero vector. Recognize that the second column of A is -2 times the first column of A. We can use this fact to construct our matrix B. Consider the multiplication of A with the vector $\vec{b} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$. This multiplication results in the zero vector. Thus, if our matrix B defined as $B = [\vec{b}\vec{b}]$, then we will have our answer:

$$\begin{bmatrix} 3 & -6 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

2.0.8 Problem 15

a. False. AB is defined as $[A\vec{b_1}A\vec{b_2}]$.

b. False. Each column of AB is a linear combination of the columns of A using weights from the corresponding column in B.

c. True. Matrix multiplication distributes over addition.

d. True, by theorem 3.

e. False. The transpose of a product of matrices is equal to the product of their transposes in reverse order.

2.0.9 Problem 16

a. True. Expanding the multiplication of matrices A and B will show that the first row of AB is $[a_{11} \cdots a_{1n}][\vec{b_1} \cdots \vec{b_n}]$.
b. True. This is the definition of matrix multiplication.

c. False. $(A^2)^T = A^T A^T$.

d. False. $(ABC)^T = C^T B^T A^T$. This generalization is stated immediately after theorem 3.

e. True. This is stated in theorem 3.

2.0.10 Problem 22

Assume that B is a matrix whose columns are linearly independent and let A be a matrix such that AB is defined. Because the columns of B are linearly independent, we know that does not exist a non-trivial solution to the equation $B\vec{x} = \vec{0}$. If we multiply both sides by A, we get that $AB\vec{x} = A\vec{0} \Rightarrow AB\vec{x} = \vec{0}$. Since there does not exist a non-trivial solution to $AB\vec{x} = \vec{0}$, the columns of AB are linearly independent.