Chapter 2.2

Problem 3

Let $A = \begin{bmatrix} 7 & 3 \\ -6 & -3 \end{bmatrix}$. The inverse of a 2x2 matrix is: $\frac{1}{ad-bc}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Thus, $A^{-1} = \frac{1}{(7)(-3)-(3)(-6)}\begin{bmatrix} -3 & -3 \\ 6 & 7 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} -3 & -3 \\ 6 & 7 \end{bmatrix}$.

Problem 6

Solve the linear system: $\begin{cases} 7\text{x}_1 + 3x_2 = -9 \\ \text{-6x}_1 - 3x_2 = 4 \end{cases}$

$$A\vec{x} = \begin{bmatrix} -9 \\ 4 \end{bmatrix} \Rightarrow \vec{x} = A^{-1}\begin{bmatrix} -9 \\ 4 \end{bmatrix} \Rightarrow \vec{x} = \frac{1}{3}\begin{bmatrix} -3 & -3 \\ 6 & 7 \end{bmatrix}\begin{bmatrix} -9 \\ 4 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 27 \\ -54 \end{bmatrix}\begin{bmatrix} -12 \\ 28 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 15 \\ -26 \end{bmatrix}$$

Problem 8

$A = PBP^{-1} \Rightarrow P^{-1}A = BP^{-1} \Rightarrow P^{-1}AP = B$.

Problem 32

Find A^{-1} if it exists when $A = \begin{bmatrix} 1 & 2 & -1 \\ -4 & -7 & 3 \\ -2 & -6 & 4 \end{bmatrix}$. We will augment by I_3 and row reduce.

$$\left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ -4 & -7 & 3 & 0 & 1 & 0 \\ -2 & -6 & 4 & 0 & 0 & 1 \end{array}\right] \backsim \left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 4 & 1 & 0 \\ 0 & -2 & 2 & 2 & 0 & 1 \end{array}\right] \backsim \left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 4 & 1 & 0 \\ 0 & 0 & 0 & 10 & 2 & 1 \end{array}\right]$$

Because A is not row equivalent with I_3, A is not invertible.

Chapter 2.3

Problem 12

(a) False. Matrix multiplication is not commutative.

(b) False. If the map T were onto or one-to-one, this would be true.

(c) Flase. Not every nxn matrix spans $\mathbb{R}^n$

(d) True. If A had n pivots, $A\vec{x} = \vec{b}$ would only have the trivial solution by the Invertible Matrix Theorem.

(e) True. If a matrix is invertible, so is its transpose. This is Theorem 6.

Problem 20

Yes, it is possible. According to the invertible matrix theorem, if an nxn matrix A spans $\mathbb{R}^n$, then $A\vec{x} = \vec{b}$ has **at least one** solution for all $\vec{b}$ in $\mathbb{R}^n$.

Problem 21

The existence of some $\vec{v}$ such that $C\vec{u} = \vec{v}$ has more than one solution does not prevent C from spanning $\mathbb{R}^n$. As in number 20, the invertible matrix theorem says that if $C\vec{u} = \vec{v}$ has **at least one** solution for all $\vec{v}$ in $\mathbb{R}^n$, then it spans $\mathbb{R}^n$.

Problem 28

Assume that AB is invertible and that B is not invertible. This means that B is not one-to-one, which implies that there exists an x and a y such that $Bx = By$. However, this would also imply that $ABx = ABy$ which cannot be true because AB is one-to-one by the invertible matrix theorem. This is a contradiction. Therefore, B must be invertible.

1 Chapter 3.1

1.0.1 Problem 10

Calculate the determinant of A when $A = \begin{bmatrix} 1 & -2 & 5 & 2 \\ 0 & 0 & 3 & 0 \\ 2 & -6 & -7 & 0 \\ 5 & 0 & 4 & 4 \end{bmatrix}$.

$$\begin{vmatrix} 1 & -2 & 5 & 2 \\ 0 & 0 & 3 & 0 \\ 2 & -6 & -7 & 0 \\ 5 & 0 & 4 & 4 \end{vmatrix} = 3 * \begin{vmatrix} 1 & -2 & 2 \\ 2 & -6 & 5 \\ 5 & 0 & 4 \end{vmatrix} = -15 * \begin{vmatrix} -2 & -2 \\ -6 & 5 \end{vmatrix} \text{-12} * \begin{vmatrix} 1 & -2 \\ 2 & -6 \end{vmatrix}$$

$det(A) = -15(-10 + 12) - 12(-6 + 4) = -30 + 24 = -6$

1.0.2 Problem 12

$$\begin{vmatrix} 1 & -2 & 5 & 2 \\ 0 & 0 & 3 & 0 \\ 2 & -6 & -7 & 0 \\ 5 & 0 & 4 & 4 \end{vmatrix} = 4 * \begin{vmatrix} -1 & 0 & 0 \\ 6 & 3 & -3 \\ -8 & 4 & -3 \end{vmatrix} = -4 * \begin{vmatrix} 3 & -3 \\ 4 & -3 \end{vmatrix}$$

= -4(-9 + 12) = -4 * 3 = -12.

1.0.3 Problem 14

$$\begin{vmatrix} 6 & 3 & 2 & 4 & 0 \\ 9 & 0 & -4 & 1 & 0 \\ 8 & -5 & 6 & 7 & 1 \\ 3 & 0 & 0 & 0 & 0 \\ 4 & 2 & 3 & 2 & 0 \end{vmatrix} = \begin{vmatrix} 6 & 3 & 2 & 4 \\ 9 & 0 & -4 & 1 \\ 3 & 0 & 0 & 0 \\ 4 & 2 & 3 & 2 \end{vmatrix} = 3* \begin{vmatrix} 3 & 2 & 4 \\ 0 & -4 & 1 \\ 2 & 3 & 2 \end{vmatrix}$$

$$= 12 * \begin{vmatrix} 3 & 4 \\ 2 & 2 \end{vmatrix} - 3* \begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix} = \text{12(6-8) - 3(9-4) = -24 - 15 = -39}$$