Homework 1

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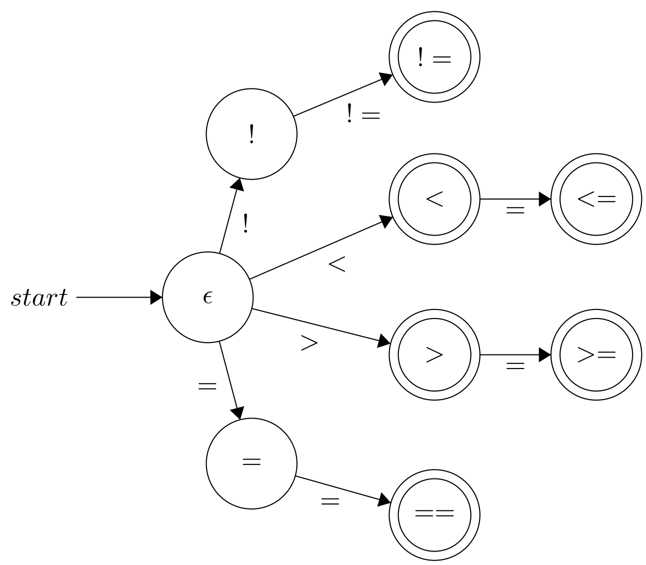
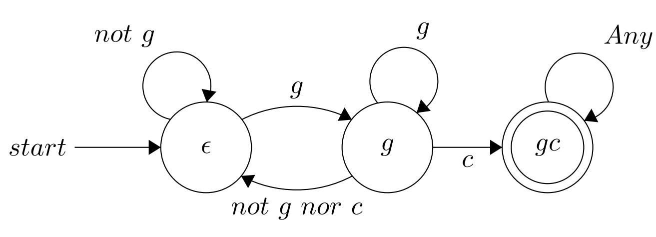
1 Problem 1

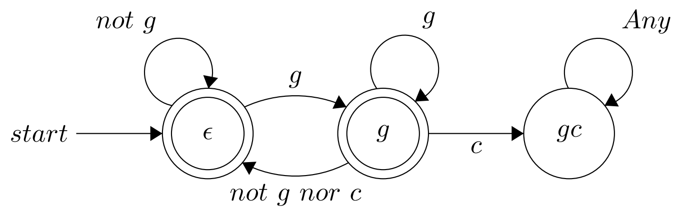


2 Problem 2

2.1 a

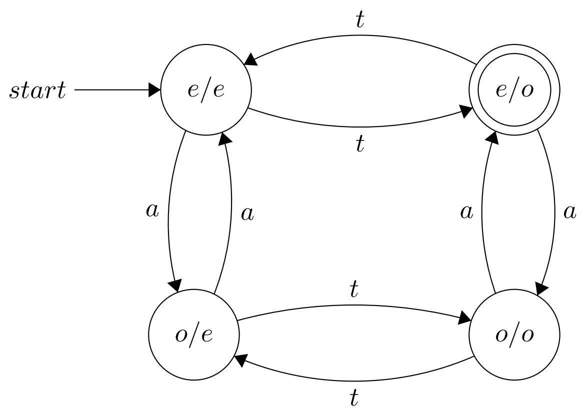


2.2 b

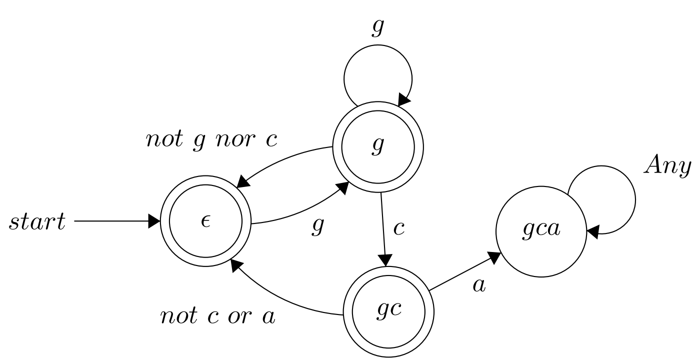


2.3 c

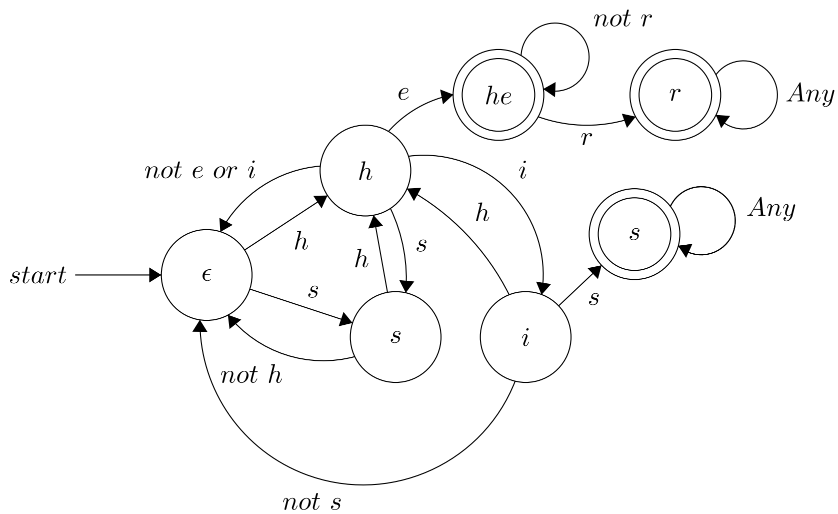
Labels are in the form of a/t where e =even and o =odd.



2.4 d



3 Problem 3



4 Problem 4

Proof.

We will induct on the length of an input string w.

Basis: The base case $|w| = 0$, i.e. $w = \epsilon$. In this case, $\delta(0, w) = 0$
. This holds with the language definition.

Inductive: Let $w = xa$ be a string where a is the last symbol of w and x is the string the precedes a. The inductive hypothesis holds for x. We must consider the transitions that might occur based on what state, S, that the automaton ended on when given the input x.

Assume that $S = 0$. There are two possible transitions out of this state. In the case where a is 0, we are essentially multiplying the binary integer by 2. Multiplying an integer that is divisible by 3 by 2 yields another integer that is divisible by 3. Therefore, the language definition holds. In the case when a is 1, this has the effect of adding 1 to the binary integer. Considering that the general form of an integer that is divisible by 3 can be represented as $3m, m \in \mathbb{R}$, then multiplying by 2 and adding 1 would yield $6m + 1$, which is not divisible by 3. In the automaton, this case correctly transfers for a non-accepting state, which holds with the language definition.

Now consider the case where $S = 1$. There are two possible transitions out of this state. Consider the transition where we add a 1 to the string. To be in this state, the integer would have something to the form would $6m + 1$. The addition of this 1 will have the effect of multiplying by 2 and adding 1, yielding an integer of the form $12m+3$, which is divisible by 3. The addition of a 0 would have the effect of mulitplying by 2, yielding $12m+2$, which is not divisible by 3. The first transition will transfer to an accepting state, which holds will the language. Similarly, the second transition will transfer to a non-accepting state, which also holds with the language.

Lastly, consider the case where $S = 2$. A transition with 1 would have the effect of multiplying an integer of the form $12m+2$ by 2 and adding 1, which yields a number that is not divisible by 3, and is also a non-accepting state. The transition with 0 has the effect of multiplying by 2, which yields a number not divisible by 3 and ends at a non-accepting state. Both of these transitions hold with the definition of the language.

Therefore, all transitions from each state in the automaton hold with the definition of the language.

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5 Problem 5