1 Chapter 1.9

1.0.1 Problem 17

Let $T : R^4 \rightarrow R^4$

$$\begin{bmatrix} \mathrm{x}_1 \\ \mathrm{x}_2 \\ \mathrm{x}_3 \\ \mathrm{x}_4 \end{bmatrix} \rightarrow \begin{bmatrix} \mathrm{x}_1 + 2x_2 \\ 0 \\ 2\mathrm{x}_2 + x_4 \\ \mathrm{x}_2 - x_4 \end{bmatrix}$$

Let's determine the image of $\vec{e_1}, \vec{e_2}, \vec{e_3}$, and $\vec{e_4}$:

$$\vec{e_1} \rightarrow \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \vec{e_2} \rightarrow \begin{bmatrix} 2 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \vec{e_3} \rightarrow \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \vec{e_4} \rightarrow \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}$$

Given these images for e_1, e_2, e_3, and e_4, we can redefine T as such:
$T : R^4 \rightarrow R^4$

$$\begin{bmatrix} \mathrm{x}_1 \\ \mathrm{x}_2 \\ \mathrm{x}_3 \\ \mathrm{x}_4 \end{bmatrix} \rightarrow A\vec{x}, \text{ where } A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 1 & 0 & -1 \end{bmatrix}$$

1.0.2 Problem 19

Let $T : R^3 \rightarrow R^2$

$$\begin{bmatrix} \mathrm{x}_1 \\ \mathrm{x}_2 \\ \mathrm{x}_3 \end{bmatrix} \rightarrow \begin{bmatrix} \mathrm{x}_1 - 5x_2 + 4x_3 \\ \mathrm{x}_2 - 6x_3 \end{bmatrix}$$

Let's determine the image of $\vec{e_1}, \vec{e_2}$, and $\vec{e_3}$,

$$\vec{e_1} \rightarrow \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \vec{e_2} \rightarrow \begin{bmatrix} -5 \\ 1 \end{bmatrix}, \vec{e_3} \rightarrow \begin{bmatrix} 4 \\ -6 \end{bmatrix}$$

Given these images for e_1, e_2, and e_3,, we can redefine T as such:
$T : R^4 \rightarrow R^4$

$$\begin{bmatrix} \mathrm{x}_1 \\ \mathrm{x}_2 \\ \mathrm{x}_3 \end{bmatrix} \rightarrow A\vec{x}, \text{ where } A = \begin{bmatrix} 1 & -5 & 4 \\ 0 & 1 & -6 \end{bmatrix}$$

1.0.3 Problem 21

Let $T : R^2 \to R^2$

$$\begin{bmatrix} \mathrm{x}_1 \\ \mathrm{x}_2 \end{bmatrix} \to \begin{bmatrix} \mathrm{x}_1 + x_2 \\ 4\mathrm{x}_1 + 5x_2 \end{bmatrix}$$

First, we must find a matrix transformation equivalent to this linear transformation:

$\vec{e_1} \to \begin{bmatrix} 1 \\ 4 \end{bmatrix}$, $\vec{e_2} \to \begin{bmatrix} 1 \\ 5 \end{bmatrix}$, $T : \vec{x} \to A\vec{x}$ where $A = \begin{bmatrix} 1 & 1 \\ 4 & 5 \end{bmatrix}$

To find a vector $\vec{x}$ where $T\vec{x} = \begin{bmatrix} 3 \\ 8 \end{bmatrix}$, we must find the solution to the matrix A augmented by $\begin{bmatrix} 3 \\ 8 \end{bmatrix}$:

$$\left[\begin{array}{cc|c} 1 & 1 & 3 \\ 4 & 5 & 8 \end{array}\right] \backsim \left[\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & 1 & -4 \end{array}\right] \mathrm{R}_2 - = 4R_1 \backsim \left[\begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & -4 \end{array}\right] \mathrm{R}_1 - = R_2$$

Therefore, $T\vec{x} = \begin{bmatrix} 3 \\ 8 \end{bmatrix}$ is consistent when $\vec{x} = \begin{bmatrix} 7 \\ -4 \end{bmatrix}$.

2 Chapter 2.1

2.0.4 Problem 2

Let: $A = \begin{bmatrix} 2 & 0 & -1 \\ 4 & -5 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 7 & -5 & 1 \\ 1 & -4 & -3 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 3 & 5 \\ -1 & 4 \end{bmatrix}$, and $E = \begin{bmatrix} -5 \\ 3 \end{bmatrix}$

$$A + 3B = \begin{bmatrix} 2 & 0 & -1 \\ 4 & -5 & 2 \end{bmatrix} + \begin{bmatrix} 21 & -15 & 3 \\ 3 & -12 & -9 \end{bmatrix} = \begin{bmatrix} 23 & -15 & 2 \\ 7 & -17 & -7 \end{bmatrix}$$

$2C - 3E$ is not defined because C and E are different sizes.

$$DB = \begin{bmatrix} (21+5) & -15-20 & (3-15) \\ (-7+4) & 5-16 & (-1-12) \end{bmatrix} = \begin{bmatrix} 26 & -35 & -12 \\ -3 & -11 & -13 \end{bmatrix}$$

EC is not defined because E has 1 column and C has 2 rows.

2.0.5 Problem 4

Let $A = \begin{bmatrix} 5 & -1 & 3 \\ -4 & 3 & -6 \\ -3 & 1 & 2 \end{bmatrix}$.

$A - 5I_3 = \begin{bmatrix} 5 & -1 & 3 \\ -4 & 3 & -6 \\ -3 & 1 & 2 \end{bmatrix}$ - $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 3 \\ -4 & -2 & -6 \\ -3 & 1 & -3 \end{bmatrix}$

$(5I_3)A = 5(I_3A) = 5A = \begin{bmatrix} 25 & -5 & 15 \\ -20 & 15 & -30 \\ -15 & 5 & 10 \end{bmatrix}$.

2.0.6 Problem 8

The matrix B would need to have 5 rows. The number of rows in the product is determined by the number of rows in the left operand of matrix multiplication, and since BC has 5 rows, so must B.

2.0.7 Problem 12

Let $A = \begin{bmatrix} 3 & -6 \\ -2 & 4 \end{bmatrix}$.

2.0.8 Problem 15

a. False. AB is defined as $[A\vec{b_1}A\vec{b_2}]$.

b. False. Each column of AB is a linear combination of the columns of A using weights from the corresponding column in B.

c. True. Matrix multiplication distributes over addition.

d. True, by theorem 3.

e. False. The transpose of a product of matrices is equal to the product of their transposes in reverse order.

2.0.9 Problem 16

a.

b. True. This is the definition of matrix multiplication.

c. False. $(A^2)^T = A^T A^T$.

d. False. $(ABC)^T = C^T B^T A^T$. This generalization is stated immediately after theorem 3.

e. True. This is stated in theorem 3.

2.0.10 Problem 22