Chapter 4.6

Problem 8

The dimension of the null space of A ($dim(Null(A))$), would be 4.

The column space of A would not be equal $\mathbb{R}^4$. The vectors in $Coll(A)$ are in $\mathbb{R}^6$ and could not span $\mathbb{R}^4$.

Problem 10

By Rank-Nullity, dimension of the column space of is 7 (the number of columns in the matrix) minus 5 (the dimension of the null space), which is 2. Thus, $dim(coll(A)) = 2$.

Problem 12

The dimension of the row space of a matrix is equivalent to the rank of the matrix. The rank of A is 4, thus the dimension of the row space of A is also 4.

Chapter 4.7

Problem 8

Let $\vec{b_1} = \begin{bmatrix} -1 \\ 8 \end{bmatrix}, \vec{b_2} = \begin{bmatrix} 1 \\ -7 \end{bmatrix}, \vec{c_1} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \vec{c_2} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

To find $P_{C\leftarrow B}$, we row reduce this matrix until the left side is equal to the identity matrix:

$$\left[\begin{array}{cc|cc} 1 & 1 & 1 & 1 \\ 2 & 1 & 8 & -7 \end{array}\right] \backsim \left[\begin{array}{cc|cc} 1 & 1 & 1 & 1 \\ 0 & 1 & -10 & 9 \end{array}\right] \backsim \left[\begin{array}{cc|cc} 1 & 0 & 9 & -9 \\ 0 & 1 & -10 & 9 \end{array}\right]$$

Thus, $P_{C\leftarrow B} = \begin{bmatrix} 9 & -9 \\ -10 & 9 \end{bmatrix}$.

There are two ways we can compute $P_{B\leftarrow C}$. We can either compute it using the same process we used to compute $P_{C\leftarrow B}$ or we can find the inverse of $P_{C\leftarrow B}$. Since $P_{C\leftarrow B}$ is a 2x2 matrix, its inverse has a simple closed form:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \Rightarrow \frac{1}{(9)(9)-(-9)(-10)} \begin{bmatrix} 9 & 9 \\ 10 & 9 \end{bmatrix} = -\frac{1}{9} \begin{bmatrix} 9 & 9 \\ 10 & 9 \end{bmatrix}$$

Thus, $P_{B\leftarrow C} = -\frac{1}{9} \begin{bmatrix} 9 & 9 \\ 10 & 9 \end{bmatrix}$.

Problem 10

Let $\vec{b_1} = \begin{bmatrix} 6 \\ -12 \end{bmatrix}, \vec{b_2} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}, \vec{c_1} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}, \vec{c_2} = \begin{bmatrix} 3 \\ 9 \end{bmatrix}$.

To find $P_{B \leftarrow C}$, we row reduce this matrix until the left side is equal to the identity matrix:

$$\left[\begin{array}{cc|cc} 6 & 4 & 4 & 3 \\ -12 & 2 & 2 & 9 \end{array}\right] \backsim \left[\begin{array}{cc|cc} 6 & 4 & 4 & 3 \\ 0 & 10 & 10 & 15 \end{array}\right] \backsim \left[\begin{array}{cc|cc} 6 & 4 & 4 & 3 \\ 0 & 2 & 2 & 3 \end{array}\right] \backsim$$
$$\left[\begin{array}{cc|cc} 6 & 0 & 0 & -3 \\ 0 & 2 & 2 & 3 \end{array}\right] \backsim \left[\begin{array}{cc|cc} 1 & 0 & 0 & -\frac{1}{2} \\ 0 & 1 & 1 & \frac{3}{2} \end{array}\right]$$

Therefore, $P_{B \leftarrow C} = \begin{bmatrix} 0 & -\frac{1}{2} \\ 1 & \frac{3}{2} \end{bmatrix}$. Once again, we can find $P_{C \leftarrow B}$ by find the inverse of $P_{B \leftarrow C}$.

$$P_{C \leftarrow B} = P_{B \leftarrow C}^{-1} = \begin{bmatrix} 0 & -\frac{1}{2} \\ 1 & \frac{3}{2} \end{bmatrix}^{-1} = \frac{1}{\frac{1}{2}} \begin{bmatrix} \frac{3}{2} & \frac{1}{2} \\ -1 & 0 \end{bmatrix}^{-1} = 2 \begin{bmatrix} \frac{3}{2} & \frac{1}{2} \\ -1 & 0 \end{bmatrix}^{-1}$$

Chapter 5.1

Problem 2

Problem 4

Problem 10

Problem 12

Problem 14

Chapter 5.2

Problem 2

Problem 4

Problem 6