Problem 1

The statement that $A \subseteq B$ implies that every element in A is also contained within B. Additionally, the statement $B \subseteq A$ implies that every element in B is also contained within A. If A contains all of the elements in B, and B contains all of the elements in A, then it follows (as night follows day), that A and B must contain the same elements.

Problem 2

The empty set is not a proper subset of itself. The definition of proper subset requires that the second operand contain elements not contained within the first operand. This would imply that no set could be a proper subset of itself, because it cannot possibly meet the requirement for additional elements to be present. This property holds for the empty set as well.

Problem 3

Theorem 1 *If* $A \subseteq B$*, then* $A \bigcup C \subseteq B \bigcup C$.

Proof. ■

Theorem 2 *If* $A \subseteq B$*, then* $A \bigcap C \subseteq B \bigcap C$.

Proof. ■

Theorem 3 *If* $A \subseteq B$*, then* $A - B \subseteq C - A$

Proof. ■

Theorem 4 $A \bigcap (B - A) = \emptyset$

Proof. ■