# Chapter 2.2

### Problem 3

Let $A = \begin{bmatrix} 7 & 3 \\ -6 & -3 \end{bmatrix}$. The inverse of a 2x2 matrix is: $\frac{1}{ad-bc}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Thus, $A^{-1} = \frac{1}{(7)(-3)-(3)(-6)}\begin{bmatrix} -3 & -3 \\ 6 & 7 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} -3 & -3 \\ 6 & 7 \end{bmatrix}$.

### Problem 6

Solve the linear system: $\begin{cases} 7\text{x}_1 + 3x_2 = 9 \\ \text{-6x}_1 - 3x_2 = ? \end{cases}$

$$A\vec{x} = \begin{bmatrix} 9 \\ ? \end{bmatrix} \Rightarrow \vec{x} = A^{-1}\begin{bmatrix} 9 \\ ? \end{bmatrix} \Rightarrow \vec{x} = \frac{1}{3}\begin{bmatrix} -3 & -3 \\ 6 & 7 \end{bmatrix}\begin{bmatrix} 9 \\ ? \end{bmatrix}$$

### Problem 8

$A = PBP^{-1} \Rightarrow P^{-1}A = BP^{-1} \Rightarrow P^{-1}AP = B$.

### Problem 32

Find $A^{-1}$ if it exists when $A = \begin{bmatrix} 1 & 2 & -1 \\ -4 & -7 & 3 \\ -2 & -6 & 4 \end{bmatrix}$. We will augment by $I_3$ and row reduce.

$$\left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ -4 & -7 & 3 & 0 & 1 & 0 \\ -2 & -6 & 4 & 0 & 0 & 1 \end{array}\right] \backsim \left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 4 & 1 & 0 \\ 0 & -2 & 2 & 2 & 0 & 1 \end{array}\right] \backsim \left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 4 & 1 & 0 \\ 0 & 0 & 0 & 10 & 2 & 1 \end{array}\right]$$

Because $A$ is not row equivalent with $I_3$, $A$ is not invertible.

# Chapter 2.3

### Problem 12

**(a)** False. Matrix multiplication is not commutative.

**(b)** False. If the map $T$ were onto or one-to-one, this would be true.

**(c)** Flase. Not every $nxn$ matrix spans $\mathbb{R}^n$

**(d)** True. If $A$ had $n$ pivots, $A\vec{x} = \vec{b}$ would only have the trivial solution by the Invertible Matrix Theorem.

**(e)** True. If a matrix is invertible, so is its transpose. This is Theorem 6.

**Problem 20**

Yes, it is possible. According to the invertible matrix theorem, if an $nxn$ matrix $A$ spans $\mathbb{R}^n$, then $A\vec{x} = \vec{b}$ has **at least one** solution for all $\vec{b}$ in $\mathbb{R}^n$.

**Problem 21**

The existence of some $\vec{v}$ such that $C\vec{u} = \vec{v}$ has more than one solution does not prevent $C$ from spanning $\mathbb{R}^n$. As in number 20, the invertible matrix theorem says that if $C\vec{u} = \vec{v}$ has **at least one** solution for all $\vec{v}$ in $\mathbb{R}^n$, then it spans $\mathbb{R}^n$.

**Problem 28**

Theorem 6 states that $(AB)^{-1} = B^{-1}A^{-1}$. This equality shows that for $AB$ to be invertible, it must be representable as the product of 2 invertible matrices.

# 1 Chapter 3.1