Errata for Geometry, Topology and Physics (Second Edition) by Mikio Nakahara

This is an unofficial and incomplete list of errors and typos found in the second edition of Geometry, Topology and Physics. If I have overlooked or misidentified an error, please contact me at `george.johnson@physics.ox.ac.uk`. For items marked *, the credit goes to the creator(s) of the errata at `http://theory.caltech.edu/classes/ph229a/`.

Quantum Physics

1. page 8, in the Lagrangian in example 1.4 $\dot{\phi}^2$ should be $\dot{\theta}^2$

2. page 9, a bra vector should be defined as an element of $\mathcal{H}^*$

3. page 17, in (1.65) ε should be 2ε

4. page 21, in (1.92) there is a missing factor of $\sqrt{m}$

5. page 21, in (1.92) the factor $(x - y)$ should be squared only once

6. page 33, many missing factors of m in equations (1.143) – (1.151)

7. page 38, (1.164) and equation beneath seem incorrect, see comments below

8. page 39, the Hamiltonian at the top of the page is missing a factor of ω

9. page 39, in (1.168) $cc^\dagger$ should be $c^\dagger c$

10. page 44, $\mathrm{d}\theta$ should be $\mathrm{d}\theta'$ in (1.187); $\mathrm{d}\theta_1$ should be $\mathrm{d}\theta'_1$ in equation for I^2

11. page 44, variable change from θ_k, θ'_k to η_k, η^*_k seems incorrect, see comments below

12. page 45, in equation at top of page, $\mathrm{d}\eta_i$ should be $\mathrm{d}\eta_1$

13. page 45, in (1.190) there should be no $2^{n/2}$ and exponent should be $-{}^t K M^{-1} K/2$

14. page 48, in second line of second equation for $Z(\beta)$, missing factor $\mathrm{e}^{-\theta^*\theta}$

15. page 50, in equation for $Z(\beta)$, first two products should run over n, not k

16. page 53, in (1.220) the $-$ in front of iε should be $+$

17. page 53, in (1.224) the $-$ in front of the integral and in front of iε should be $+$

18. page 55, in (1.237) the x in the argument of the exponential should be $(x - y)$

19. page 56, in (1.241d) the second $\mathbf{E}$ should be $\mathbf{B}$

20. page 64, in (1.293) there should be no factor of i

21. page 65, the LHS of (1.298) should be traced over

22. page 65, (1.298) should have only one Hodge star in the second term

23. page 65, in (1.300), (1.304) there should be no factor of i

24. page 65, (1.301b) should have a $+$ instead of a $-$ for consistency with (10.117b)

Homology Groups

1. page 109, above (3.41), should have $Z_2(K) = \{0\}$ instead of $Z_1(K) = \{0\}$

Homotopy Groups

1. page 130,* in (4.9), should have $1/2 \leq r \leq 1$

2. page 130,* in example 4.1 the homotopy interpolates between id_Y and $f \circ g$

Manifolds

1. page 196, in (5.62) the $\wedge$ on the RHS should all be $\otimes$

2. page 199, beneath (5.70), the sum should run from $i = 1$ to $r + 1$

3. page 202,* (5.83) should read $\mathrm{i}_{[X,Y]}\omega = [\mathcal{L}_X, \mathrm{i}_Y]\omega$

4. page 203, in (5.94) there should be a $+$ after $\partial/\partial p_\mu$

5. page 204, in (5.96) f and g should be interchanged in the second term

6. page 213, in (5.125), should have $\phi(s)$ instead of $\phi(t)$ on the LHS

7. page 216, beneath (5.138), should have $T_g^* G$

8. page 217,* beneath (5.130), should have $-x^t \eta x$ instead of $-X^t \eta X$

De Rham Cohomology Groups

1. page 234, in corollary 6.1 the $[c_i]$ need to be linearly independent, not just different

2. page 235, in (6.27), should have d_{r-1} instead of d_r and d_r instead of d_{r+1}

3. page 230, in third paragraph, should have $r = 3$ instead of $m = 3$

4. page 235, in theorem 6.3 we must have $r \geq 1$

5. page 237,* in section 6.4.1 we must have that M is orientable

6. page 238, in (6.40) the sum should run from $r = 0$

7. page 239, in the first line of (6.46) the sums should run from $r = 0$

Riemannian Geometry

1. page 253, beneath (7.29), should have Y^μ in the second term on the RHS

2. page 253, above (7.30), should have $V^\kappa \nabla_\kappa X = V^\kappa \nabla_\kappa Y = 0$

3. page 255, in (7.42), should have $\Gamma^\eta{}_{\mu\lambda}$ in second term of second line

4. page 264, in (7.63), should have $\sin^2 \theta$

5. page 265, in (7.68b), should have $-y'^2$ instead of $+3y'^2$

6. page 268,* in exercise 7.11, fourth term of last line, should have $g_{\lambda\nu}$ instead of $g_{\mu\nu}$

7. page 269, in (7.82), second equality, the first $-$ should be a $+$

8. page 270, penultimate line should read $R_{\kappa\lambda\mu\nu} = -R_{\lambda\kappa\mu\nu}$ (credit S. Hernandez Cuenca)

9. page 275, in proof of proposition 7.1, should have $\bar{\nabla}_X Y$, $\nabla_X Y$ instead of $\bar{\nabla}_X$, ∇_X

10. page 277, in (7.115) should have a $+$ instead of a $-$ after $R_{\kappa\lambda\mu\nu}$

11. page 277, beneath (7.115), should have $C^\kappa{}_{\lambda\mu\nu} = \bar{C}^\kappa{}_{\lambda\mu\nu}$

12. page 284, in (7.144) there is a missing bracket on the second line

13. page 288, beneath (7.162), should have $R^1{}_{212} = 1$, $R^2{}_{112} = -1$

14. page 288, beneath second paragraph, should have $R^\theta{}_{\phi\theta\phi} = \sin^2\theta R^1{}_{212} = \sin^2\theta$

15. page 289, in (7.167) the equation for $R^0{}_2$ should have M instead of $2M$

16. page 294, in (7.188), fourth line, there should be no factor of g^{-1}

17. page 294, (7.190) applies to a compact orientable manifold without boundary

18. page 295, in exercise 7.23 the projector P must be an orthogonal projector

19. page 296, in fifth paragraph, should have $B^r(M) \cap \mathrm{Harm}^r(M) = \{0\}$ instead of $\varnothing$

20. page 298, in proof of (c) of proposition 7.2, should have $\delta\Gamma^\kappa{}_{\kappa\mu}$ instead of $\Gamma^\kappa{}_{\kappa\mu}$

21. page 299, beneath first paragraph in third equality, should have $\delta\Gamma^\kappa{}_{\kappa\mu}$

22. page 301, in (7.220), should have $\Lambda_\alpha{}^\beta$ instead of $\Lambda_\alpha{}^\eta$

23. page 302, (7.229a$'$) should have $2m$ instead of m

24. page 305, in exercise 7.28 we make use of symmetries (i) and (ii)

25. page 305, in (7.238) the RHS should have a conformal factor $2/(\partial^\alpha X^\mu \partial_\alpha X_\mu)$

26. page 306,*in exercise 7.29 should have $h_{\alpha\beta}$ instead of $G_{\alpha\beta}$ for consistency with (7.243)

27. page 306, in (7.242) the indices on the ϕ fields in the second term should be β and γ

Complex Manifolds

1. page 309, beneath first paragraph, indices on x and y should be μ instead of ν

2. page 311, usage of a, b, c, d and a', b', c', d' in text is inconsistent with usage in matrices

3. page 311, beneath fourth paragraph, should have $\mathrm{Im}(\omega_2/\omega_1)$ in final equality

4. page 312, beneath (8.4) and beneath (8.5), should have $(\tilde{\omega}_1, \tilde{\omega}_2)$ instead of (ω_1', ω_2')

5. page 316, beneath first paragraph, after first equality, $\partial y^\mu/\partial y'^\lambda$ should be $\partial y^\mu/\partial x'^\lambda$

6. page 318, the matrix in (8.18) should be transposed (credit M. Vall)

7. page 319, last line, should have $(1/2)\dim_\mathbb{C} T_p M^\mathbb{C} = \dim_\mathbb{C} M$ (NB $(1/2)\dim_\mathbb{C} M \notin \mathbb{Z}$)

8. page 323, in proof of (b) of theorem 8.1, $\partial\omega$ should be $\partial\overline{\omega}$

9. page 328, in (8.68), should have $\Gamma^{\bar{\lambda}}{}_{\overline{\kappa\nu}}$ in the second equality

10. page 330, in (8.76d), should have $R^{\bar{\xi}}{}_{\overline{\lambda}\bar{\mu}\nu}$

11. page 330, in (8.79), should have $\mathcal{R} = -\mathrm{i}\partial\overline{\partial}\log G$ for consistency with (8.78)

12. page 330, beneath (8.79), should have $\overline{\mathcal{R}} = +\mathrm{i}\overline{\partial\overline{\partial}} \log G$

13. page 330, in proof of proposition 8.2, missing factors of -1 due to the error in (8.79)

14. page 333, in (8.86) there should be no $\mathrm{i}/2$ on the RHS of the second equality

15. page 334, fact (a), note that there is no expert consensus on whether S^6 is complex

16. page 334, fact (b), note that exceptionally $S^1 \times S^1$ *is* Kähler

17. page 339, in section 8.6.2, should have $\mathrm{Harm}_{\overline{\partial}}^{r,s}(M) \cap B_{\overline{\partial}}^{r,s}(M) = \{0\}$ instead of $\varnothing$

18. page 340, in proof of (b) of theorem 8.10, should have $\omega \in H_{\overline{\partial}}^{r,s}(M)$

Fibre Bundles

1. page 353, in last line, should have $f' = t_{ji}(p)f$

2. page 357, in first paragraph, should have $y = \psi(p)$ instead of $y = \phi(p)$

3. page 362, beneath (9.36), should have $\pi_1^{-1}(p)$ instead of $(\pi \times \pi')^{-1}(p)$

4. page 364, in second paragraph, should have $s_i(p)$ instead of $s_1(p)$

5. page 365, final paragraph, the trivial bundle would be $S^4 \times \mathrm{SU}(2)$

6. page 370, beneath (9.66), would perhaps be clearer to write $(P \times F)/G$

7. page 370, in example 9.11, T_pM should be TM

Connections on Fibre Bundles

1. page 375, beneath (10.1), should have that $A^{\#}$ is tangent to G_p at u

2. page 378, first line, should have $T_{\sigma_i(p)}P$ instead of $T_{\sigma_i}P$

3. page 378, first paragraph, should have $g_i \equiv e$

4. page 378, in verifying axiom (i), should have $\mathrm{d}g_i(u\exp(tA))/\mathrm{d}t|_{t=0}$

5. page 388, beneath (10.36), should have $u \in P$

6. page 390, (10.43), (10.44) should have factors of $1/2$ in front of the $f_{\beta\gamma}{}^{\alpha}$.

7. page 390, (10.44) should have a minus sign in front of the second term

8. page 393,* in second line, should have $E = P \times_{\rho} V$

9. page 396, first paragraph, should have $\mathrm{Ad}_g V = gVg^{-1}$

10. page 396, in second line of (10.68b) $\mathcal{A}$ should be A in accordance with (10.64)

11. page 397, in theorem 10.6 the $\omega^{\gamma}{}_{\alpha}$ should all be $\mathcal{A}^{\gamma}{}_{\alpha}$ in accordance with (10.56b)

12. page 398, the index α at the end of the last and penultimate lines should be a β

13. page 400, (10.85b) should have prefactor $1/2$, not $-1/4$, for consistency with (10.85a)

14. page 400, (10.87) should have $\pm 4\mathbf{B} \cdot \mathbf{E}$ on RHS ($\pm$ depending on convention)

15. page 401, (10.91) differs in convention to (10.9), which has t_{SN} instead of t_{NS}

16. page 403, (10.96) should have a squared modulus to ensure $\mathcal{H}$ is real

17. page 405, conventions in (10.108) and (10.110) are atypical, see comments below

18. page 406, (10.116) also differs in convention to (10.9)

19. page 406, in discussing the diffeomorphism $\text{SU}(2) \simeq S^3$, should have $t^4 I_2 + \mathrm{i} t^i \sigma_i$

20. page 406, second paragraph should reference example 9.8 instead of example 9.11

21. page 406, in (10.117b), (10.117c), should have $x^4 I_2 + \mathrm{i} x^i \sigma_i$

22. page 412, in (10.141) there should be a subscript i on $|\mathbf{R}\rangle$ at the end of the first line

23. page 413, seemingly major error in example 10.7, see comments below

24. page 414, beneath (10.148), $|\Phi(\mathbf{R})\rangle$ should just be $\Phi(\mathbf{R})$

25. page 414, in (10.153), should have $H(\mathbf{R})$ instead of $\mathcal{H}(\mathbf{R})$

26. page 415, in (10.159b) there should be no loop on the integral

27. page 417, beneath (10.167), charge should be $+1/2$ for consistency with (10.89)

Characteristic Classes

1. page 420, beneath (11.1), should have $a_p \in M(k, \mathbb{C})$

2. page 420, in the second line of (11.4) there should be no commas in the trace

3. page 421, beneath (11.5), the tensor product $\bigotimes$ should be a direct sum $\bigoplus$

4. page 422, first line, $P(M, \mathbb{C})$ should be $P(M, G)$

5. page 425, beneath (11.21), should have $\dim M = 2m$

6. page 425, final equation is missing a sum over permutations that cancels $1/(r+s)!$

7. page 427, in (11.29) the final term should be a product of x_j, with no sums

8. page 428, in (11.31) the two instances of $\mathcal{A}$ should be A

9. page 431, the two instances of $\chi(G_k)$ beneath (11.45) should be $\chi(L_k)$

10. page 431, in (11.46b) the direct sums should be ordinary sums

11. page 431, in (11.47) the sum should run from $j = 0$

12. page 434,* in (11.62b), should have $\mathrm{ch}(E) + \mathrm{ch}(F)$ on the RHS

13. page 434,* in the expression for $\mathrm{ch}(B \otimes C)$ the sum should run from $m = 0$

14. page 434, final equation should have $(\mathrm{i}/2\pi)^j$

15. page 435,* in (11.63) the RHS should be an ordinary sum instead of a direct sum

16. page 435, (11.64) should be a sum $\sum$ not a product $\prod$

17. page 438, in (11.75e) the second equality is true iff k is even

18. page 439, in (11.79b) the RHS isn't a p-form if $\mathcal{R}_{\alpha\beta}$ is defined as in above paragraph

19. page 439, beneath (11.80), should have $p_l(\mathcal{R})$ instead of $p_1(\mathcal{R})$

20. page 439, seemingly major error in example 11.6, see comments below

21. page 440, beneath (11.83) there is a reference to seemingly unrelated (6.39)

22. page 442, first line, should have λ_j instead of λ_i

23. page 444, in (11.107) there should be a 4 on the RHS instead of a 2

24. page 447, third paragraph, should have $Q_{2j+1}(g^{-1}\mathrm{d}g, 0)$

25. page 449, last line, product should run from $j, k = 0$ to $r + 2$ (and $j \neq k$)

Index Theorems

1. page 453, second paragraph, the Dirac operator is a map $\Gamma(M, E) \to \Gamma(M, E)$

2. page 454, beneath (12.2), should let E, not F, be a spin bundle over M

3. page 455, the indices on ξ in example 12.1 should all be downstairs, since $\xi \in T_p^* M$

4. page 458, beneath (12.25), should have $H^r(M; \mathbb{R})$ instead of $H_r(M; \mathbb{R})$

5. page 459, beneath (12.29), the words RHS and LHS should be interchanged

6. page 459, in (12.32) A_+ and A_- should be Δ_+ and Δ_- respectively

7. page 460, seemingly major error with the de Rham complex, see comments below

8. page 463, in (12.46) the sum $\sum$ should be a direct sum $\bigoplus$

9. page 463, in (12.46) there is a missing factor of $(-1)^{m(m+1)/2}$

10. page 463, (12.48) and (12.50) should have $|_{\mathrm{vol}}$ subscripts on the RHS

11. page 464, in (12.51), should have $(1 - g)$; in equation beneath, should have $(2 - 2g)$

12. page 464, in (12.51), should have $\mathrm{tr}\,\mathcal{F}$ instead of $\mathcal{F}$

13. page 470, missing factors of -1 in (12.83), see comments below

14. page 471, in (12.84) $\mathcal{D}_+$ should be $\mathcal{P}_+$

15. page 472, the unnumbered equation applies when the connection is flat, with $\omega_\mu = 0$

16. page 472, $\bar{\sigma}_{\mu\nu}$ should be defined as $(1/4\mathrm{i})(\bar{\alpha}^\mu \alpha^\nu - \bar{\alpha}^\nu \alpha^\mu)$

17. page 472, penultimate line of section 12.6 should read $\bar{\sigma}^{\mu\nu} = - * \bar{\sigma}^{\mu\nu}$

18. page 474, the first line should have $\tilde{h}(t)$ rather than $h(t)$

19. page 474, above (12.105) and in (12.106b) the $|\mu), |m)$ should be $|\mu\rangle, |m\rangle$

20. page 474, the exponents in (12.106) are missing a factor t

21. page 476, (12.114) is valid providing $\langle 0, i | f \rangle = 0$

22. page 478, in (12.122a) there should be a $\otimes$ between σ_1 and $\partial/\partial t$

23. page 478, the definitions of D and $D^\dagger$ in (12.122b) should be interchanged

24. page 481, in second equation, should have d_n instead of $b_n(0)$

Page 38: if the infinite product in (1.164) is evaluated correctly, it yields the wrong partition function $\sqrt{\pi}/2\sinh(\beta\omega/2)$. Equivalently, the infinite product formula deduced directly above (1.165) is in fact false — there should be no factor of π in the numerator on the right hand side. I suspect this factor of π can be traced back to the numerical factors ignored on page 36 in evaluating the Gaussian integrals, but I'm not sure.

There is also a simple typo in (1.164) — the $\beta\pi$ in the curly brackets of the infinite product should instead be $\beta\omega$.

Page 44: I think the change of variables at the bottom of the page should instead be

$$\eta_k = (\theta_k + \mathrm{i}\theta'_k)/\sqrt{2}, \qquad \eta_k^* = (\theta_k - \mathrm{i}\theta'_k)/\sqrt{2}.$$

This makes the equation on the following page correct up to terms symmetric in i, j, which vanish on contraction with a_{ij} to yield the correct final expression for the Pfaffian. This change of variables also yields a Jacobian $(-1)^n$ which cancels the factor $(-1)^{n^2}$.

Page 137*: in theorem 4.7 we should talk of *a* maximal tree rather than *the* maximal tree, since a maximal tree is far from unique.

Page 234: in the proof of (a) of corollary 6.1, we should have that the intersection of the kernels of the $\Lambda([c_i], \cdot)$ is $\{0\}$ — each individually can have non-trivial kernel.

Page 327: in (8.61) we should view $\tilde{V}^\mu(z)$ as a function of z and $\overline{z}$, and consider a perturbation just of z; in general the $\tilde{V}^\mu$ are not holomorphic functions. We could then also consider $\tilde{V}^\mu(z, \overline{z} + \Delta\overline{z})$ — if we choose our connection to have no mixed indices, then for V^μ a holomorphic vector field, this is just equal to $V^\mu(z, \overline{z})$.

Page 333: example 8.7 specifically refers to *orientable* complex manifolds. But we don't need this condition, since all complex manifolds are orientable.

Page 334: in defining a Hermitian metric from Ω, we need to show also that the metric is symmetric. This can be shown to follow from the J-invariance of any (1,1)-form. The symmetry of the metric is then necessary to prove the Hermiticity of g.

Page 357: in the definition of a vector bundle, it is stated that "transition functions belong to $\mathrm{GL}(k, \mathbb{R})$, since it maps a vector space onto another vector space". The definition of a fibre bundle only requires that this map is a diffeomorphism, however; the linearity of the map is an additional part of the definition of a vector bundle.

Page 370: in defining associated vector bundles we should talk of *a* k-dimensional representation of G rather than *the* k-dimensional representation, since there are generally many representations of a group of a given dimension.

Page 371*: in (9.72) the structure group should be $\mathrm{SL}(2, \mathbb{C})$ instead of the direct sum $\mathrm{SL}(2, \mathbb{C}) \oplus \overline{\mathrm{SL}(2, \mathbb{C})}$ — a Dirac spinor does transform under a direct sum representation (note that the fibre is $\mathbb{C}^4$), but a direct sum representation of the group $\mathrm{SL}(2, \mathbb{C})$. Further, to be completely clear, note that Weyl and Dirac spinors are sections of vector bundles associated to the $\mathrm{SPIN}(k)$ bundle (itself 'lifted' from the orthonormal frame bundle), as opposed to sections of the $\mathrm{SPIN}(k)$ bundle itself.

Page 405: in (10.108) there should be a prefactor of $-1/2$, not $-1/4$, in the first equality, as well as an integration measure $\sqrt{|g|}\mathrm{d}^4x$, and prefactor of -1, not $1/2$, in the second equality (cf. equations (1.268) and (10.85a)). This follows from

$$\mathcal{F} \wedge *\mathcal{F} = \frac{1}{2}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}(\text{vol}) \qquad \text{and} \qquad \mathrm{tr}(T_a T_b) = -\frac{1}{2}\delta_{ab}\,.$$

Equation (10.110) is modified accordingly, retaining the extra sign from the Wick rotation.

Page 413: in (10.147) we seek eigenstates of the total Hamiltonian, including both fast and slow degrees of freedom. The energies should thus not be labelled by $\mathbf{R}$, since the eigenstates are functions of $\mathbf{R}$. Further, though the energy is labelled by an integer, this need not be the same integer n as that specifying the eigenstate of the fast degrees of freedom. Thus $E_n(\mathbf{R})$ should more appropriately be E_m.

Page 413: in deriving (10.147) from the equation above it, we need to turn $\langle\mathbf{R}|\nabla_{\mathbf{R}}^2|\mathbf{R}\rangle$ into $(\langle\mathbf{R}|\nabla_{\mathbf{R}}|\mathbf{R}\rangle)^2$. This involves inserting a resolution of the identity between the two derivatives. According to (10.148), this amounts merely to inserting $|\mathbf{R}\rangle\langle\mathbf{R}|$, though this is in fact the *adiabatic* approximation and not the *Born-Oppenheimer* approximation.

In inserting this operator, however, we generate two terms, by the Leibniz rule. One is the desired $\mathcal{A}(\mathbf{R})^2$ term. The other is a term $\nabla_{\mathbf{R}}\cdot\mathcal{A}(\mathbf{R})$, which is missing from (10.147). In fact, the inclusion of this term is necessary to ensure (10.149) is correct (and it is indeed correct), since upon expanding the brackets we see that (10.149) includes the very derivative of $\mathcal{A}(\mathbf{R})$ that ought to have been present in (10.147).

Page 435: the comments at the very end of section 11.3.2 seem misleading. Characteristic classes themselves certainly *can* differentiate between two vector bundles of the same space and the same fibre dimension. This is precisely the point of characteristic classes — they classify different bundles with a given fibre over a given base space. Indeed, if the *integral* of a characteristic class over the base space can distinguish different bundles, then the classes themselves must also be able to also.

Page 439: the errors in example 7.14 have propagated here. Firstly note that $g_{\theta\theta} \neq \sin^2\theta$ as claimed. Further, the trace of $\mathcal{R}^2$ should be a trace over Lie algebra indices, that is,

$$\mathrm{tr}(\mathcal{R}^2) = \mathcal{R}^\alpha{}_\beta \mathcal{R}^\beta{}_\alpha \qquad \text{where} \qquad \mathcal{R}^\alpha{}_\beta = \frac{1}{2}R^\alpha{}_{\beta\mu\nu}\theta^\mu \wedge \theta^\nu\,,$$

as in (7.146b), and not as in the paragraph at the top of the page. In computing the trace, we should not lower the indices on $\mathcal{R}^\alpha{}_\beta$, as in this example and in (11.79b). The error in $R^\theta{}_{\phi\theta\phi}$ from example 7.14 (missing a factor $\sin^3\theta$), the error in $g_{\theta\theta}$ (an erroneous factor $\sin^2\theta$), and the error in lowering a ϕ and a θ index (an erroneous factor $\sin^2\theta$ in $\mathcal{R}_{\theta\phi}\mathcal{R}_{\phi\theta}$) conspire so that the expression for $\mathrm{tr}(\mathcal{R}^2)$ is in fact correct.

Page 445: if we include δt both in the definition of $l_t\mathcal{F}_t$ in (11.109) and in the definition of the homotopy operator k_{01} in (11.113), we will get two factors of δt in our integrals, such as in the second line of (11.114), which is nonsense.

Page 448: the Bianchi identity (11.124) should hold off-shell, and indeed it does from the definition of $*F$ directly above. Invoking the equations of motion isn't necessary.

Page 454: note that ξ_M is shorthand for $(\xi_1)^{\mu_1}(\xi_2)^{\mu_2}\cdots(\xi_m)^{\mu_m}$.

Page 455: there is one caveat in exercise 12.1. We should have that D is elliptic if and only if $\sigma(D, \xi) = \pm 1$ for at least one choice of sign. For instance, consider the Laplacian multiplied by -1. This is surely elliptic, but $\sigma(D, \xi) = 1$ has empty solution set.

Page 460: the symbol $\sigma(\mathrm{d}, \xi) = \xi \wedge$ obtained in (12.36) is *not* non-singular for $\xi \neq 0$ as claimed (indeed, the fibre dimensions k and k' are generally different), and d is *not* an elliptic operator. To apply the index theorem, we should consider the operator $\mathrm{d} + \mathrm{d}^\dagger$, which is elliptic, and the two-term complex $\Omega^\pm(M)$, as in example 12.2. As argued in section 12.1.3, this complex has the same index as the de Rham complex.

Since elliptic operators on a compact manifold are Fredholm (see section 12.1.2), we don't need to worry about restricting $\Omega^r(M)$ to $H^r(M)$, as stated at the bottom of page 460, since $\mathrm{d} + \mathrm{d}^\dagger$ has finite-dimensional kernel and cokernel when acting on $\Omega^\pm(M)$. This is just as well, since the AS index theorem as applied in (12.38) involves the characteristic classes of the r-form bundle $\wedge^r T^* M^\mathbb{C}$, not some bundle of closed forms.

Put another way, the defining feature of an elliptic complex is *not* that the differential operators involved are elliptic, as suggested in section 12.1.3. Instead, the defining feature is that the symbol complex is *exact*, which is to say that the image of $\sigma(\mathrm{D}_r, \xi)$ coincides with the kernel of $\sigma(\mathrm{D}_{r+1}, \xi)$. This can be shown to hold for the map $\xi \wedge$. Such a definition is motivated by the (non-trivial) result that a complex (E, D) is elliptic if and only if the operator $\mathrm{D} + \mathrm{D}^\dagger$, which acts on the even and odd bundles of (12.21), is elliptic. These same comments apply to the Dolbeault complex and equation (12.45).

Page 463: the first line would read better as 'take a $(0, r)$-form $\omega \in \Omega_p^{0,r}(M)$'. Firstly, ω is not a section but rather the value of a section $\tilde{s}$ at the point p. Secondly, the words 'holomorphic' and 'anti-holormophic' as applied to differential forms typically concern the derivatives of the components, as in definition 8.3. Here, however, the word is being used merely to identify ω as a $(0, r)$-form, as opposed to a general r-form. This usage is analogous to that in the context of tangent vectors; in this case, 'holomorphic' refers to the subspace of the tangent space the vector belongs to (see section 8.2.3).

Page 470: in the expression for the topological index in (12.83) there is a missing factor of $(-1)^{m(m+1)/2}$ on the RHS (cf. (12.33)). Likewise, on page 467, in the unnumbered equation for the topological index of the signature complex, I believe the factor $(-1)^l$ should instead be $(-1)^{l(2l+1)}$ (cf. (12.38)). Both of these factors are inconsequential, however, since both indices are zero unless $m = 0 \bmod 4$.

Page 496: in (12.183) it is stated that $e^{-\beta\omega/2} \sinh(\beta\omega/2)$ tends to one as ω goes to zero. This is plainly false. Indeed it appears that the determinant of $\mathrm{d}/\mathrm{d}t$ is in fact zero. The only resolution I can see is to use the independence of the index of β to take the limit $\beta \to \infty$ before taking $\omega \to 0$, which gives the desired limit for the determinant.