

## AN ASSESSMENT OF LINEAR SUPERCONDUCTING MOTORS FOR MAGLEV

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## INTRODUCTION

There is at present considerable interest in magnetic suspensions for high speed ground transport and in the complementary development of linear motors for propulsion. In particular, electrodynamic levitation, or maglev, in which d.c. excited magnets (superconducting) on the vehicle interact with a conducting (aluminium) track naturally lends itself to linear variants of the well-known synchronous or d.c. commutator machines. Although linear synchronous machines (LSM) have received most attention hitherto (Thornton (1), Slemon (2)), in the following some reasons for considering the commutator machine (LCM) are outlined and a preliminary comparison of the machine types is made.

## The Wolfson Project

The work at Warwick is centred around the project, funded by the Wolfson Foundation, to build a test vehicle (weighing 150 kg) to run on a 550m long track on the University campus. To provide speeds of  $30\text{ms}^{-1}$  or more for a useful experimental time requires accelerations of  $1g$  - an order of magnitude greater than is possible in revenue systems. The consequence for the test model machine is high armature (track) current loading, which can lead to excessive power losses and reactive power consumption.

## Linear Commutator Motors

The low efficiency and power factor of an LSM with high armature current can be overcome to a large extent by reducing the excited length. In the limit only the length actually interacting with the vehicle magnets need be excited though to achieve this, power switches must be located at comparatively short intervals along the track. A possible configuration is shown in Fig. 1; this can be regarded as a linear stepping device, with the switching of each track pole being controlled by the vehicle position, speed, and required acceleration. Alternatively, the machine may be regarded as a cycloconverter fed LSM, with the converter switching elements spaced along the trackside, but with control and power distribution equipment at much more widely spaced points.

## MACHINE DESIGN

## Specific Thrust

The specific thrust,  $F_s$ , defined as the ratio of thrust to plan area of the machine, is proportional to the average flux density at the track,  $B_{av}$ , and the armature track current per unit length,  $J_{ac}$ , and is independent of machine type. For a first order analysis the effects of harmonics in electric and magnetic waves can be neglected - e.g.  $B_{av}$  includes an allowance for winding pitch, chording, etc.  $F_s$  is, however, essentially independent of the pole pitch. Table I lists some values for the machines designed at MIT (1), The Canadian Institute for Guided Ground Transport (CIGGT) (2) for revenue service and the Warwick experimental machine.

TABLE I - Specific Thrust of Linear Machines

Machine	Full Scale		Wolfson Model		
	MIT (1)	CIGGT (2)	cruise	acceleration	
Speed	99	140	70	70	$\text{ms}^{-1}$
$B_{av}$	0.33	0.19	0.48	0.48	T
$J_{ac}$	1.25	4.85	1.36	6.44	kA/m
$F_s$	0.46	1.02	0.72	3.43	$\text{kN/m}^2$

## Machine Optimisation

One approach to optimisation is to minimise the overall costs of the transport system. To do this requires assumptions about operating procedures and parameters but the variation between the different published studies is too great for meaningful comparison. Therefore, in this comparative study, the figures used by the Canadian group (Eastham (3)) have been taken as a basis and the MIT design re-evaluated accordingly.

The costs are divided into (i) fixed costs which, in a first analysis, are the capital costs of track conductor and of power conversion equipment and (ii) running costs, i.e. energy costs. The variables are, for simplicity, restricted to the cross sectional area of the track conductor,  $A$ , the excited length of track,  $G$ , and the machine pole pitch,  $\lambda/2$  (or magnetic wavelength,  $\lambda$ ). The last of these has been optimised separately from a technical standpoint in previous studies (1, 2).

## Pole Pitch and Flux Density

Separate optimisation of pole pitch has been based on maximising the machine thrust. If the vehicle magnets are approximated by a one-dimensional sinusoidal current sheet of strength  $I_v$  amps/pole, the amplitude of the flux density at the track for a levitation height,  $h$ , is given by  $(\mu_0 I_v \pi / 2\lambda) \exp(-2\pi h / \lambda)$ . Hence the total thrust,  $F$ , in the track conductors extending over a width,  $w$ , is proportional to  $(m N I_m I_v w / \lambda) \exp(-2\pi h / \lambda)$ , where there are  $N$  active poles and  $m$  phase windings each carrying  $I_m$  amps.

Also of significance are the armature power loss,  $P_a$ , and mass of conductor,  $M_a$ , per metre of track, given by:

$$P_a = m I_m^2 \rho (\lambda + 2w) / 2A\lambda \dots\dots\dots (1)$$

$$\text{and } M_a = m(\lambda + 2w)A\sigma / \lambda \dots\dots\dots (2)$$

respectively, where  $\sigma$  is the density and  $\rho$  the resistivity of the track conductor. By using equations (1) and (2) to eliminate  $I_m$  and  $A$  the thrust can be expressed by:

$$F \propto N I_v \sqrt{\frac{2P_a M_a}{\rho \sigma}} \frac{w}{\lambda + 2w} \exp(-2\pi h / \lambda) \dots\dots\dots (3)$$

Assuming  $N$ ,  $P_a$ ,  $M_a$  fixed, and determining  $\lambda$  to maximise  $F$ , the MIT group (1) have maximised the thrust per pole. Since, however, for a given vehicle length, the number of poles will be inversely proportional to wavelength,

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it may be preferable to maximise the thrust per unit of active conductor times current, i.e.  $F/(Nm(\lambda+2w)I_m)$ . This expression can be reduced (3) to a form similar to equation (3), but with a further factor  $1/\lambda$ . The optimum wavelength is then found to be only  $\frac{1}{3}$  that of the MIT design, i.e. approximately 1m.

### Integrated System

The validity of the optimisation of wavelength may, however, be questioned in so far as the vehicle excitation has been regarded as fixed. In the scheme being studied at Warwick (4), in which the same magnets are used both for levitation and propulsion, the levitation determines the flux density at the track. Some variation is possible by altering the fraction of the vehicle area covered by the magnets, though for lowest armature loading, the area should be large and the flux density relatively modest. The lift is proportional to  $B_{av}^2 \times$  active area and is fixed, while the thrust per unit armature current, which is to be maximised, is proportional to  $B_{av} \times$  area. Some modification of the magnet geometry to vary the relative proportions of flux available for lift and propulsion may also be possible. Nevertheless, it seems preferable to assume a flux density,  $B_{av}$ , independent of  $\lambda$ . Analysis equivalent to that of the preceding section then shows that the thrust varies as  $(\lambda+w)^{-1}$ , and hence there is no optimum. The pole pitch must therefore be determined by a different procedure.

### Cost Optimisation

In an attempt to optimise  $\lambda$  the cost estimate of the system outlined above was examined. The installed cost of the track conductor was taken to be  $C_c$  (\$/kg) and of the power conversion equipment  $C_I$  (\$/kVA). The total power requirement is made up of the mechanical power transferred to the vehicle,  $P_B$ , plus the track losses  $GP_A$ . The inverters may be arranged to supply either of two excited lengths, so the cost per metre becomes  $C_I(P_B+GP_A)/2G$ . The energy cost also depends on the installed power, on the unit power cost,  $C_p$  (\$/kW-hr), average vehicle frequency  $\bar{n}$  (per hour) and speed  $\bar{v}$  (km/hr). At an annual amortisation rate of  $k\%$  the annual cost,  $C$ , per metre of track, can then be expressed by:

$$C = \frac{kC_c M_a}{100} + \frac{(P_B+GP_A)}{G} \left( \frac{C_I k}{200} + \frac{8.76 C_p \bar{n} G}{\bar{v}} \right) \dots \dots \dots (4)$$

and the cost per passenger-km is proportional to  $(C/\bar{n}) \times$  (vehicle capacity).  $M_a$ ,  $P_A$  are given by equations (1) and (2), with  $I_m$  expressed in terms of  $F$  and  $B_{av}$ , which are fixed.  $C$  can be differentiated with respect to  $A$ ,  $G$  and  $\lambda$  and the conditions for an optimum found. As the resulting simultaneous equations cannot be solved algebraically, two were solved for  $A$  and  $G$  for given values of  $\lambda$ . Loci of these optima with  $C_p$ ,  $C_I$  as parameters are shown in Fig. 2, and the corresponding total costs plotted against  $\lambda$  are given in Fig. 3. The most significant result is that cost increases monotonically with  $\lambda$ , though so slowly as to be relatively unimportant in determining  $\lambda$ .

This conclusion is especially important for the integrated system of levitation and propulsion, since other factors strongly influence the choice of pole pitch. These include the variation of lift force with speed, the magnitude and speed range of the 'drag peak', and the supply frequency of the linear machine - all of which tend to favour longer wavelengths.

### LINEAR COMMUTATOR MOTOR

The graphs illustrating the cost optimisation include curves for  $C_I$  as low as 2\$/kVA, compared with

published values of typically 25. To further justify this thyristor costs for the configuration of Fig. 1 have been estimated on the assumption that the excited length exceeds the vehicle length, so that the switch units control the full power. The savings over conventional inverter equipment arise partly in the possibility of using relatively low grade thyristors (turn off time should not be critical), partly because of the short duty cycle (though the scope for over-rating thyristors is limited in view of 'on' times of several seconds), and partly by the elimination of transformers, filters and the rectifiers of an inverter (though some of this equipment would have to be provided at widely spaced distribution points). Nevertheless, a list price of thyristors of 0.2\$/kVA suggests a value of  $C_I$  of 2 is justified for a preliminary design.

The results of this analysis are summarised in Table II, and illustrate the short excited length,  $G$ , of the LCM. For the MIT and the Canadian machines the values of  $\lambda$  are those proposed by the respective groups.

TABLE II - Machine Costs and Parameters

Machine	$\lambda$ (m)	$G$ (km)	$A$ (cm <sup>2</sup> )	$C_I$	Cost(£/pass.km)	Vehicle Capacity (60% load factor)
MIT	2.85	5	1.0	25	1.0	84
CIGGT	1.14	3	0.6	25	0.79	60
LCM	3.0	0.3	0.5	2	0.60	60

$$C_p = 0.02 \text{ \$/kW.hr; } \bar{v} = 475 \text{ km/hr; } 360 \text{ passengers/hr.}$$

### CONCLUSION

This preliminary analysis suggests that overall costs of linear machines are relatively insensitive to pole pitch, thus allowing this parameter to be chosen on other grounds. The overall costs of an LCM appear to be comparable with those of the LSM, provided that the simple machine being investigated at Warwick is technically acceptable.

### REFERENCES

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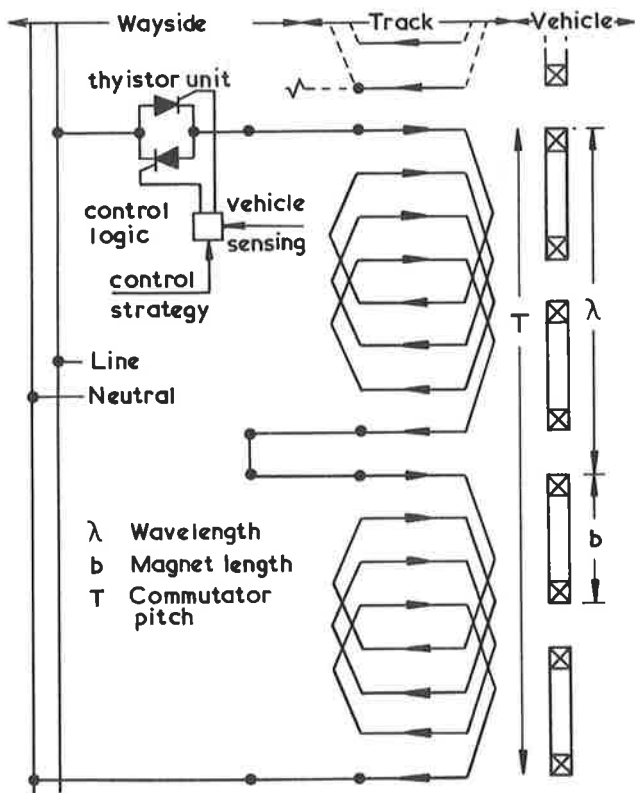


Fig. 1 - Scheme of Possible LCM.  
(only one of several similar windings shown)

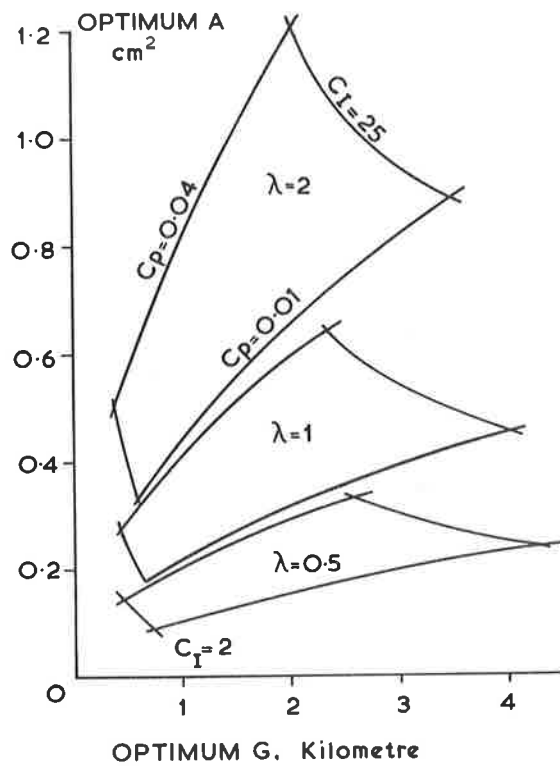


Fig. 2 - Loci of Optimum G, A.

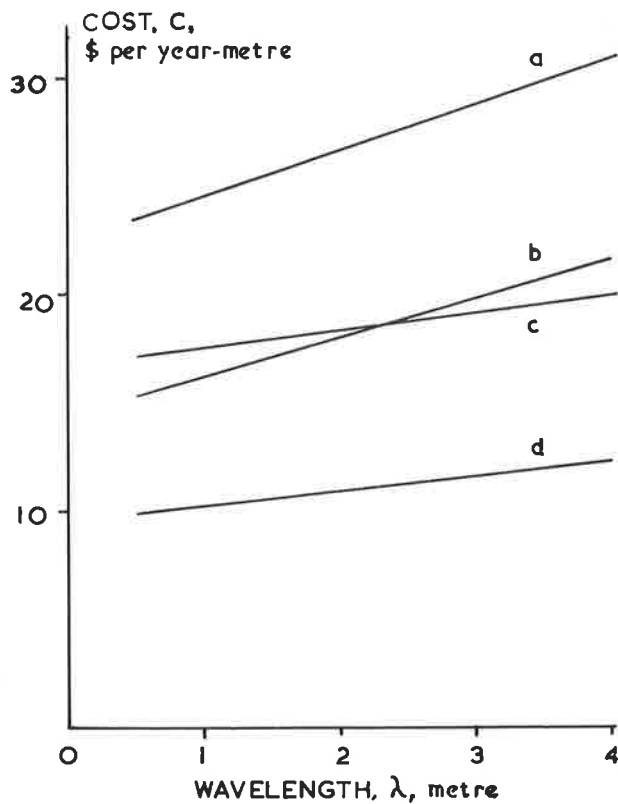


Fig. 3 - Costs as function of wavelength

TABLE III—Unit Costs for Fig. 3.

Curve	$C_I$ , \$/kVA	$C_P$ , \$/kW-hr.
a	25	0.02
b	25	0.01
c	2	0.02
d	2	0.01

## NORMAL FORCE VARIATION IN SINGLE-SIDED LINEAR INDUCTION MACHINES

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INTRODUCTION

It is really only within the last decade that attention has been given to the study of the normal, or 'levitation', force in single-sided linear induction machines. The initial studies were for simplified models of infinite length and width. These will be referred to as one-dimensional, or 1-D models (Laithwaite (1), Freeman and Lowther (2)). Subsequently transverse edge effects were taken into account. Two methods are commonly used. Either a resistivity multiplier is employed or the current waveform in the transverse direction is modulated. This is commonly referred to as the two-dimensional, or 2-D model, see ref. (2) and the work of Oberretl (3).

Longitudinal, or entry and exit effects are commonly allowed for by modulating the current waveform in the longitudinal direction, with some allowance for transverse edge effects. This is often referred to as a 3-D model (3). A word of caution about the above mentioned 2-D and 3-D models. Strictly these models are only 1-D models with current modulation in one or two directions. There is no variation in material properties except in the direction normal to the plane containing the air-gap.

The object of this paper is to draw attention to certain basic factors, which can be established from relatively simple formulae, rather than try to account for the detailed performance of a particular design. So often, one only needs to have an approximate idea of how a device will behave. A detailed study can always be made at a later stage. For example, when contemplating the possibility of using the normal force to levitate a vehicle, or simply a conducting mass, the complex power input per newton of normal force can be a key factor. Induction levitation, by its very nature, is energy consuming, although some energy might be used for propulsion purposes. In this paper we present results which can give a designer an initial guide to the feasibility of using single-sided linear induction motors for levitation.

In the following is discussed a basic 1-D model having a single harmonic current sheet supported on infinitely permeable iron, an air-gap, a conducting layer and a backing region of air or iron. The model is shown in Fig. 1. By making certain simplifying assumptions, two circle diagrams can be derived, one for the model with backing iron, the other without. The circle diagrams have been found to be extremely useful when discussing the behaviour of SLIMs. To our knowledge, only Poloujadoff (4) has discussed the use of a circle diagram in this context. We show how the idea might be extended to include normal force and machines having air-backed rotors. Simple formulae are then derived for certain factors such as complex power/newton of normal force.

THEORY 1-D MODEL

The theory is given in earlier work (2), only final results will be quoted here. For the 1-D model the normal force is given by:

$$F_L = F_Z = 0.25 \mu_0 |K|^2 \left(1 - \frac{|Z_2|^2}{|Z_3|^2}\right) \text{ newtons per square metre} \quad \dots (1)$$

where  $Z_2$  is the surface impedance 'looking up' from the current sheet. It is defined as the electric-field-strength/magnetic-field-strength ratio at a particular point.

$$Z_2 = Z_2' (Z_3 + Z_2' \tanh(\gamma_2 g_2)) / (Z_2' + Z_3 \tanh(\gamma_2 g_2))$$

$$Z_3 = Z_3' (Z_4 + Z_3' \tanh(\gamma_3 g_3)) / (Z_3' + Z_4 \tanh(\gamma_3 g_3))$$

$$Z_n' = j\omega \mu_0 \mu_n / \gamma_n$$

$$\gamma_n = (k^2 + j s_n / d_n^2)^{1/2} = k(1 + j s_n G_n)^{1/2}$$

$$d_n = \text{depth of penetration} \times (2)^{-1/2} = (\rho_n / \mu_0 \mu_n \omega)^{1/2}$$

$$k = 2\pi/\lambda; \quad s_n = \text{slip of region } n$$

$$K = \text{line current density} = 4\pi N_{\text{eff}} I / (\lambda p) \text{ amperes per metre}$$

$$M = \text{number of phases}$$

$$N_{\text{eff}} = \text{effective turns in series per phase}$$

$$I = \text{phase current, time maximum value}$$

$$\lambda = \text{wavelength}; \quad p = \text{number of poles}$$

Equation (1) expresses the normal force in circuitual terms. It can be seen that when  $|Z_2| > |Z_3|$  the normal force is one of attraction. This would occur if regions 3 and 4 consisted of non-conducting magnetic material. Alternatively if 3 and 4 were both non-magnetic conducting regions, then  $|Z_2| < |Z_3|$ , the characteristic impedance of air, and repulsion would occur.

From a knowledge of the input wave impedance, the complex power input from the current sheet can also be calculated:

$$S = P + jQ = 0.5 K^2 Z_2 \text{ VA/N} \dots \dots \dots (2)$$

N.B. This does not include stator winding loss, slot leakage or end leakage reactance. P and Q are the real and complex components of power/m<sup>2</sup>.

At this point it is convenient to consider two separate models (i) iron-backed and (ii) air-backed. Possibly the most commonly encountered example of the first occurs in traction applications, the second is of interest in materials handling.

Iron-Backed 1-D Model

In reference (2) it was shown that a simplified form of presentation was possible by 'blending' regions 2 and 3 together. The new region 2-3 is then assumed to have a resistivity  $\rho_{23} = (g_2 + g_3)\rho_3/g_3$ . The input wave impedance is then given by:

$$Z_2 = Z_{23}' \coth(\gamma g) \dots \dots \dots (3)$$

where  $Z_{23}' = j\omega \mu_0 / \gamma$ . This simplified form makes it possible to present the normal force as a function of two variables only, see Fig 3 in reference (2). Hence, limiting values of the normal force could be obtained as well as the slip at which the normal force changed sign from positive, lifting, to negative, attracting.

$$\text{The criterion for this was: } kg s_x G = 1.0 \dots \dots \dots (4)$$

where  $g = (g_2 + g_3)$  and  $s_x = \text{slip at cross-over}$ .