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Linear Synchronous Machine Design for Minimum Terminal Rating

Summary

When operating linear synchronous machines, three main design options emerge. For a particular mechanical loading the machine can be run at either maximum efficiency, unity or maximum power factor, or minimum terminal rating. Each of the options are analysed, using the machine performance equations, and their effects on machine terminal requirements are compared, using the CIGGT motor as an example.

1. Introduction

Linear Synchronous Machine (LSM) analysis shows that machine efficiency is maximised when the track and vehicle current sheet magneto motive forces are displaced by $\frac{\pi}{2}$ radians. At this operational position the machine behaves (when viewed from the terminals) as an inductive load. Because of the phase reactance, the machine draws lagging reactive current from the distribution system, and the terminal rating is that deduced from the real power obtained from the machine and its overall efficiency and power factor.

Conventional control of synchronous machines usually involves local open loop operation with the current angle (or torque angle) less that $\frac{\pi}{2}$ radians. Increased loading on the machine which forces the current angle to exceed this value will result in slippage and loss of synchronism. The method of control is to vary the field current on the rotor by means of an automatic voltage regulator (AVR) and if this is sufficiently controlled in closed loop the machine may be made to operate beyond its previous torque limit. If current forcing of the machine armature is used with a rotor position feedback loop, then four quadrant operation of the machine is possible. The latter method of control for LSM's is more promising, since the possibility of operating the on-board superconducting magnets in other than the persistant current mode is remote at present, because of the excessive a.c. loss, and would incur payload and auxiliary power penalties.

Provided the machine phase reactance is small, the machine may be operated to correct or increase its own powerfactor, by balancing the induced voltage component against the reactive drop, so that from the terminals the load appears to be largely resistive, depending on the magnitude of back emf available. Usually this will give a lower terminal power requirement over the maximised efficiency operation, and will also reduce the terminal phase voltage. Accompanying these improvements is a necessary increase in phase current requirement.

A third design option is to operate the machine so that the total complex power requirement (volt-ampères) is minimised, which usually will occur with a current angle between the first two options. Since the distribution network and associated switchgear, transformers and convertors will always be rated in terms of complex power, this is perhaps the most reasonable of the three approaches.

Each of the three alternative operating conditions will be analysed and then the CIGGT machine will be examined and the effect on terminal rating demonstrated.

2. Nomenclature

E_B machine induced back emf, volts

E_T machine terminal phase voltage, volts

F_B machine thrust, newtons

I	r.m.s. phase current, ampères
m	number of machine phases
P_B	real power output (developed mechanical power), watts
P_T	terminal active "input" power, watts
Q_T	terminal reactive "input" power volt-ampères reactive
R	phase resistance, ohms
S_T	machine terminal V-A rating (input complex power) volt-ampères
v	machine speed, meters/second
V_1	active terminal phase voltage, volts
V_2	reactive terminal phase voltage, volts
X	phase reactance, ohms
α	current angle, field mmf lag on stator mmf, radians
η	efficiency
θ	power factor angle, current lag on terminal voltage, radians.

3. Analysis

3.1 Condition for maximum efficiency

By definition the machine efficiency η is the ratio of developed mechanical power to input power, so

$$\eta = \frac{P_B}{P_T} \tag{1}$$

Also, the machine input power is made up of the developed power and the track losses,

$$P_T = P_B + mI^2R \tag{2}$$

so

$$\eta = \frac{P_B}{P_B + mI^2R}$$

$$\eta = \frac{1}{1 + \frac{mI^2R}{P_B}} \tag{3}$$

The developed power P_B is the product of machine thrust and speed, and is linked to the back emf by

$$P_B = vF_B = mE_B I \sin\alpha \tag{4}$$

Using (4) to eliminate I in (3)

$$\eta = \frac{1}{1 + \dfrac{P_B R}{mE_B^2 \sin^2\alpha}} \tag{5}$$

The maximum of (5) is at a value of α which satisfies $\cot\alpha \ (1 + \cot^2\alpha) = 0$, which has only one non-complex solution at $\alpha_c = \dfrac{\pi}{2}$.

The value of maximum efficiency is given by

$$\eta\Big|_{\alpha=\frac{\pi}{2}} = \frac{1}{1 + \dfrac{P_B R}{mE_B^2}} \tag{6}$$

3.2 Power factor effects

Exact compensation of the LSM power factor will occur if the resolved back emf exactly matches the leakage inductance voltage drop in the armature phase winding, and unity power factor at the terminals will ensue. If the phase reactance is too large then the back emf phasor can only be manipulated to maximise the machine power factor.

3.2.1. Condition for unity power factor

The machine terminal voltage can be expressed in terms of the voltages V_1 and V_2 that are in phase and quadrature respectively with the phase current.

$$V_1 = E_B \sin \alpha + IR \tag{7}$$

$$V_2 = E_B \cos \alpha + IX \tag{8}$$

$$E_T^2 = V_1^2 + V_2^2 \tag{9}$$

$$\theta = \tan^{-1} \frac{V_2}{V_1} \tag{10}$$

Exact compensation and unity power factor occur if $\theta = 0$, or $V_2 = 0$. From (8)

$$\cos \alpha = -\frac{IX}{E_B} \tag{11}$$

From (4)

$$\sin \alpha = \frac{P_B}{mE_B I} \tag{12}$$

Combining (11) and (12)

$$\sin 2\alpha = \frac{-2P_B X}{m E_B^2} \tag{13}$$

α must be in the second quadrant since $\sin \alpha$ is positive and $\cos \alpha$ is negative. Evaluating the right hand side of (13) will provide a solution for the principal value only, i.e. $-\frac{\pi}{2} \leqslant 2\alpha \leqslant \frac{\pi}{2}$, so using $\sin 2\alpha = \sin(\pi - 2\alpha)$, the particular solution for α will be produced.

$$\sin(\pi - 2\alpha) = \frac{-2P_B X}{mE_B^2} \tag{14}$$

or

$$\alpha = \frac{\pi}{2} + \frac{1}{2} \operatorname{Sin}^{-1} \frac{2P_B X}{mE_B^2} \tag{15}$$

3.2.2. Condition for maximum power factor

Maximum power factor imples a minimum $\tan \theta$. From (10),

$$\frac{\partial}{\partial \theta} (\tan \theta) = \frac{V_2' V_1 - V_2 V_1'}{V_1^2} = 0 \text{ for a minimum}$$

or maximum.

i.e.

$$V_2' V_1 - V_2 V_1' = 0 \tag{16}$$

Using (4), (7) and (8) in (16), a quadratic in cot α is produced,

$$\cot^2\alpha - \frac{2X}{R}\cot\alpha - \left(\frac{mE_B^{\,2}}{P_B R} + 1\right) = 0 \qquad (17)$$

The appropriate solution to (17) is

$$\cot\alpha = \frac{X}{R}\left[1 - \left(1 + \frac{R^2}{X^2}\left(\frac{mE_B^{\,2}}{P_B R} + 1\right)\right)^{\frac{1}{2}}\right] \qquad (18)$$

Calculator solutions will give $-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$, and using the expression $\cot\alpha = -\cot(\pi - \alpha)$

$$\alpha = \pi - \cot^{-1}\left\{\frac{X}{R}\left[\left(1 + \frac{R^2}{X^2}\left(\frac{mE_B^{\,2}}{P_B R} + 1\right)\right)^{\frac{1}{2}} - 1\right]\right\} \qquad (19)$$

will generate $\frac{\pi}{2} < \alpha < \pi$. If the negative sign in (18) was changed to give the positive root solution for (17), the minimum power factor value of α would be obtained.

3.3 Condition for minimum terminal rating

Using (7), (8), (9)

$$E_T = \left(E_B^{\,2} + I^2(R^2+X^2) + 2E_B I(R\sin\alpha + X\cos\alpha)\right)^{\frac{1}{2}} \qquad (20)$$

The terminal volt-ampere rating is given by

$$S_T = m\,E_T\,I$$

$$S_T = m\,I\,E_B\left(1 + \frac{I^2}{E_B^2}\left(R^2 + X^2\right) + \frac{2I}{E_B}(R\sin\alpha + X\cos\alpha)\right)^{\frac{1}{2}} \qquad (21)$$

using (4) to substitute for mI, I^2/E_B^2, I/E_B,

$$S_T = \frac{P_B}{\sin\alpha}\left[1 + \frac{P_B^{\,2}(R^2+X^2)}{m^2E_B^{\,4}\sin^2\alpha} + \frac{2P_B}{E_B^{\,2}m}(R + X\cot\alpha)\right]^{\frac{1}{2}} \qquad (22)$$

so $$\frac{\partial S_T}{\partial \alpha} = -\frac{\cos\alpha}{\sin^2\alpha} P_B \left[\quad\right]^{\frac{1}{2}}$$

$$+ \frac{P_B}{\sin\alpha}\cdot\frac{1}{2}\left[\quad\right]^{-\frac{1}{2}} \frac{\partial}{\partial\alpha}\left[\quad\right] \tag{23}$$

$= 0$ for minimum

i.e. $$2\frac{\cos\alpha}{\sin\alpha}\left[\quad\right] = \frac{\partial}{\partial\alpha}\left[\quad\right] \tag{24}$$

or

$$2\frac{\cos\alpha}{\sin\alpha}\left[1 + \frac{P_B^2(R^2+X^2)}{m^2 E_B^4 \sin^2\alpha} + \frac{2P_B}{E_B^2 m}(R + X\cot\alpha)\right]$$

$$= \frac{-2P_B^2\cos\alpha\,(R^2+X^2)}{m^2 E_B^4 \sin^3\alpha} - \frac{2P_B X}{E_B^2 m \sin^2\alpha} \tag{25}$$

rearranging, and using the relationships $2\frac{\cos\alpha}{\sin\alpha} + \frac{1}{\sin\alpha\cos\alpha}$

$= 3\cot\alpha + \tan\alpha$

(25) becomes

$$\tan^3\alpha + \left[\frac{2P_B(R^2+X^2)}{m E_B^2 X} + \frac{2R}{X} + \frac{E_B^2 m}{X P_B}\right]\tan^2\alpha + 3\tan\alpha$$

$$+ 3\tan\alpha + \frac{2P_B(R^2+X^2)}{m E_B^2 X} = 0 \tag{26}$$

The real solution to this cubic gives the value of α at which S_T, the terminal input complex power is a minimum.

If (26) is rewritten as

$$t^3 + pt^2 + qt + r = 0 \tag{27}$$

where

$$t = \tan\alpha \tag{28}$$

$$p = \frac{2P_B(R^2+X^2)}{m E_B^2 X} + \frac{2R}{X} + \frac{E_B^2 m}{X P_B} \tag{29}$$

$$q = 3 \tag{30}$$

$$r = \frac{2P_B(R^2+X^2)}{m E_B^2 X} \tag{31}$$

with a further substitution

$$t = y - \frac{p}{3} \qquad (32)$$

(27) can be transformed to its reduced form

$$y^3 + ay + b = o \qquad (33)$$

where

$$a = q - \frac{p^2}{3} \quad \left(= 3\left(\frac{q}{3} - \left(\frac{p}{3}\right)^2\right)\right) \qquad (34)$$

$$b = \frac{2p^3}{27} - \frac{pq}{3} + r$$

$$= 2\left(\frac{p}{3}\right)^3 - 3\left(\frac{p}{3}\right)\left(\frac{q}{3}\right) + r \qquad (35)$$

The discriminant D is given by

$$D = \left(\frac{b}{2}\right)^2 + \left(\frac{a}{3}\right)^3 \qquad (36)$$

In the ranges concerning LSM's, D is positive, which gives one real and two complex conjugate roots for the reduced equation. The real root y, is given by

$$y_1 = \left[-\frac{b}{2} + D^{\frac{1}{2}}\right]^{\frac{1}{3}} + \left[-\frac{b}{2} - D^{\frac{1}{2}}\right]^{\frac{1}{3}}$$

or

$$y_1 = \left[D^{\frac{1}{2}} - \frac{b}{2}\right]^{\frac{1}{3}} - \left[D^{\frac{1}{2}} + \frac{b}{2}\right]^{\frac{1}{3}} \qquad (37)$$

Equations (32) and (28) can then be used to establish the required value of α for minimum terminal rating.

4. Effect of choice of current angle on a particular design

The numerical values used to evaluate the different operating conditions are based on the CIGGT design. This LSM develops 40 kN thrust at its cruise speed of 480 km/h, which corresponds to a developed mechanical output power of 5.34 MW. The phase inductance and resistance are 2.42 mH/km and 0.56 Ω/km respectively, which for a 5km block length give values of terminal reactance and resistance of 8.89 Ω and 2.8 Ω . The back emf is 4 kV. The armature is meander wound into three phases.

4.1 Maximum efficiency

Using (6) the maximum efficiency, at $\alpha = \frac{\pi}{2}$ is

$$\eta = \frac{1}{1 + \frac{P_B R}{m E_B^{\,2}}} = 0.7625$$

4.2 Power factor

Since in (15) $\frac{2 P_B X}{m E_B^2} = 1.982 > 1$, there is no balance point for unity power factor operation, and equation (19) must be used to find maximum power factor value of α .

Substituting into (19)

$$\alpha^c = \pi^c - \cot^{-1}\left\{3.175\left[(1.41766)^{\frac{1}{2}} - 1\right]\right\}^c$$

which solves to give $\alpha = 121^\circ$ as the position of maximum power factor. Using (7) - (10) $\cos\theta = 0.8860$ at its maximum.

4.3 Minimum terminal rating

A H.P. 9100A calculator program has been written to evaluate α for minimum terminal rating, for inputs P_B, E_B, m, R and X. The print out is of the form (for the CIGGT machine)

$$\left.\begin{array}{lcl} P_B & = & 5.34 \quad 06 \\ E_B & = & 4\,000. \\ m & = & 3. \\ R & = & 2.8 \\ X & = & 8.89 \end{array}\right\} \text{Input data}$$

$r/2$	=	1.087123	
$p/3$	=	1.271759	
$a/3$	=	-.617370	Output data
$b/2$	=	1.236390	
$D^{\frac{1}{2}}$	=	1.137256	
α	=	108.050268	

So the current angle for minimum terminal rating is 108°.

5.0 Per-unit values

By definition,

$$\text{a per-unit (pu) value} = \frac{\text{actual value (in any unit)}}{\text{base value (in the same unit)}}$$

It is common synchronous machine practice to refer parameters to the expected operating condition base. If the CIGGT machine performance is plotted out (Figure 1), the values of α for the various conditions calculated in the previous sections are confirmed, and the variation of terminal phase power, current and voltage can be normalized with respect to the base α values. The normalization of values is not only for graphical simplicity; similar machine types or design variations are much more easily compared, especially with respect to sensitivity to parameter variations.

6.0 Comparison of design options

The minimum complex power base value is 8.27 MVA, with termihal voltage of 5.89 kV and phase current of 468 amps. The three design options and relevant performance are shown in Table I.

Condition	α^{o}	S_T	E_T	I	η	cos θ	$\eta\cos\theta$
Maximum efficiency	90	1.06	1.12	0.951	0.763	0.798	0.609
Min. complex power	108	1.00	1.00	1.00	0.744	0.868	0.646
Max. power factor	121	1.04	0.936	1.11	0.702	0.886	0.622

Table I. Relative performance of design options

It is not immediately apparent, for this particular design, whether the choice to operate at minimum terminal rating offers any real advantage over $\alpha = \pi/2^c$ - there is only a 6% reduction in VA rating. This is because of the broadness of the variation of S_T about its minimum point, in this case. Table II shows the change in active and complex powers for per unit base of a) $\alpha = 90^o$ and b) $\alpha = 108^o$.

α	S_T, complex		P_T, active		Q_T, reactive	
	a	b	a	b	a	b
90	1	1.06	1	0.975	1	1.29
108	0.943	1	1.03	1	0.777	1
121	0.978	1.04	1.09	1.06	0.754	0.971

Table II Per unit terminal powers for bases of $\alpha = 90^o$ and 108^o

In going from $\alpha = 90^o$ to $\alpha = 108^o$, a reduction of 22% in terminal reactive power can be achieved, with only a 3% increase in active power; the total power also drops by 6%. From this base the effect of going to the maximum power factor $\alpha = 121^o$ point only reduces reactive power by 3% and requires a 6% and 4% increase in active and total power respectively. The associated increase in the armature track loss would amount to 23% because the machine efficiency is beginning to drop off more rapidly at $\alpha > 110^o$, (Figure 1).

Operation at minimum S_T will generally mean a terminal voltage that has reduced by more than the phase current has increased. The implication is that the phase current affects the track heating and conductor temperature rise and so determines the type of insulation and its ageing, and that terminal voltage affects the insulation and inverter semiconductor required blocking voltage. The CIGGT machine terminal voltage reduces by 10% and phase current increases by only 5%, in going from $\alpha = 90^o$ to $\alpha = 108^o$.

Generally, it is more important in designing a linear superconducting machine to be fully aware of the variations that will be present in the system performance, rather than to strictly specify precise operating regimes, since these will be dictated by the installed system economics. It is sufficient to note that the LSM is not as restricted in terms of available tractive power as for example a linear induction machine. Because the armature winding is in the guideway, specific loading of the machine can be changed by simply changing armature conductor dimensions to suit particular guideway grades, acceleration requirements etc., without penalizing vehicle pay load or cruise capability.

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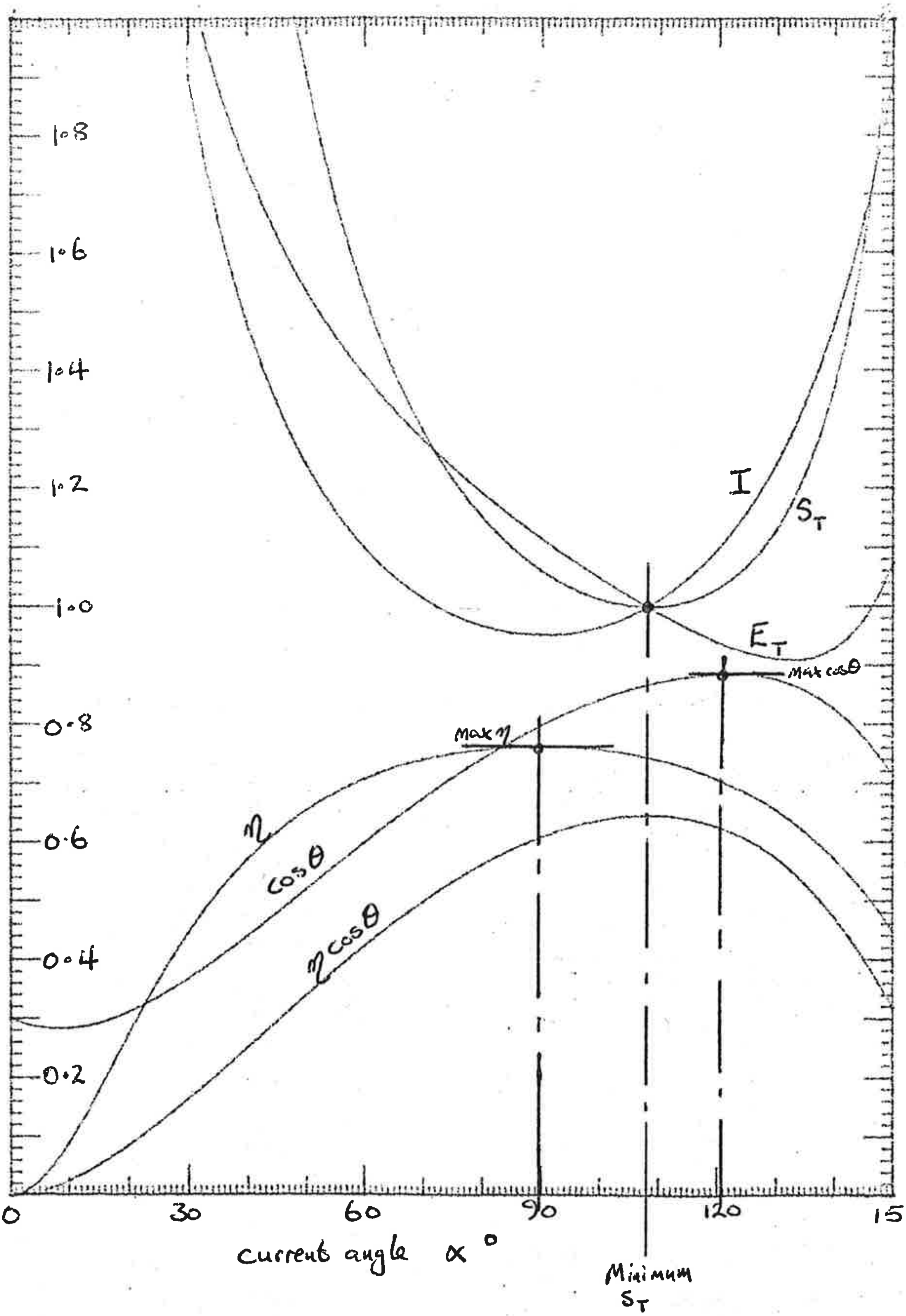

Fig.1 Performance characteristics of LSM at cruise.