Consider $\nabla \times \psi = \frac{\partial \psi}{\partial t}$ with initial data $\psi(\vec{x}, 0) = g(\vec{x})$.
We then have that $[k]_\times \hat{\psi} = \frac{\partial \hat{\psi}}{\partial t}$ so that $\hat{\psi} = e^{[k]_x t} \hat{g}(k)$.
Then, $\psi(\vec{x}, t) = \frac{1}{(2\pi)^3} \int_{\mathbb{R}^3} e^{[k]_x t} \hat{g}(k) e^{i(k \cdot \vec{x})} dk$.
With Rodrigues' formula, we have that $e^{[k]_\times t} = I + [k]_\times \sin(t) + [k]_\times^2 (1 - \cos(t))$.

$$\begin{aligned}
\implies \psi(\vec{x}, t) = \frac{1}{(2\pi)^3} \int_{\mathbb{R}^3} \hat{g}(k) e^{i(k \cdot \vec{x})} dk + \frac{\sin(t)}{(2\pi)^3} \int_{\mathbb{R}^3} [k]_\times \hat{g}(k) e^{i(k \cdot \vec{x})} dk + \frac{1 - \cos(t)}{(2\pi)^3} \int_{\mathbb{R}^3} [k]_\times^2 \hat{g}(k) e^{i(k \cdot \vec{x})} dk \\
= \mathcal{F}^{-1}(\hat{g}) + \sin(t) \mathcal{F}^{-1}(k \times \hat{g}) + (1 - \cos(t)) \mathcal{F}^{-1}(k \times (k \times \hat{g})) \\
= \hat{g} + \sin(t)(\nabla \times g) + (1 - \cos(t))(\nabla \times \mathcal{F}^{-1}(k \times \hat{g})) \\
= g(\vec{x}) + \sin(t)(\nabla \times g) + (1 - \cos(t))(\nabla \times \nabla \times g)
\end{aligned}$$