1. Consider $\nabla \times \psi = i\frac{\partial \psi}{\partial t}$ with initial data $\psi(\vec{x}, 0) = g(\vec{x})$.
We then have that $i[k]_\times \hat{\psi} = i\frac{\partial \hat{\psi}}{\partial t}$ so that $\hat{\psi} = e^{[k]_x t}\hat{g}(k)$.
Then, $\psi(\vec{x}, t) = \frac{1}{(2\pi)^3}\int_{\mathbb{R}^3} e^{[k]_x t}\hat{g}(k)e^{i(k\cdot\vec{x})}dk$.
With Rodrigues' formula, we have that $e^{[k]_\times t} = I + [k]_\times \sin(t) + [k]_\times^2(1 - \cos(t))$.

$$\begin{aligned}
\implies \psi(\vec{x}, t) &= \frac{1}{(2\pi)^3}\int_{\mathbb{R}^3} \hat{g}(k)e^{i(k\cdot\vec{x})}dk + \frac{\sin(t)}{(2\pi)^3}\int_{\mathbb{R}^3} [k]_\times \hat{g}(k)e^{i(k\cdot\vec{x})}dk + \frac{1-\cos(t)}{(2\pi)^3}\int_{\mathbb{R}^3} [k]_\times^2 \hat{g}(k)e^{i(k\cdot\vec{x})}dk \\
&= \mathcal{F}^{-1}(\hat{g}) + \sin(t)\mathcal{F}^{-1}(k \times \hat{g}) + (1 - \cos(t))\mathcal{F}^{-1}(k \times (k \times \hat{g})) \\
&= g - i\sin(t)(\nabla \times g) - i(1 - \cos(t))(\nabla \times \mathcal{F}^{-1}(k \times \hat{g})) \\
&= g - i\sin(t)(\nabla \times g) + (1 - \cos(t))(\nabla \times (\nabla \times g))
\end{aligned}$$

2. Consider for a function $\psi(\vec{x}, t)$ the equation $\nabla^2\psi + x\psi = \frac{\partial \psi}{\partial t}$.

$$\begin{aligned}
\implies -|k|^2\hat{\psi} + i\frac{\partial \hat{\psi}}{\partial k} &= \frac{\partial \hat{\psi}}{\partial t} \\
\implies \nabla_k^2\hat{\hat{\psi}} - k\hat{\hat{\psi}} &= \frac{\partial \hat{\hat{\psi}}}{\partial t}
\end{aligned}$$

So then, with $f = \psi + \hat{\hat{\psi}}$, we not only have that $\hat{\hat{f}} = f$ but also that f satisfies the heat equation since $\nabla^2 f = \frac{\partial f}{\partial t}$.