Here, we let $\mathbb{N}^+ = \{1, 2, \cdots\}$ and $\mathbb{R}^+ = [0, \infty)$.
Consider the following equation.

$$\nabla \times \psi = i\frac{\partial \psi}{\partial t} \tag{1}$$

with boundary value $\psi(\vec{x}, 0) = g(\vec{x})$ for $\psi : [-\pi, \pi]^3 \times \mathbb{R}^+ \to \mathbb{R}^3$ and $g : [-\pi, \pi]^3 \to \mathbb{R}^3$.

Let $\langle \cdot, \cdot \rangle_1$ be the inner product on $L_1([-\pi, \pi]^3, \mu_C)$ with μ_C the Lebesgue measure on $\mathbb{C}$. Then, define $\langle f, g \rangle = \vec{v}$ for a function $f \in (L_1([-\pi, \pi]^3, \mu_C))^3$ and $g \in L_1([-\pi, \pi]^3, \mu_C)$, where $\vec{v}_k = \langle f_k, g \rangle_2$.

Assume ψ is a solution, so that $\psi \in (L_1([-\pi, \pi]^3 \times \mathbb{R}^+, \mu_C^3 \times \mu_L))^3$ where μ_L is the Lebesgue measure on $\mathbb{R}$.

As each component of ψ is in $L_1([-\pi, \pi]^3 \times \mathbb{R}^+, \mu_C^3 \times \mu_L)$, we can write ψ as below.

$$\psi(\vec{x}, t) = \sum_{\alpha \in \mathbb{Z}^3} \sum_{n=-\infty}^{\infty} e^{i(tn + (\alpha \cdot \vec{x}))} C_{n\alpha}$$

Now, we have the following.

$$\begin{aligned}
i\frac{\partial \psi}{\partial t} &= \nabla \times \psi \\
\implies \sum_{\alpha \in Z^3} \sum_{n=-\infty}^{\infty} ine^{i(tn+(\alpha\cdot\vec{x}))} C_{n\alpha} &= \sum_{\alpha \in Z^3} \sum_{n=-\infty}^{\infty} \nabla \times (ie^{i(tn+(\alpha\cdot\vec{x}))} C_{n\alpha}) \\
&= \sum_{\alpha \in Z^3} \sum_{n=-\infty}^{\infty} e^{i(tn+(\alpha\cdot\vec{x}))} [\alpha]_\times C_{n\alpha}) \\
\implies inC_{n\alpha} &= [\alpha]_\times C_{n\alpha}
\end{aligned}$$

Then, $C_{n\alpha}$ must be an eigenvector of $[\alpha]_\times$ with eigenvalue in.
Let $\phi_-(\alpha) = (\alpha_1\alpha_2 + i\alpha_3|\alpha|, \alpha_1^2 + \alpha_3^2, \alpha_2\alpha_3 - i\alpha_1|\alpha|)$ and $\phi_+(\alpha) = (\alpha_1\alpha_2 - i\alpha_3|\alpha|, \alpha_1^2 + \alpha_3^2, \alpha_2\alpha_3 + i\alpha_1|\alpha|)$.
Then, $[\alpha]_\times$ has the 3 eigenvectors α and $\phi_\pm(\alpha)$ with corresponding eigenvalues $0, \pm i|\alpha|$.
Note that if $|\alpha| \notin \mathbb{N}^+$, then the only nonzero $C_{n\alpha}$ must be $C_{0\alpha}$.
With this, we can rewrite ψ's Fourier Series as follows, with E_α the Eigenvector matrix of $[\alpha]_\times$ and $\lambda_\alpha, \Lambda_\alpha \in \mathbb{C}^3$ constants.

$$\psi(\vec{x}, t) = \sum_{\alpha \in \mathbb{Z}^3} \sum_{n=-\infty}^{\infty} e^{i(tn + (\alpha \cdot \vec{x}))} C_{n\alpha}$$

$$= \sum_{\alpha \in \mathbb{Z}^3 \wedge |\alpha| \in \mathbb{N}^+} e^{i(\alpha\cdot\vec{x})} \Big((\lambda_\alpha)_1 \alpha + (\lambda_\alpha)_2 e^{-it|\alpha|} \phi_-(\alpha) + (\lambda_\alpha)_3 e^{it|\alpha|} \phi_+(\alpha) \Big) + \sum_{\alpha \in \mathbb{Z}^3 \wedge |\alpha| \notin \mathbb{N}^+} e^{i(\alpha\cdot\vec{x})} \alpha \Lambda_\alpha \tag{2}$$

$$\implies \psi(\vec{x}, 0) = \sum_{\alpha \in \mathbb{Z}^3 \wedge |\alpha| \in \mathbb{N}^+} e^{i(\alpha\cdot\vec{x})} E_\alpha \lambda_\alpha + \sum_{\alpha \in \mathbb{Z}^3 \wedge |\alpha| \notin \mathbb{N}^+} e^{i(\alpha\cdot\vec{x})} \alpha \Lambda_\alpha \tag{3}$$

Theorem 0.1 *A solution ψ exists with boundary value g if and only if for all $\alpha \in \mathbb{Z}^3$ with $|\alpha| \notin \mathbb{N}^+$, we have that $\langle g, e^{i(\alpha\cdot\vec{x})} \rangle = \Lambda_\alpha \alpha$ for some constant Λ_α.*

Proof of Sufficiency. We will construct such a ψ.
Define ψ as in (2). From (3), we see that for the boundary value condition $\psi(\vec{x}, 0) = g(\vec{x})$ to be satisfied we need $E_\alpha \lambda_\alpha = \langle g, e^{i(\alpha\cdot\vec{x})} \rangle$ for α with $|\alpha| \in \mathbb{N}^+$, so define $\lambda_\alpha = E_\alpha^{-1} \langle g, e^{i(\alpha\cdot\vec{x})} \rangle$.
By hypothesis, we already have that $\langle g, e^{i(\alpha\cdot\vec{x})} \rangle = \Lambda_\alpha \alpha$ if $|\alpha| \notin \mathbb{N}^+$. We must verify that ψ is a solution.

$$\nabla \times \psi = \sum_{\alpha \in \mathbb{Z}^3 \wedge |\alpha| \in \mathbb{N}^+} \nabla \times \left(e^{i(\alpha \cdot \vec{x})} \left((\lambda_\alpha)_1 \alpha + (\lambda_\alpha)_2 e^{-it|\alpha|} \phi_-(\alpha) + (\lambda_\alpha)_3 e^{it|\alpha|} \phi_+(\alpha) \right) \right) + \sum_{\alpha \in \mathbb{Z}^3 \wedge |\alpha| \notin \mathbb{N}^+} \nabla \times (e^{i(\alpha \cdot \vec{x})} \alpha \Lambda_\alpha)$$

$$= \sum_{\alpha \in \mathbb{Z}^3 \wedge |\alpha| \in \mathbb{N}^+} \left(\nabla e^{i(\alpha \cdot \vec{x})} \right) \times \left((\lambda_\alpha)_1 \alpha + (\lambda_\alpha)_2 e^{-it|\alpha|} \phi_-(\alpha) + (\lambda_\alpha)_3 e^{it|\alpha|} \phi_+(\alpha) \right) + \sum_{\alpha \in \mathbb{Z}^3 \wedge |\alpha| \notin \mathbb{N}^+} (\nabla e^{i(\alpha \cdot \vec{x})} \times \alpha \Lambda_\alpha)$$

$$= \sum_{\alpha \in \mathbb{Z}^3 \wedge |\alpha| \in \mathbb{N}^+} i e^{i(\alpha \cdot \vec{x})} [\alpha]_\times \left((\lambda_\alpha)_1 \alpha + (\lambda_\alpha)_2 e^{-it|\alpha|} \phi_-(\alpha) + (\lambda_\alpha)_3 e^{it|\alpha|} \phi_+(\alpha) \right) + \sum_{\alpha \in \mathbb{Z}^3 \wedge |\alpha| \notin \mathbb{N}^+} i e^{i(\alpha \cdot \vec{x})} (\alpha \times \alpha) \Lambda_\alpha$$

$$= i \sum_{\alpha \in \mathbb{Z}^3 \wedge |\alpha| \in \mathbb{N}^+} e^{i(\alpha \cdot \vec{x})} \left(-i|\alpha| (\lambda_\alpha)_2 e^{-it|\alpha|} \phi_-(\alpha) + i|\alpha| (\lambda_\alpha)_3 e^{it|\alpha|} \phi_+(\alpha) \right)$$

We do the same for $i\frac{\partial \psi}{\partial t}$.

$$i\frac{\partial \psi}{\partial t} = i \sum_{\alpha \in \mathbb{Z}^3 \wedge |\alpha| \in \mathbb{N}^+} \frac{\partial}{\partial t} \left(e^{i(\alpha \cdot \vec{x})} \left((\lambda_\alpha)_1 \alpha + (\lambda_\alpha)_2 e^{-it|\alpha|} \phi_-(\alpha) + (\lambda_\alpha)_3 e^{it|\alpha|} \phi_+(\alpha) \right) \right) + i \sum_{\alpha \in \mathbb{Z}^3 \wedge |\alpha| \notin \mathbb{N}^+} \frac{\partial}{\partial t} (e^{i(\alpha \cdot \vec{x})} \alpha \Lambda_\alpha)$$

$$= i \sum_{\alpha \in \mathbb{Z}^3 \wedge |\alpha| \in \mathbb{N}^+} \left(e^{i(\alpha \cdot \vec{x})} \left(-i|\alpha| (\lambda_\alpha)_2 e^{-it|\alpha|} \phi_-(\alpha) + i|\alpha| (\lambda_\alpha)_3 e^{it|\alpha|} \phi_+(\alpha) \right) \right)$$

So, we have verified that $i\frac{\partial \psi}{\partial t} = \nabla \times \psi$.
Note that this ψ is guaranteed to converge because $|\psi| \leq |g|$ from the Fourier Series, and since $g \in L_1$ we have that ψ is then too.

Proof of Necessity. If there were some α with $|\alpha| \notin \mathbb{N}^+$ but $\langle g, e^{i(\alpha \cdot \vec{x})} \rangle \neq \Lambda_\alpha \alpha$ for any constant Λ_α, then we have that the boundary value condition could not possibly hold since any solution ψ must satisfy (3).