ME455 HW4 Graham Clifford

Problem #1:

$$v(t)^{[k]} = \underset{v(t)}{\operatorname{argmin}} \int_0^T D_1 \ell(x(t)^{[k]}, u(t)^{[k]})\, z(t) + D_2 \ell(x(t)^{[k]}, u(t)^{[k]}) \cdot v(t)\, dt$$

$$+ D_x m(x(T)^{[k]}) \cdot z(T)$$

$$+ \int_0^T z(t)^T Q_z z(t) + v(t)^T R_v v(t)\, dt$$

$$= \int_0^T D_1 \ell(x(t)^{[k]}, u(t)^{[k]}) z(t) + z(t)^T Q_z z(t) + D_2 \ell(x(t)^{[k]}, u(t)^{[k]}) v(t) + v(t)^T R_v v(t)\, dt$$

$$+ D_x m(x(T)^{[k]}) \cdot z(T)$$

$$= \int_0^T z(t) \underbrace{\left[D_1 \ell(x(t)^{[k]}, u(t)^{[k]}) + z(t)^T Q_z \right]}_{a_z(t)^T} + v(t) \underbrace{\left[D_2 \ell(x(t)^{[k]}, u(t)^{[k]}) + v(t)^T R_v \right]}_{b_v(t)^T} dt$$

$$+ D_x m(x(T)^{[k]}) \cdot z(T)$$

now we can sub in $a_z(t)$ & $b_v(t)$...

$$0 = \rho(t)^T B(t) + v(t)^T R_v + D_2 (\ell(x(t)^{[k]}, u(t)^{[k]}))$$

solve for $v(t)$

$$v(t) = -R_v^{-1} \left[B(t)^T \rho(t) + D_2 (\ell(x(t), u(t))) \right]$$

and sub in $v(t)$

$$\dot{z}(t) = A(t) z(t) - B(t) R^{-1} \left[B(t)^T \rho(t) + D_2 (\ell(x(t)^{[k]}, u(t)^{[k]})) \right]$$

$$\dot{\rho}(t) = -A(t)^T \rho(t) - Q_z z(t) - D_1 (\ell(x(t)^{[k]}, u(t)^{[k]}))$$

$$\boxed{\begin{bmatrix} \dot{z} \\ \dot{\rho} \end{bmatrix} = \begin{bmatrix} A(t) & -B(t) R^{-1} B(t)^T \\ -Q_z & -A(t)^T \end{bmatrix} \begin{bmatrix} z \\ \rho \end{bmatrix} + \begin{bmatrix} -B(t) R^{-1} D_2 (\ell(x(t)^{[k]}, u(t)^{[k]})) \\ -D_1 (\ell(x(t)^{[k]}, u(t)^{[k]})) \end{bmatrix}}$$