

# ME455 Active Learning HW4

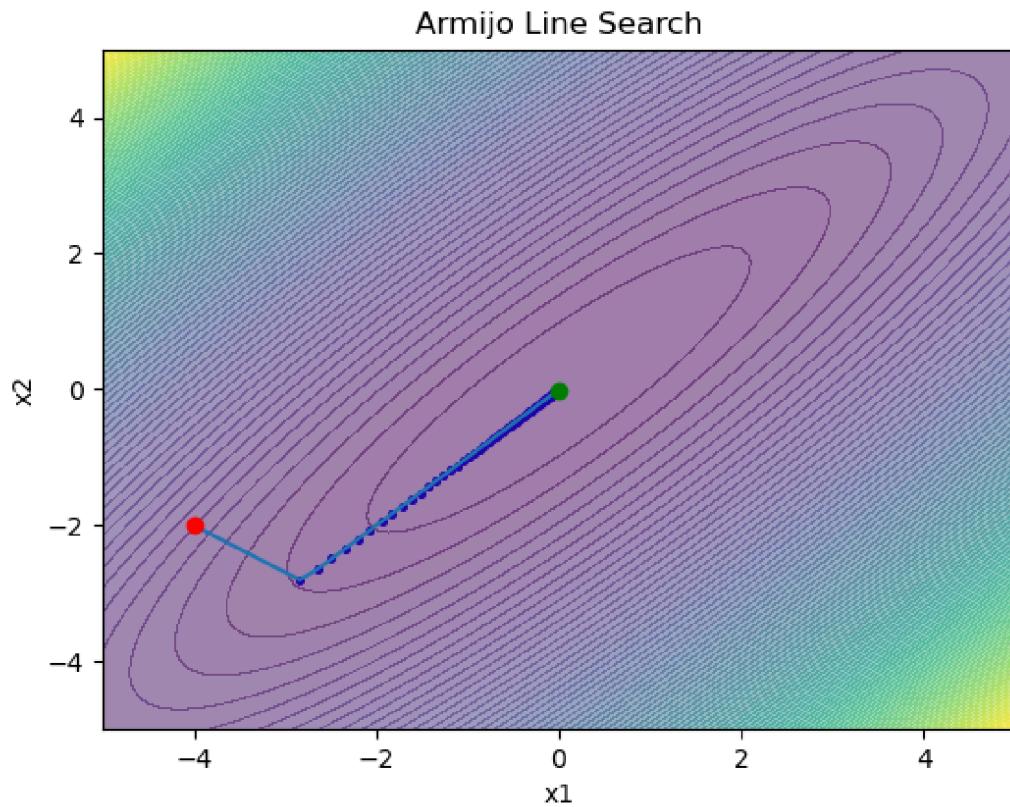
Graham Clifford

5/9/2024

## Problem 1

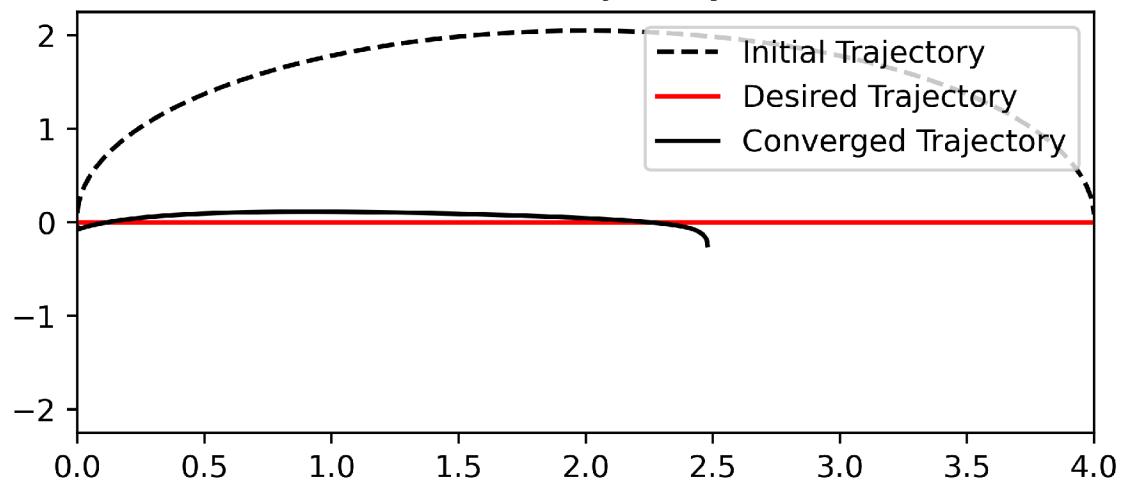
Please see handwritten pages for problem 1.

## Problem 2

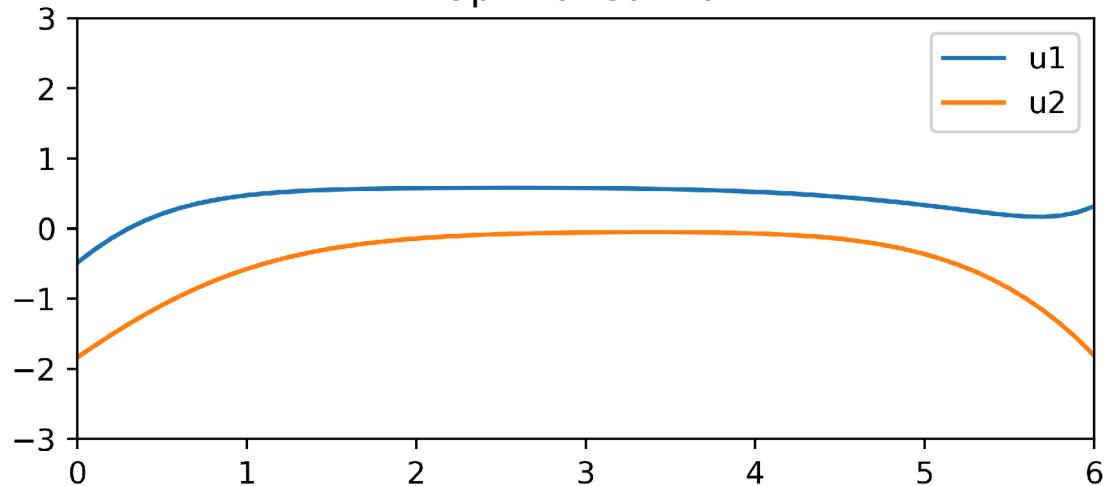


## Problem 3

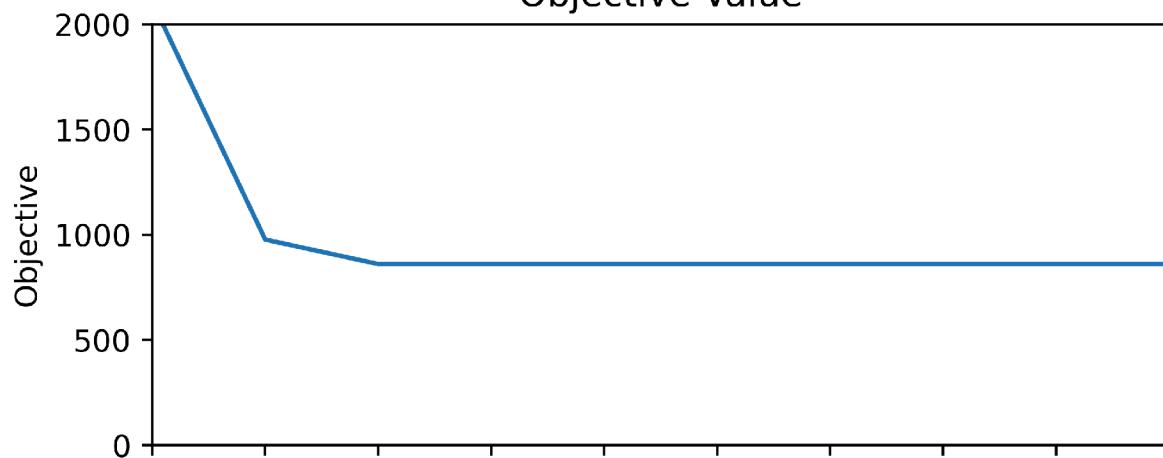
### State Trajectory



### Optimal Control



### Objective Value





I'm not really sure why I couldn't get my plots to look like the example plots, however I think there must be something wrong with either my iLQR algorithm or the way I calculate my loss. I'm doing my iLQR algorithm where I calculate the cumulative loss for the entire trajectory with the current control sequence, as well as the cumulative loss for the entire trajectory with the modified control sequence ( $u + \alpha * \gamma$ ). Maybe this is wrong, but it seemed to me like the example code was encouraging this strategy. This leaves me with a loss that plateaus at 1000, which is kinda weird.

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## Problem #1:

$$\begin{aligned}
 v(t)^{(k)} &= \underset{v(t)}{\operatorname{argmin}} \int_0^T D_1 \ell(x(t)^{(k)}, u(t)^{(k)}) z(t) + D_2 \ell(x(t)^{(k)}, u(t)^{(k)}) \cdot v(t) dt \\
 &\quad + D_{\partial x} (x(t)^{(k)}) \cdot z(t) \\
 &\quad + \int_0^T z(t)^T Q_z z(t) + v(t)^T R_v v(t) dt \\
 &= \int_0^T D_1 \ell(x(t)^{(k)}, u(t)^{(k)}) z(t) + z(t)^T Q_z z(t) + D_2 \ell(x(t)^{(k)}, u(t)^{(k)}) v(t) + v(t)^T R_v v(t) dt \\
 &\quad + D_m (x(t)^{(k)}) \cdot z(t) \\
 &= \int_0^T z(t) \left[ D_1 \ell(x(t)^{(k)}, u(t)^{(k)}) + z(t)^T Q_z \right] + v(t) \left[ D_2 \ell(x(t)^{(k)}, u(t)^{(k)}) + v(t)^T R_v \right] dt \\
 &\quad + D_m (x(t)^{(k)}) \cdot z(t)
 \end{aligned}$$

now we can sub in  $a_z(t)$  &  $b_v(t)$ ...

$$0 = q(t)^T \beta(t) + v(t)^T R_v + D_2 (\ell(x(t)^{(k)}, u(t)^{(k)}))$$

solve for  $v(t)$

$$v(t) = -R_v^{-1} [\beta(t)^T \eta(t) + D_2 (\ell(x(t), u(t)))]$$

and sub in  $v(t)$

$$\dot{z}(t) = A(t) z(t) - \beta(t) R^{-1} [\beta(t)^T \eta(t) + D_2 (\ell(x(t)^{(k)}, u(t)^{(k)}))]$$

$$\dot{\eta}(t) = -A(t)^T q(t) - Q_z z(t) - D_1 (\ell(x(t)^{(k)}, u(t)^{(k)}))$$

$$\boxed{\begin{bmatrix} \dot{z} \\ \dot{\eta} \end{bmatrix} = \begin{bmatrix} A(t) & -\beta(t) R^{-1} \beta(t)^T \\ -Q_z & -A(t)^T \end{bmatrix} \begin{bmatrix} z \\ \eta \end{bmatrix} + \begin{bmatrix} -\beta(t) R^{-1} D_2 (\ell(x(t)^{(k)}, u(t)^{(k)})) \\ -D_1 (\ell(x(t)^{(k)}, u(t)^{(k)})) \end{bmatrix}}$$