



Zebra Logic

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Outline

- Basic Logic Refresh
- Zebra Puzzle

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First-Order Logic

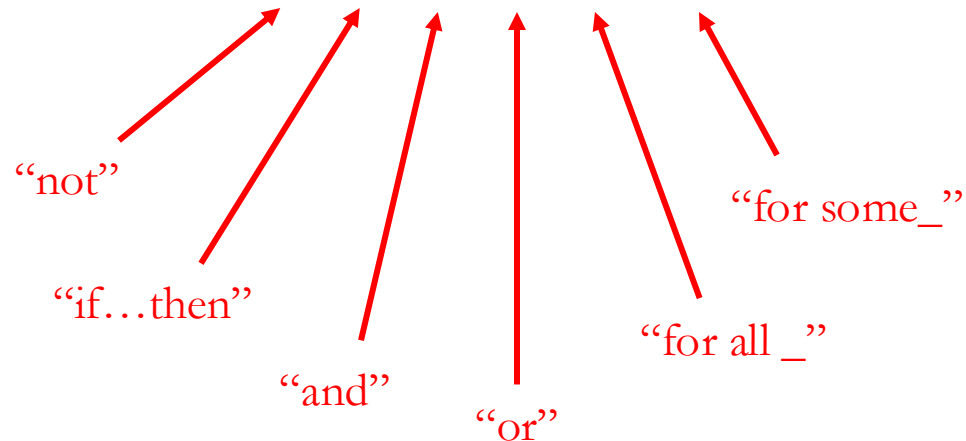
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- FOL consists of the following vocabulary:
 - Logical operators and connectives
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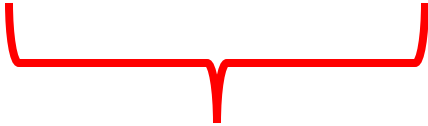
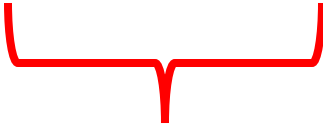
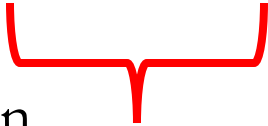
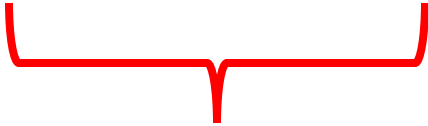
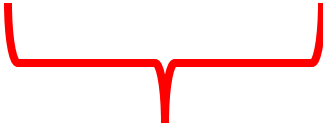
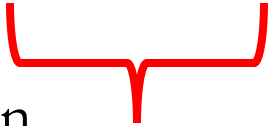
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- 
- “_ is P” “_ is P to _” “_ is P to _ and _”
“_ is R” “_ is R to _” “_ is R to _ and _”

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variables fill in these slots

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 - Punctuation
- variables fill in slots for predicates too

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Example

- English sentence: “All bald men are happy.”

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-  The variable associated with “V” binds variables associated with predicates within its *scope*
- $V_x((B_x \ \& \ M_{_}) \rightarrow H_{_})$


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-  $V_x((B_x \ \& \ M_x) \rightarrow H_x)$ In this case all the variables are within the scope of “V”

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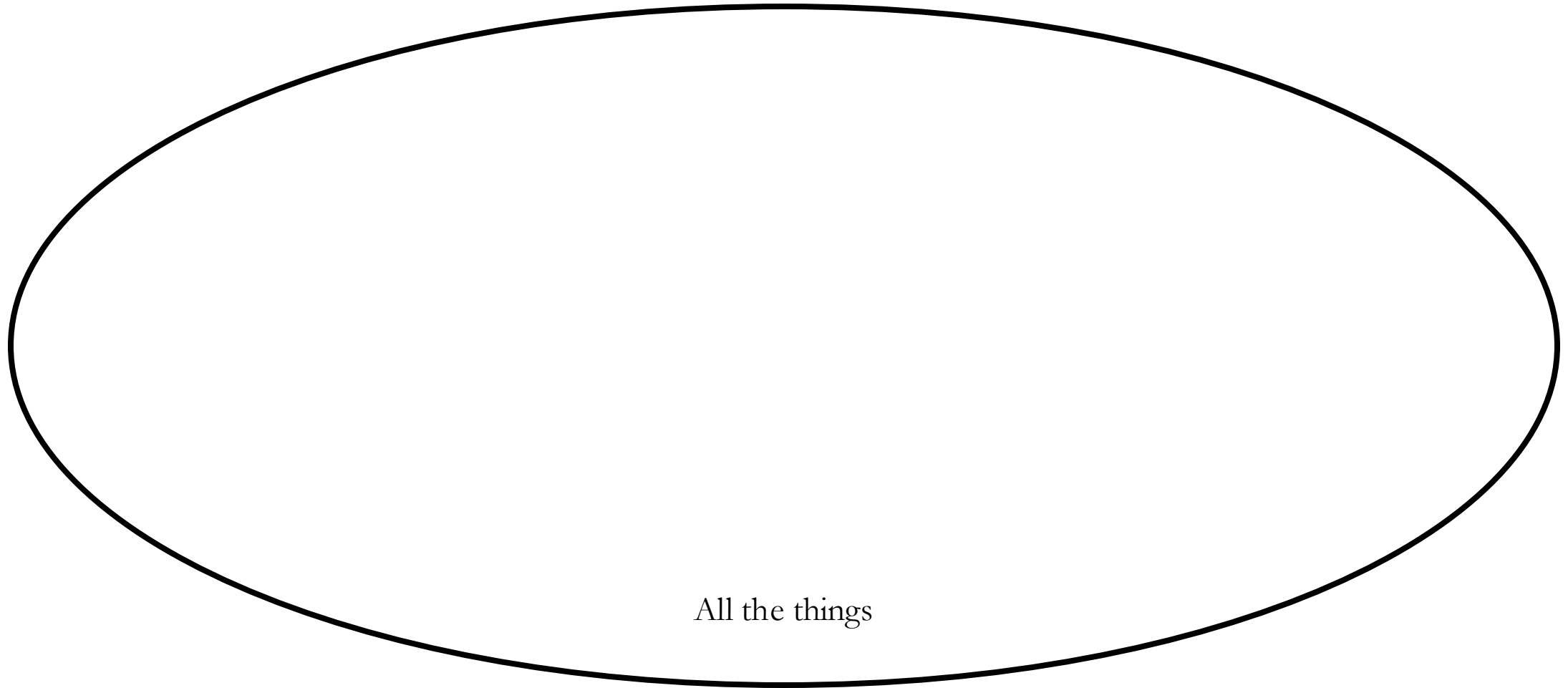
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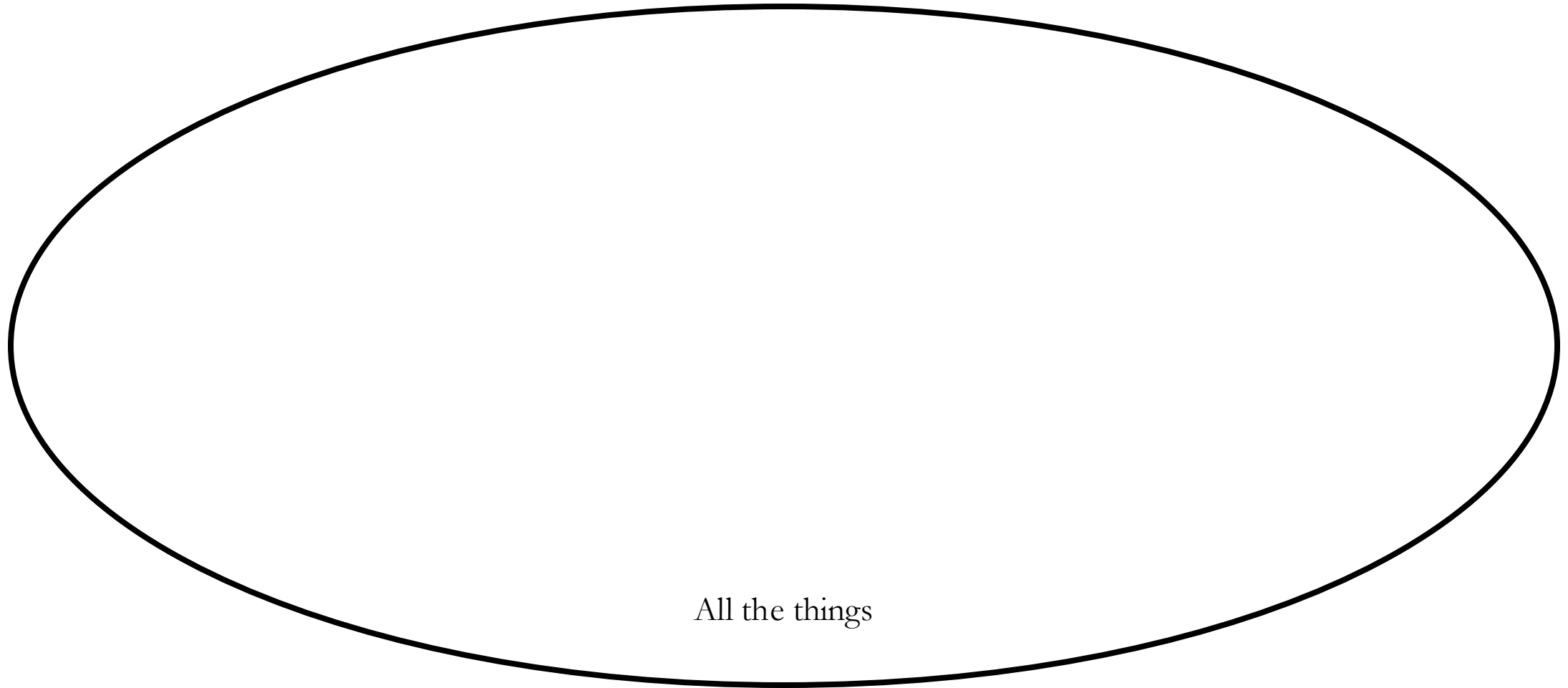
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All the things

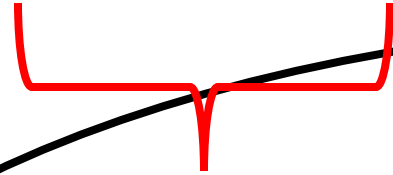
Interpretation

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Interpretation

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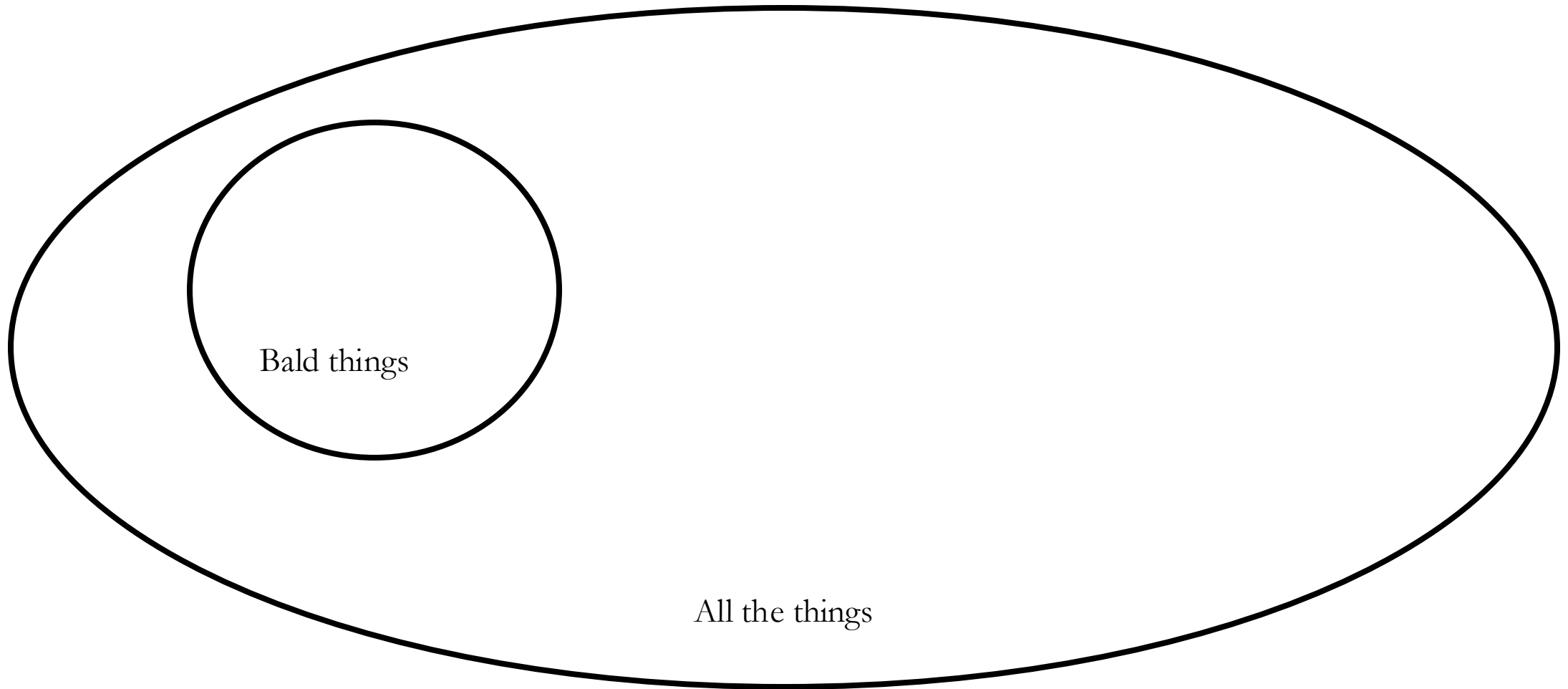


This restricts the domain
to just bald men

All the things

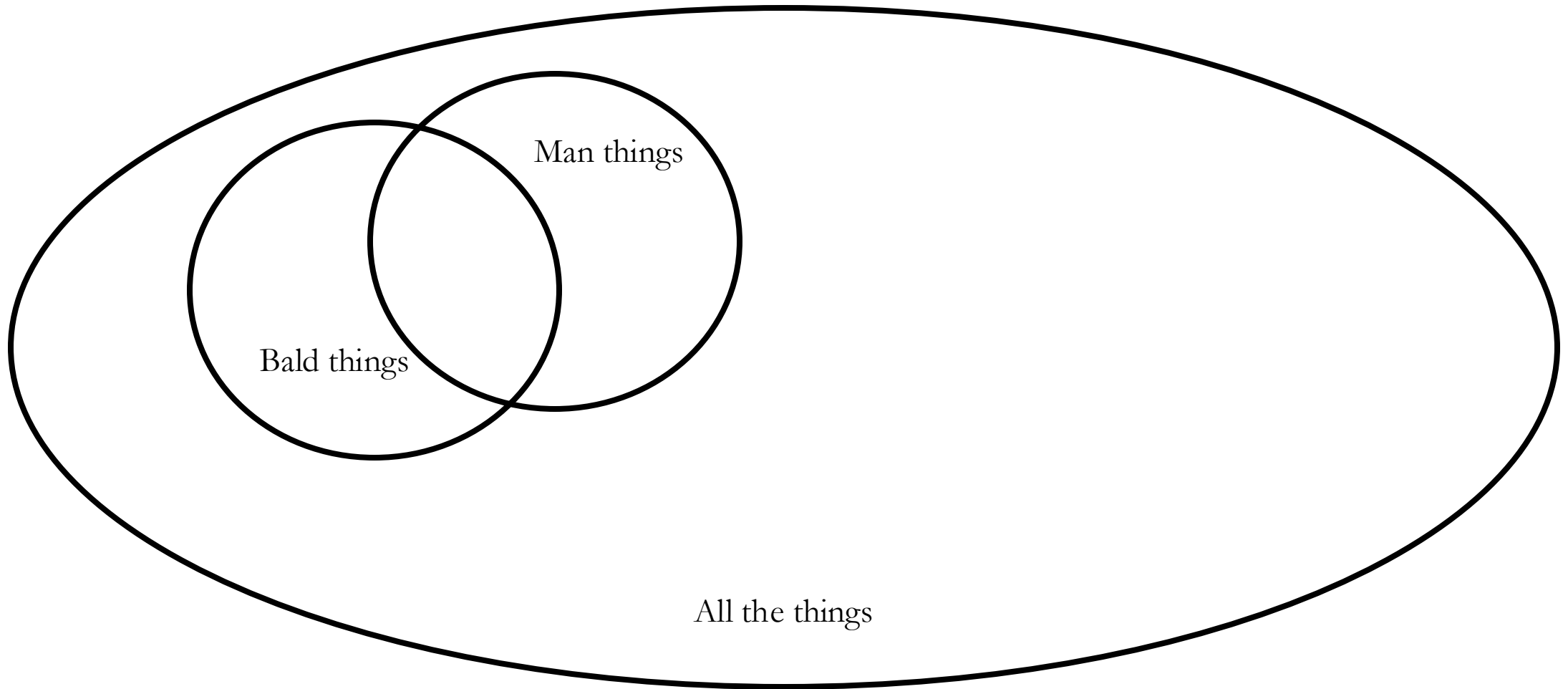
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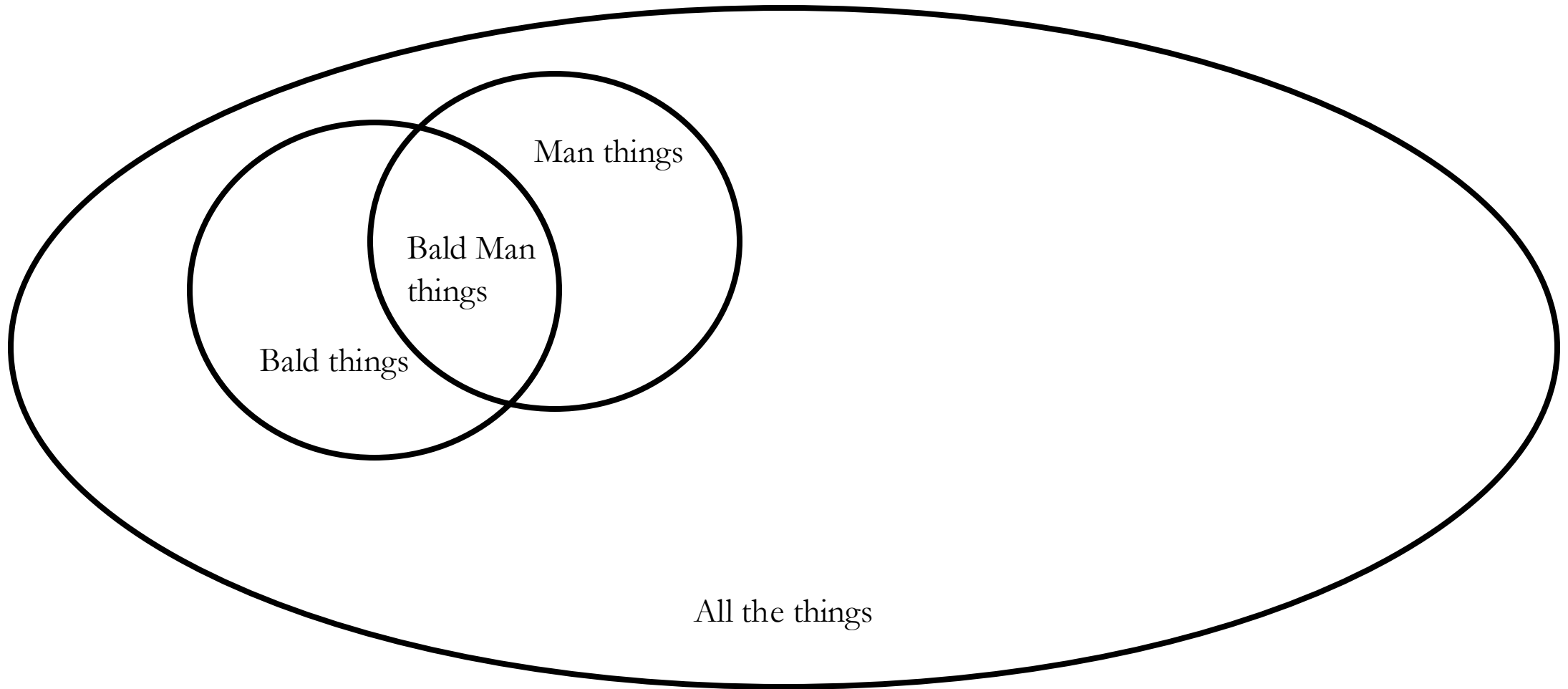
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For every x, if x is bald and a **man**, then x is happy



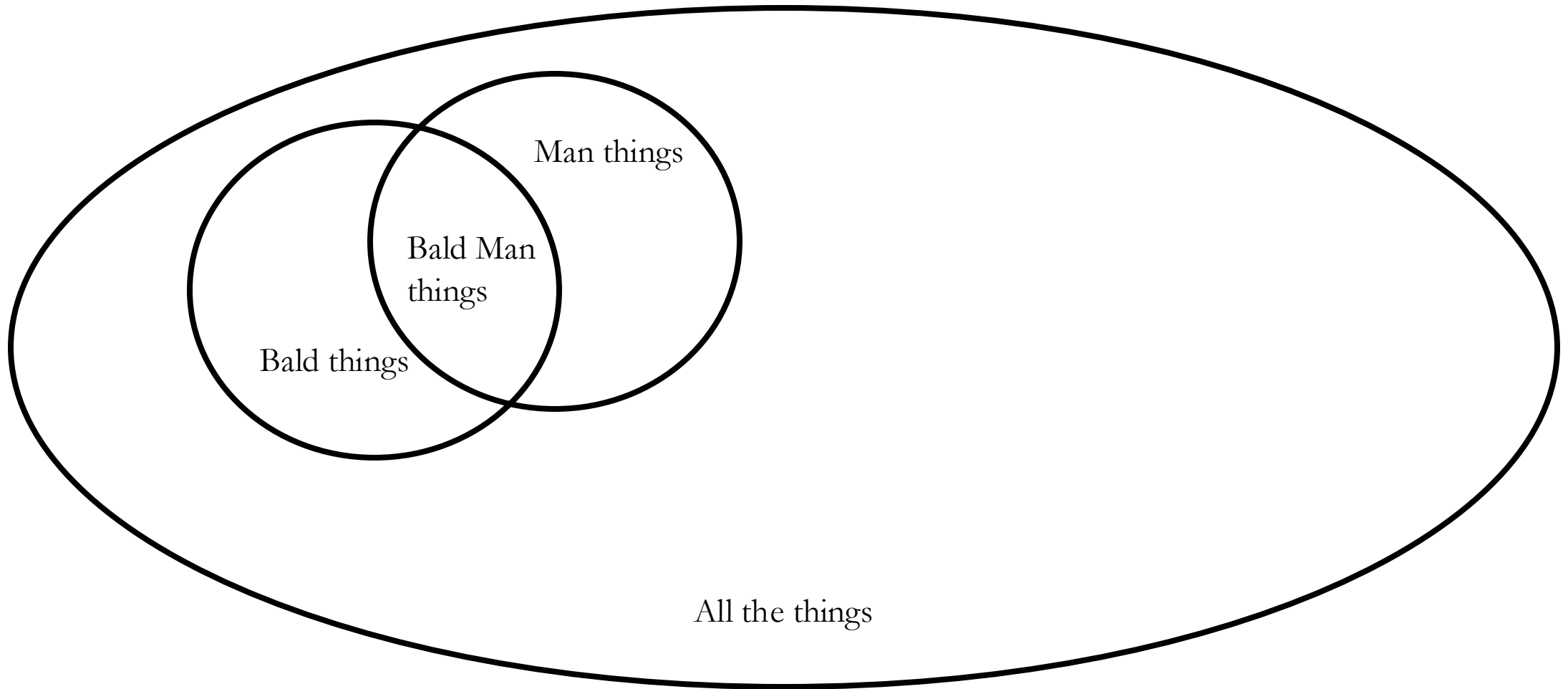
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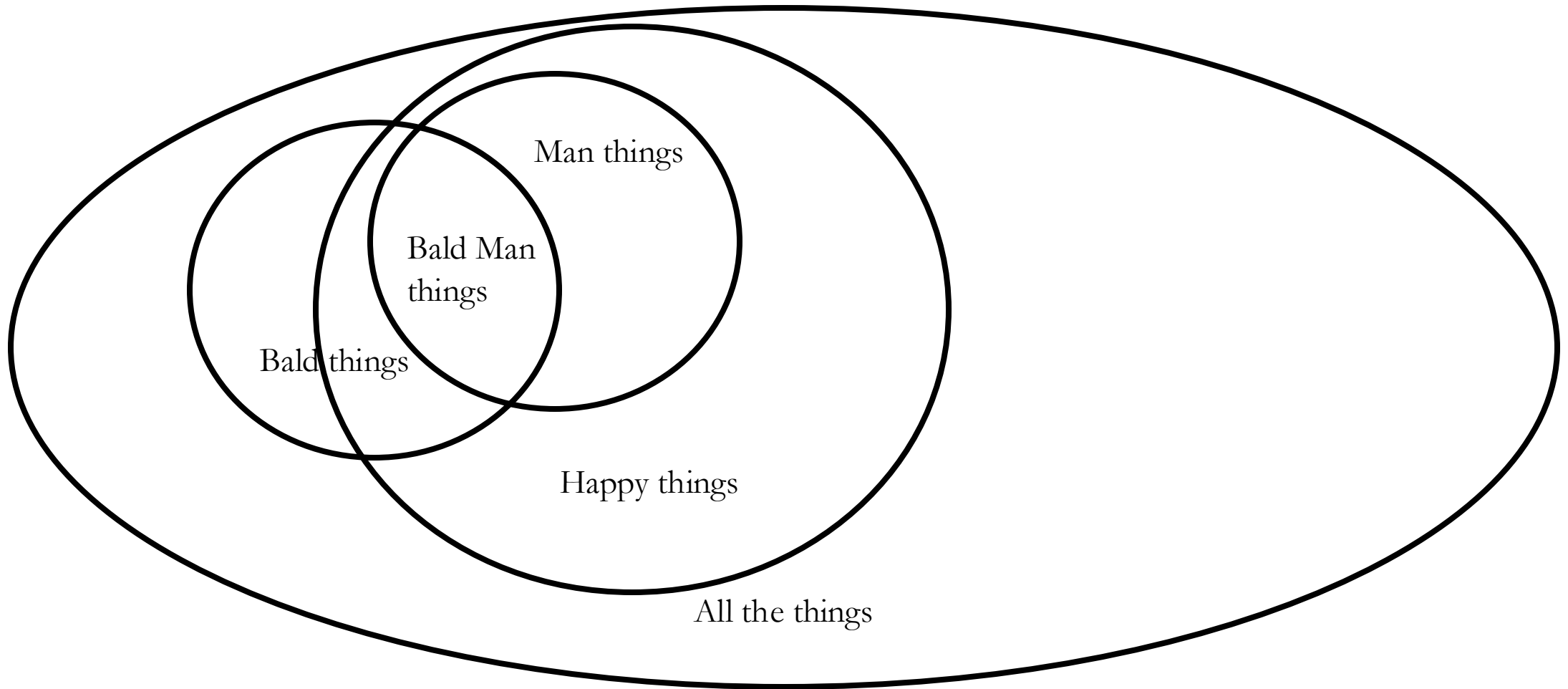
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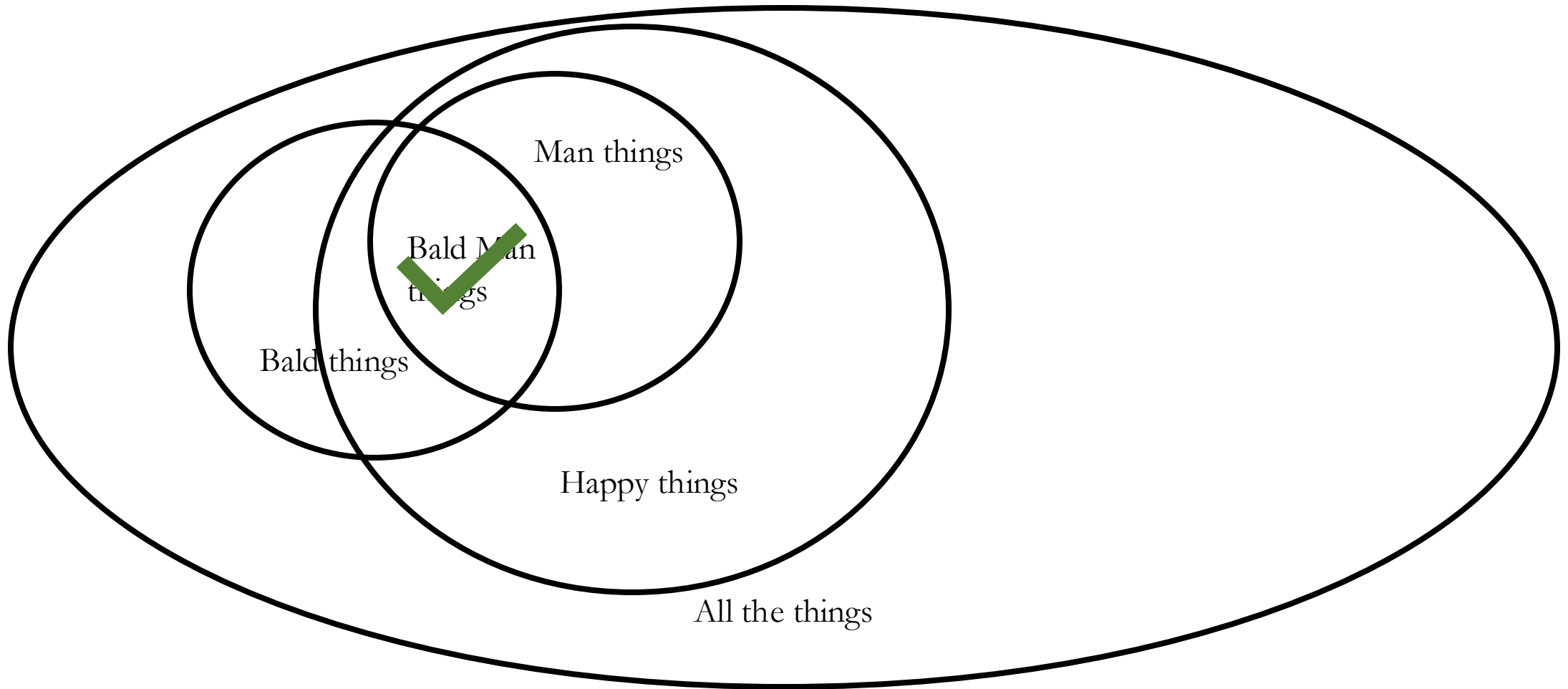
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Supplemented FOL

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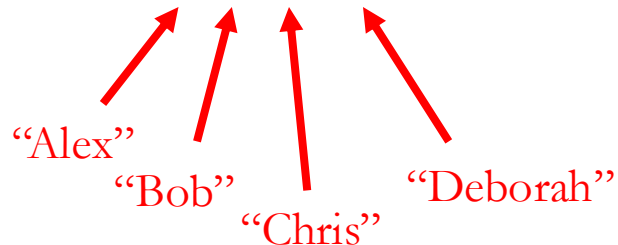
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 - Names (a , b , c , d , ...)



“Alex”
“Bob”
“Chris”
“Deborah”

Supplemented FOL

- Compare:
- Someone is bald and happy
 - $\text{Ex}(\text{Bx} \ \& \ \text{Hx})$
- John is bald and happy
 - $(\text{Bj} \ \& \ \text{Hj})$

Binary Relations

- FOL includes predicates, e.g. *is red*, *is bald*, and relations, e.g. *is part of*, *is between*, *is next to*, etc.
- For example (give John's arm the name 'a'):
 - John has an arm and it is part of John
 - $\text{part of}(a, j)$
 - John has a sister, Kellye
 - $\text{is related to}(j, k)$
 - John is between Sam and Deborah
 - $\text{is between}(j, s, d)$

FOL is Too Powerful!

- Once we add ternary relations, the formal language becomes *undecidable*
- What this means – roughly – is that we can't determine in FOL for every expression whether that expression is false or we just haven't found a counterexample to it
- In other words, it's impossible in FOL hard to prove every negative

Restricting FOL

- Because we don't want our computers to loop forever when we ask them to check our expressions, researchers have turned to more restrictive versions of FOL
- These are known as **description logics** (or 'guarded fragments' of FOL)
- The logical language you see in Protégé is a description logic

Description Logic & Protégé

- Basically, description logics start with FOL, and add the following constraints:
- Only binary relations are allowed
- Names for particulars are in the language
- The same object may have multiple aliases

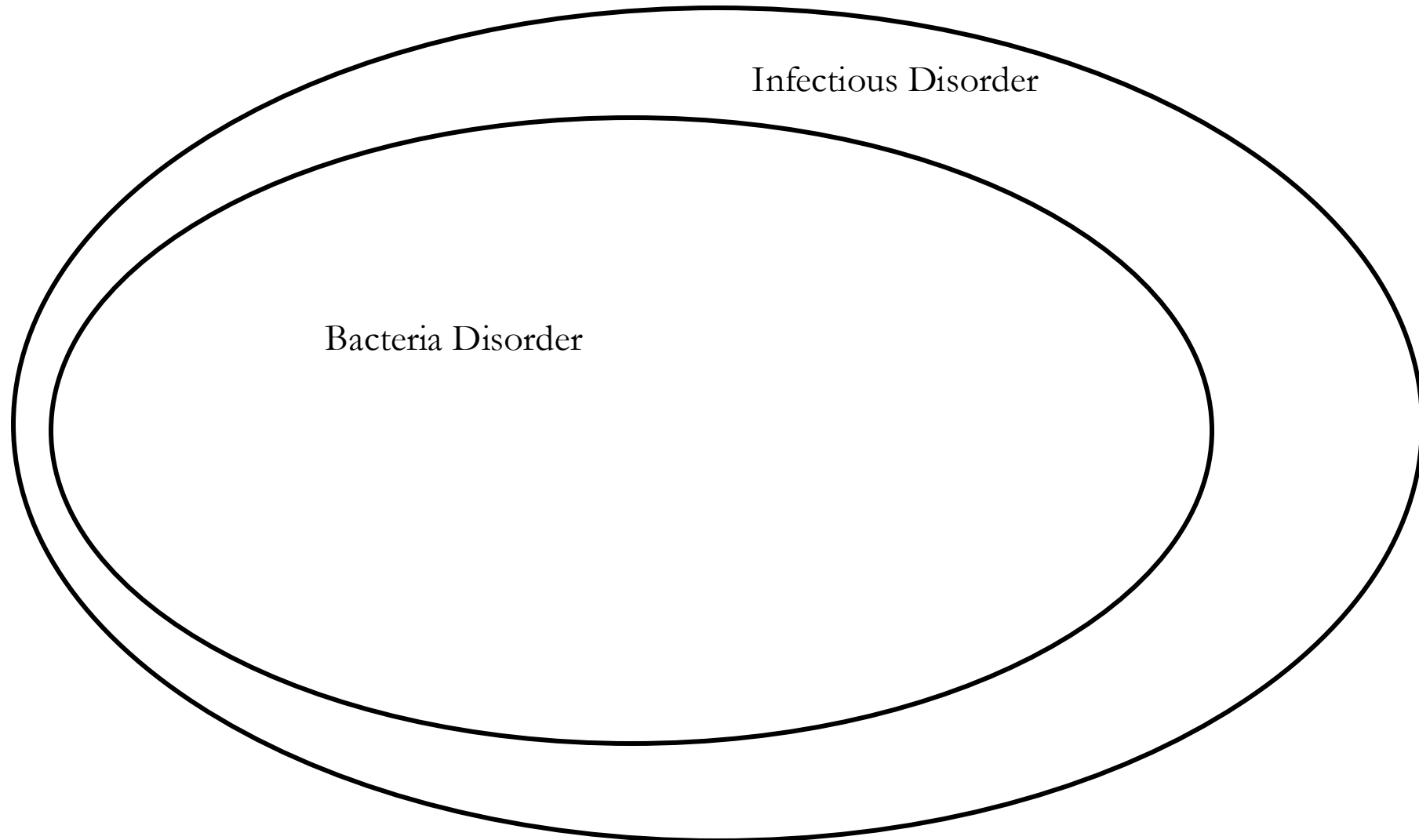
Importance

- Why care about the logic? Because that's what computers will read
- When we introduce terms into an ontology, there is an implicit hierarchy logic involved
- But it's very simple; the real strength of ontologies is in linking parts of hierarchies to other parts of hierarchies
- That allows for many inferences to be drawn automatically

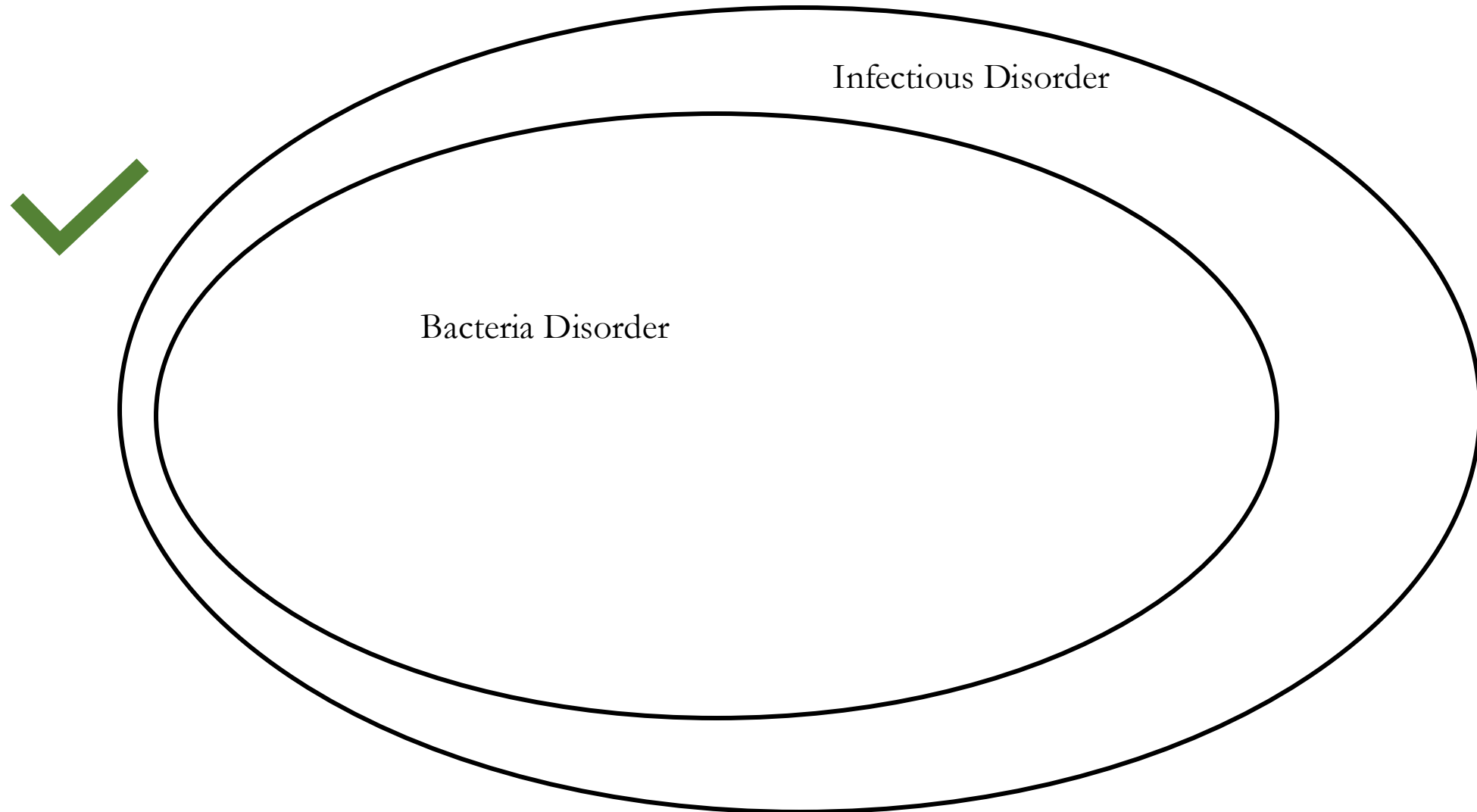
Example: Bacteria Disorder

- Suppose we just add the term ‘bacteria disorder’ as a subclass of ‘infectious disorder’ to protégé but don’t add anything else
- The only thing a computer would know then is that anything that’s a bacteria disorder is an infectious disorder
- It wouldn’t even know bacteria disorders must have bacteria involved in them!

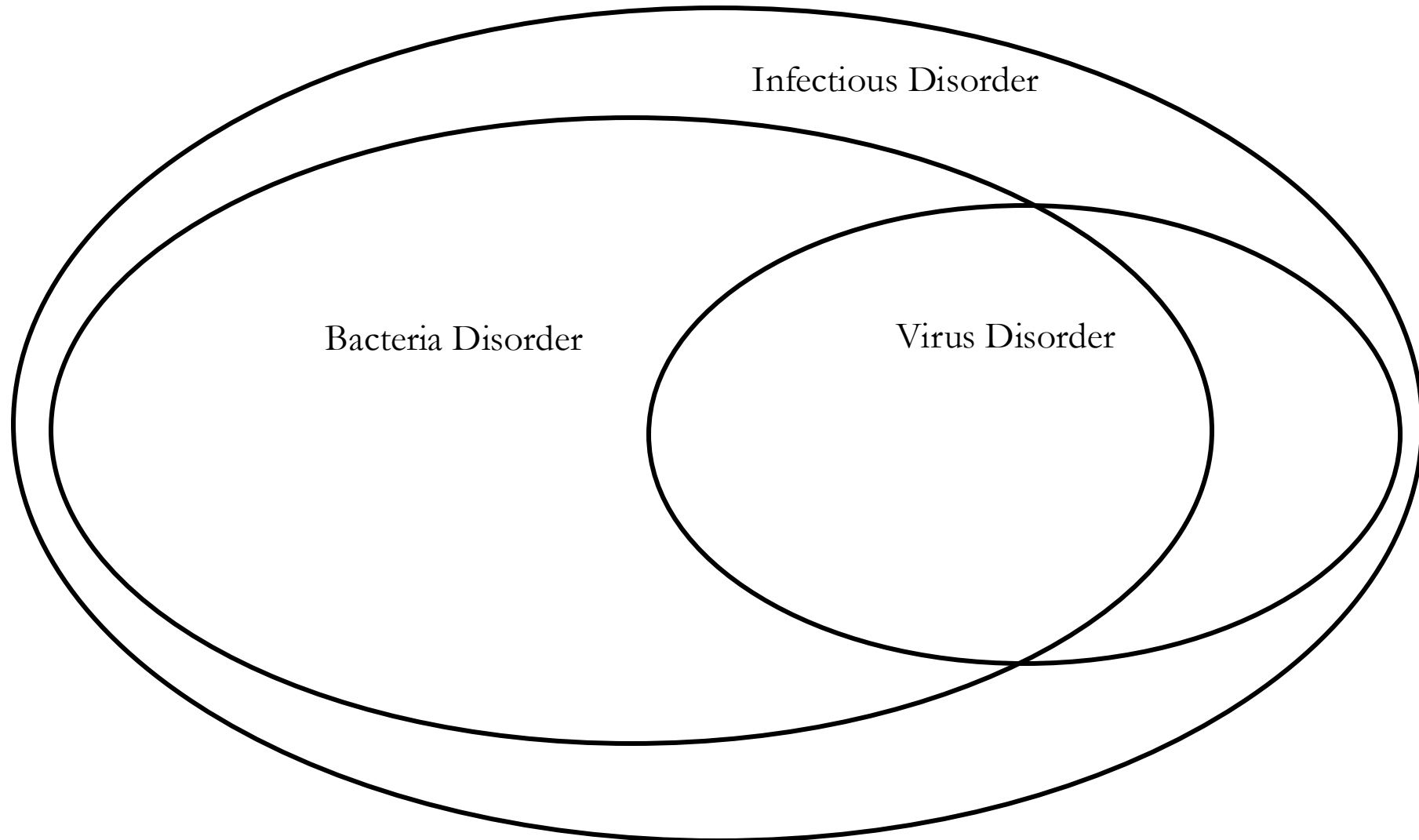
Possibilities



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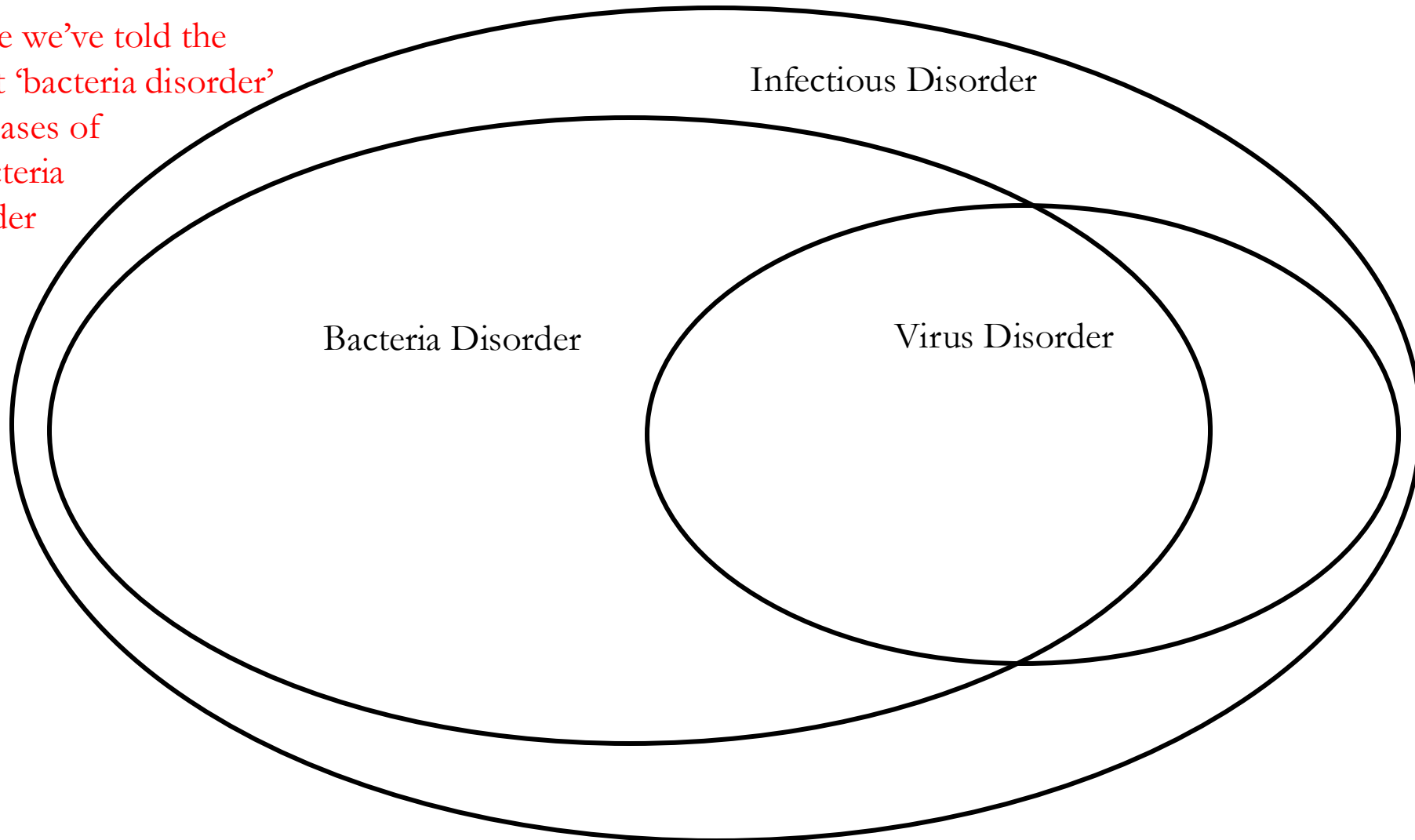


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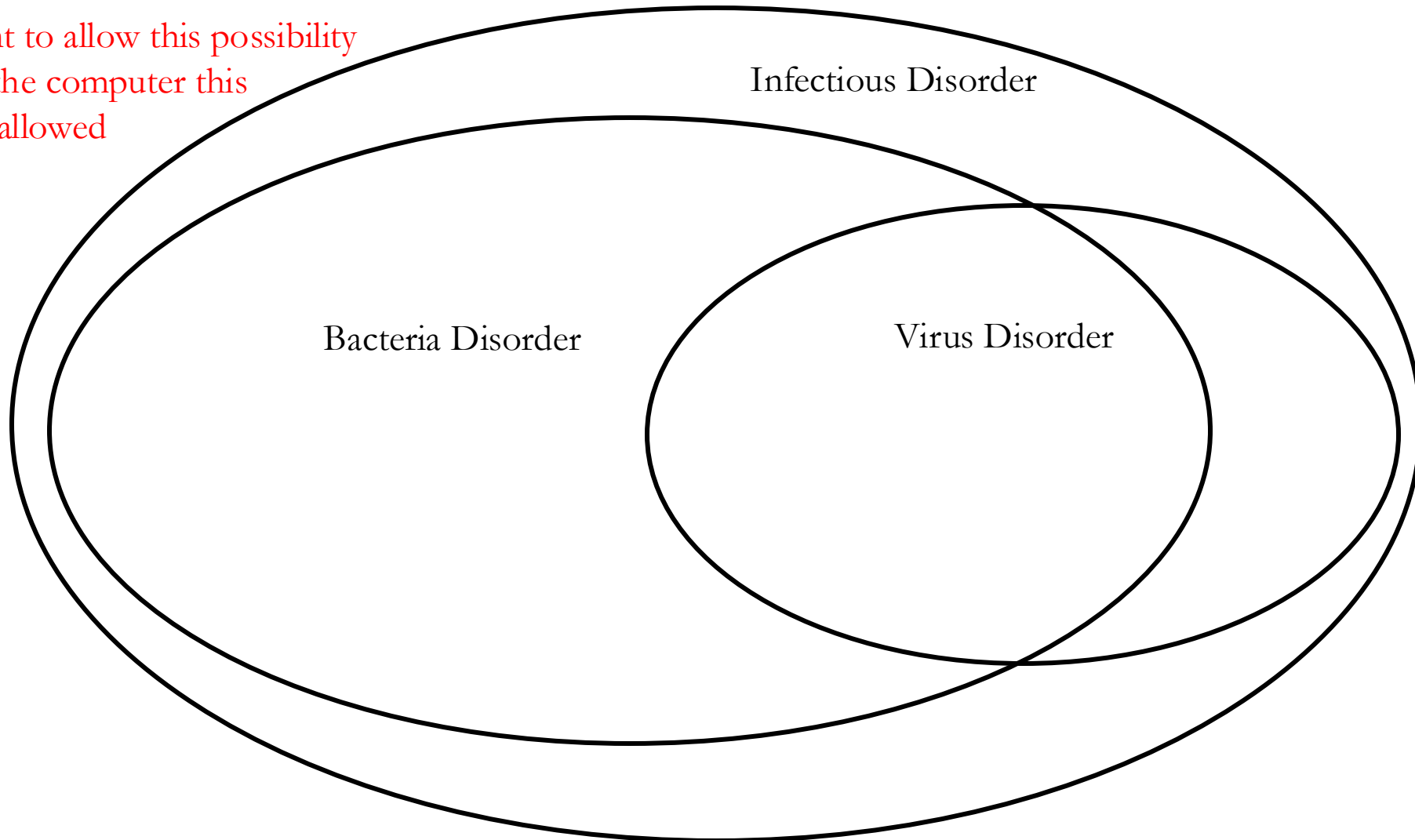
Possibilities

Given what little we've told the
computer about 'bacteria disorder'
it would allow cases of
overlapping bacteria
and virus disorder

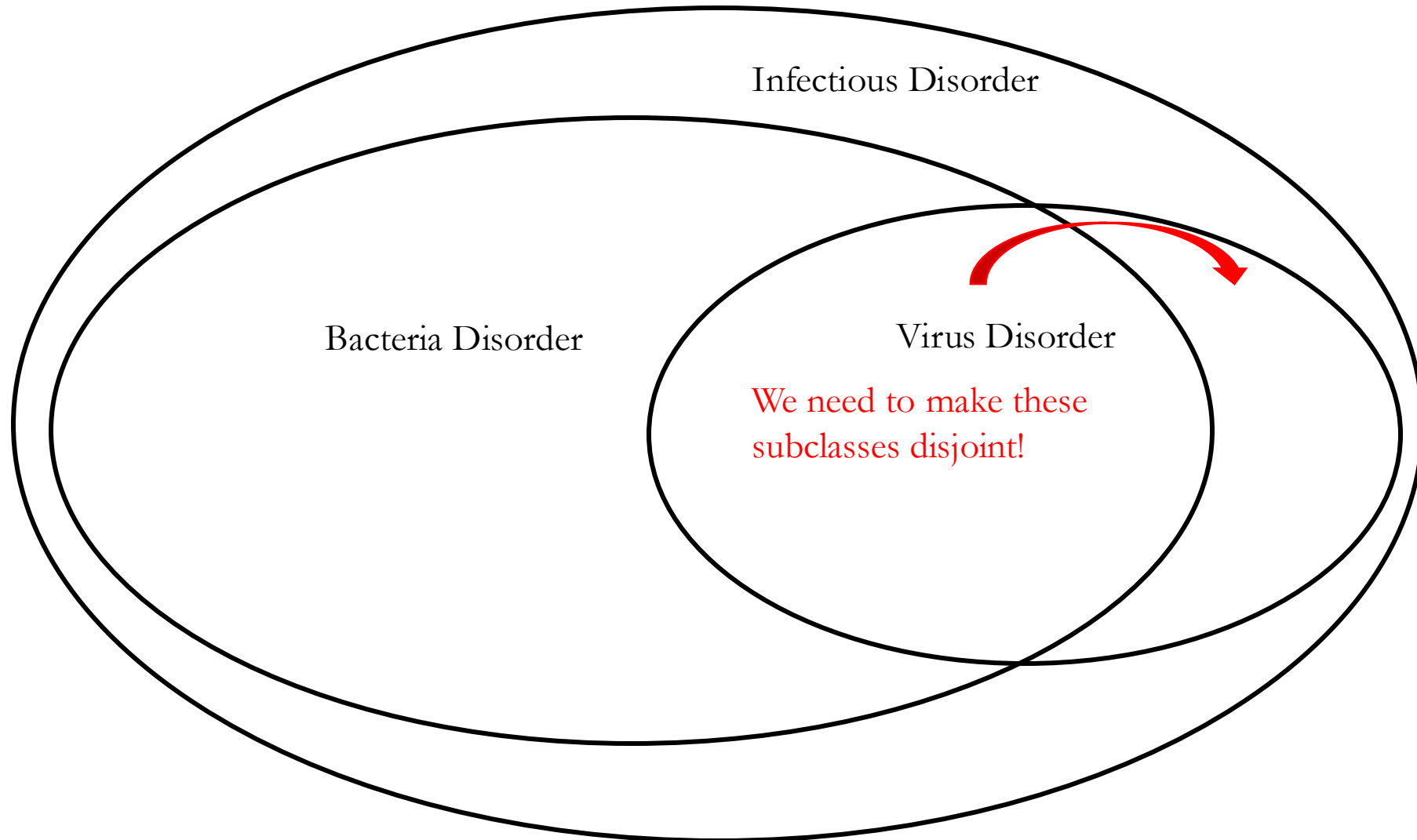


Possibilities

If we don't want to allow this possibility
we have to tell the computer this
situation is not allowed



Possibilities



Protégé Disjoint

- immunodeficiency
- ▼ ● infectious disorder
 - acute infection
 - **bacteria disorder**
 - primary infection
 - secondary infection
- ▶ ● virus disorder
- geographical entity
- ▼ ● infection
 - community-acquired infection
 - ▶ ● extracellular infection
 - hospital-acquired infection
 - ▼ ● infectious disorder
 - acute infection
 - primary infection
 - secondary infection
 - ▼ ● virus disorder
 - ▶ ● coronavirus disorder
- intracellular infection

Description: bacteria disorder

● 'infectious disorder'

General class axioms +

SubClass Of (Anonymous Ancestor)

● infection
and disorder

Instances +

Target for Key +

Disjoint With +

● 'virus disorder'

Disjoint Union Of +

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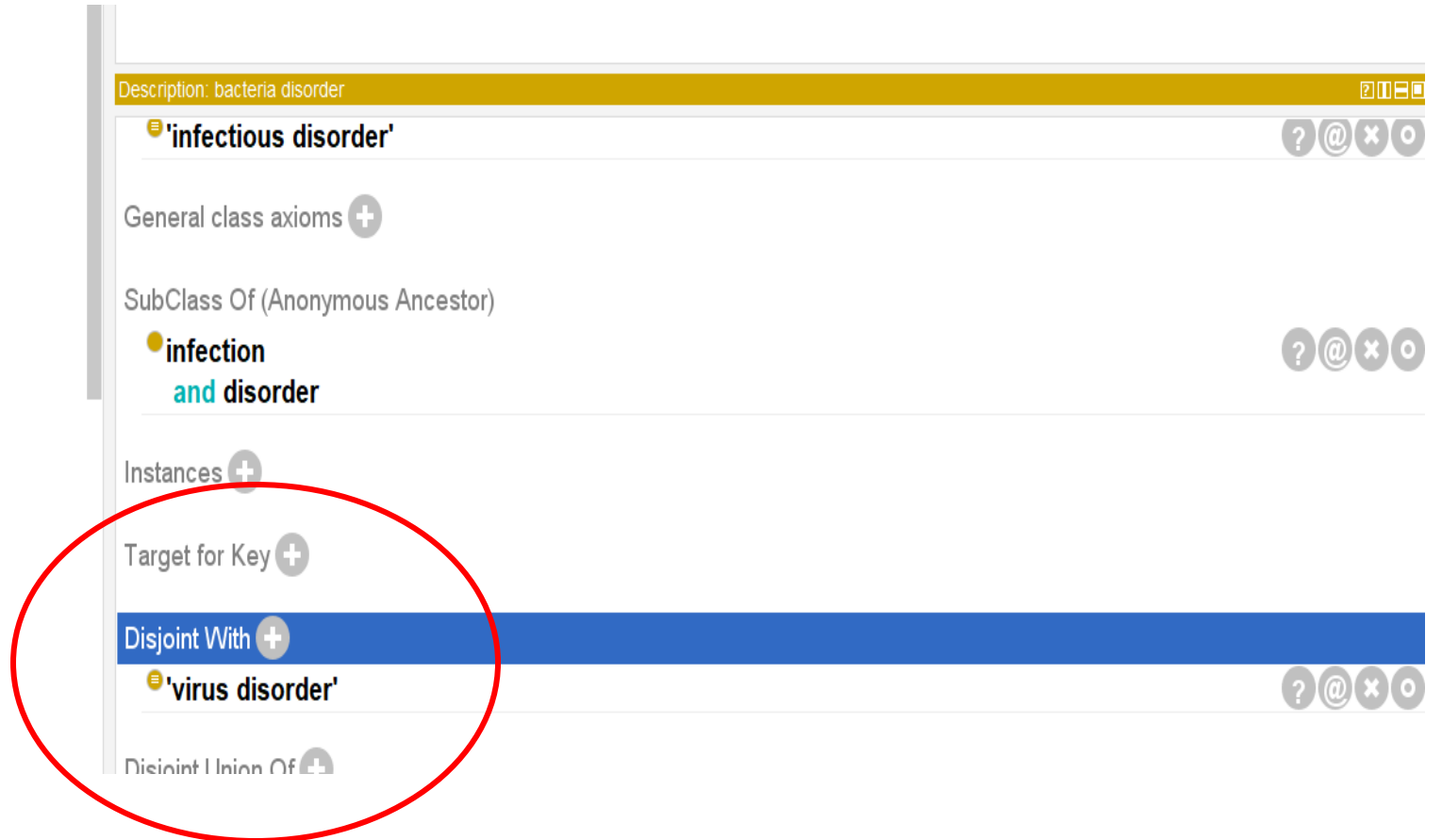
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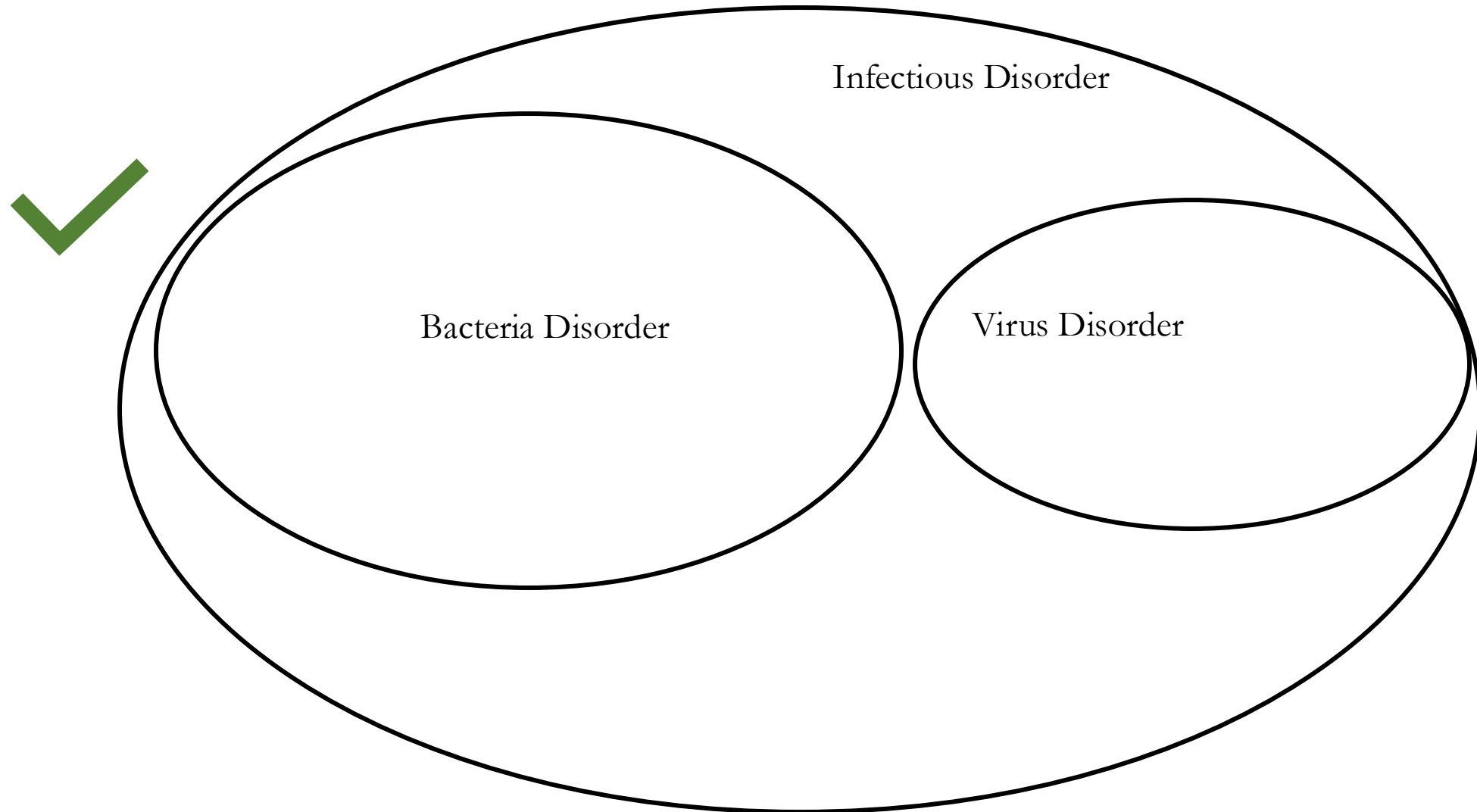
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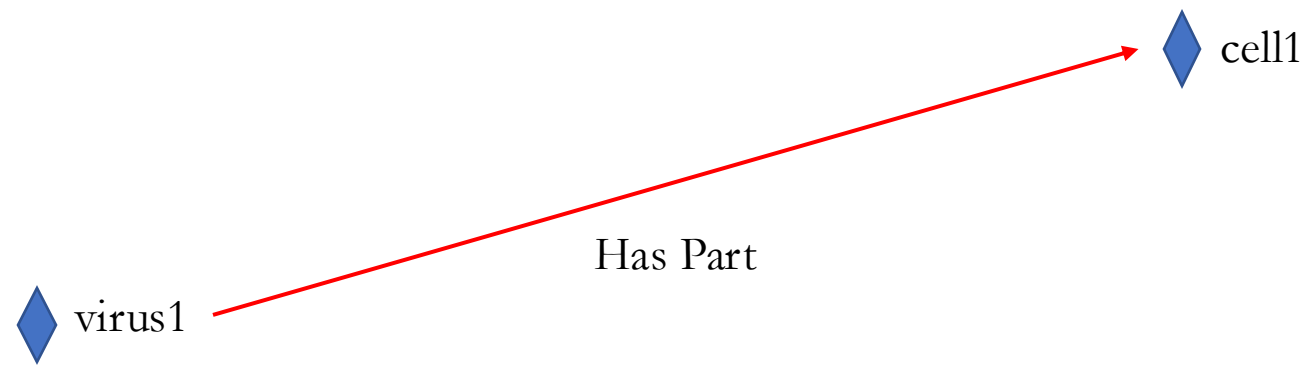
Highlight bacteria disorder in the hierarchy, then click the “+”, then in the expression editor type ‘virus disorder’

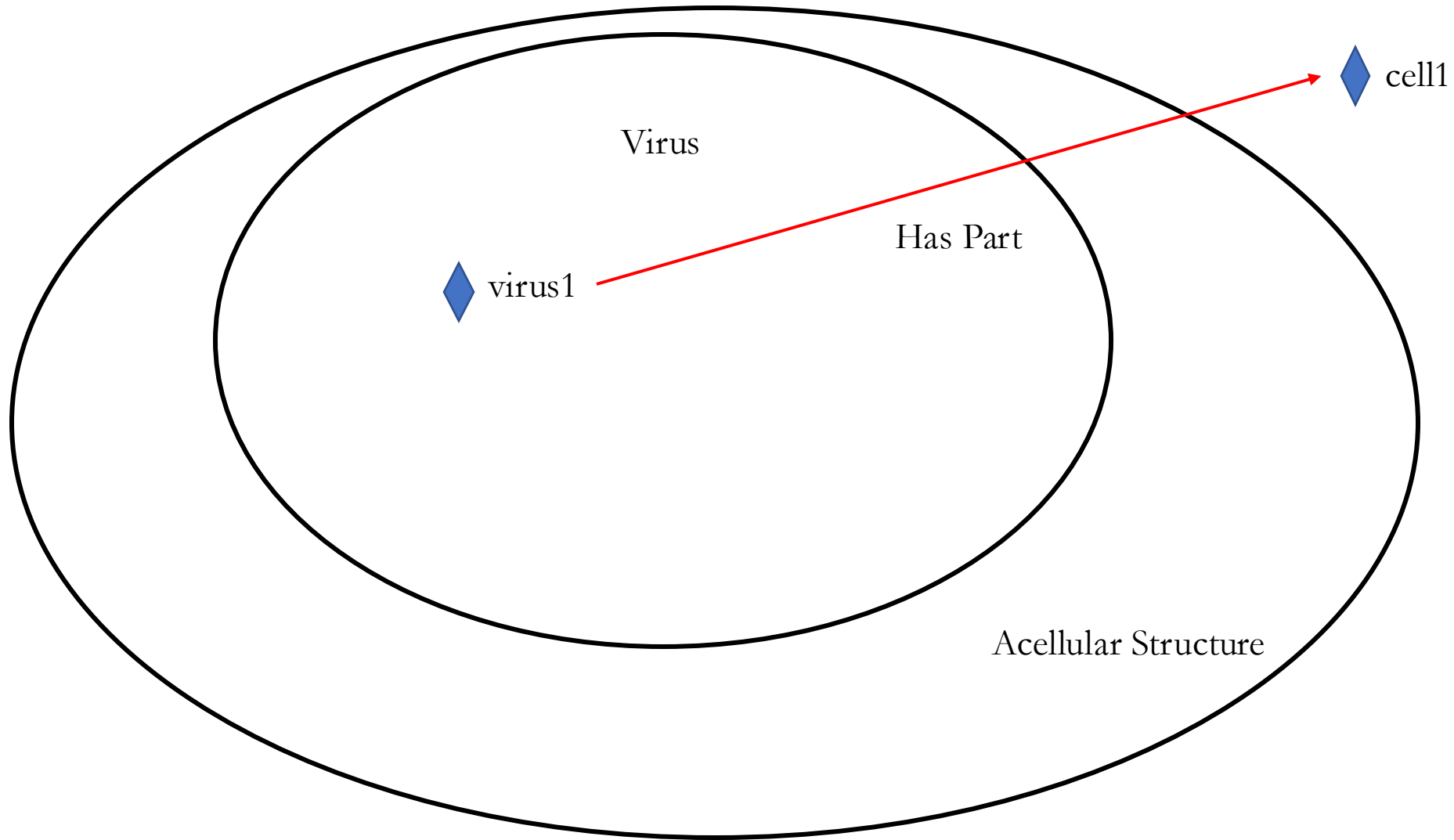
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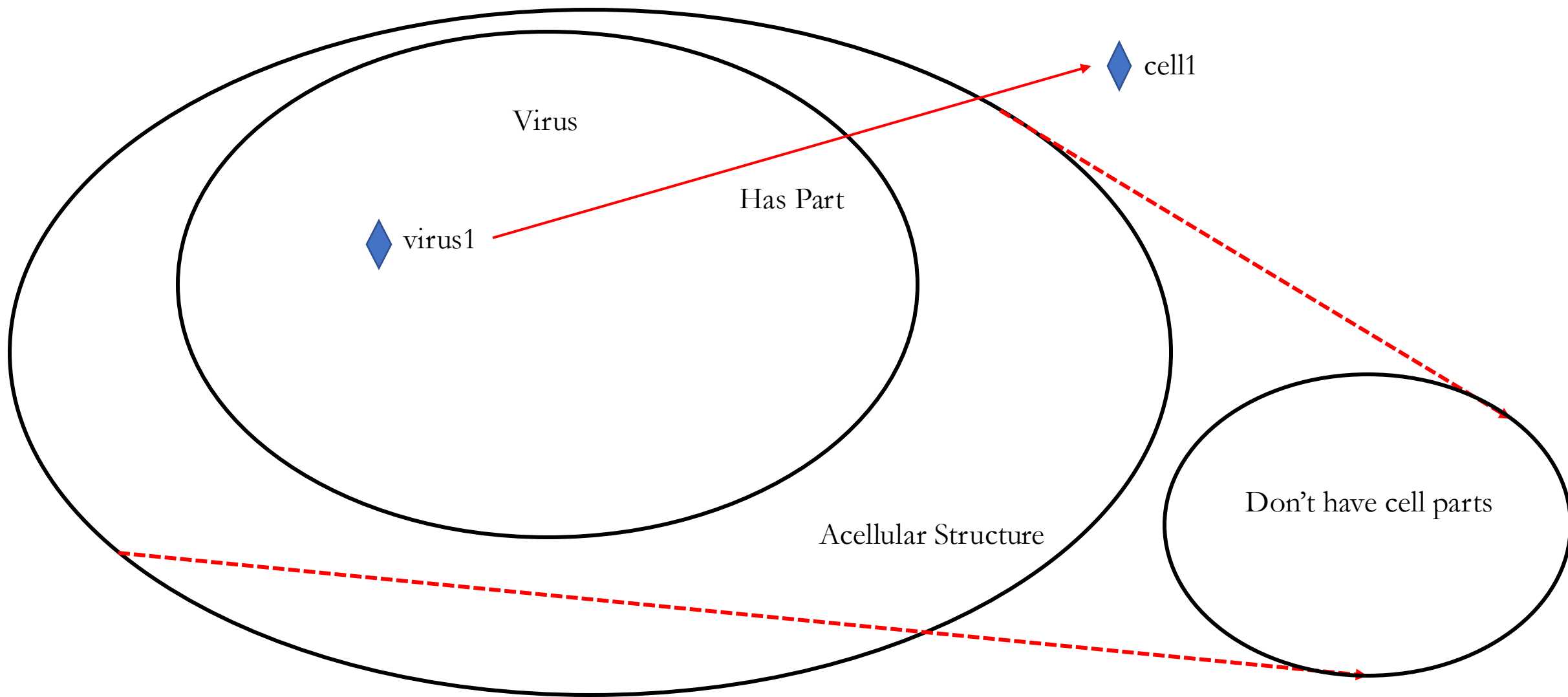


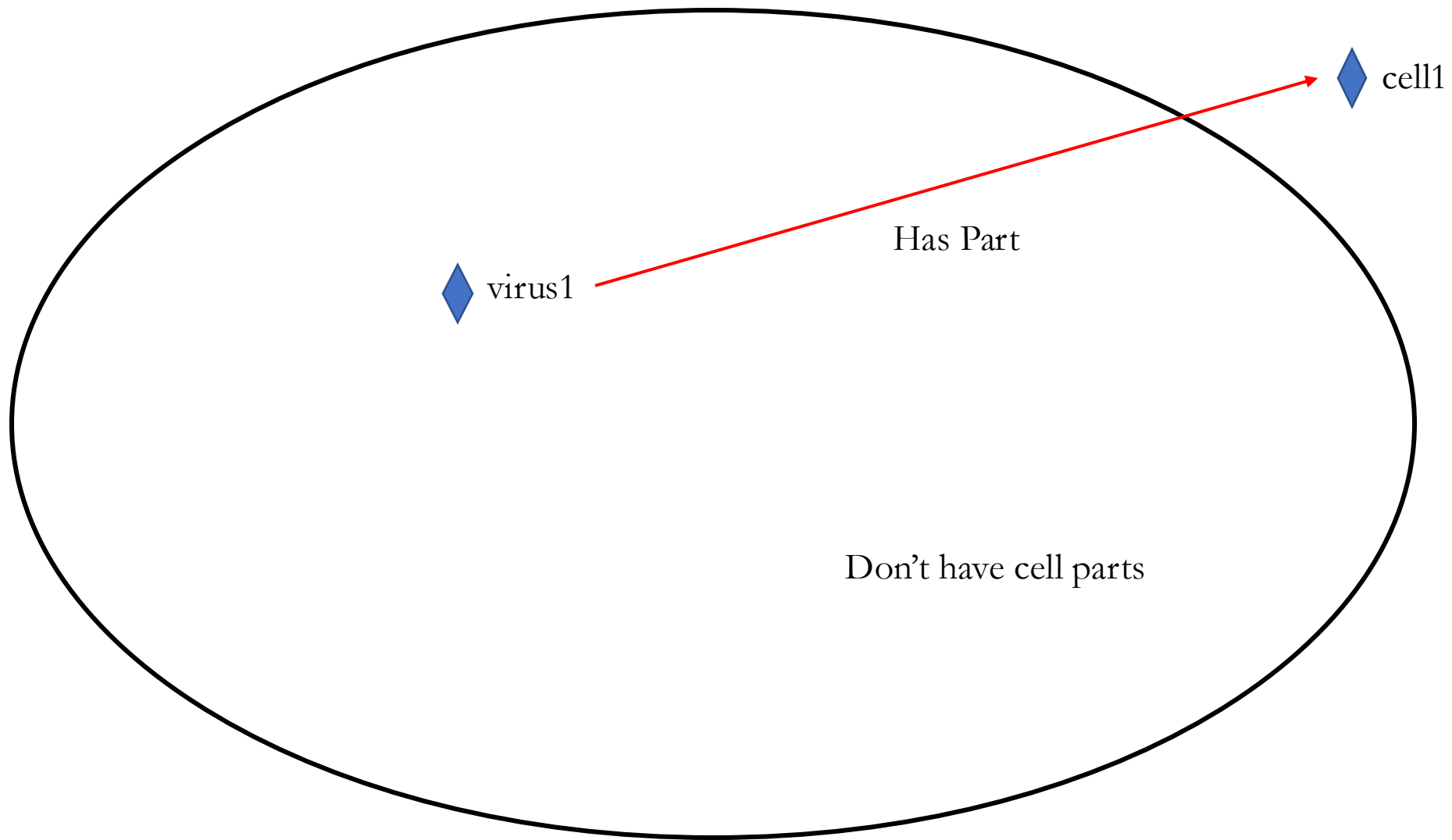
Individuals

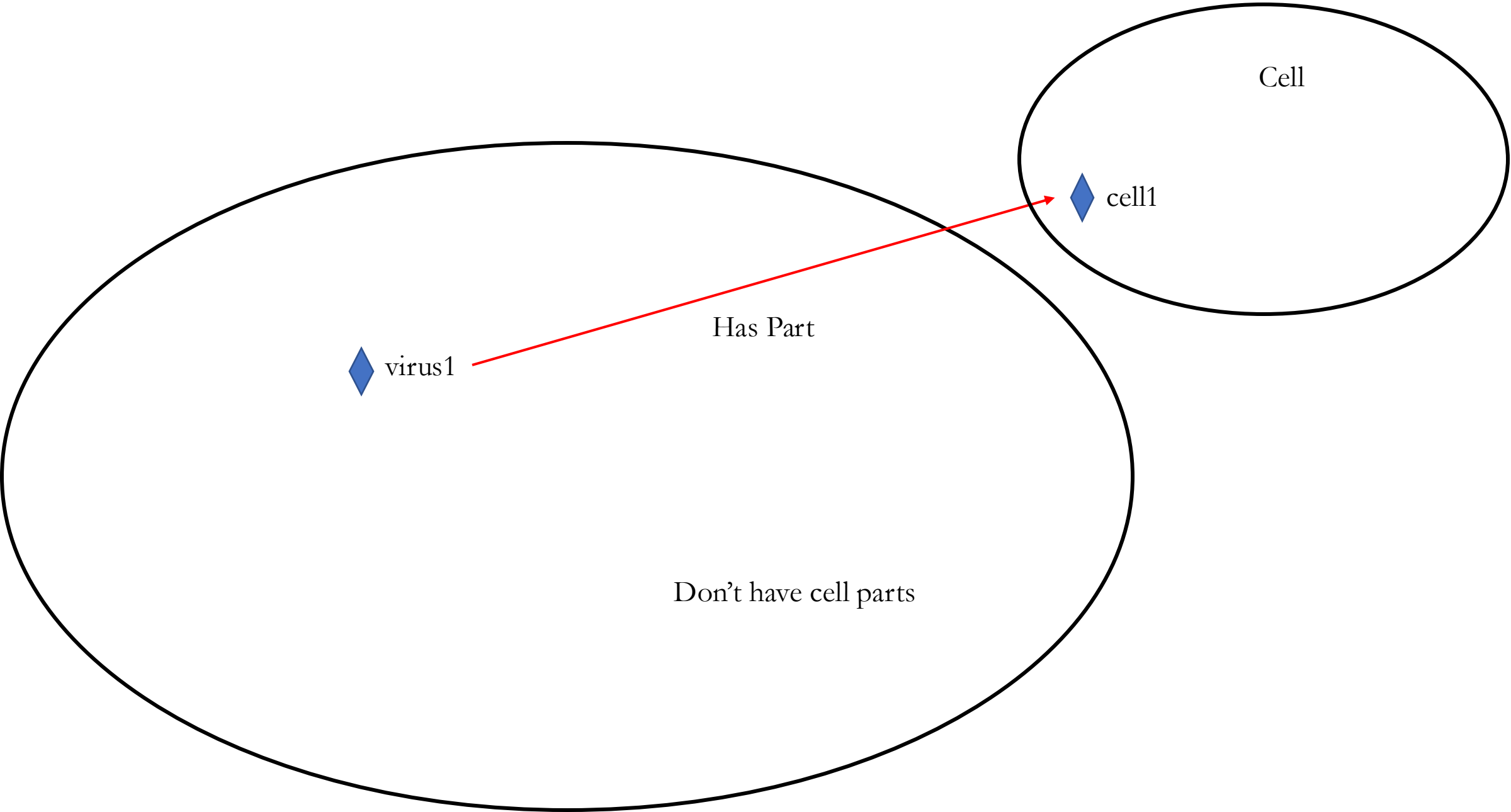
- I'm going to introduce an individual 'virus1' that is a virus
- And an individual 'cell1' that is a cell
- I do this in the interest of showing you how protégé will check your work for you, and protect you from introducing inconsistent items to the ontology
- Specifically, I'll assert (falsely) that virus1 has cell1 as a part...

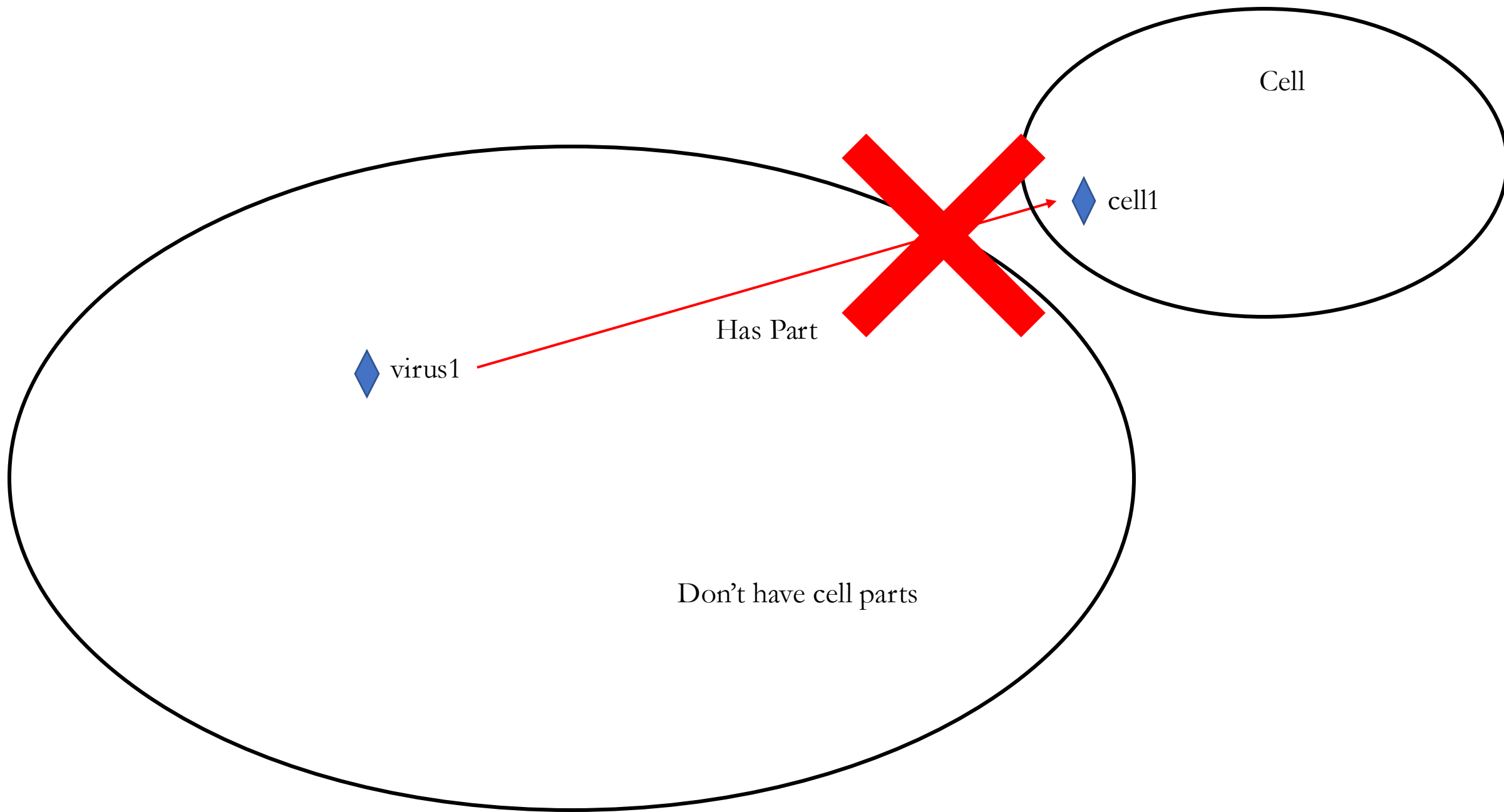












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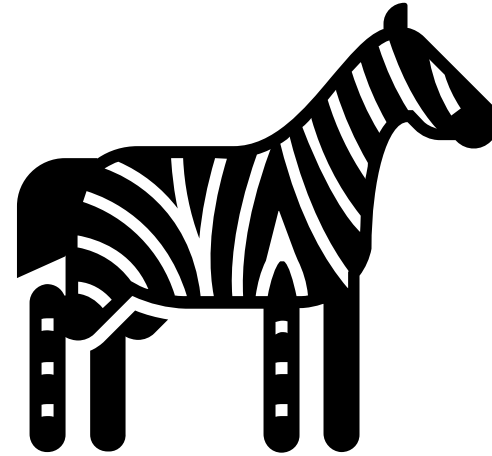
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Zebra Puzzle

1. There are five houses.
2. The Englishman lives in the red house.
3. The Spaniard owns the dog.
4. Coffee is drunk in the green house.
5. The Ukrainian drinks tea.
6. The green house is immediately to the right of the ivory house.
7. The Old Gold smoker owns snails.
8. Kools are smoked in the yellow house.
9. Milk is drunk in the middle house.
10. The Norwegian lives in the first house.
11. The man who smokes Chesterfields lives in the house next to the man with the fox.
12. Kools are smoked in a house next to the house where the horse is kept.
13. The Lucky Strike smoker drinks orange juice.
14. The Japanese man smokes Parliaments.
15. The Norwegian lives next to the blue house.

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WHO OWNS THE ZEBRA?

[rdfs:comment](#) [language: en]

- Note 1: Each house is painted exactly one color; each house is painted a different color.
- Note 2: Each house has exactly one human occupant of distinct nationality and exactly one distinct pet is owned by that human.
- Note 3: Each human occupant drinks exactly one distinct beverage and smokes exactly one distinct brand of cigarettes.
-

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Zebra Puzzle

Classes allow you to represent uncertainty; you know that there are five men, you are trying to uncover instance-level facts about them

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- ☰ Beverage
- ☰ Cigarette
- ☰ Color
- ☰ House
- ☰ Man
- ☰ Pet

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 **color_of**
 **drinks**
 **drunk_by**
 **has_color**
 **home_of**
 **left_of**
 **lives_in**
 **owned_by**
 **owns**
 **right_of**
 **smoked_by**
 **smokes**

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We do this by representing
logical relationships between
instances of classes using object
properties

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12. Kools are smoked in a house next to the house where the horse is kept.
13. The Lucky Strike smoker drinks orange juice.
14. The Japanese man smokes Parliaments.
15. The Norwegian lives next to the blue house.

 **color_of**
 **drinks**
 **drunk_by**
 **has_color**
 **home_of**
 **left_of**
 **lives_in**
 **owned_by**
 **owns**
 **right_of**
 **smoked_by**
 **smokes**

Zebra Puzzle

1. There are five houses.
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If R is the inverse of S, then for
any $\langle x, y \rangle$ in R, $\langle y, x \rangle$ is in S

Inverse Of +

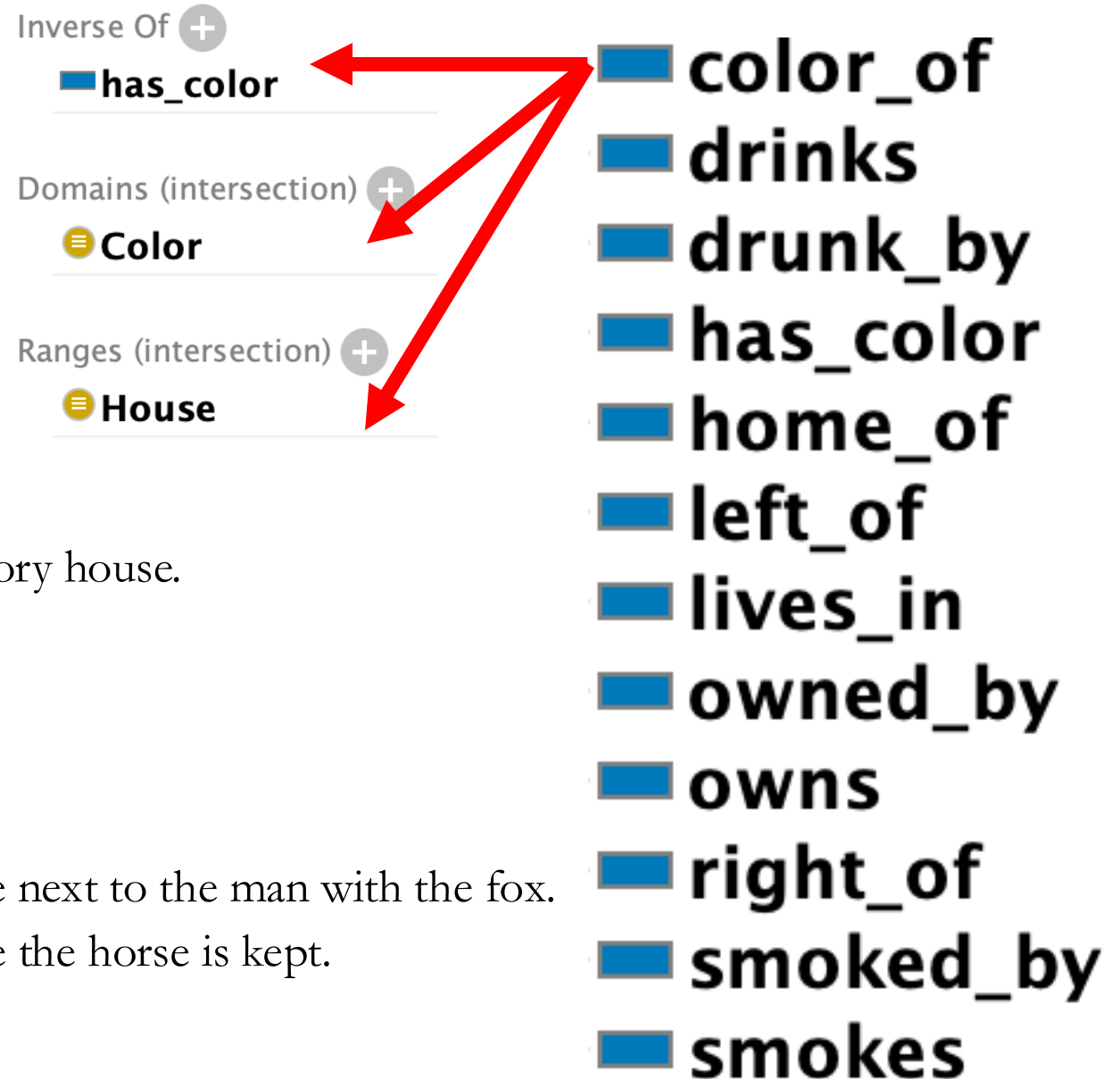
has_color



color_of
drinks
drunk_by
has_color
home_of
left_of
lives_in
owned_by
owns
right_of
smoked_by
smokes

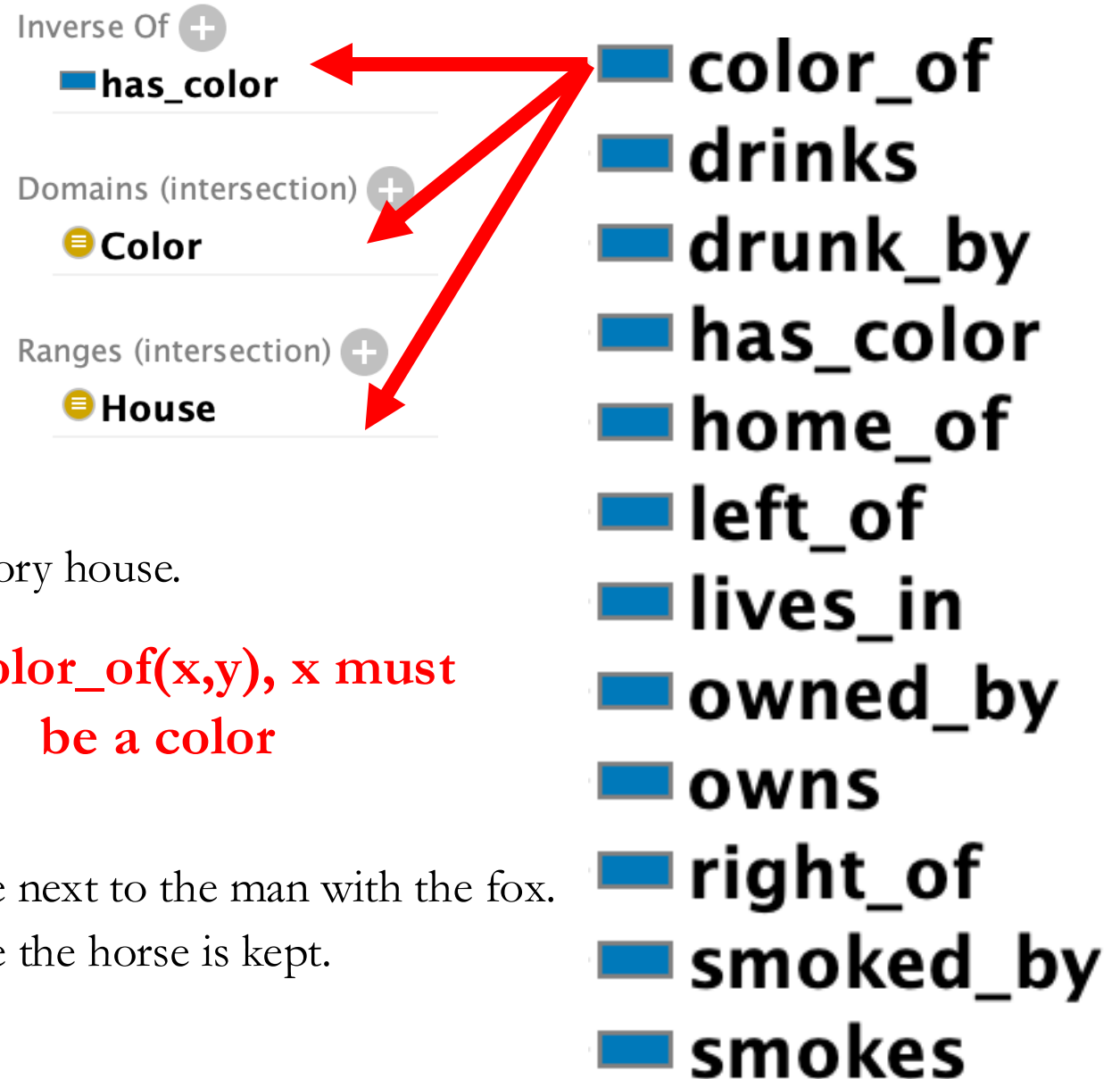
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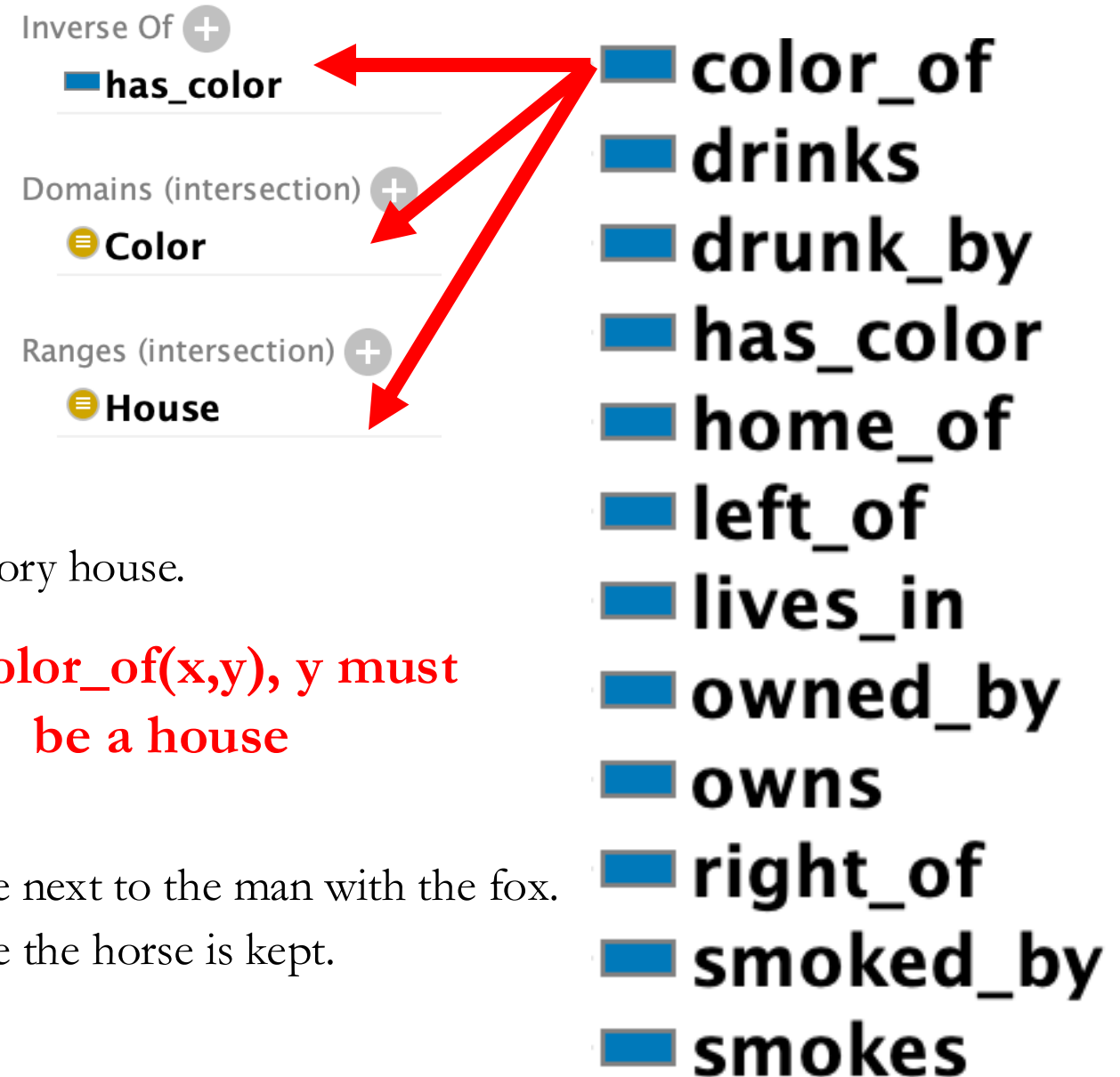
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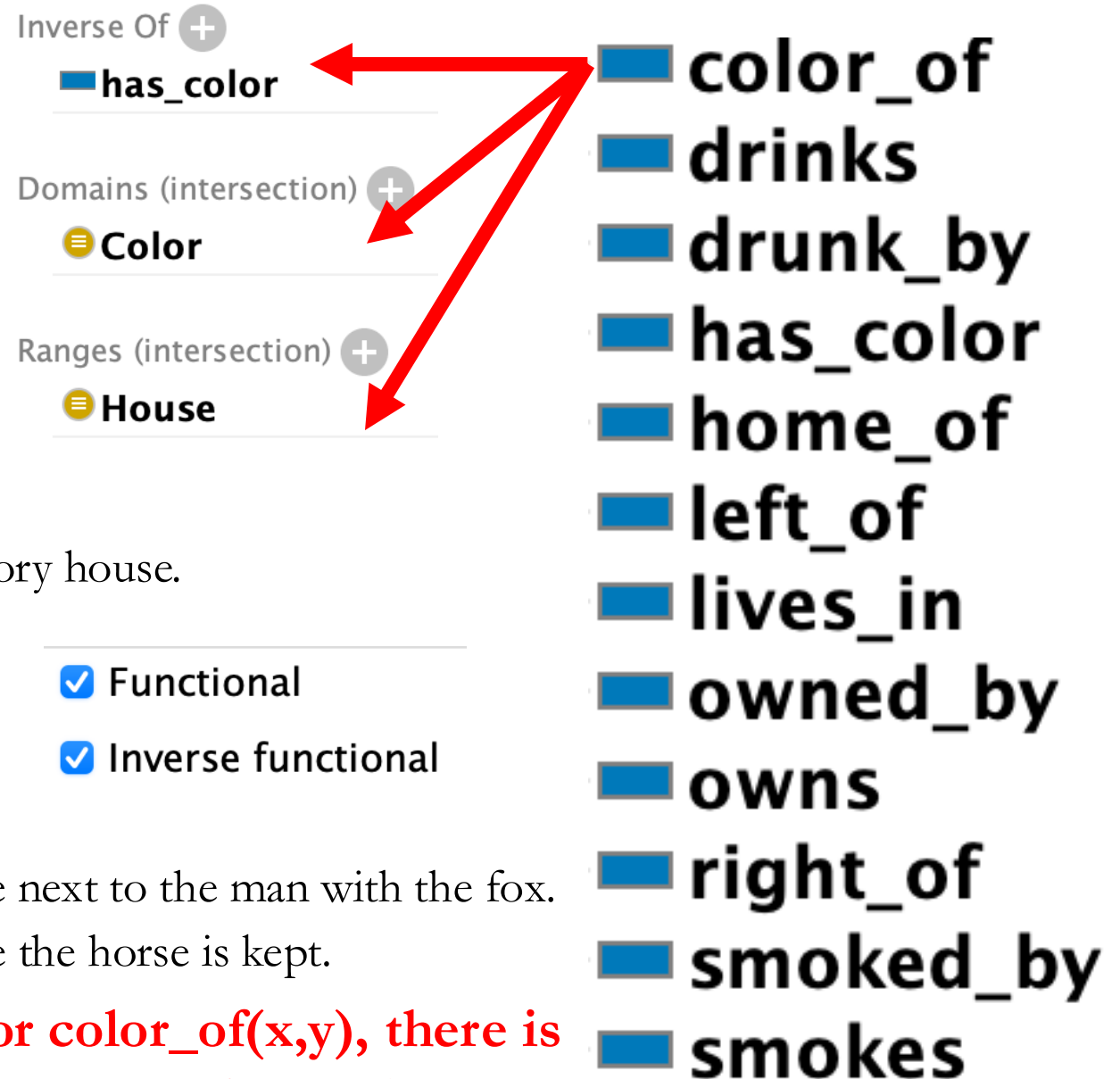
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**For color_of(x,y), there is
a 1-1 relationship
between color and house**

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 **drunk_by**
 **has_color**
 **home_of**
 **left_of**
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Zebra Puzzle

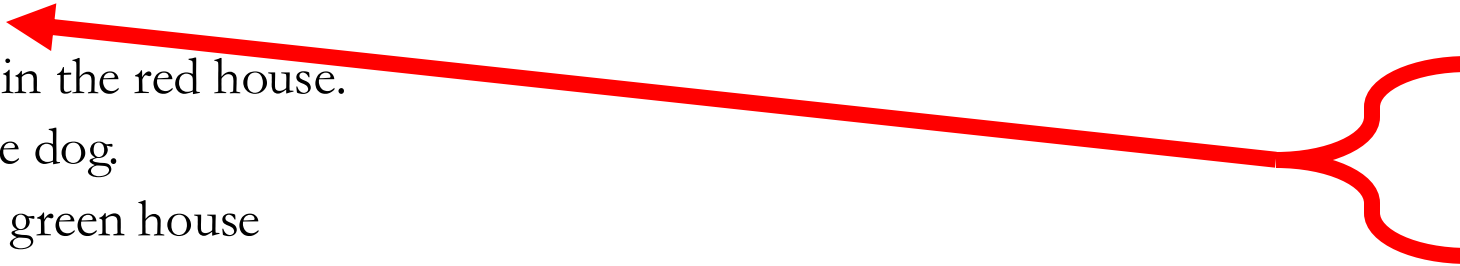
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- ◆ fox
- ◆ green
- ◆ horse
- ◆ house_1
- ◆ house_2
- ◆ house_3
- ◆ house_4
- ◆ house_5
- ◆ ivory
- ◆ japanese
- ◆ kools
- ◆ lucky_strikes
- ◆ milk
- ◆ norwegian
- ◆ old_gold
- ◆ orange_juice
- ◆ parliaments
- ◆ red
- ◆ snail
- ◆ spaniard
- ◆ tea
- ◆ ukrainian
- ◆ water
- ◆ yellow
- ◆ zebra

Zebra Puzzle

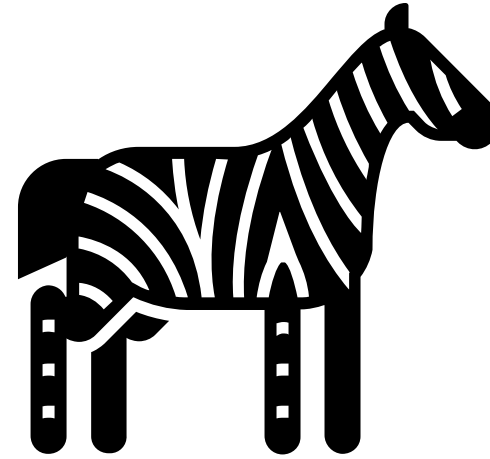
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◆ house_4
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- ◆ red
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- ◆ spaniard
- ◆ tea
- ◆ ukrainian
- ◆ water
- ◆ yellow
- ◆ zebra

Zebra Puzzle

☰ Color

● color_of some (left_of some (has_color value ivory))

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- ◆ water
- ◆ yellow
- ◆ zebra

Zebra Puzzle

● color_of some (right_of some (has_color value ivory))



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green is the color of some x

- ◆ blue
- ◆ chesterfields
- ◆ coffee
- ◆ dog
- ◆ englishman
- ◆ fox
- ◆ green
- ◆ horse
- ◆ house_1
- ◆ house_2
- ◆ house_3
- ◆ house_4
- ◆ house_5
- ◆ ivory
- ◆ japanese
- ◆ kools
- ◆ lucky_strikes
- ◆ milk
- ◆ norwegian
- ◆ old_gold
- ◆ orange_juice
- ◆ parliaments
- ◆ red
- ◆ snail
- ◆ spaniard
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- ◆ ukrainian
- ◆ water
- ◆ yellow
- ◆ zebra

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...and since the domain of
color_of is colors and the
range is houses, it follows
that x is a house

- ◆ blue
- ◆ chesterfields
- ◆ coffee
- ◆ dog
- ◆ englishman
- ◆ fox
- ◆ green
- ◆ horse
- ◆ house_1
- ◆ house_2
- ◆ house_3
- ◆ house_4
- ◆ house_5
- ◆ ivory
- ◆ japanese
- ◆ kools
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- ◆ milk
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- ◆ parliaments
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- ◆ water
- ◆ yellow
- ◆ zebra

Zebra Puzzle

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- green is the color of some house

- ◆ blue
- ◆ chesterfields
- ◆ coffee
- ◆ dog
- ◆ englishman
- ◆ fox
- ◆ green
- ◆ horse
- ◆ house_1
- ◆ house_2
- ◆ house_3
- ◆ house_4
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- ◆ ivory
- ◆ japanese
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- ◆ lucky_strikes
- ◆ milk
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- ◆ orange_juice
- ◆ parliaments
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- ◆ spaniard
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- Where that house is to the right of some x

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- ◆ chesterfields
- ◆ coffee
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- ◆ englishman
- ◆ fox
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- ◆ horse
- ◆ house_1
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Inverse Of +

■ left_of

Domains (intersection) +

■ House

Ranges (intersection) +

■ House

☒ Functional

☒ Inverse functional

■ color_of
■ drinks
■ drunk_by
■ has_color
■ home_of
■ left_of
■ lives_in
■ owned_by
■ owns
■ right_of
■ smoked_by
■ smokes

Zebra Puzzle

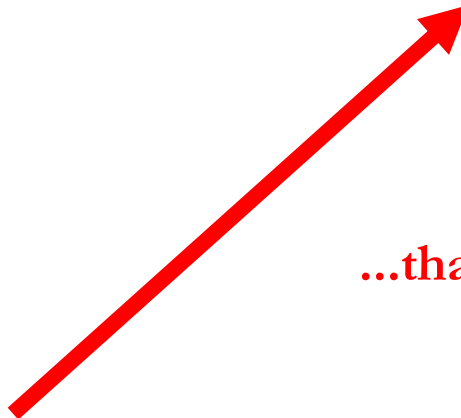
● color_of some (right_of some (has_color value ivory))

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- Where that house is to the right of some house

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- ◆ chesterfields
- ◆ coffee
- ◆ dog
- ◆ englishman
- ◆ fox
- ◆ green
- ◆ horse
- ◆ house_1
- ◆ house_2
- ◆ house_3
- ◆ house_4
- ◆ house_5
- ◆ ivory
- ◆ japanese
- ◆ kools
- ◆ lucky_strikes
- ◆ milk
- ◆ norwegian
- ◆ old_gold
- ◆ orange_juice
- ◆ parliaments
- ◆ red
- ◆ snail
- ◆ spaniard
- ◆ tea
- ◆ ukrainian
- ◆ water
- ◆ yellow
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Zebra Puzzle

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 2. The Englishman lives in the red house.
 3. The Spaniard owns the dog.
 4. Coffee is drunk in the green house
 5. The Ukrainian drinks tea.
 6. The green house is immediately to the right of the ivory house.
 7. The Old Gold smoker owns snails.
 8. Kools are smoked in the yellow house.
 9. Milk is drunk in the middle house.
 10. The Norwegian lives in the first house.
 11. The man who smokes Chesterfields lives in the house next to the man with the fox.
 12. Kools are smoked in a house next to the house where the horse is kept.
 13. The Lucky Strike smoker drinks orange juice.
 14. The Japanese man smokes Parliaments.
 15. The Norwegian lives next to the blue house.
- 
- ...that is ivory

- ◆ blue
- ◆ chesterfields
- ◆ coffee
- ◆ dog
- ◆ englishman
- ◆ fox
- ◆ green
- ◆ horse
- ◆ house_1
- ◆ house_2
- ◆ house_3
- ◆ house_4
- ◆ house_5
- ◆ ivory
- ◆ japanese
- ◆ kools
- ◆ lucky_strikes
- ◆ milk
- ◆ norwegian
- ◆ old_gold
- ◆ orange_juice
- ◆ parliaments
- ◆ red
- ◆ snail
- ◆ spaniard
- ◆ tea
- ◆ ukrainian
- ◆ water
- ◆ yellow
- ◆ zebra

Zebra Puzzle

● color_of some (right_of some (has_color value ivory))

1. There are five houses.
 2. The Englishman lives in the red house.
 3. The Spaniard owns the dog.
 4. Coffee is drunk in the green house
 5. The Ukrainian drinks tea.
 6. The green house is immediately to the right of the ivory house.
 7. The Old Gold smoker owns snails.
 8. Kools are smoked in the yellow house.
 9. Milk is drunk in the middle house.
 10. The Norwegian lives in the first house.
 11. The man who smokes Chesterfields lives in the house next to the man with the fox.
 12. Kools are smoked in a house next to the house where the horse is kept.
 13. The Lucky Strike smoker drinks orange juice.
 14. The Japanese man smokes Parliaments.
 15. The Norwegian lives next to the blue house.
- Altogether, green is the color of some house x that is to the right of some house y that has color ivory
-

- ◆ blue
- ◆ chesterfields
- ◆ coffee
- ◆ dog
- ◆ englishman
- ◆ fox
- ◆ green
- ◆ horse
- ◆ house_1
- ◆ house_2
- ◆ house_3
- ◆ house_4
- ◆ house_5
- ◆ ivory
- ◆ japanese
- ◆ kools
- ◆ lucky_strikes
- ◆ milk
- ◆ norwegian
- ◆ old_gold
- ◆ orange_juice
- ◆ parliaments
- ◆ red
- ◆ snail
- ◆ spaniard
- ◆ tea
- ◆ ukrainian
- ◆ water
- ◆ yellow
- ◆ zebra