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VOLTAGE LIFTS OF GRAPHS FROM A CATEGORY THEORY VIEWPOINT

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ABSTRACT. We prove that the notion of a voltage graph lift comes from an adjunction between the category of voltage graphs and the category of group labeled graphs.

1. Introduction

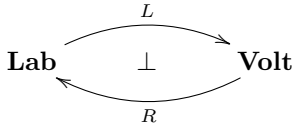
In this paper, a *graph* means a structure sometimes called a *symmetric multidigraph* – that means that it may have multiple darts with the same source and target, and the set of all darts of the graph is equipped with an involutive mapping λ that maps every dart to a dart with source and target swapped.

A *voltage graph* is a graph in which every dart is labeled with an element of a group in a way that respects the involutive symmetry λ, so that the label of a dart d is inverse to the label of $\lambda(d)$. Similarly, a *group labeled graph* has all vertices labeled with elements of a group.

In [8], Gross introduced the construction of a *derived graph of a voltage graph*. Nowadays, derived voltage graphs are called *(ordinary) voltage graph lifts* – this is the terminology we will use in the present paper. Let us mention in passing that in [9], voltage graphs were generalized to a more general notion of *permutation voltage graphs*, in which the darts are labelled with permutations.

After their discovery, voltage graph lifts were extensively investigated in many papers. Voltage graph lifts were applied for example in the research concerning the degree-diameter problem [3, 4], lifting graph automorphisms [15] and several other areas of graph theory.

In the present paper, we prove that there is an adjunction



between the category **Volt** of voltage graphs and a category **Lab** of group labeled graphs. We prove that for every object G of **Volt**, the underlying graph of the voltage graph $LR(G)$ is isomorphic to the voltage graph lift of G.

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