1. Preliminaries

1.1. Notation.

- **EA** is category of effect algebras.
- **Conv** is the category of convex sets (whatever that is).
- $\Sigma : \mathbf{EA} \to \mathbf{Conv}^{op}$ is the state space functor.
- $E : \mathbf{Conv}^{op} \to \mathbf{EA}$ is the effects functor.

1.2. The adjunction. Let A be an effect algebra. We write $\Sigma(A)$ for the convex set consisting of all states on A, equipped with convex combinations defined pointwise. If $f : A \to B$ is a morphism of effect algebras, then $\Sigma(f) : \Sigma(B) \to \Sigma(A)$ is given by the rule $\Sigma(f)(s) = s \circ f$. The mapping $\Sigma(f)(s)$ is clearly a state on A and, whenever s, s' are two states on B, $\theta \in [0,1]$ and $a \in A$,

$$\begin{aligned}\big(\Sigma(f)\big(\theta s + (1-\theta)s'\big)\big)(a) = \\ \big(\big(\theta(s) + (1-\theta)s'\big) \circ f\big)(a) = \big(\big(\theta(s) + (1-\theta)s'\big)\big(f(a)\big)\big) = \\ \theta\big(s\big(f(a)\big)\big) + (1-\theta)\big(s'\big(f(a)\big)\big) = \theta\big((s \circ f)(a)\big) + (1-\theta)\big((s' \circ f)(a)\big) = \\ \theta\big(\big(\Sigma(f)(s)\big)(a)\big) + (1-\theta)\big(\big(\Sigma(f)(s')\big)(a)\big)\end{aligned}$$

meaning that

$$\Sigma(f)(\theta s + (1-\theta)s') = \theta(\Sigma(f)(s)) + (1-\theta)(\Sigma(f)(s'))$$

so $\Sigma(f)$ is a morphism in $\mathbf{Conv}^{op}$.

Let K be a convex set. An effect $\phi : K \to [0,1]$ on K is an affine mapping (that means, a morphism in **Conv**) into a line segment $[0,1]$. Clearly, the set of all effects on K (denoted by $E(K)$) can be equipped with a pointwise partial addition inherited from the effect algebra $[0,1]$. In detail, if $\phi, \psi \in E(K)$, then $\phi \oplus \psi$ exists in $E(K)$ if and only if, for all points $x \in X$, $\phi(x) + \psi(x) \leq 1$ and then $(\phi \oplus \psi)(x) = \phi(x) \oplus \psi(x)$.

If $f : K \to K'$ is a morphism in **Conv**, then $E(f) : E(K') \to E(K)$ is given by the rule $E(f)(\phi) = \phi \circ f$. Let us prove that this is indeed a morphism of effect algebras. Suppose that $\phi \oplus \psi$ exists in $E(K')$. Then, for all $x \in K$,

$$(E(f)(\phi \oplus \psi))(x) = ((\phi \oplus \psi) \circ f)(x) = (\phi \oplus \psi)(f(x)) = \phi(f(x)) + \psi(f(x)) \leq 1,$$

because $\phi \oplus \psi$ exists in $E(K')$. Thus,

$$\phi(f(x)) + \psi(f(x)) = (\phi \circ f)(x) + (\psi \circ f)(x) = ((E(f))(\phi))(x) + ((E)(f))(\psi)(x) \leq 1$$

Therefore, $E(f)(\phi) \oplus E(f)(\psi)$ exists in $E(K)$ and $E(f)(\phi) \oplus E(f)(\psi) = E(f)(\phi \oplus \psi)$. Moreover, it is easy to see that $E(f)(1) = 1$, so $E(f)$ is a morphism of effect algebras.