VE281 Writing Assignment One

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Ex. 1

The strategy is:

Input: maximum integer N
Output: correct integer x

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begin ← 1, end ← N
while True do
    x ← ⌊(begin + end)/2⌋
    result ← make a guess with x
    if result = "equal to" then break
    else if result = "less then" then end ← x − 1
    else begin ← x + 1
    end if
end while
```

In the worst case, I never directly guess the correct number in the process of the strategy. After a guess of N, the next range of numbers become $(N-1)/2$. When $N = 2^m - 1$, the last guess will contain $2^2 - 1 = 3$ numbers, and I'll get the correct one after the guesses. So the number of guess in the worst case is $m - 1$.

In the average case, if there are M numbers left, I've got the probability of $1/M$ to guess the number directly. Let the probability of guessing n times be P_n,

$$P_1 = \frac{1}{2^m - 1}$$

$$P_n = \frac{1 - P_{n-1}}{2^{m-n+1} - 1} = \frac{2^{n-1}}{2^m - 1}$$

Then we can get the equation

$$T_m = \sum_{n=1}^{m-1} nP_n = \frac{1 \cdot 2^0 + 2 \cdot 2^1 + \cdots + n \cdot 2^{m-2}}{2^m - 1} = \frac{(m-2)2^{m-1} + 1}{2^m - 1}$$

Ex. 2

In the best situation, if $n = 2^m$, only cm steps (c is a constant) is necessary. So $f(n) = \log n$.

Ex. 3

$$\lim_{n\to\infty}\frac{n^{100}}{1.001^n}=\lim_{n\to\infty}\frac{100n^{99}}{1.001^n\ln 1.001}=\cdots=\lim_{n\to\infty}\frac{100!}{1.001^n\ln^{100}1.001}=0$$

So $n^{100}=O(1.001^n)$ is false.

Ex. 4

Since $f_1(n)=\Theta(g_1(n)),\ f_2(n)=\Theta(g_2(n))$, we know

$$c_1g_1(n)\leqslant f_1(n)\leqslant c_2g_1(n), n\geqslant n_1$$

$$c_3g_2(n)\leqslant f_2(n)\leqslant c_4g_2(n), n\geqslant n_2$$

Then

$$c_1g_1(n)+c_3g_2(n)\leqslant f_1(n)+f_2(n)\leqslant c_2g_1(n)+c_4g_2(n), n\geqslant \max\{n_1,n_2\}$$

Now we should find $h(n)$ so that

$$\lim_{n\to\infty}\frac{c_1g_1(n)+c_3g_2(n)}{h(n)}=C_1$$

$$\lim_{n\to\infty}\frac{c_2g_2(n)+c_4g_2(n)}{h(n)}=C_2$$

So

$$h(n)=\max\{g_1(n),g_2(n)\}$$

Ex. 5

The outer loop will be called $\lfloor 1+\log_a n\rfloor$ times, in k^{th} loop, $i=a^k$, where $k\in\Big[0,\lfloor\log_a n\rfloor\Big]\cap Z$. The inner loop will be called ib times in each loop.

$$T=b\sum_{k=0}^{\lfloor\log_a n\rfloor}a^k=\frac{b\left(1-a^{\lfloor\log_a n\rfloor}\right)}{1-a}$$

Ex. 6

(a)

$$T_1(n)=\lceil n/2\rceil$$

$$T_1(n)=\lceil n/20\rceil$$

$$T_3(n)=\lceil 1+\log_2 n\rceil$$

(b)

$$C_1(n)=4n+2\lceil n/2\rceil$$

$$C_2(n)=10+4n+\lceil n/20\rceil$$

$$C_3(n)=40+4n+2\lceil 1+\log_2 n\rceil$$

(c) Method 1 is the cheapest, it costs $40.

(d) Method 2 is the cheapest, it costs $529.

(e) Method 3 is the cheapest, it costs $8256.