Modelling of Complex Systems

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Project 2

One-dimensional random walks

Part 1 — One-dimensional symmetrical Random Walks

Perform simulations of random walks of a particle on a one-dimensional chain. The probabilities of jumping to the left p and to the right q are equal, $p = q = 0.5$.

Task 1.1. At $t = 0$ the particle starts the walk at $x = 0$ and makes a total of $t_f = 50$ jumps.

Plot the trajectories of three random walks, i.e., plot the position x of the particle as a function of time t, from $t = 0$ to $t = t_f$ (we define time t as the number of jumps):

$$x(t) = \sum_{i=1}^{t} S_i$$

Here $S_i = \pm 1$ is the random jump made by the particle at step i.

Task 1.2. Compute the probability $P(x, t_f)$ of finding the particle at site x at a given time t_f. The particle starts the walk (at $t = 0$) from site $x = 0$. Use (i) $t_f = 40, 41$; (ii) $t_f = 400, 401$; (iii) $t_f = 4000, 4001$, then average over even and odd t_f and find the averaged probability $\bar{P}(x, t_f)$. Compare the distribution obtained experimentally from your simulations with the theoretical distribution function:

$$P(x,t) = \frac{1}{\sqrt{2\pi t}} e^{-\frac{x^2}{2t}}$$

Algorithm:

1. For each time t_f generate $N = 50\,000$ trajectories (or more, if possible, to get batter statistics). Find $P(x, t_f) = N(x, t_f)/N$, where $N(x, t_f)$ is the number of trajectories that finish at point x. Average over even and odd times:

   $$\bar{P}(x,t) = \frac{1}{2}[P(x,t) + P(x,t+1)].$$

   Plot the averaged distribution function $\bar{P}(x,t)$. Compare with the theoretical distribution function $P(x,t)$.

2. Check that

   $$\sum_x \bar{P}(x,t) = 1, \quad \langle x \rangle = \sum_x x\bar{P}(x,t) = 0, \quad \text{and} \quad \langle (x - \langle x \rangle)^2 \rangle = \sum_x (x - \langle x \rangle)^2 \bar{P}(x,t) = t.$$

3. Plot on the same graph your results for (i)-(iii).

Part 2 — One-dimensional Random Walks with a drift

Task 2.1. Simulate random walks of a particle on a one-dimensional chain with asymmetric jump probabilities. The probabilities to jump on the left and on the right equal to $p = 0.5 - \delta$ and $q = 0.5 + \delta$, respectively. Take $\delta = 0.015$. Compute the probability $P(x, t_f)$ to find this particle after time t_f (i.e., after t_f jumps) at site x. At $t = 0$ the particle starts the walk at $x = 0$. Use (i) $t_f = 40, 41$; (ii) $t_f = 400, 401$; (iii) $t_f = 4000, 4001$, then average over even and odd t_f and find the averaged probabilities $\bar{P}(x, t)$. Compare the distribution obtained experimentally with the theoretical distribution function:

$$P(x, t) = \frac{1}{\sqrt{2\pi t}} e^{-\frac{(x-2t\delta)^2}{2t}}.$$

Algorithm:

1. For each time t_f generate $N = 50\,000$ trajectories (or more, if possible, to get batter statistics). Find $P(x, t_f) = N(x, t_f)/N$, where $N(x, t_f)$ is the number of trajectories that finish at point x. Then average over even and odd times:

   $$\bar{P}(x, t) = \frac{1}{2}[P(x, t) + P(x, t + 1)].$$

   Plot the averaged distribution function $\bar{P}(x, t)$. Compare with the theoretical distribution function $P(x, t)$.
2. Check that

   $$\sum_x \bar{P}(x, t) = 1, \quad \langle x \rangle = \sum_x x\bar{P}(x, t) = 2t\delta, \quad \text{and} \quad \langle (x - \langle x \rangle)^2 \rangle = \sum_x (x - \langle x \rangle)^2 \bar{P}(x, t) = t.$$
3. Plot on the same graph your results for (i)-(iii).