Modelling of Complex Systems

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Project 5

Complex networks

Erdős-Rényi networks

Generate Erdős-Rényi networks as follows. Start with N isolated nodes, and then insert $L = \frac{N}{2}c$ edges between pairs of randomly chosen nodes. Make sure that each edge connects two distinct nodes (no self-loops are allowed) and that there are no multiple edges connecting the same pair of nodes.
We can store and represent the network using the adjacency matrix $\boldsymbol{A}$. Start with a matrix $\boldsymbol{A}$ of 0's, then, when an edge is added between nodes i and j replace the entries A_{ij} and A_{ji} by a 1 (we can also use the adjacency matrix to avoid connecting the same pair of nodes more than once).
The degree of vertex i is defined as

$$q_i = \sum_{j=1}^{N} A_{ij}.$$

The degree distribution is defined as

$$P(q) = \frac{N(q)}{N},$$

where $N(q)$ is the number of vertices of degree q.

Task 1 — Calculate the mean degree $\langle q \rangle$, the branching coefficient B, and second and third moments:

$$\langle q \rangle = \frac{1}{N}\sum_{i=1}^{N} q_i = \sum_{q=0}^{N-1} qP(q),$$

$$B = \frac{1}{N\langle q \rangle}\sum_{i=1}^{N} q_i(q_i - 1) = \frac{1}{\langle q \rangle}\sum_{q=0}^{N-1} P(q)q(q-1) = \frac{\langle q(q-1) \rangle}{\langle q \rangle},$$

$$\langle q^2 \rangle = \frac{1}{N}\sum_{i=1}^{N} {q_i}^2,$$

$$\langle q^3 \rangle = \frac{1}{N}\sum_{i=1}^{N} {q_i}^3.$$

Show that $B/_{\langle q \rangle} \approx 1$.

Average the data over m realizations of the random graph. Compare with the theoretical results:

$$P(q) = e^{-c}\frac{c^q}{q!},$$

$$\langle q \rangle = c,$$

$$B = c,$$

Task 2 — Calculate the clustering coefficient

$$C = \frac{n_{tr}}{n_{pt}}.$$

Here $n_{pt} = \frac{1}{6}N\langle q(q-1)\rangle$ is the number of possible triangles, and n_{tr} is the number of triangles in the network:

$$n_{tr} = \frac{1}{6}\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N} A_{ij}A_{jk}A_{ki}.$$

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Show that $C \approx \langle q\rangle / N$.

Task 3 — Calculate the Pearson coefficient that characterizes degree-degree correlations in the network:

$$\rho = \frac{\sum_{i=1}^{N}\sum_{j=1}^{N} A_{ij}(q_i - Q)(q_j - Q)}{N\langle q\rangle\sigma^2},$$

where

$$Q = \frac{\langle q^2\rangle}{\langle q\rangle} \qquad \text{and} \qquad \sigma^2 = \frac{\langle q^3\rangle}{\langle q\rangle} - \frac{\langle q^2\rangle^2}{\langle q\rangle^2}.$$

Show that the Pearson coefficient is small.

In your report describe the network based on the degree distribution, the clustering coefficient, and the Pearson coefficient.

Parameters for simulations:
N =10000, c =50, m =100.

Algorithm:
1. Initialize a $N \times N$ matrix $\boldsymbol{A}$ with all entries set to 0.
2. For each edge generate two integers i and j uniformly at random between 1 and N.
3. If $i = j$ or $A_{ij} = 1$ repeat step 2, otherwise proceed to the next step.
4. Update matrix $\boldsymbol{A}$ with the new edge by setting $A_{ij} = A_{ji} = 1$.
5. Repeat steps 2-4 L times.
6. Calculate degrees, $q_i = \sum_{j=1}^{N} A_{ij}$.
7. Calculate the number of vertices with degree q, i.e., $N(q)$, and then find degree distribution $P(q)$.
8. Repeat steps 1-7 m times and average the degree distribution $P(q)$ over the m realizations of the network.
9. Calculate the mean degree $\langle q\rangle = \sum_{q=1}^{N-1} qP(q)$.
10. Calculate the branching coefficient,
$$B = \frac{1}{\langle q\rangle}\sum_{q=0}^{N-1} P(q)q(q-1).$$
11. Calculate second and third moments, $\langle q^2\rangle$ and $\langle q^3\rangle$.
12. Calculate the clustering coefficient $C = \frac{n_{tr}}{n_{pt}}$.