MATH 354: PROBLEM SET 3

RICE UNIVERSITY, FALL 2019

Due date: Friday, September 20th, by 5pm in my office (you can slide it under the door). You are welcome to turn in your work during lecture on Friday.

Parts A, and B should be handed in **separately**. They will be graded by different TAs.

Please staple your homework!

Reminder from the syllabus: "The homework is not pledged and you can collaborate with other students in the class. In fact, you are very much *encouraged* to do so. However, you are not allowed to look up solutions in any written form; in particular, you are not allowed to look up solutions online. **Students caught violating this rule will be reported to the Honor Council.** You should write up your solutions individually."

1. Part A

Hand in the following exercises from Chapter 1 of Axler's book:

2A: 11, 14, 15.   2B: 2(a)+(f)+(g), 5.

2. Part B

Hand in the following exercises from Chapters 1 and 2 of Axler's book:

2B: 3, 8.   2C: 1, 3, 8

3. Comments

(1) Problem 2A.15 is best solved by applying the result of Problem 2A.14. For the latter, the Linear Dependence Lemma and 2.23 should help.

(2) The result of Problem 2C.1 is extremely useful. We will use this over and over in the course. I remember it by the slogan "a subspace of full dimension is the entire space." This slogan is only good inasmuch as it makes you recall the precise statement of this problem!

Date: September 13th, 2019.

(3) Now that we have the concept of basis and dimension in hand, Problem 2C.3 is not meant to be long and difficult. You just have to run through the possibilities for the dimension of a subspace U of $\mathbb{R}^3$, e.g., if $\dim U = 1$, then a basis of U consists of a single vector, so U is the set of scalar multiples of this vector, i.e., a line through the origin. If $\dim U = 2$, then a basis of U consists of two vectors, so U is...

(4) For 2C.8, part (a), what I would suggest is you consider a general polynomial $p(x) = ax^4 + bx^3 + cx^2 + dx + e$. The condition $\int_{-1}^{1} p(x)\,dx = 0$ imposes a relation among the coefficients a, b, c, d and e. We have 5 variables and 1 relation. Intuitively, one might thus expect that the vector space at hand is 4-dimensional. Pick 4 polynomials whose coefficients satisfy the linear relation you have uncovered, and *show* that they are linearly independent and span the space in question.