Recall that if a function f is meromorphic with a pole at z_0 of order k, then f has a Laurent expansion at z_0 of the form

$$f(z) = \sum_{n=-k}^{\infty} a_n (z - z_0)^n$$

and we refer to the partial sum

$$\sum_{n=-k}^{-1} a_n (z - z_0)^n$$

as the principal part of f at z_0.

Theorem 0.0.1: Mittag-Leffler Theorem

Let $\{z_n\} \subset \mathbb{C}$ be a sequence of distinct complex numbers with $\lim_{n\to\infty} |z_n| = \infty$. Let $\{p_n(z)\}$ be a sequence of polynomials with $p_n(0) = 0$. Then there exists a function f meromorphic on the complex plane with poles exclusively at $\{z_n\}$ whose principal part at each z_n is $p_n(\frac{1}{z-z_n})$. In fact, every meromorphic function f with poles at $\{z_n\}$ is of the form

$$f(z) = \sum_{n=1}^{\infty} \left[p_n \left(\frac{1}{z - z_n} \right) - q_n(z) \right] + \varphi(z)$$

for some polynomials $\{q_n\}$ and an entire function φ. The sum converges absolutely and uniformly on all $K \subset \mathbb{C}$ compact with $K \cap \{z_n\} = \emptyset$.

Proof. ■