Math 357: Undergraduate Abstract Algebra 2.   Instructor: Dr. C. Walton

Homework #2 on lecture material from M 2/1, W 2/3, F 2/5
Due: Wednesday, February 10, 2021 at noon *sharp* *(better to submit early)*

Include full statements of problems in your solution set.
See syllabus for grading guide and teaching page for writing tips

Please include your full name at the top of your homework set.

Practice Problems (discussed during class time)

(1) [Mon]

  (a) Show that $x^4 + 10x + 5$ is irreducible over $\mathbb{Z}$.

  (b) Show that $x^6 + 30x^5 - 15x^3 + 6x - 120$ is irreducible over $\mathbb{Z}$.

  (c) Show that $x^{p-1} + x^{p-2} + \cdots + x + 1$ is irreducible over $\mathbb{Z}$ for p prime.

(2) [Wed] Goodman, Exercise 8.1.1

(3) [Fri] Let R be a ring, and let M be a left R-module. Take $\{N_i\}_{i \in I}$ to be a nonempty collection of (left R-)submodules of M.

  (a) Show that $\bigcap_{i \in I} N_i$ is a submodule of M.

  (b) Is $\bigcup_{i \in I} N_i$ is a submodule of M? Prove this statement or provide a counterexample.

Advanced Problems (to be completed outside of class time)

(A) Show that $(x - 1)(x - 2)(x - 3) \cdots (x - n) - 1$ is irreducible over $\mathbb{Z}$, for each $n \in \mathbb{N}$.

(B) Dummit-Foote, Exercise §10.1 #15

(C) Dummit-Foote, Exercise §10.1 #8

(D) Goodman, Exercise 8.2.1