A lot of geometric theorems can be visualized in the complex plane, in which the complex operations are immensely useful for proving theorems.

It is important to establish that the complex plane indeed inherits the metric properties of $\mathbb{R}^2$.

Definition 0.0.1: Scalar Product and Angles

Let $z, w \in \mathbb{C}$. The **scalar product** of z and w is defined by

$$\langle z, w \rangle := \mathrm{Re}(z\overline{w}).$$

We define the **angle between** z **and** w to be

$$\theta(z, w) := \cos^{-1}\left(\frac{\langle z, w \rangle}{|z|\,|w|}\right).$$

The angle formula is chosen so that

$$\cos\theta(z, w) = \frac{\langle z, w \rangle}{|z|\,|w|}$$
$$\sin\theta(z, w) = \frac{\langle z, -iw \rangle}{|z|\,|w|}.$$

Exercise 0.0.1

Show that the scalar product defined above indeed satisfies the necessary properties for a general scalar product.