Math 357: Undergraduate Abstract Algebra 2. Instructor: Dr. C. Walton

Homework #3 on lecture material from M 2/8, W 2/10, F 2/12

Due: ~~Friday, February 19~~ Wednesday, February 24, 2021 at noon *sharp*

(change due to Winter storm)

Include full statements of problems in your solution set.
See syllabus for grading guide and teaching page for writing tips

Please include your full name at the top of your homework set.

Practice Problems (discussed during class time)

(1) [Mon] Prove the Proposition in Lecture 7, Video 1.

(2) [Wed] Read, and write out in your own words, the proof of Goodman's Proposition 8.1.32: equivalence of (a) and (b). Then complete Goodman, Exercise 8.1.10.

(3) [Fri] Let $C_m = \langle g \mid g^m = e \rangle$ be the cyclic group of order m. For an element $A \in GL_n(\mathbb{C})$, consider the map $\rho : C_m \to GL_n(\mathbb{C})$ defined by $\rho(g^i) = A^i$, for $i = 0, \dots, m-1$. Show that ρ is a representation of G over $\mathbb{C}$ if and only if $A^m = I$.

Advanced Problems (completed outside of class time, and *can discuss in class if time permits)

(A) [Mon*] Goodman, Exercise 8.2.7 (can assume corresponding Isom. Theorem for Groups)

(B) [Mon*] Goodman, Exercise 8.2.4 (can assume corresponding Isom. Theorem for Groups)

(C) [Wed*] Goodman, Exercise 8.1.7 (see Goodman Example 8.19, can provide an example for this problem)

(D) [Fri*] (a) Let ρ be a representation of a group G of degree 1. Show that the quotient group $G/\ker\rho$ is abelian.

(b) Let (V, ρ) be a representation of a group G. Suppose that there are elements $g, h \in G$ so that the operators ρ_g and ρ_h on V commute (that is, $(\rho_g \circ \rho_h)(v) = (\rho_h \circ \rho_g)(v)$ for all $v \in V$). Do we then have that g and h commute in G?