Math 357: Undergraduate Abstract Algebra 2. Instructor: Dr. C. Walton

Homework #8 on lecture material from M 3/22, W 3/24

Due: Wednesday, March 31, 2021 at noon *sharp* *(better to submit early)*

Include full statements of problems in your solution set.
See syllabus for grading guide and teaching page for writing tips

Please include your full name at the top of your homework set.

Practice Problems (discussed during class time)

(1) [Mon] (a) Describe the cyclotomic extension $\mathbb{Q}(\zeta_8)$ in the same manner as done in the Lecture 23, Video 2 notes. Compute the degree of $\mathbb{Q}(\zeta_8)$ over $\mathbb{Q}$ without using the Euler-Phi formula, and compute the cyclotomic polynomial $\Phi_8(x)$.

(b) Repeat part (a) for $n = 12$.

(2) [Mon] Dummit-Foote, Exercise §13.6 #3

(3) [Wed] Prove that the following statements are equivalent for a field K.

(a) K is algebraically closed.

(b) Every nonconstant polynomial in $K[x]$ splits completely over K.

(c) Every irreducible polynomial in $K[x]$ has degree 1.

(d) There is no algebraic extension of the field K other than K itself.

Advanced Problems (completed outside of class time, and *can discuss in class if time permits)

(A) [Mon*] Dummit-Foote, Exercise §13.6 #6

(B) [Mon*] Dummit-Foote, Exercise §13.6 #7
(The Möbius Inversion formula is stated on page 588.)

(C) [Wed*] Let K be an extension field of F, and let E be the subfield of K consisting of all elements that are algebraic over F. Show that if K is algebraically closed, then E is the algebraic closure of F.

(Aside, not part of the problem: Think about the case when $F = \mathbb{Q}$ and $K = \mathbb{C}$.)