THE UNIVERSITY OF
NEW MEXICO

Introduction to Functional Analysis

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Office Hours:	N/A	**Class Time:**	N/A
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Course Description

Functional analysis is the study of (potentially infinite-dimensional) vector spaces and linear functions on them. We will begin by discussing the definition of Banach (normed) and Hilbert (inner product) spaces, then get into the meat of linear operators– boundedness, extensions, etc. Functional vector spaces are pretty key examples throughout this and so we will spend time constructing these examples. The goal will be to build up to spectral theory, a subfield of functional analysis that concerns with defining a more abstract form of "eigenvalue" for general vector spaces. Spectral theory proves ubiquitous because we can learn a lot about differential operators and more on function spaces by studying their spectrum.

Course Objectives

At the end of this course you should

1. Be familiar with –
2. Be able to state, understand, and apply –

Texts

Introductory Functional Analysis, Kreyszig (edition).

Applied Analysis, Hunter (edition).

Course Policies

N/A

Class Schedule

Week 1, 10/27 - 11/3: Preliminaries: Metric Spaces

Be able to prove:

- Whether a bilinear map is a metric
- Other formulations of the triangle inequality
- Whether simple metric spaces are separable
- The equivalence between closure and limits of convergent sequences
- The equivalence between topological continuous functions and sequential continuous functions
- Basic convergence properties about Cauchy and convergent sequences

Be able to state, understand, and apply:

- Definition of a metric and know some standard metrics
- Holder inequality for sums (and the Cauchy-Schwarz inequality for sums)
- Minkowski inequality for sums
- Definitions and basic properties of open/closed sets and their variants (interior, closure)
- Definition of continuity
- Definition of dense/separable sets
- Definitions and basic properties of Cauchy and convergent sequences
- Definition of completeness

Be familiar with:

- The definition of a sequence space l^p
- The definition of a function space $C[a,b]$
- Important examples of open/closed sets (X, $\varnothing$, B, $\overline{B}$, etc.)
- The proofs of non-separability of l^∞ and separability of l^p
- Important examples of dense/separable and complete spaces

Read:
Chapter 1.1-1.4 of Kreyszig (Primary)
Chapter 1 of Hunter (Supplementary)

Turn in:
Exercise 1.1 #12 of Kreyszig
Exercises 1.2 #4, #13, #15 of Kreyszig
Exercises 1.3 #8, #12 of Kreyszig
Exercise 1.4 #1 of Kreyszig

Week 2, 11/8 - 11/12: Preliminaries: More Metric Spaces and Vector Spaces

Be able to prove:

- That a given set is (not) compact, by both open covers and sequential compactness
- Facts about subsets of compact/complete/closed sets
- Facts about subspaces of vector spaces
- That a given basis is linearly (in)dependent

Be able to state, understand, and apply:

- Definition of uniform convergence
- Both definitions of compactness
- Heine-Borel Theorem
- Bolzano-Weierstrass theorem
- Theorems relating continous functions and compact sets
- Definition of vector spaces
- Definition of linear independence and bases (and dimension)
- Definition of quotient spaces

Be familiar with:

- Important examples of (in)complete spaces (with intuition as to why)
- The intuition as to how completions of metric spaces are constructed
- The proofs that $[0, 1]$ is compact and $(0, 1)$ is not compact
- The equivalence of sequential compactness and compactness in metric spaces
- Examples of finite and infinite-dimensional vector spaces (including quotient spaces)

Read:
Chapter 1.5-1.6, 2.1 of Kreyszig (Required)
Chapter 1.7 of Hunter (Required) Chapter 1 of Hunter (Optional)

Turn in:
Exercises 1.5 #13-15 of Kreyszig
Exercises 1.6 #6 of Kreyszig
Exercises 2.1 #5, #14 of Kreyszig

Week 3, 11/15 - 11/19: Preliminaries: More Metric Spaces and Vector Spaces

Be able to prove:

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Be able to state, understand, and apply:

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Be familiar with:

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Read:
Chapter 2.2-2.4 of Kreyszig (Required)
Chapter 5 of Hunter (Optional)

Turn in:
Exercises 2.2 #8, 15
Exercises 2.3 #9, 15
Exercises 2.4 #1, 6
