Math 357: Undergraduate Abstract Algebra 2. Instructor: Dr. C. Walton

Homework #9 on lecture material from M 3/29, W 3/31, F 4/2

Due: Wednesday, April 7, 2021 at noon *sharp* *(better to submit early)*

Include full statements of problems in your solution set.
See syllabus for grading guide and teaching page for writing tips

Please include your full name at the top of your homework set.

Practice Problems (discussed during class time)

(1) [Mon] Suppose that K is a separable field extension of F, and E is an intermediate field so that $F \subseteq E \subseteq K$. Prove that:
(i) K is separable over E, and
(ii) E is separable over F.

(2) [Wed] Dummit-Foote, Exercise §13.5 #11

(3) [Fri] Dummit-Foote, Exercises §13.5 #3, 4

Advanced Problems (completed outside of class time, and *can discuss in class if time permits)

(A) [Mon*] Dummit-Foote, Exercise §13.5 #5

(B) [Wed*] Let F be the quotient field of the polynomial ring $\mathbb{F}_2[t]$, that is, F consists of fractions $f(t)/g(t)$, for $f(t), g(t) \in \mathbb{F}_2[t]$ with $g(t) \neq 0$, with addition and multiplication performed as one typically adds and multiplies fractions. Consider the polynomial $f(x) = x^2 - t \in F[x]$. Show that:
(i) $f(x)$ is irreducible in $F[x]$;
(ii) $f(x)$ is not separable in $F[x]$.

(C) [Fri*] Prove Fermat's Little Theorem: If p is prime and c is an integer, then $c^p \equiv c \bmod p$.