A Cardinality Minimization Approach to Security-Constrained Economic Dispatch

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April 8, 2021

Abstract

We present a threshold-based cardinality minimization formulation to model the security-constrained economic dispatch problem. The model aims to minimize the operating cost of the system while simultaneously reducing the number of lines operating in emergency operating zones during contingency events. The model allows the system operator to monitor the duration for which lines operate in emergency zones and ensure that they are within the acceptable reliability standards determined by the system operators. We develop a continuous difference-of-convex approximation of the cardinality minimization problem and a solution method to solve the problem. Our numerical experiments demonstrate that the cardinality minimization approach reduces the overall system operating cost as well as avoids prolonged periods of high electricity prices during contingency events.

1 Introduction

The independent system operators (ISOs) and regional transmission organizations (RTOs) oversee the non-discriminatory access to transmission assets, operate the transmission system independently, and foster competitive generation among wholesale market participants. The ISO/RTOs use bid-based markets arranged hierarchically over multiple timescales. The lowest timescale of the market scheduling process is the real-time Security-Constrained Economic Dispatch (SCED). SCED is used to determine generation across the power system to satisfy the electricity demand at the least cost while meeting the system reliability requirements.

SCED aims to balance the intrinsically competing goals of system efficiency and reliability. While system efficiency is realized by fully utilizing the available transmission capacity, system reliability requires conserving transmission capacity to handle contingency scenarios. Maintaining this balance in real-time poses a significant challenge to the system operators in normal operating conditions, let alone system operations in extreme weather events such as the Texas winter storm in February 2021. The large-scale integration of intermittent renewable resources such as wind and solar also exasperates system operations.

SCED is modeled as a single-period or a multiperiod optimization model. Since it was first proposed in [3], the SCED optimization models have evolved significantly. See [9] for a review of early and [7, 13] for a review of more recent works. This evolution is driven by the desire to include more detailed system operations (e.g., inclusion of AC optimal power flow [33]),

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reliability requirements (e.g., transient stability [2]), and market considerations (e.g., flexible ramping [32]), as well as computational considerations.

The "security" criterion in SCED is enforced by ensuring that the economic dispatch solution simultaneously meets a set of contingency scenarios corresponding to system component failures. Typically, operators consider sets of contingencies that include scenarios with at most one component failure (known as $N-1$ contingency analysis). A feasible solution to SCED must, upon a component failure, be able to redistribute the power flows across the system without overloading the remaining components.

SCED can be performed either in preventive or corrective forms [26]. The preventive approach aims to determine a base-case dispatch solution that can withstand contingency scenarios without any adjustments. On the other hand, the corrective approach allows the base-case dispatch solution to deviate cost-effectively. While the preventive approaches are more secure, they tend to be overly conservative. Therefore, approaches that aim to determine a base-case dispatch solution that minimizes deviation under presumed contingency are preferred. In these approaches, it is also desirable to treat transmission as a flexible asset. In this regard, SCED base-case dispatch solutions can be determined by considering corrective transmission switching or corrective rescheduling.

Corrective transmission switching allows a transmission element to be switched out of service shortly after a contingency occurs to avoid post-contingency violations. Transmission switching has many benefits such as improved reliability [21, 29], congestion management [18], and ease the incorporation of renewable resources [20]. In the context of SCED, corrective transmission switching has been studied recently in [22].

Corrective rescheduling using mathematical optimization was first proposed in [24]. The underlying assumption of this approach is that the operational limit violations (e.g., thermal limits of transmission lines) can be endured for limited periods. For example, a given power line can have a Normal rating (denoting the most desirable thermal rating), a long-term emergency (LTE) rating, and a short-term emergency (STE) rating [11]. Prior corrective rescheduling works have focused on the contingency filtering that aims to reduce the number of contingency scenarios to be considered for determining the base-case dispatch solution [8, 30]. More recently, a multistage contingency response model was proposed in [23]. Since corrective rescheduling is undertaken as a reactive measure, the models proposed in these works implicitly consider the knowledge of when the contingency occurs. Moreover, these models impose a bound on the deviation from the base-case at a given period and ignore the duration of time for which the limit violations occur. For instance, New England ISO requires the line flow to return to less than the LTE rating within 15 minutes [1]. Incorporating these reliability requirements in SCED results in a non-convex optimization problem (even when we consider linear/direct current approximations or convex relaxations of the power flow). In light of these, the main contributions of our work are as follows:

1. *SCED as cardinality minimization problem.* We introduce an SCED formulation with an objective function that includes the operational costs and a reliability term that penalizes operating the transmission lines in emergency zones. The model accommodates differential penalties for operating in tiered emergency operating zones and includes strict constraints on the duration of time a particular line operates in an emergency zone. The resulting model is a threshold-based *cardinality minimization problem* (CMP), a non-convex, non-smooth optimization problem.

2. *Difference-of-convex approximation and algorithm.* We develop a continuous approximation of the threshold-based CMP formulation of SCED using the principles of difference-of-convex optimization (DCO). We express the differential thresholds of individual emergency operating zones as a difference of two piecewise convex functions. Such an approximation allows for the use of a difference-of-convex algorithm to solve the program that exhibits de-

sirable convergence and performance guarantees. Our approximation in conjunction with our DCO formulation for the SCED problem resolves the complexity typically present in threshold-based CMP settings. To the best of our knowledge, our work is the first application of DCO to a power systems problem.

3. *Numerical experiments.* The computational experiments are the first of its kind that demonstrate the advantages of using the CMP formulation of SCED. The experiments reveal that the CMP-based model reduces the overall operating cost and avoids prolonged periods of high electricty prices during contingency events.

Our approach formulates the cardinality of power lines operating under different ratings and controls the distribution by solving a CMP. The CMP exploits the discrete indicator function to flag a logical condition (e.g., presence of emergency rating) and models the cardinality exactly. In this paper, we propose to solve an approximation problem involving a difference-of-convex function using difference-of-convex algorithm [15] to avoid computational intractability of the CMP formulation of SCED. Such modeling of cardinality and implementing approximation methods have received much attention in wide range of applications including signal recovery [28] and artificial intelligence [5] in engineering, gene expression study [17] in bioinformatics, and portfolio selection in finance [31]. Our work is motivated by recent advances made in the variable selection literature supporting the use of difference-of-convex programming. The remainder of the paper is organized as follows. In §2 we present the details of our CMP formulation of SCED. We first present the formulation in a single-period setting and later address the multiperiod setting through a rolling-horizon implementation. In §3 we develop a difference-of-convex approximation of the SCED formulation and present a solution method. Finally in §4 we illustrate the performance of the proposed model and solution method though numerical experiments.

2 Problem Formulation

In this section we present a formulation of the economic dispatch problem that captures the operational as well as system reliability requirements. The formulation results in a cardinality minimization problem for which we propose a novel approach involving a difference-of-convex approximation. We will first present a single-period formulation of the dispatch problem, and then extend it to a multi-period setting.

2.1 Single-period Formulation

We consider a day-ahead or an hour-ahead operations problem of a power system denoted by $(\mathcal{B}, \mathcal{L})$, where $\mathcal{B}$ and $\mathcal{L}$ are the sets of buses and lines, respectively. The the goal of the operations problem is to minimize the total cost of operations. This problem is often stated with the following nominal (base-case) objective function:

$$F_1(\mathbf{x}, \xi) := \sum_{g\in\mathcal{G}} c_g p_g + \sum_{g\in\mathcal{R}} s_g(\xi_g - p_g)_+ + \sum_{d\in\mathcal{D}} s_d(\xi_d - p_d)_+. \tag{1}$$

Here, $\mathbf{x}$ is a consolidated decision vector that includes decisions corresponding to generation quantities $(p_g)_{g\in\mathcal{G}}$, utilized renewable generation $(p_g)_{g\in\mathcal{R}}$, satisfied demand $(p_d)_{d\in\mathcal{D}}$, line power flows $(f_{ij})_{(i,j)\in\mathcal{L}}$, and any additional variables necessary to represent the power flows. The total cost of operations in (1) includes three terms corresponding to the total generation cost, opportunity cost associated with renewable curtailment, and load shedding penalties, respectively. We consider a setting where curtailing renewable generation is undesirable, and hence, the unutilized renewable generation $(\xi_g - p_g)_+ = \max\{\xi_g - p_g, 0\}$ is penalized at a rate of s_g (\$/MWh) for all $g \in \mathcal{R}$. Similarly, unmet demand $(\xi_d - p_d)_+ = \max\{\xi_d - p_d\}$ is also penalized at a rate of s_d

(\$/MWh) for all $d \in \mathcal{D}$. From an operational point of view, s_g can be interpreted as the cost of lost opportunity for renewable generators and s_d as the loss of load penalty.

The operations problem is considered in light of several requirements. These requirements are modeled as constraints in an optimization problem. The first set of constraints includes the following.

$$\sum_{j:(j,i)\in\mathcal{L}} f_{ji} - \sum_{j:(i,j)\in\mathcal{L}} f_{ij} + \sum_{g\in\mathcal{G}_i\cup\mathcal{R}_i} p_g - \sum_{i\in\mathcal{D}_i} p_d = 0 \qquad \forall i \in \mathcal{B} \tag{2a}$$

$$p_g^{\min} \leq p_g \leq p_g^{max} \qquad g \in \mathcal{G} \tag{2b}$$

$$0 \leq p_g \leq \xi_g \qquad g \in \mathcal{R} \tag{2c}$$

$$0 \leq p_d \leq \xi_d \qquad d \in \mathcal{D} \tag{2d}$$

$$(f_{ij})_{(i,j)\in\mathcal{L}} \in \mathfrak{F}. \tag{2e}$$

The flow balance equation in (2a) ensures that the net injection at all buses $\mathcal{B}$ in the power network is zero. The constraint (2b) ensures that the generation amounts of all the generators that are operational, indexed by the set $\mathcal{G}$, are within their respective minimum and maximum capacities. The constraint (2c) restricts the amount of generation from renewable resources (indexed by the set $\mathcal{R}$) utilized to be less than the total available generation ξ_g. Similarly, the demand met at load location d is bounded from above by the actual demand ξ_d for all $d \in \mathcal{D}$. This is captured by constraint (2d).

The set $\mathfrak{F}$ captures the physical requirements of the network. These include the active and the reactive flows, and the active and reactive power capacities for each line $(i, j) \in \mathcal{L}$ in the power network. These requirements also include the lower and upper limits on the voltage magnitude and voltage phase angle $(V_i\angle\theta_i)$ at each bus $i \in \mathcal{B}$ of the power network. For instance, when one uses the linear direct-current approximation of the power flow, the set $\mathfrak{F}$ is a polyhedron given by

$$\mathfrak{F} = \left\{ \begin{array}{c} ((f_{ij})_{(i,j)\in\mathcal{L}} \\ (V_i, \theta_i)_{i\in\mathcal{B}}) \end{array} \middle| \begin{array}{ll} f_{ij} = \frac{V_iV_j}{X_{ij}}(\theta_i - \theta_j) & \forall(i,j) \in \mathcal{L} \\ f_{ij}^{\min} \leq f_{ij} \leq f_{ij}^{\max} & \forall(i,j) \in \mathcal{L} \\ \theta_i^{\min} \leq \theta_i \leq \theta_i^{\max} & \forall i \in \mathcal{B} \end{array} \right\}. \tag{3}$$

Alternatively, one could employ the recently developed convex relaxations of the power flow such as the quadratic convex relaxation [10], the second-order conic relaxation [19], and the semidefinite programming [16]. When these convex relaxations are employed, the set $\mathfrak{F}$ reduces to a convex compact set. The solution method presented in §3.1 is designed for convex feasible sets, and therefore, it is impervious to the particular modeling approach employed for the power flows.

The power flow on transmission lines are additionally limited by the thermal ratings that are determined by the of the ISO/TO's reliability standards. Under normal conditions, the system operates such that the transmission line and the corresponding equipment loading do not exceed a normal thermal rating. However, in the event of a contingency, the ISO operating procedures allow the use of less restrictive ratings for brief periods of time. These ratings are represented as operating zones marked by increasing levels of threshold. For instance, the California ISO imposes a 24 hour (normal), 4 hour (STE), and 15 minute (LTE) ratings [1]. Similar operating practices are in place at other ISO, albeit, the exact duration approved for operating in a zone and the threshold levels may differ based on rating methodologies. Along these practices, we adopt a similar three-tier operating zones for line $(i, j) \in \mathcal{L}$ that are characterized by upper thresholds $\zeta_{ij}^n < \zeta_{ij}^\ell < \zeta_{ij}^s$.

- *Normal*: Flow is within the upper threshold of ζ^n. Flow in this range, i.e., $|f_{ij}| \leq \zeta^n_{ij}$ is considered acceptable system performance.
- *Long-term emergency*: Flow is beyond ζ^n_{ij}, but within the upper threshold of ζ^ℓ_{ij}, i.e., $\zeta^n_{ij} \leq |f_{ij}| \leq \zeta^\ell_{ij}$. Flow in this range for at most 4 hours is acceptable.
- *Short-term emergency*: Flow is beyond ζ^ℓ_{ij}, but within the threshold of ζ^s_{ij}. This flow is captured by $\zeta^\ell_{ij} \leq |f_{ij}| \leq \zeta^s_{ij}$. Flow in this range for at most 15 minutes is acceptable.

These operating zones are illustrated in Figure 1.

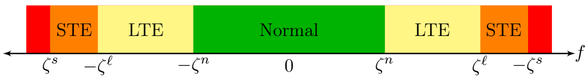


Figure 1: Operating zones based on thermal ratings of lines

Ordinarily, the SCED problem is formulated by restricting the flows to be within the normal operating zone. This is done by including constraints of the form: $|f_{ij}| \leq \zeta^n$ for all $(i, j) \in \mathcal{L}$. However, such an approach may prove to be less economical in a power system where a significant portion of the generation is from intermittent generators such as wind and solar. Due to large variability in intermittent generation, the flows within the system exhibit large fluctuations, thereby increasing the probability of lines operating in emergency zones. To address this, we present a SCED formulation which aims to minimize the number of lines operating in the emergency zones in addition to minimizing the operating cost.

In this regard, we consider that the flow beyond ζ^s_{ij} is unacceptable from a system reliability perspective. This restriction is imposed in our dispatch model as explicit constraints:

$$|f_{ij}| \leq \zeta^s_{ij} \qquad \forall (i, j) \in \mathcal{L}. \tag{4}$$

To capture the number of lines operating in the emergency zones, we denote by $\mathcal{E}^\ell$ and $\mathcal{E}^s$ the sets of lines that are operating beyond the normal and LTE threshold values, respectively. These sets are defined as

$$\mathcal{E}^\ell := \{(i, j) \in \mathcal{L} \mid |f_{ij}| \geq \zeta^n_{ij}\}, \text{ and} \tag{5a}$$
$$\mathcal{E}^s := \{(i, j) \in \mathcal{L} \mid |f_{ij}| \geq \zeta^\ell_{ij}\}, \tag{5b}$$

respectively. The additional system reliability requirement of minimizing the number of lines that do not operate in the normal zone is captured by the following function.

$$F_2(\mathbf{x}, \xi) = \gamma^\ell |\mathcal{E}^\ell| + \gamma^s |\mathcal{E}^s|. \tag{6}$$

Here, $\gamma^\ell, \gamma^s > 0$ are parameters that ensure that the STE zone is less desirable compared to LTE zone. Using the operational cost $F_1(\cdot)$ and system reliability objective $F_2(\cdot)$, defined in (1) and (6), respectively, the single period dispatch model can be stated as follows

$$\begin{aligned} &\min \; F_1(\mathbf{x}, \xi) + F_2(\mathbf{x}, \xi) \\ &\text{subject to } (2), (4). \end{aligned} \tag{7}$$

When flows are limited to operate within the normal threshold ζ^n, the resulting SCED models are either convex or linear programs based on the approach adopted to model the power flows (i.e., the set $\mathcal{F}$). However, in the presence of reliability objective $F_2(\cdot)$, the optimization program in (7) is a *cardinality minimization problem* (CMP). The CMP is a non-smooth non-convex optimization problem, and therefore, directly solving (7) is a computationally challenging undertaking. In order to tackle this difficulty, we present a computationally viable approximation of (7) in §3.

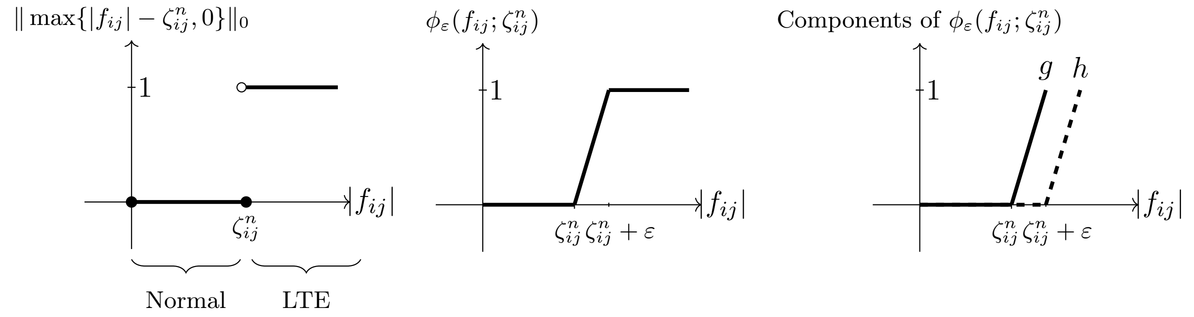


Figure 2: Illustration of difference-of-convex approximation

2.2 CMP-based Rolling-horizon SCED

The system reliability requirements additionally mandate that an equipment return to normal operating zone with a fixed period of time. In order to incorporate these requirements, we extend the CMP-based SCED formulation to a multiperiod setting.

Denote by $\mathcal{T} := \{1,\dots,T\}$ the set of decision epochs in the multiperiod horizon. A time index t will appear in the subscript for the applicable parameters and variables defined previously. Let T^ℓ and T^s denote the acceptable number of time periods that a line can operate in LTE and STE zones, respectively. At any decision epoch $t \in \mathcal{T}$ we denote the state of the system by a vector s_t that includes the following components: (i) the generation level for conventional generators, $(p_{gt-1})_{g\in\mathcal{G}}$, (ii) the flow on each line $(f_{ijt-1})_{(i,j)\in\mathcal{L}}$, and (iii) the number of epochs since entering an emergency operating zone $z = \ell$ or s, denoted as $(\tau^z_{ijt-1})_{(i,j)\in\mathcal{L}}$. A line operating in normal zone in time period t will have τ_{ijt} set to zero.

Across multiple time periods, a SCED model must capture the generator ramp rate restrictions, by including constraints of the form:

$$\Delta^{\min}_g + p_{gt-1} \le p_{gt} \le \Delta^{\max}_g + p_{gt-1} \quad \forall g \in \mathcal{G}, t \in \mathcal{T}, \tag{8}$$

where $\Delta^{\min}_g/\Delta^{\max}_g$ are minimum/maximum ramp limits. The system reliability requirements are captured by the following constraint:

$$|f_{ijt}| \le \zeta^n \mathbf{1}_{(\tau^\ell_{ijt-1}=T^\ell)} + \zeta^\ell \mathbf{1}_{(\tau^s_{ijt-1}=T^s)} \quad \forall (i,j) \in \mathcal{L}, \tag{9}$$

where $\mathbf{1}_{(\cdot)}$ is the indicator function. The above constraint enforces that the flow on line (i,j) is within the LTE upper threshold, i.e., $|f_{ijt}| \le \zeta^\ell$ if the time since entering the STE zone is equal to the acceptable amount T^s. Similarly, the flow is forced to return to normal zone once the acceptable amount of time for operating in LTE (T^ℓ) is reached.

For a given state s_t and realization ξ_t, the CMP-based SCED problem for time period t can be stated as

$$\begin{aligned} h_t(s_t,\xi_t) = \min\ & F_{1,t}(\mathbf{x},\xi_t) + F_{2,t}(\mathbf{x},\xi_t) \\ \text{subject to } & (2),(4),(8),(9). \end{aligned} \tag{10}$$

Notice that, given the state of the system s_t, the bounds in (8) and the right-hand side quantity in (9) can be computed easily and the constraints appear as simple as variable bounds. In the rolling-horizon setting, an instance of the model in (10) is solved for time period t and the state vector is updated with the optimal solution f^*. In particular, the third component of the state vector is updated as follows:

$$\tau^\ell_{ijt+1} = \begin{cases} 0 & \text{if } |f^\star_{ijt}| \le \zeta^n_{ij}, \\ \tau^\ell_{ijt} + 1 & \text{if } |f^\star_{ijt}| > \zeta^n_{ij} \end{cases} \tag{11}$$

for all $(i, j) \in \mathcal{L}$. The component τ^s_{ijt} is updated in a similar manner. A model instances is then setup using the updated state vector s_{t+1} and solved for time period $(t + 1)$. The procedure is continued until the end of the horizon.

3 Difference-of-Convex Approximation

The CMP-based SCED models in (7) and (10) are non-smooth and non-convex optimization problems. The sets $\mathcal{E}^\ell$ and $\mathcal{E}^s$ in (7) consist of all lines with flow beyond the desired capacity that the cardinality of the sets $|\mathcal{E}^\ell|$ and $|\mathcal{E}^s|$ represent the number of lines operating in the emergency zones. In this section, we present an exact formulation of such cardinalities, and introduce a computationally viable approximation of the quantities for the CMP formulation of SCED.

We illustrate the principal technique on a line (i, j) in the set $\mathcal{E}^\ell$. For line (i, j), it is not difficult to see that the line is operating in the LTE zone if $|f_{ij}| - \zeta^n_{ij} > 0$, or equivalently, if $\max\{|f_{ij}| - \zeta^n_{ij}, 0\} > 0$. For such a scenario, we can express the emergency operation of the line exactly by $\| \max\{|f_{ij}| - \zeta^n_{ij}, 0\}\|_0 = 1$, where the ℓ_0-function, denoted by $\| \cdot \|_0$, is defined as $\|t\|_0 = 1$ if $t \neq 0$ and $\|t\|_0 = 0$ if $t = 0$. The right-side of the equation becomes 0 if the line is operating under the normal zone. Hence the formulation indicates whether or not a flow exceeds the normal threshold ζ^n; we refer the reader to the leftmost pane in Figure 2 for an illustration. A similar technique is applied to other lines in the set $\mathcal{E}^\ell$ and the lines in the set $\mathcal{E}^s$. The cardinality of the sets containing lines operating in the emergency zones can then be expressed as

$$|\mathcal{E}^\ell| = \sum_{(i,j)\in\mathcal{L}} \| \max\{|f_{ij}| - \zeta^n_{ij}, 0\}\|_0 \tag{12a}$$

$$|\mathcal{E}^s| = \sum_{(i,j)\in\mathcal{L}} \| \max\{|f_{ij}| - \zeta^\ell_{ij}, 0\}\|_0. \tag{12b}$$

As depicted in Figure 2, the exact formulation of the cardinalities requires employing a discontinuous function, making any optimization problem involving such expressions a discrete problem. We propose to approximate each summands in (12) by a piecewise linear function to remove the discontinuity of the original formulation by connecting the two disjoint pieces of the function; the central pane of Figure 2 illustrates the continuous piecewise linear approximation for a single line. An advantage of using a surrogate, as opposed to directly solving a discrete optimization problem, is computational efficiency. Many existing methods involving the ℓ_0-function, e.g., best subset selection and cardinality constraint, introduce auxiliary binary integer variables [4, 14] or reformulate the problem using complementarity constraints [6, 12, 34]. The potential burden of increased computational time from such reformulations could be addressed by using continuous functions instead, and we will demonstrate the effectiveness of the proposed approach in the § 4.

Let us denote the approximation function for the summands in (12) by $\phi_\varepsilon(\cdot;\cdot)$. The approximation function is defined for a variable (flow) and a parameter (threshold). For a given line $(i, j) \in \mathcal{E}^\ell$ and the threshold ζ^n_{ij}, we formally introduce the function:

$$\begin{aligned} \phi_\varepsilon(f_{ij}; \zeta^n_{ij}) &= \max\left\{\frac{1}{\varepsilon}(|f_{ij}| - \zeta^n_{ij}), 0\right\} \\ &\qquad - \max\left\{\frac{1}{\varepsilon}(|f_{ij}| - \zeta^n_{ij}) - 1, 0\right\} \\ &= \begin{cases} 0 & \text{if } |f_{ij}| \le \zeta^n_{ij} \\ \frac{1}{\varepsilon}(f_{ij} - \zeta^n_{ij}) & \text{if } \zeta^n_{ij} < |f_{ij}| \le \zeta^n_{ij} + \varepsilon \\ 1 & \text{if } \zeta^n_{ij} + \varepsilon < |f_{ij}|. \end{cases} \end{aligned} \tag{13}$$

The positive scalar ε in the above definition is an approximation parameter that can be pre-selected or tuned in practice. The function is defined by three pieces – the constant value of 0 indicates that the flow is with the threshold, a value of 1 indicates that the flow clearly exceeds the threshold, and a value between 0 and 1 indicates that the line just entered the emergency zone. Table 1 lists the exact and the approximate functions for all operating zones. We note that each of the above approximation is a difference-of-convex function; a function $f(x)$

Zone	Exact formulation	Approximation
Normal	$\\|\max\{\|f\| - \zeta^n, 0\}\\|_0 = 0$	$\phi_\varepsilon(f;\zeta^n) = 0$
LTE	$\\|\max\{\|f\| - \zeta^n, 0\}\\|_0 = 1$	$0 < \phi_\varepsilon(f;\zeta^n) \leq 1$
	$\\|\max\{\|f\| - \zeta^\ell, 0\}\\|_0 = 0$	$\phi_\varepsilon(f;\zeta^\ell) = 0$
STE	$\\|\max\{\|f\| - \zeta^n, 0\}\\|_0 = 1$	$\phi_\varepsilon(f;\zeta^n) = 1$
	$\\|\max\{\|f\| - \zeta^\ell, 0\}\\|_0 = 1$	$0 < \phi_\varepsilon(f;\zeta^\ell) \leq 1$

Table 1: Exact and Approximate Formulations for different operating zones

is a difference-of-convex function if there exist two convex functions $g(x)$ and $h(x)$ such that $f(x) = g(x) - h(x)$. To see this, consider the definition of $\phi_\varepsilon(\cdot;\cdot)$ given in (13). Each term in the right-side is a convex function since the max-operator preserves convexity given by the absolute value function.

Applying the approximate function to all lines, the surrogates for the set cardinalities in (12) can be written as

$$|\mathcal{E}^\ell| \approx \sum_{(i,j)\in\mathcal{L}} \phi_\varepsilon(f_{ij};\zeta^n_{ij}) \quad \text{and} \quad |\mathcal{E}^s| \approx \sum_{(i,j)\in\mathcal{L}} \phi_\varepsilon(f_{ij};\zeta^\ell_{ij}).$$

Using the above construction, we introduce the approximation problem that reformulates (7) by applying the surrogates of the cardinalities:

$$\begin{aligned} \min\ & F_1(\mathbf{x},\xi) \\ & + \gamma^\ell \sum_{(i,j)\in\mathcal{L}} \phi_\varepsilon(f_{ij};\zeta^n_{ij}) + \gamma^s \sum_{(i,j)\in\mathcal{L}} \phi_\varepsilon(f_{ij};\zeta^\ell_{ij}) \\ & \text{subject to } (2),(4). \end{aligned} \tag{14}$$

We recall that $\gamma^\ell > 0$ and $\gamma^s > 0$ are the weighting parameters that control the weight of minimizing the total cost of operations captured by $F_1(\cdot,\cdot)$ and the number of lines operating in the emergency zones.

3.1 Solution Method

The problem (14) has a difference-of-convex objective function and convex constraints. A popular approach to solve a problem with such structure is to apply difference-of-convex algorithm (DCA). Introduced by Le Thi and Phan [27], the DCA iteratively solves a convex program which is given by a local approximation of the objective function. At each iteration, the algorithm linearizes concave components of the objective function using the current point, and solves the resulting convex problem producing decreasing sequence of iterates.

Employing DCA to solve the problem (14), we linearize each concave part of the objective given by $\phi_\varepsilon(\cdot;\cdot)$. For better presentation of the algorithm, let us denote g and h for the two

Algorithm 1 Difference-of-convex algorithm for (14)

Input: Reliability parameters $\{\zeta_{ij}^n\}$ and $\{\zeta_{ij}^\ell\}$ for all $(i,j) \in \mathcal{L}$, and hyperparameters $\varepsilon, c, \gamma^\ell, \gamma^s > 0$.
Initialization: Set $k = 0$. $f^k = (f_{ij}^k)_{(i,j)\in\mathcal{L}}$
while (termination criteria is not met) **do**
  Compute the subgradient:

$$v_{ij}^{k,n} = \begin{cases} -\frac{1}{\varepsilon} & \text{if } f_{ij}^k < \zeta_{ij}^n - \varepsilon \\ \left[-\frac{1}{\varepsilon}, 0\right] & \text{if } f_{ij}^k = \zeta_{ij}^n - \varepsilon \\ 0 & \text{if } \zeta_{ij}^n - \varepsilon < f_{ij}^k < \zeta_{ij}^n + \varepsilon \\ \left[0, \frac{1}{\varepsilon}\right] & \text{if } f_{ij}^k = \zeta_{ij}^n + \varepsilon \\ \frac{1}{\varepsilon} & \text{if } \zeta_{ij}^n + \varepsilon < f_{ij}^k. \end{cases} \tag{15}$$

  Solve the subproblem

$$\begin{aligned} \min \ & F_1(\mathbf{x}, \xi) + \frac{c}{2}\|\mathbf{f} - \mathbf{f}^k\|_2^2 \\ & + \gamma^\ell \sum_{(i,j)\in\mathcal{L}} \left\{ g(f_{ij}; \zeta_{ij}^n) - v_{ij}^{k,n} f_{ij} \right\} \\ & + \gamma^s \sum_{(i,j)\in\mathcal{L}} \left\{ g(f_{ij}; \zeta_{ij}^s) - v_{ij}^{k,\ell} f_{ij} \right\} \\ \text{subject to } & (2), (4); \end{aligned} \tag{16}$$

  Let $\mathbf{x}^k$ denote the optimal solution of (16) with vector $\mathbf{f}^k = (f_{ij}^k)_{(i,j)\in\mathcal{L}}$ corresponding to the flow variables.
  $k = k + 1$
end while
Output: The optimal solution $\mathbf{x}^* = \mathbf{x}^k$.

functions shown in the definition of $\phi(\cdot;\cdot)$ in (13), i.e.,

$$\begin{aligned} g(f_{ij}; \zeta_{ij}^n) &= \max\left\{ \frac{1}{\varepsilon}(|f_{ij}| - \zeta_{ij}^n), 0 \right\} \\ h(f_{ij}; \zeta_{ij}^n) &= \max\left\{ \frac{1}{\varepsilon}(|f_{ij}| - \zeta_{ij}^n) - 1, 0 \right\}. \end{aligned} \tag{17}$$

Given a current point f_{ij}^k at the k-th iteration, we approximate the latter function using its subgradient.

$$h(f_{ij}; \zeta_{ij}^n) \approx h(f_{ij}^k; \zeta_{ij}^n) + v_{ij}^k (f_{ij} - f_{ij}^k)$$

where $v_{ij}^{k,n} \in \partial h(f_{ij}^k, \zeta_{ij}^n)$. We note that the approximation shown on the right-side is a linear function with some constant terms. Since minimizing without constant terms does not affect the solution of the problem, we discard the constant terms in the algorithm. Incorporating the linear approximation, we present Algorithm 1 to solve the problem (14).

4 Numerical Experiments

In this section we report the results from the numerical experiments with the CMP formulation of SCED. These experiments were conducted on an updated version of the RTS-96 test system [25]. The test system comprises of 73 buses, 108 lines, and 158 generators. The experiments were conducted on a computer using a 2.7 GHz Intel Core i5 processor with 8GB of RAM, running macOS Sierra version 10.12.6. Gurobi Optimizer version 9.0.0 through CVX was used in MATLAB.

We consider a time resolution of 15 minutes that corresponds to the acceptable time limits for the LTE rating. Following this choice, the parameters $T^\ell = 16$ and $T^s = 1$ (number of 15-minute time intervals in 4 hours). The original load data that has a time resolution of 5-minutes was transformed into 15-minute intervals by averaging over corresponding three time intervals. Additionally, the load time series was multiplied by a constant factor to emulate a contingency scenario in the system. For our experiments, we consider scenarios where the system is stressed for a prolonged period of time (multiple hours). We assume that the set of generators is committed and no additional resources can be brought online in real time. This assumption is made to focus the experiments on illustrating the ability of the CMP to efficiently utilize the available line capacities. While the generation cost data is included in the data set, we set the generation-shedding penalty (loss-of-opportunity) cost to $s_g = \$300$ and load-shedding penalty cost (value of lost-load) to $s_d = \$1000$. These penalties are applied uniformly for all the generators and loads, respectively.

In our experiments, we use an alternative model that does not use cardinality minimization to serve as a comparison for the CMP formulation of SCED. We refer to this model as the "strict" model as it imposes all the lines to operate within the normal zone for every time period. This model can be stated as min $\{F_1(\mathbf{x}, \xi) \mid (2), (8), |f_{ij}| \leq \zeta^n_{ij}\}$. Since both the CMP and the strict SCED models adhere to the physical requirements and limitations, viz., (2) and (8), the corresponding total operating costs can be fairly compared. Before we present these comparison results, we will present the steps undertaken to identify the hyperparameters used in the difference-of-convex approximation.

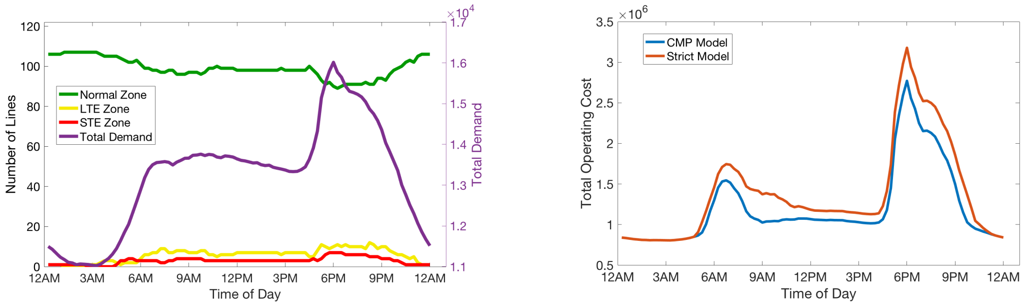
4.1 Hyperparameter Tuning

Recall that the difference-of-convex Algorithm 1 requires hyperparameters ε, γ^ℓ, and γ^s as input. A 3-dimensional grid search between ε, γ^ℓ, and γ^s was performed to choose hyperparameter values for the CMP formulation of SCED. Five values of ε were compared across 10 different values for both γ^ℓ and γ^s. The 5 tested values for ε were powers of 10 evenly spaced from 10^{-4} to 10^0. Tested values for γ^ℓ, and γ^s ranged from 0.1 to 1.0 in increments of 0.1. For each combination of ε, γ^ℓ, and γ^s, the total operating cost that includes the cost of generation plus the shedding costs, detailed in equation (1), was recorded. The number of power lines within each operating zones at the end of each 15-minute interval were also recorded.

Our analysis reveals a tradeoff between number of lines in the Normal zone and total operating cost. When $\varepsilon = 10^{-4}$, the total operating cost greatly increases, but there are more power lines operating withing the desirable thermal rating. We see the operating cost and subsequent number of lines operating in desirable thermal ratings both decrease as ε increases. The harsher penalties associated with small ε values leads to more demand shedding, as the penalty for each threshold violation begin to outweigh shedding penalties. Because the overall objective of the SCED problem is to minimize the total cost while still satisfying physical and thermal rating requirements, hyperparameter values associated with low costs are chosen for our experiments. The hyperparameter values for the CMP formulation of SCED subsequently discussed uses $\varepsilon = 10^{-1}$, $\gamma^\ell = 0.5$, and $\gamma^s = 0.5$.

4.2 Comparison of Cost Performance and Prices

For the chosen values of the hyperparameters, Algorithm 1 was used to solve the CMP formulation of rolling horizon SCED over 24 hours. The algorithm converges within five iterations for all the instances solved. The average time to solve a single 15-minute time interval problem was 1.57 minutes.



(a) Number of lines in each operating zone for the CMP-based SCED

(b) Total operating cost comparison

Figure 3: Performance of CMP model.

Day	Total Cost for the Day		Total Units of Demand Shed		Average Lines in Normal / LTE / STE Zone	
	CMP	Strict	CMP	Strict	CMP	Strict
Winter-1	1.163×10^8	1.330×10^8	2.546×10^4	4.350×10^4	99.250 / 5.781 / 2.969	108 / 0 / 0
Winter-2	8.922×10^7	9.811×10^7	4.859×10^3	1.447×10^4	103.063 / 3.291 / 1.646	108 / 0 / 0
Spring-1	1.007×10^8	1.157×10^8	1.215×10^4	2.833×10^4	101.594 / 4.146 / 2.260	108 / 0 / 0
Spring-2	8.200×10^7	8.778×10^7	1.003×10^3	7.358×10^3	104.760 / 2.167 / 1.073	108 / 0 / 0
Summer-1	5.557×10^8	5.818×10^8	4.601×10^5	4.883×10^5	93.917 / 7.250 / 6.833	108 / 0 / 0
Summer-2	5.772×10^8	6.077×10^8	4.823×10^5	5.153×10^5	89.792 / 8.677 / 9.531	108 / 0 / 0
Autumn-1	3.177×10^8	3.412×10^8	2.228×10^5	2.483×10^5	97.230 / 6.437 / 4.333	108 / 0 / 0
Autumn-2	2.226×10^8	2.453×10^8	1.337×10^5	1.583×10^5	97.230 / 5.510 / 5.260	108 / 0 / 0

Table 2: Table summarizing the CMP model with the strict model across various days

Figure 3a shows the number of lines that are operating in each of the operating zones. We see a decrease in the number of power lines in the Normal Zone around 8AM and 5PM when power system is stressed due to contingency event. The constraints (9) ensure that the duration time that an individual line operates outside the normal zone are within the acceptable reliability parameters. The added flexibility of allowing the lines to operate outside the normal zone results in efficient utilization of resources to address contingency. This is reflected in the lower operating costs when compared to the strict model that lacks this flexibility as seen in Figure 3b. The total operating costs include the generation and shedding costs (if any). The CMP formulation of SCED especially lowered total cost for periods of highest demand (5 P.M. – 9 P.M.).

We performed the experiment on eight different contingency scenarios, with two days corresponding to each of the four seasons. The results are shown in Table 2. The table shows the total operating cost, the units of demand shed, and the average number of lines operating outside the normal zone. The table shows operating costs are the highest during summer contingency

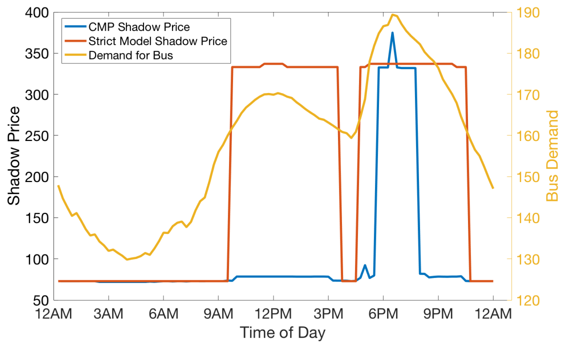


Figure 4: Location marginal price at Bus-50

events and the least in winter. The CMP formulation of SCED results in a lower cost uniformly across all scenarios with the highest decrease of 12.92% on day Spring-1. The high contingency summer days also result in the highest number of lines operating in the emergency zones (14.083 and 18.208, respectively).

Figure 4 shows the location marginal price (LMP) obtained from the strict and CMP models at Bus 50 which corresponds to the median demand in the network. The figure also shows the system demand over the 24 hour horizon under the scenario corresponding to Winter-2. The LMPs are the optimal dual solution (shadow prices) associated with the flow balance equation (2a). The strict model results in scarcity price of $333 for prolonged periods of time, once between 9:30 A.M. – 3:15 P.M. and again between 4:30 P.M. – 10:15 P.M. On the other hand, the CMP model completely avoids the first interval of scarcity prices and reduces the duration of the second interval to 5:30 P.M. – 7:30 P.M. These results illustrate the potential of avoiding prolonged periods of scarcity prices by using the CMP model.

5 Conclusion and Future Work

In this paper, we presented the CMP formulation of SCED that explicitly accounts for the number of transmission lines that are operating in the emergency zones during a contingency event. The objective function in this new formulation included the total operating cost as well as differential penalties on the number of lines operating in different emergency zones. Constraints ensured that the duration of operation in emergency zones was within the acceptable reliability standards set by the system operators. The CMP formulation is a non-convex, non-smooth optimization problem. We developed a difference-of-convex approximation of the formulation and used the DCA to obtain its solution. To the best of our knowledge, this is first study to apply use the highly effective difference-of-convex optimization approximation and solution method for a power systems operations problem. The numerical experiments illustrated the advantages of using the CMP model in reducing the total operating cost as well as avoiding prolonged periods of scarcity prices.

The current CMP formulation of SCED considers static thermal ratings and therefore results in a deterministic optimization problem. Several recent studies have shown the advantage of using dynamic thermal rating of transmission lines. Inclusion of dynamic thermal rating in the CMP model has the potential to further improve system operations. However, this inclusion will result in non-convex, non-smooth stochastic optimization problems. The stochastic difference-of-convex optimization is in its infancy and is the subject of our ongoing work. We will undertake the CMP formulation of SCED with dynamic thermal rating in our future research endeavors.

References

[1] Establishing system operating limits forthe operations horizon. Technical report, Califirnia Independent System Operator, 2020.

[2] Shrirang Abhyankar, Guangchao Geng, Mihai Anitescu, Xiaoyu Wang, and Venkata Dinavahi. Solution techniques for transient stability-constrained optimal power flow–part i. *IET Generation, Transmission & Distribution*, 11(12):3177–3185, 2017.

[3] O Alsac and Brian Stott. Optimal load flow with steady-state security. *IEEE Transactions on Power Apparatus and Systems*, (3):745–751, 1974.

[4] Dimitris Bertsimas, Angela King, and Rahul Mazumder. Best subset selection via a modern optimization lens. *The Annals of Statistics*, 44(2):813–852, April 2016.

[5] Kevin Bui, Fredrick Park, Shuai Zhang, Yingyong Qi, and Jack Xin. Structured sparsity of convolutional neural networks via nonconvex sparse group regularization. *Frontiers in Applied Mathematics and Statistics*, 6, February 2021.

[6] Oleg P. Burdakov, Christian Kanzow, and Alexandra Schwartz. Mathematical programs with cardinality constraints: Reformulation by complementarity-type conditions and a regularization method. *SIAM Journal on Optimization*, 26(1):397–425, January 2016.

[7] Florin Capitanescu, JL Martinez Ramos, Patrick Panciatici, Daniel Kirschen, A Marano Marcolini, Ludovic Platbrood, and Louis Wehenkel. State-of-the-art, challenges, and future trends in security constrained optimal power flow. *Electric Power Systems Research*, 81(8):1731–1741, 2011.

[8] Florin Capitanescu and Louis Wehenkel. A new iterative approach to the corrective security-constrained optimal power flow problem. *IEEE Transactions on Power Systems*, 23(4):1533–1541, 2008.

[9] B H Chowdhury and Salfur Rahman. A review of recent advances in economic dispatch. *IEEE Transactions on Power Systems*, 5(4):1248–1259, 1990.

[10] Carleton Coffrin, Hassan L Hijazi, and Pascal Van Hentenryck. The qc relaxation: A theoretical and computational study on optimal power flow. *IEEE Transactions on Power Systems*, 31(4):3008–3018, 2015.

[11] Dale Douglass, Jake Gentle, Huu-Minh Nguyen, William Chisholm, Charles Xu, Tip Goodwin, Hong Chen, Sarma Nuthalapati, Neil Hurst, Ian Grant, Jose Antonio Jardini, Robert Kluge, Paula Traynor, and Cody Davis. A review of dynamic thermal line rating methods with forecasting. *IEEE Transactions on Power Delivery*, 34(6):2100–2109, 2019.

[12] M. Feng, J.E. Mitchell, Jong-Shi Pang, A. Waechter, and X. Shen. Complementarity formulations of l0-norm optimization problems. *Pacific Journal of Optimization*, 14(2):273–305, 2018.

[13] Stephen Frank, Ingrida Steponavice, and Steffen Rebennack. Optimal power flow: a bibliographic survey i. *Energy Systems*, 3(3):221–258, 2012.

[14] Andrés Gómez and Oleg A. Prokopyev. A mixed-integer fractional optimization approach to best subset selection. *INFORMS Journal on Computing*, March 2021.

[15] R Horst and N V Thoai. Dc programming: Overview. *Journal of Optimization Theory and Applications*, 103(1):1–43, 1999.

[16] Rabih A Jabr. Radial distribution load flow using conic programming. *IEEE Transactions on Power Systems*, 21(3):1458–1459, 2006.

[17] C. Ke, M. Ahn, S. Shin, and Y. Lou. Iteratively reweighted group lasso based on log-composite regularization. *Optimization Online.*, 2020.

[18] Amin Khodaei and Mohammad Shahidehpour. Transmission switching in security-constrained unit commitment. *IEEE Transactions on Power Systems*, 25(4):1937–1945, 2010.

[19] Burak Kocuk, Santanu S Dey, and X Andy Sun. Strong socp relaxations for the optimal power flow problem. *Operations Research*, 64(6):1177–1196, 2016.

[20] Akshay S Korad and Kory W Hedman. Robust corrective topology control for system reliability. *IEEE Transactions on Power Systems*, 28(4):4042–4051, 2013.

[21] Xingpeng Li, Pranavamoorthy Balasubramanian, Mostafa Sahraei-Ardakani, Mojdeh Abdi-Khorsand, Kory W Hedman, and Robin Podmore. Real-time contingency analysis with corrective transmission switching. *IEEE Transactions on Power Systems*, 32(4):2604–2617, 2016.

[22] Xingpeng Li and Kory W Hedman. Enhanced energy management system with corrective transmission switching strategy—part i: Methodology. *IEEE Transactions on Power Systems*, 34(6):4490–4502, 2019.

[23] Yanchao Liu, Michael C Ferris, and Feng Zhao. Computational study of security constrained economic dispatch with multi-stage rescheduling. *IEEE Transactions on Power Systems*, 30(2):920–929, 2014.

[24] A Monticelli, MVF Pereira, and S Granville. Security-constrained optimal power flow with post-contingency corrective rescheduling. *IEEE Transactions on Power Systems*, 2(1):175–180, 1987.

[25] National Renewable Energy Laboratory. Reliability Test System - Grid Modernization lab consortium. Online (Github). Accessed: 2010-09-30.

[26] Vijay Pappu, Marco Carvalho, and Panos Pardalos, editors. *Optimization and Security Challenges in Smart Power Grids*. Springer Berlin Heidelberg, 2013.

[27] D.T. Pham and H.A.L̃e Thi. Convex analysis approach to d.c. programming: Theory, algorithms and applications. *ACTA Mathematica Vietnamica*, 22(1):289–355, 1997.

[28] Yaghoub Rahimi, Chao Wang, Hongbo Dong, and Yifei Lou. A scale-invariant approach for sparse signal recovery. *SIAM Journal on Scientific Computing*, 41(6):A3649–A3672, January 2019.

[29] Wei Shao and Vijay Vittal. Corrective switching algorithm for relieving overloads and voltage violations. *IEEE Transactions on Power Systems*, 20(4):1877–1885, 2005.

[30] Goran Strbac, Syed Ahmed, Daniel Kirschen, and Ron Allan. A method for computing the value of corrective security. *IEEE Transactions on Power Systems*, 13(3):1096–1102, 1998.

[31] Yoshimasa Uematsu and Shinya Tanaka. High-dimensional macroeconomic forecasting and variable selection via penalized regression. *The Econometrics Journal*, 22(1):34–56, January 2019.

[32] Beibei Wang and Benjamin F Hobbs. A flexible ramping product: Can it help real-time dispatch markets approach the stochastic dispatch ideal? *Electric Power Systems Research*, 109:128–140, 2014.

[33] Xuan Wu and Antonio J Conejo. Security-constrained acopf: Incorporating worst contingencies and discrete controllers. *IEEE Transactions on Power Systems*, 35(3):1936–1945, 2019.

[34] Yue Xie and Uday V. Shanbhag. Tractable ADMM schemes for computing KKT points and local minimizers for $l0$-minimization problems. *Computational Optimization and Applications*, 78(1):43–85, October 2020.