Defn: Set

A set is a collection of objects.

Ex. $S = \{1, 2, 3\}$

$\mathbb{N} = \{1, 2, 3, 4, \ldots\}$ = natural/counting numbers

$\mathbb{Q} = \{ \frac{m}{n}$ where $m, n \in \mathbb{N} \}$

Defn: Set Membership

We say "x is in S" denoted

$$x \in S$$

if S contains x as an element

Ex. $5 \in \mathbb{N} = \{1, 2, 3, 4, 5, 6, \ldots\}$

here!

Ex. $2/3 \in \mathbb{Q}$

Ex. $2/3 \notin \mathbb{N}$

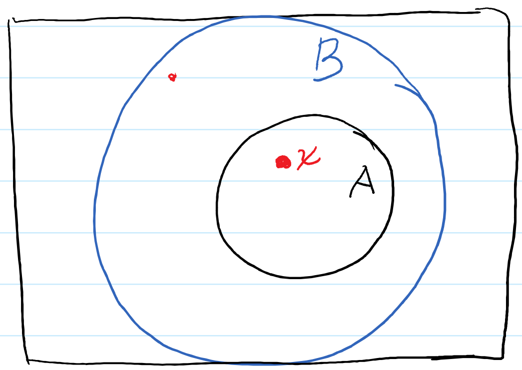
read: "not in"

Defn: Containment

We say "A is a subset of B" denoted

$$A \subset B$$

if $x \in A$ implies $x \in B$



Ex. $\{1,2,3\} \subset \mathbb{N}$

$\mathbb{Q} \subset \mathbb{R}$ real numbers

$\mathbb{N} \not\subset \{1,2,3\}$

Ex. $A \subset B$, $B \not\subset A$

Defn: Set Equality

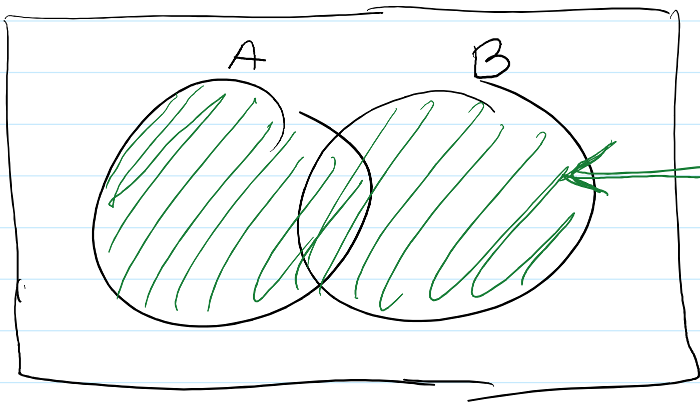
We say "A is equal to B" if both $A \subset B$ and $B \subset A$. We write "$A = B$".

Set Operations

Defn: Union

The union of A and B denoted "$A \cup B$" is defined as

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$



$A \cup B$

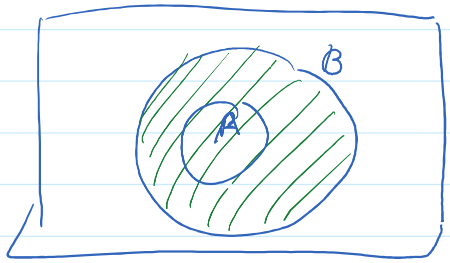
Ex. $A = \mathbb{N} = \{1, 2, 3, 4, \ldots\}$

$B = \{-1, -2, -3, \ldots\}$

then $A \cup B = \{\pm 1, \pm 2, \pm 3, \ldots\}$

Ex. $\mathbb{Q} \cup \mathbb{R} = \mathbb{R}$ b/c $\mathbb{Q} \subset \mathbb{R}$

→ Fact: $A \subset B$ then $A \cup B = B$

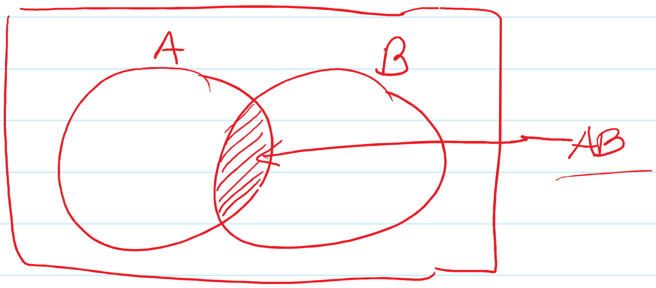


Ex. $\mathbb{N} \cup \mathbb{N} = \mathbb{N}$
(Idempotency)
Fact: $A \cup A = A$

Defn: Intersection

We define the intersection of A and B as

$$AB = A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$



Ex. $A = \mathbb{N}$, $B = \{-1, -2, -3, \dots\}$

$AB = \emptyset$ empty set

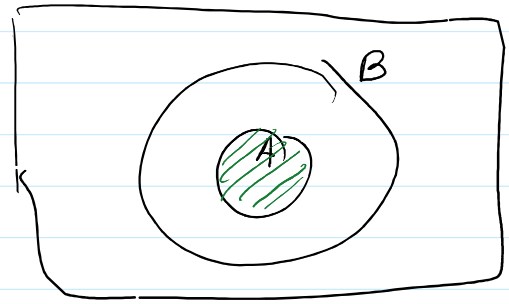
Ex. $\mathbb{Q}\mathbb{N} = \mathbb{N}$

b/c $\mathbb{N} \subset \mathbb{Q}$

Fact! $A \subset B$ then $AB = A$

Idempotency:

$AA = A$



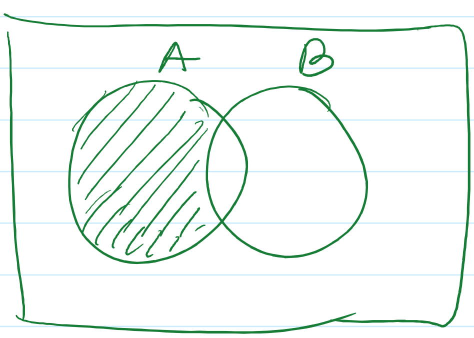
Defn: Set Difference

The say the "difference" between two set A ad B denoted

$$A \setminus B$$

is defined

$$A \setminus B = \{x \mid x \in A \text{ ad } x \notin B\}$$



Ex. $A = \{1, 2, 3\}$

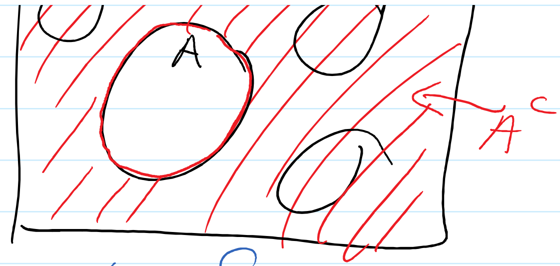
$B = \{3, 4, 5\}$

$A \setminus B = \{1, 2\}$

$B \setminus A = \{4, 5\}$

Defn: Set Complement

Want: $A^c = \{x \mid x \notin A\}$



Need: Universe set S where $A \subset S$

$$\boxed{A^c = S \setminus A = \{x \in S \mid x \notin A\}}$$

Ex. $A = \{5, 6\}$, $S = \mathbb{N}$ $(A \subset S)$

then $A^c = \{1, 2, 3, 4, 7, 8, \ldots\}$

Basic Theorems:

(1) Commutivity: $A \cup B = B \cup A$

$AB = BA$

(2) Associativity: $A \cup (B \cup C) = (A \cup B) \cup C$

$A(BC) = (AB)C$

(3) Distributivity: $A(B \cup C) = AB \cup AC$

$A \cup (BC) = (A \cup B)(A \cup C)$

(4) De Morgan's Law:

(i) $(A \cup B)^c = A^c \cap B^c$

(ii) $(AB)^c = A^c \cup B^c$.

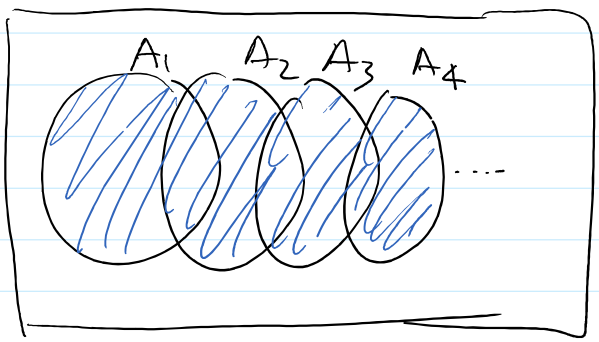
Countably Infinite Set Operations

Let $A_1, A_2, A_3, \ldots$ be subsets of S

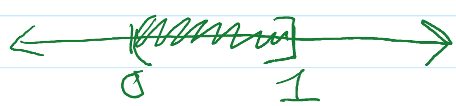
denoted: $(A_i)_{i=1}^{\infty}$

Defn: The union of these sets denoted

$$\bigcup_{i=1}^{\infty} A_i = \{x \in S \mid x \in A_i \text{ for some } i\}$$

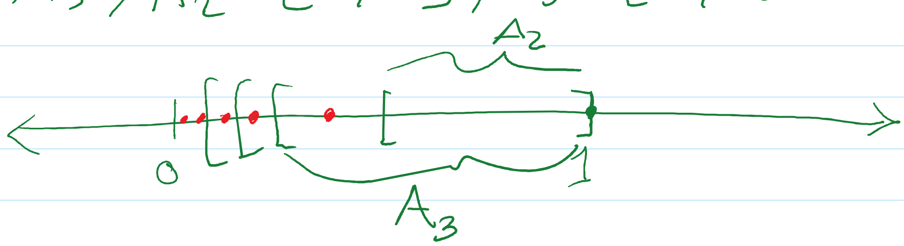


Ex. $S = (0, 1]$



$A_i = [1/i, 1]$ for $i = 1, 2, 3, \ldots$

$A_1 = \{1\}$, $A_2 = [1/2, 1]$, $A_3 = [1/3, 1]$



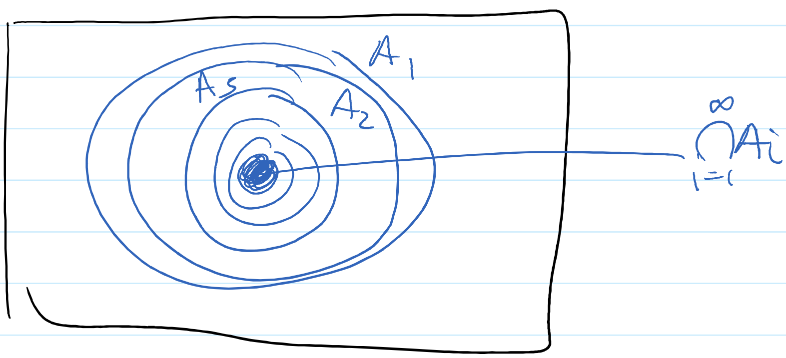
$$\bigcup_{i=1}^{\infty} A_i = (0, 1] = S$$

Defn: Infinite Intersection

Defn: Infinite Intersection

The infinite intersection of $(A_i)_{i=1}^{\infty}$ denoted

$$\bigcap_{i=1}^{\infty} A_i = \{x \in S \mid x \in A_i \text{ for all } i\}$$

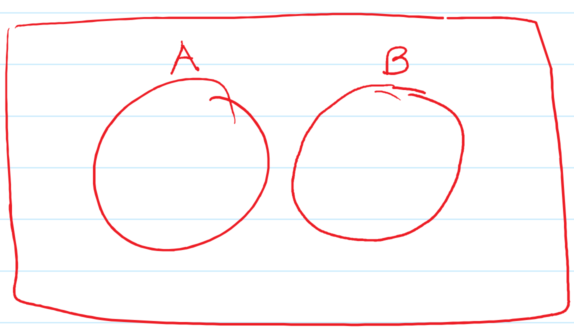


Ex. $A_i = [1/i, 1]$

$$\bigcap_{i=1}^{\infty} A_i = \{1\}$$

Defn: Disjoint

We say A, B are <u>disjoint</u> if $AB = \emptyset$.



no overlap

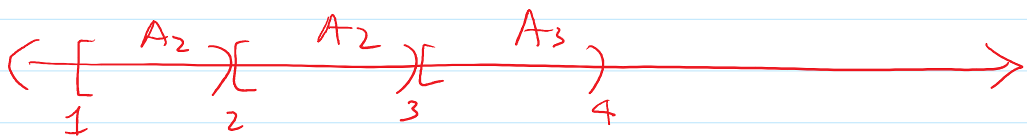
Ex. $A = \{1,2,3\}$, $B = \{4,5,6\}$) disjoint $AB = \emptyset$.

Defn: Pairwise Disjoint

If we have a collection $(A_i)_{i=1}^{\infty}$ we say they are pairwise disjoint if

$$A_i A_j = \emptyset \quad \forall\, i \neq j$$

Ex. $A_i = [i, i+1)$ $i = 1, 2, 3, \ldots$

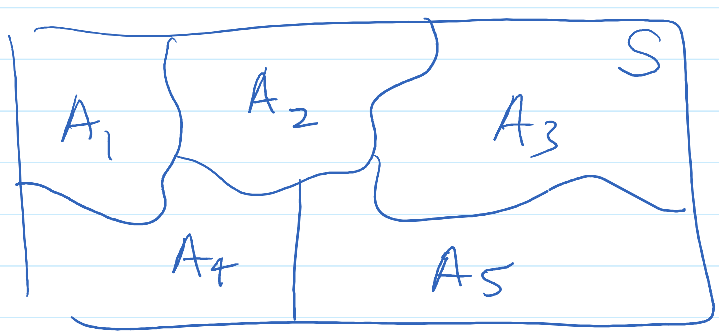


these are pairwise disjoint.

Defn: Partition

Given a set S and a seq. $(A_i)_i$ where $A_i \subseteq S$ then we say the the collection (A_i) are a partition of S if

1. (A_i) are (pairwise) disjoint
2. $\bigcup_i A_i = S$



Ex. $A_i = [i, i+1)$ these partition $[1, \infty) = S$

Defn: Power Set

For a set A the power set of A denoted

$$2^A = \mathcal{P}(A) = \{B \mid B \subset A\}$$

Ex. $A = \{1, 2\}$

$$2^A = \{\{1\}, \{2\}, A, \emptyset\}$$

notice: $|2^A| = 2^{|A|}$