Lecture 1 - Basic Set Notation

Thursday, September 2, 2021 2:02 PM

Defn: Set

A set is a collection of objects.

Ex. $S = \{1, 2, 3\}$

$\mathbb{N} = \{1, 2, 3, \ldots\}$ "natural numbers"

$\mathbb{Q} = \{m/n$ where $m, n \in \mathbb{N}\}$

Defn: Set Membership

We say "x is in S" denoted

$$x \in S$$

if S contains x as an element.

Ex. $5 \in \mathbb{N} = \{1, 2, 3, 4, 5, \ldots\}$

here it is!

Ex. $2/3 \in \mathbb{Q}$

Ex. $2/3 \notin \mathbb{N}$

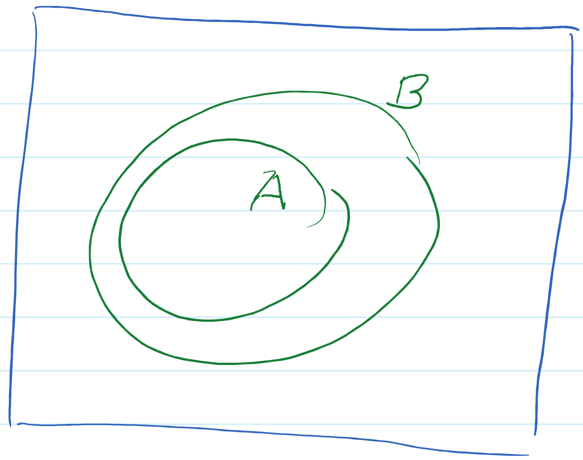
read: not in

Defn: Containment

We say "A is a subset of B" denoted

$$A \subset B$$

if $x \in A$ implies $x \in B$.



Ex. $\{1,2,3\} \subset \mathbb{N}$

Ex. $\mathbb{Q} \subset \mathbb{R}$ ← reals

Ex. $\mathbb{N} \not\subset \{1,2,3\}$ ← not subset

Defn: Set Equality

We say "A is equal to B" if both

$A \subset B$ and $B \subset A$.

We write $A = B$.

Set Operations

Defn: Union

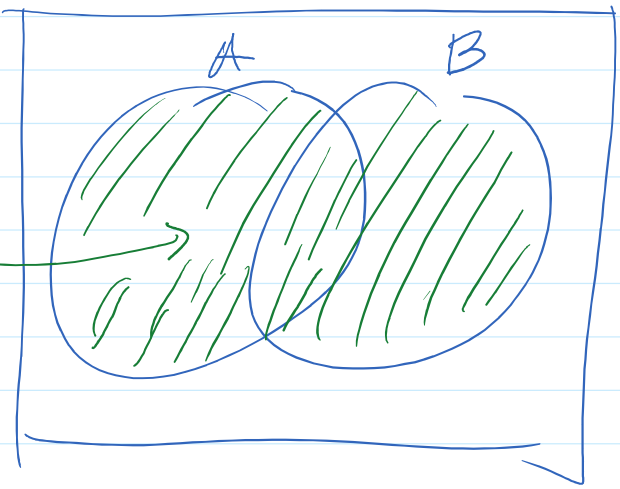
The union of A and B denoted $A \cup B$

is defined as

$$A \cup B = \{x \mid x \in A \ \underline{\text{or}} \ x \in B\}$$

Ex. $A = \mathbb{N}$
$B = \{-1, -2, -3, \dots\}$ $A \cup B$

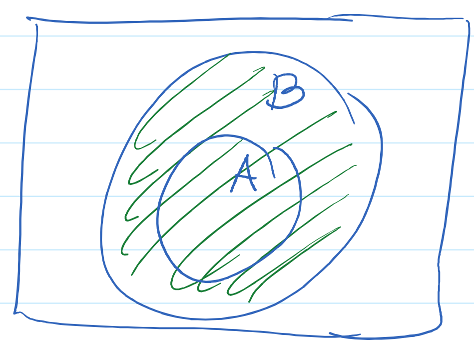
$A \cup B = \{\pm 1, \pm 2, \pm 3, \dots\}$



Ex. $\mathbb{Q} \cup \mathbb{R} = \mathbb{R}$
b/c $\mathbb{Q} \subset \mathbb{R}$

Fact: If $A \subset B$ then $A \cup B = B$

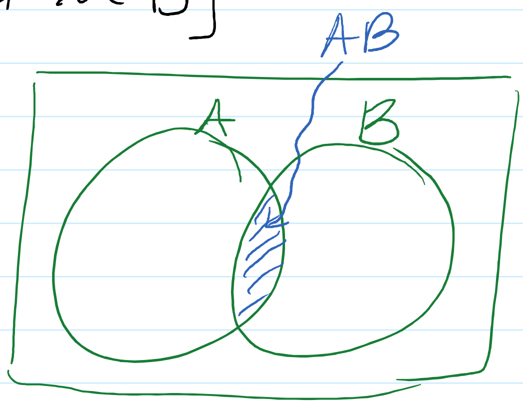
Ex. $\mathbb{N} \cup \mathbb{N} = \mathbb{N}$



Fact: $A \cup A = A$
(idempotency)

Defn: Intersection
We define the intersection of A and B denoted $A \cap B$ or AB as

$$AB = \{ x \mid x \in A \text{ and } x \in B \}$$



Ex, $A = \mathbb{N}$

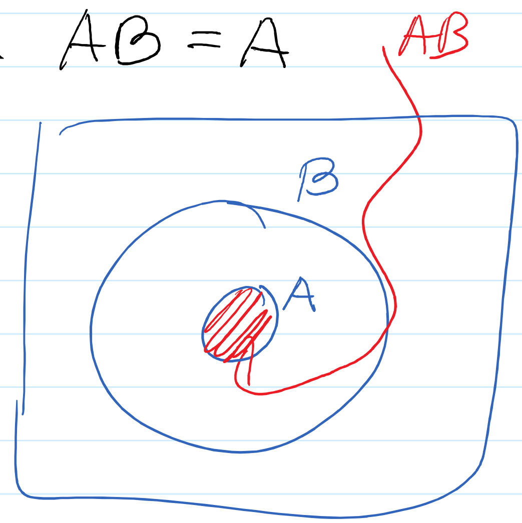
$B = \{-1, -2, -3, \dots\}$

then

$AB = \emptyset$ empty set

Ex. $\mathbb{Q}\mathbb{N} = \mathbb{N}$ b/c $\mathbb{N} \subset \mathbb{Q}$

Fact: If $A \subset B$ then $AB = A$



Fact: $AA = A$

(idempotency)

Defn: Set Difference

We say the "difference" between A and B denoted

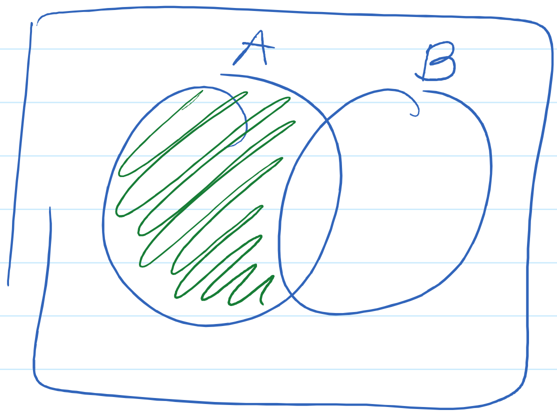
$$A \setminus B$$

is defined as

$$A \setminus B = \{x \mid x \in A \text{ and } x \notin B\}$$

Ex. $A = \{1, 2, 3\}$

$B = \{3, 4, 5\}$ $\quad A \setminus B$

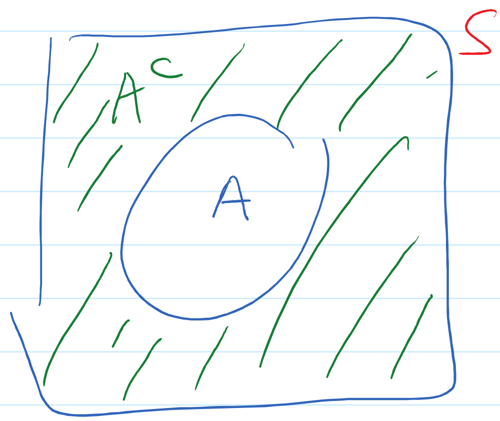


$A \setminus B = \{1, 2\}$

$B \setminus A = \{4, 5\}$

Defn: Complement

Want: $A^c = \{x \mid x \notin A\}$



$A = \{1, 2, 3\}$

A^c

Need: universe set S where $A \subset S$, then

$$A^c = S \setminus A = \{x \in S \mid x \notin A\}$$

Ex. $A = \{5, 6\} \subset \mathbb{N} = S$ then

$$A^c = \{1, 2, 3, 4, 7, 8, \dots\}$$

Basic Theorems

① Commutivity: $A \cup B = B \cup A$

$$AB = BA$$

② Associativity: $A \cup (B \cup C) = (A \cup B) \cup C$

$$A(BC) = (AB)C$$

③ Distributivity: $A(B \cup C) = AB \cup AC$

$$A \cup (BC) = (A \cup B)(A \cup C)$$

④ De Morgan's Laws:

① $(A \cup B)^c = A^c B^c$

② $(AB)^c = A^c \cup B^c$

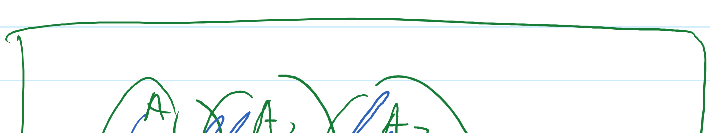
Countably Infinite Set Operations

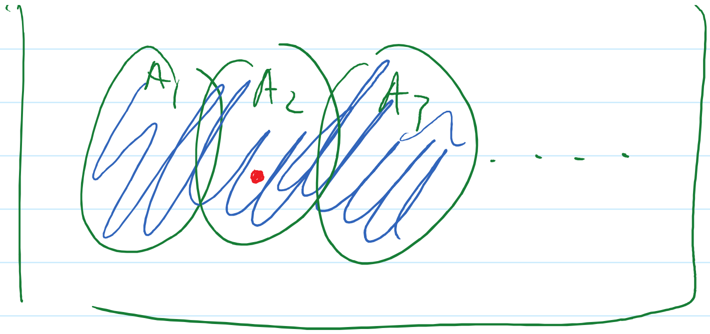
Let $A_1, A_2, A_3, \ldots$ be subsets of S

(notation $(A_i)_{i=1}^{\infty}$)

Defn: Countable Union

$$\bigcup_{i=1}^{\infty} A_i = \{x \in S \mid x \in A_i \text{ for some } i\}$$

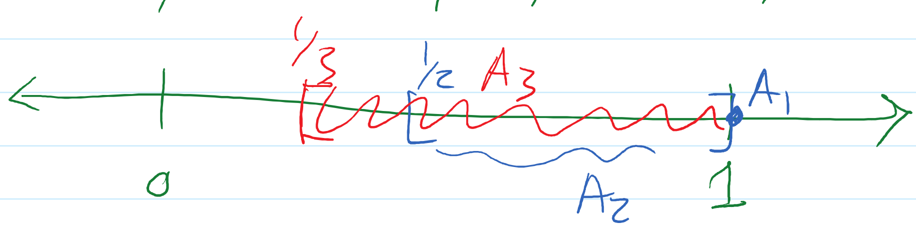




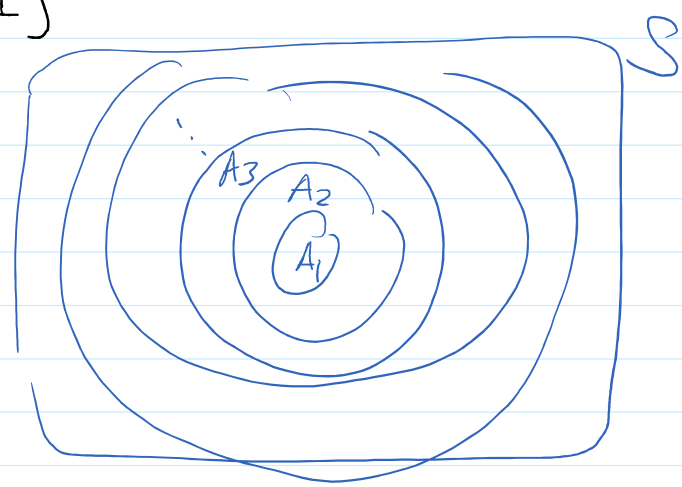
<u>Ex.</u> Let $S=(0,1]\subset\mathbb{R}$

and $A_i=[1/i,1]$

$A_1=\{1\}$, $A_2=[1/2,1]$, $A_3=[1/3,1]$



$$\bigcup_{i=1}^{\infty} A_i = S = (0,1]$$



<u>Defn: Countable Intersection</u>

The intersection of $(A_i)_{i=1}^{\infty}$ is

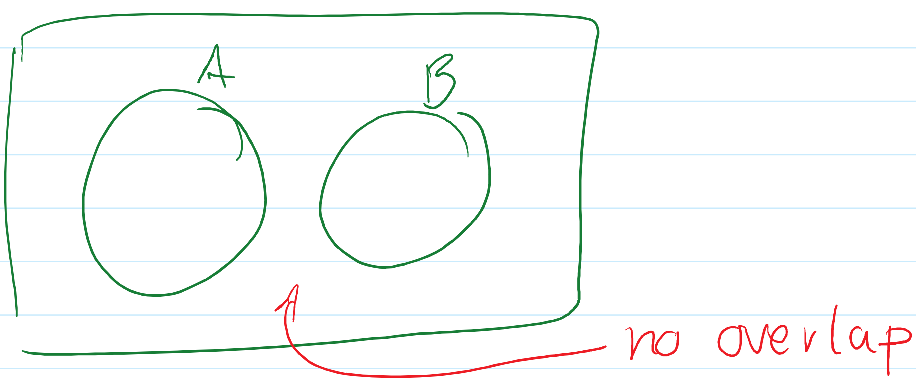
$$\bigcap_{i=1}^{\infty} A_i = \{ x \in S \mid x \in A_i \ \forall i \}$$

Ex, $A_i = [1/i, 1]$ then

$$\bigcap_{i=1}^{\infty} A_i = \{1\}$$

Defn: Disjoint

We say A and B are disjoint if $AB = \emptyset$.



Ex, $A = \{1,2,3\}$
$B = \{4,5,6\}$ then $AB = \emptyset$ so they are disjoint.

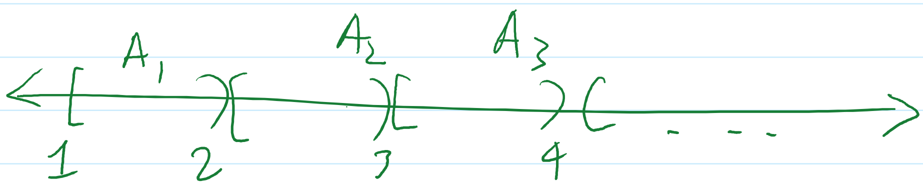
Defn: Pairwise Disjoint

If we have a sequence (A_i), we say

they are pairwise disjoint if

$$A_i A_j = \emptyset \quad \text{for } i \neq j$$

Ex. $A_i = [i, i+1)$ are pairwise disjoint

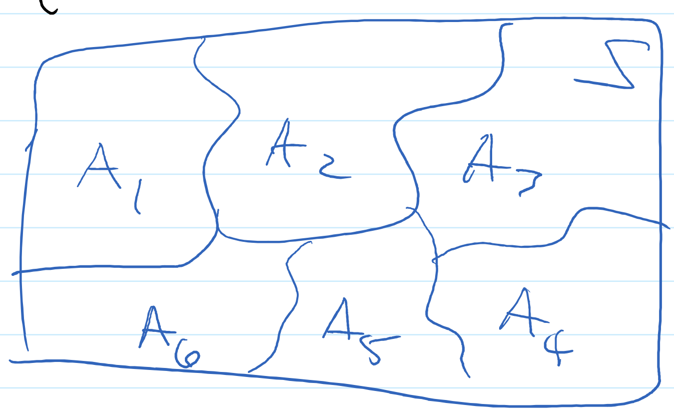


Defn: Partition

We say a seq (A_i) where $A_i \subset S$ partition S if

① the A_i are pairwise disjoint

② $\bigcup_i A_i = S$



Defn: Power Set

The power set of a set A denoted

$$\mathcal{P}(A) \quad \text{or} \quad 2^A$$

is defined as the set of all subsets of A

$$2^A = \{ B \mid B \subset A \}$$

<u>Ex.</u> $A = \{1, 2\}$ then $2^A = \{\{1\}, \{2\}, A, \emptyset\}$

<u>Fact:</u> $|2^A| = 2^{|A|}$