$1(sfatements) = \begin{cases} 0 & statement false \\ 1 & statement frue \end{cases}$   $Ex. 1(x \in A) = \begin{cases} 0 & x \notin A \\ 1 & x \in A \end{cases}$   $PDF = \begin{cases} a & D \end{cases}$ 

PDF of a RV X~ Exp(X)

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x > 0 \\ 0 & x \leq 0 \end{cases}$$

 $f(x) = \lambda e^{-\lambda x}$  for x > 0

$$f(x) = \lambda e^{-\lambda x} 1(x > 0)$$

More generally:

$$f(x) = m 1(x \in Support)$$

Independence:

$$X \perp Y \Leftrightarrow f(x,y) = f(x)f(y)$$

$$\frac{\xi_{X_{1}}}{f(x_{1}y)} = ce^{-x} - y = for x > 0 \text{ and } y > 0$$

$$= ce^{-x} - y = 1(x > 0) = ce^{-x} - y = 1(x > 0) =$$

$$\chi = (\chi_1, \dots, \chi_n)$$

$$\chi = (\chi_1, \dots, \chi_n) \in \mathbb{R}^n$$

$$f(X) = f(x_1, ..., x_n)$$

$$= f(x_1) f(x_2) - - - f(x_n)$$

$$= \prod_{i=1}^{n} f(x_i)$$

## Defn: Statistic

If (Xi) i=1 one a tond. sample and

$$T: \mathbb{R}^n \to \mathbb{R}^d$$

then T(X) is called a statistic.

$$\overline{X} = \frac{1}{n} (X_1 + X_2 + X_3 + \dots + X_n)$$

$$S^{2} = \frac{1}{2} \left( \chi_{i} - \chi_{j} \right)^{2}$$

Defn: Sampling Distribution of a Statistic

The samply dist. of T(X) is Simply the dist of T(X).

Theorem: Central Limit Theorem

Intro Stats:  $X \approx N(\mu, 6^2 h)$ for large n

451 statement:

If 
$$X_i$$
 are a random sample

when

 $Y = \sqrt{n} \left( \frac{X - \mu}{6} \right) \xrightarrow{d} N(0, 1)$ 
 $X \approx N(\mu, 6/n) \xrightarrow{f} N(0, 1)$ 
 $CDF/MGF of left$ 

Convers to  $CDF/MGF of Right$ 
 $X_i \stackrel{iid}{\sim} Bernaul(i(p))$ 
 $\mu = FX_i = p$  and  $Var X_i = p(1-p) = 6^2$ 

CLT says:

 $Var \left( \frac{X - p}{P(1-p)} \right) \xrightarrow{d} N(0, 1)$ 

practically:  $X \approx N(p, p(1-p))$ 

practically: 
$$X \approx N(p, \frac{P(1-p)}{p})$$
 $X - p \approx N(0, \frac{P(1-p)}{p})$ 

Var(aX)  $= \frac{X - P}{N} \approx N(0, 1)$ 

Intro stats:  $= \frac{75\%}{CT}$ :  $= \frac{X + 2\sqrt{X(1-X)}}{N}$ 

$$Y = \sqrt{n} \left( \frac{X - u}{6} \right)^{n} \cdot EX_{i} = u$$

$$= \sqrt{n} \sum_{i=1}^{n} X_{i} - u$$

$$= \left( \frac{\sqrt{n}}{n} \sum_{i=1}^{n} X_{i} - \sqrt{n} u \right) \quad \text{the } EY_{i} = 0$$

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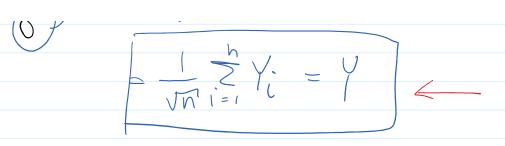
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Going to show that MGF of Y -> MGF of N(0,1)

 $M^{\sum X_{i}(t)} = M^{X_{i}(t)}M^{X_{i}(t)}$ 

 $M_{ax}(t) = M_{x}(at)$ 

 $M_{\gamma}(t) = M_{\frac{1}{\sqrt{n}}} \sum_{i=1}^{n} \gamma_{i}(t) = M(t/\sqrt{n})^{n}$ 

Taylor Series

g: R -> R +lut is k-times differentable

then  $T_{k}(\chi) = \sum_{r=0}^{k} \frac{g^{(r)}(0)}{r!} \chi^{r}$ 

then  $2 = g(x) - T_k(x) \rightarrow 0$ as  $x \rightarrow 0$ 

Approx M w/ a taylor series

M(1) = (1) + (1) + (1) + (1) + (2)

$$M(t) = \frac{(1)t^{6}}{0!} + \frac{M(1)(0)t^{1}}{(1!)} + \frac{M(2)(0)t^{2}}{2!}$$

$$= 1 + \frac{E(1)t^{2}}{2!} + \frac{E(1)t^{2}}{2!} + \cdots$$

$$= 1 + t^{2}/2 + \cdots$$

$$= 1 +$$