Lecture 1 - Basic Set Notation

Thursday, September 2, 2021 8:06 AM

Defn: Set

A set is a collection of objects.

Ex. $S = \{1, 2, 3\}$

$\mathbb{N} = \{1, 2, 3, \ldots\}$ "natural numbers"

$\mathbb{Q} = \{ m/n \text{ where } m, n \in \mathbb{N} \}$

Defn: Set Membership

We say "x is in S" denoted

$x \in S$

if S contains x as an element.

Ex. $5 \in \mathbb{N} = \{1, 2, 3, 4, 5, 6, \ldots\}$

here!

Ex. $2/3 \in \mathbb{Q}$

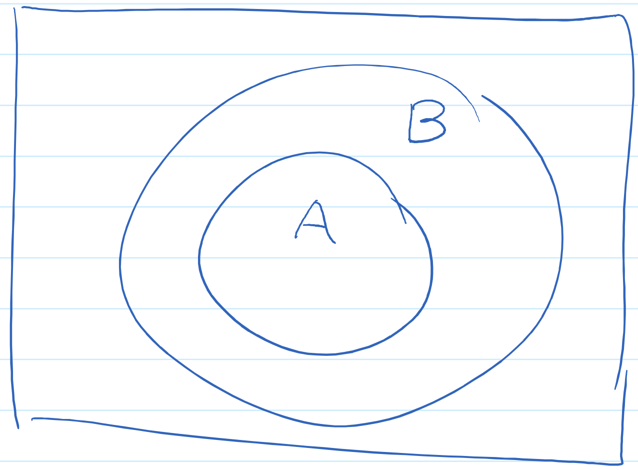
Ex. $2/3 \notin \mathbb{N}$

"not in"

Defn: Containment

We say "A is a subset of B" denoted

$$A \subset B$$



if $x \in A$ implies $x \in B$.

Ex, $\{1,2,3\} \subset \mathbb{N}$

Ex, $\mathbb{Q} \subset \mathbb{R}$

real numbers

Ex, $\mathbb{N} \not\subset \{1,2,3\}$

not a subset

Defn: Set Equality

We say "A is equal to B" if both

$A \subset B$ and $B \subset A$.

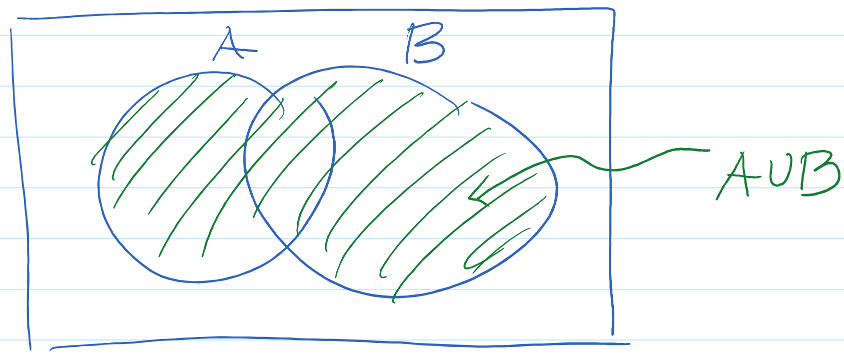
We write $A = B$.

Set Operations

Defn: Union

The union of A and B denoted $A \cup B$ is defined as

$$A \cup B = \{ x \mid x \in A \text{ or } x \in B \}$$



Ex. $A = \mathbb{N}$ and $B = \{-1, -2, -3, \dots\}$

$$A \cup B = \{\pm 1, \pm 2, \pm 3, \dots\}$$

Ex. $\mathbb{Q} \cup \mathbb{R} = \mathbb{R}$

Fact: $A \subset B$ then $A \cup B = B$

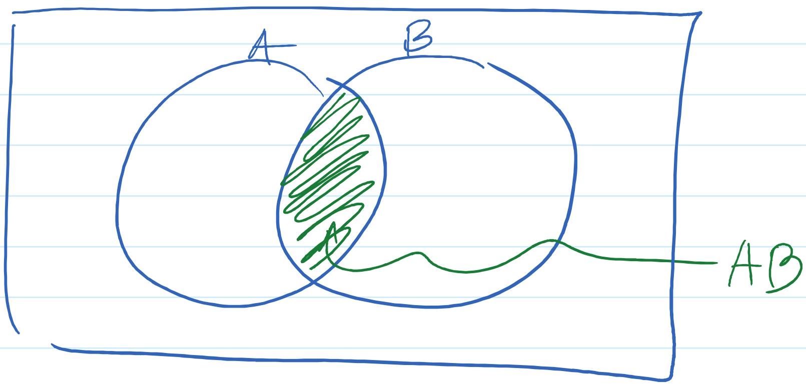
Ex. $\mathbb{N} \cup \mathbb{N} = \mathbb{N}$ [b/c $\mathbb{N} \subset \mathbb{N}$]

Fact: $A \cup A = A$ (idempotency)

Defn Intersection

We define the intersection of A and B denoted $A \cap B$ or AB

$A \cap B = AB = \{x \mid x \in A \text{ and } x \in B\}$



Ex. $A = \mathbb{N}$, $B = \{-1, -2, -3, \ldots\}$

$AB = \emptyset$ empty set containing no elements

Ex. $\mathbb{Q}\mathbb{N} = \mathbb{N}$ b/c $\mathbb{N} \subset \mathbb{Q}$

Fact: If $A \subset B$ then $AB = A$

Fact: $AA = A$ (idempotency)

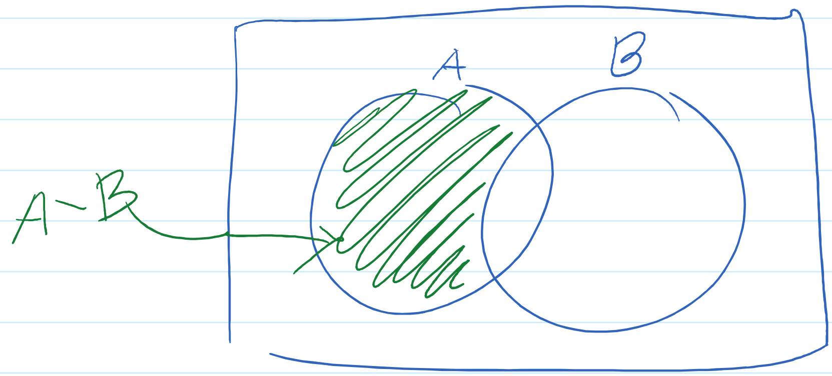
Defn: Set Difference

We say the difference btwn A and B denoted

$$A \setminus B$$

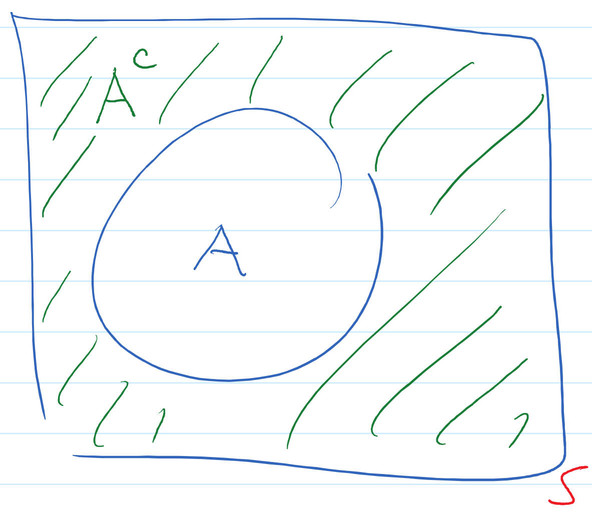
is defined as

$$A \setminus B = \{x \mid x \in A \text{ and } x \notin B\}$$



Ex. $A = \{1, 2, 3\}$
$B = \{3, 4, 5\}$ then $A \setminus B = \{1, 2\}$
$B \setminus A = \{4, 5\}$

Defn: Complements



Want:

$$A^c = \{x \mid x \notin A\}$$

Need same universe of possible sets S
where $A \subset S$

Then the complement of A denoted A^c is

$$A^c = \{x \in S \mid x \notin A\} = S \setminus A$$

Basic Theorems

Basic Theorems

(1) Commutativity: $A \cup B = B \cup A$

$AB = BA$

(2) Associativity: $A \cup (B \cup C) = (A \cup B) \cup C$

$A(BC) = (AB)C$

(3) Distributive: $A(B \cup C) = AB \cup AC$

$A \cup (BC) = (A \cup B)(A \cup C)$

(4) De Morgan's Law

(1) $(A \cup B)^c = A^c B^c$

(2) $(AB)^c = A^c \cup B^c$

Countably Infinite Set Operations

Let $A_1, A_2, A_3, \ldots$ be subsets of S

denoted $(A_i)_{i=1}^{\infty}$

Defn: Countable Union

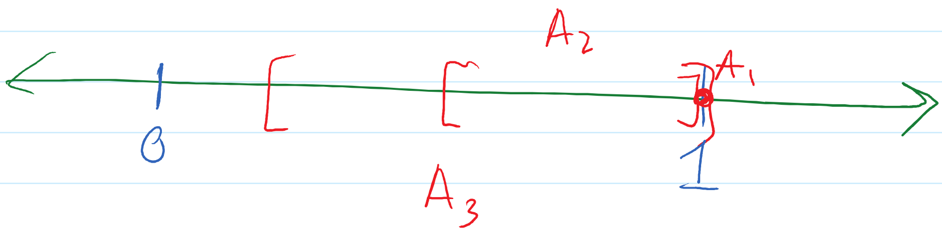
The union of these sets is

$$\bigcup_{i=1}^{\infty} A_i = \{x \in S \mid x \in A_i \text{ for some } i\}$$

Ex. $S = (0, 1] \subset \mathbb{R}$

$A_i = [1/i, 1]$ for $i = 1, 2, 3, \ldots$

$A_1 = \{1\}$, $A_2 = [1/2, 1]$, $A_3 = [1/3, 1]$



So $\displaystyle\bigcup_{i=1}^{\infty} A_i = (0, 1] = S$

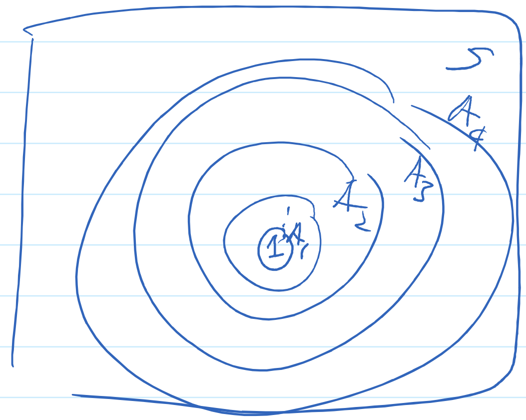
Defn: Countable Intersection

The countable intersection of $(A_i)_{i=1}^{\infty}$ is

$$\bigcap_{i=1}^{\infty} A_i = \{x \in S \mid x \in A_i \text{ for all } i\}$$

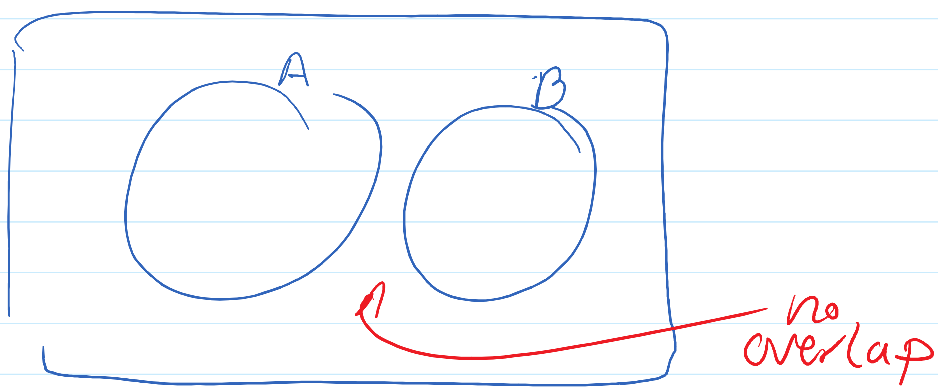
Ex. $A_i = [1/i, 1]$

$$\bigcap_{i=1}^{\infty} A_i = \{1\}$$



Defn: Disjoint

We say A and B are disjoint if $AB = \emptyset$.



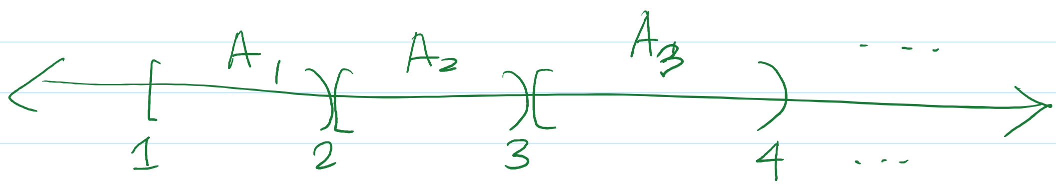
Ex. $A = \{1, 2, 3\}$
$B = \{4, 5, 6\}$ then $AB = \emptyset$
So they are disjoint.

Defn: Pairwise Disjoint

We say a collection (A_i) is pairwise disjoint if

$$A_i A_j = \emptyset \quad \text{for all } i \neq j$$

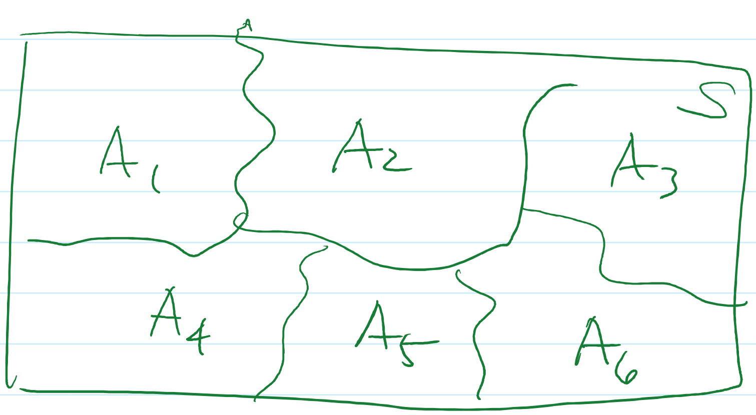
Ex. $A_i = [i, i+1) \subset \mathbb{R}$



Defn: Partition

Given a set S and a seq (A_i) where $A_i \subset S$ we say the collection are a partition of S if

1. A_i are pairwise disjoint
2. $\bigcup_i A_i = S$



Ex. $A_i = [i, i+1)$ then these partition $[1, \infty)$

Defn: Power Set

For a set A the powerset of A

$$\mathcal{P}(A) = 2^A = \{B \mid B \subset A\}$$

Ex. $A = \{1, 2\}$ then

$$2^A = \{\{1\}, \{2\}, A, \emptyset\}$$

notice $\quad |2^A| = 2^{|A|}$

card of 2^A