Laymen's defu:

-> things don't affect each other

-> events are independent if knowing

the occurrence (or not) of one event, doesn't

change the prob. of the other

Defn: Independence (of Events)

If A, B CS, we say "A is independent of B", denoted A LB, if P(AB) = P(A)P(B).

-> distributive Law fer intersection

> justifies product notation for intersection

Theorem:

If ALB then

P(A|B) = P(A).

Pf.

$$\frac{PF.}{P(A|B)} = \frac{P(AB)}{P(B)} = \frac{P(A)P(B)}{P(B)} = P(A).$$
Ex. Consider rolling two dice. Cincle Demo

$$P(at | east one (0))$$

$$= (-P(no (0s))$$

$$A = na (0s)$$

$$A_1 = no 6$$
 on first roll
$$A_2 = no 6 \text{ on second roll}$$

$$= (-P(A_1A_2))$$
 Assuming $A_1 \downarrow A_2$
$$= (-P(A_1)P(A_2))$$

Counting perspective

Sample twice
$$(r=2)$$
 from $\S1,...,6\S$ $(n=6)$ $w/$ replacement.

Ordered:

$$= \{(1,6), (2,6), (3,6), (4,6), (5,6), (6,6), (6,6), (6,6), (6,7$$

$$P(E) = \frac{|E|}{|S|} = \frac{11}{36}$$

Unoidered:

$$|S| = (n+r-1) = (0+2-1) = (\frac{4}{2}) = 21$$

$$P(E) = \frac{6}{21}.$$

Ex. Roll two dice (incle pardently)

$$E = \{(1,3), (1,4), (1;5) - (2,3), (2,4), (7,5)\}$$

note:
$$E = $1,23 \times $3,4,53$$
 castesian product

$$|\pm| = |51,23| \cdot |53,4,53|$$

= 2 · 3

Overall'.

$$P(E) = \frac{2.3}{6.6} = (\frac{2}{6})(\frac{3}{6})$$

prds. of a 1 or 2 a 3, 4, 5

Theorem: Complements and Independence

If A ILB then Gard

P(AB) = P(A) - P(AB)

O A I B

P(A) - P(A)P(B)

(2) A 11 R

$$= P(A)(I-P(B))$$

$$= P(A)P(B^{c})$$

Defn: Mutval Independence

Generalize independence to multiple events.

If (Ai) is a seg of events we say they are (mutually) independent if

for all subsequences

Ai, Aiz/Aiz/---; Air

 $P\left(\bigcap_{j=1}^{k} A_{ij} \right) = P(A_{i_1}) P(A_{i_2}) - P(A_{i_k})$ $= \frac{k}{|P(A_{i_j})|}$ $= \frac{1}{|P(A_{i_j})|}$

Q: Do I really read to check all subsequences?

(ould I just check:

No.

Ex. Roll two dice.

$$A = "doubles" = S(1,1),(2,2),...,(6,6) \} - |A| = 6$$

$$B = "Sum is between + and (0")$$

$$= \{(1,0),(7,5),(3,4),(4,3),(5,2),(6,1)\}$$

$$(2,6),(3,5),(4,4),(5,3),(6,2),(6,3),(6,3),(6,3),(6,4),(5,5),(4,6)\}$$

$$C = \frac{1}{5} \text{ sum is } 2 + \frac{1}{7} \frac{1}{1}$$

$$= \frac{3}{5} (1,1), \qquad \frac{1}{3} \frac{1}{1} \frac{1$$

Mutrally Independent?

P(ABC) = P(A)P(B)P(C)?

1/36

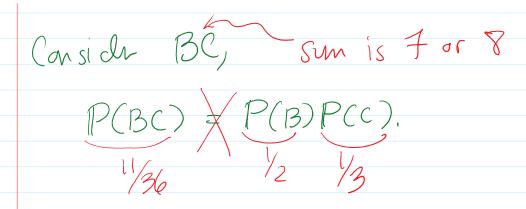
1/36

1/6

1/2

1/3

Maila Bon Bon San is for 8



Defn: Pairwise Independence

If (Ai) is a seg of events we say they are pairwise independent if

P(A; Aj) = P(Aj)P(Aj) for ifj.

Can ALA?

 $P(A) = P(AA) = P(A)P(A) = P(A)^2$

 $P(A) = P(A)^2$

recall: P(A) € [0,1]

This works if P(A) = 0 or 1.

Pairwise Independence & Motual Indep.

٤٤, S = {abc, acb, bac, bca, cab, cba, aaa, bbb, ccc 3. |S|=9, all outcomes egrally likely Ai = i Spot is an a A = gabc, acb, aaa} /A | = |A2 |= |A3 | = 3 Az = { bac, cab, aaa} Az = {bca, cba, aaa} Pairwise Independent? Saaa?

Pairwise Independent?

P(AiAj) = P(Ai) P(Aj)

1/9 = (3/9) (3/9)

Mutual Independence

P(A, A, A, A, 3) = P(A,) P(A, 2) P(A, 3)

1/9 + 1/3 /3 1/3

Not mutally independent,

Ex. JWST had ~ 400 single points of failure.

JWST fails if ony fail, Only works if all fasks succeed.

Assure flut all tasks are independent.

$$P(JWST works)$$

$$= P(()W_i)$$

$$= 1 P(W_i)$$

$$= 1 = 1$$

$$= TT (1 - P(W_i^c))$$

$$= TT (1 - 1000)$$

$$= (1 - 1000)$$

$$= (1 - 1000)$$

Ex. Flip a coin 3 times.

X = # heads among my 3 flips.

2 e S	$X(\mathcal{L})$	
H H H	<i>'</i> 5	
HHT	2	
HTH	2	
HTT	1 a fuction	
THH	2	
T 11 T		
T T H		
7:7		
·		