Lecture 1 - Basic Set Notation

Defn: Set

A set is a collection of objects.

Ex. $S = \{1, 2, 3\}$

$\mathbb{N} = \{1, 2, 3, 4, \ldots\}$ "natural numbers"

$\mathbb{Q} = \{ m/n : m, n \text{ are integers and } n \neq 0 \}$

Defn: Set Membership

We say that "x is in S" denoted

$$x \in S$$

if S contains x as an element.

Ex. $5 \in \mathbb{N}$

$2/3 \in \mathbb{Q}$

$2/3 \notin \mathbb{N}$ read: not in

Defn: Containment

We say "A is a subset of B" denoted

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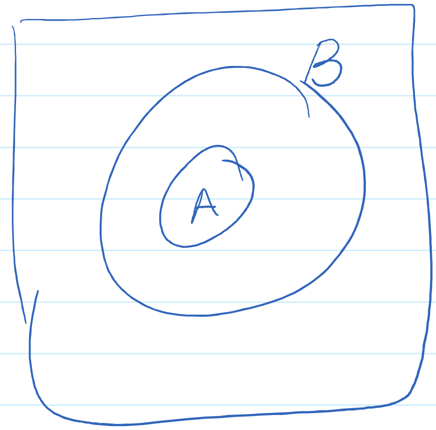
$$A \subset B$$



if $x \in A$ implies $x \in B$.

Ex. $\{1,2,3\} \subset \mathbb{N}$

$\mathbb{Q} \subset \mathbb{R}$ real numbers

$\mathbb{N} \not\subset \{1,2,3\}$ not a subset

Defn: Set Equality

We say "A is equal to B" if

$$A \subset B \quad \text{and} \quad B \subset A$$

We write $A = B$.

Set Operations

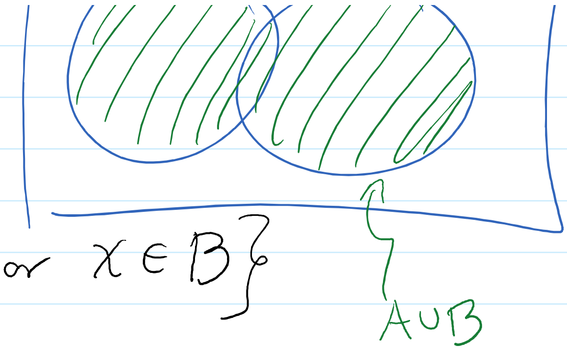
Defn: Union

The union of A and B denoted

$$A \cup B$$

$A \cup B$

is defined as



$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

<u>Ex.</u> $A = \mathbb{N}$, $B = \{-1, -2, -3, \ldots\}$

$$A \cup B = \{\pm 1, \pm 2, \pm 3, \ldots\}$$

<u>Ex.</u> $\mathbb{Q} \cup \mathbb{R} = \mathbb{R}$

b/c $\mathbb{Q} \subset \mathbb{R}$

<u>Fact</u>: If $A \subset B$ then $A \cup B = B$.

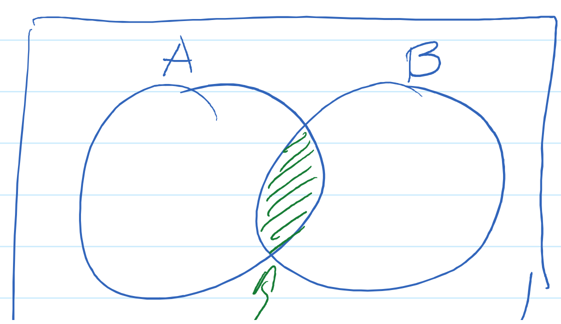
<u>Ex.</u> $\mathbb{N} \cup \mathbb{N} = \mathbb{N}$

<u>Fact</u>: $A \cup A = A$

<u>Defn: Intersection</u>

The intersection of A and B denoted

$$A \cap B \quad \text{or} \quad AB$$



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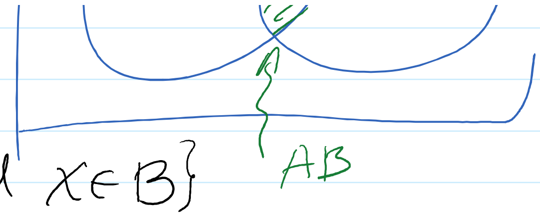


$$AB = \{x \mid x \in A \text{ and } x \in B\}$$

<u>Ex.</u> $A = \mathbb{N}$

$B = \{-1, -2, -3, \dots\}$

then $AB = \emptyset$ ← empty set

<u>Ex.</u> $\mathbb{Q}\mathbb{N} = \mathbb{N}$ b/c $\mathbb{N} \subset \mathbb{Q}$

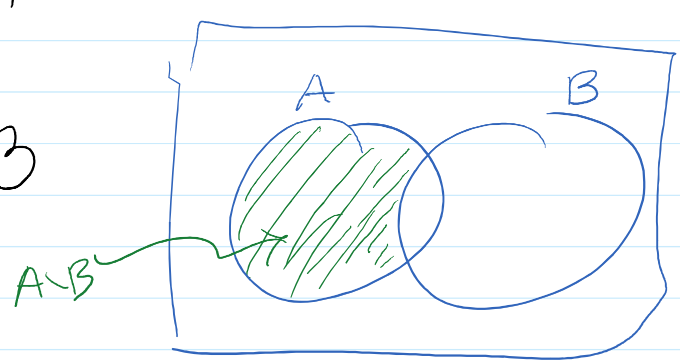
<u>Fact:</u> If $A \subset B$ then $AB = A$.

<u>Fact:</u> $AA = A$

<u>Defn: Set Difference</u>

We say the difference between A and B denoted

$$A \setminus B$$



is defined as

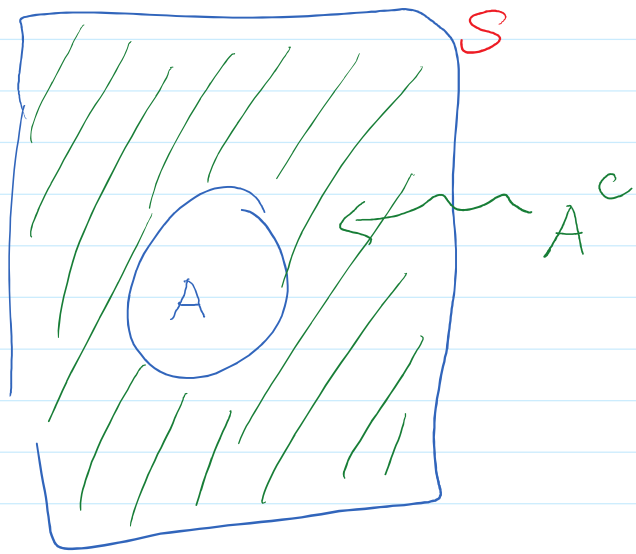
$$A \setminus B = \{x \mid x \in A \text{ and } x \notin B\}$$

Ex. $A = \{1, 2, 3\}$

$B = \{3, 4, 5\}$

then $A \setminus B = \{1, 2\}$, $B \setminus A = \{4, 5\}$

Defn: Complement



Want:

$$A^c = \{x \mid x \notin A\}$$

Need: universe set S

then

$$A^c = \{x \in S \mid x \notin A\} = S \setminus A$$

Ex. $A = \{1, 2\}$, $S = \mathbb{N}$

then $A^c = \{3, 4, 5, 6, \ldots\}$

Basic Theorems

① Commutivity: $A \cup B = B \cup A$

$AB = BA$

② Associativity: $A \cup (B \cup C) = (A \cup B) \cup C$

$$A(BC) = (AB)C$$

③ Distributivity: $A(B \cup C) = AB \cup AC$

$$A \cup (BC) = (A \cup B)(A \cup C)$$

④ De Morgan's Law:

① $(A \cup B)^c = A^c B^c$

② $(AB)^c = A^c \cup B^c$

Countably Infinite Set Operations

Let $A_1, A_2, A_3, \ldots$ be subsets of S

notation: $(A_i)_{i=1}^{\infty}$

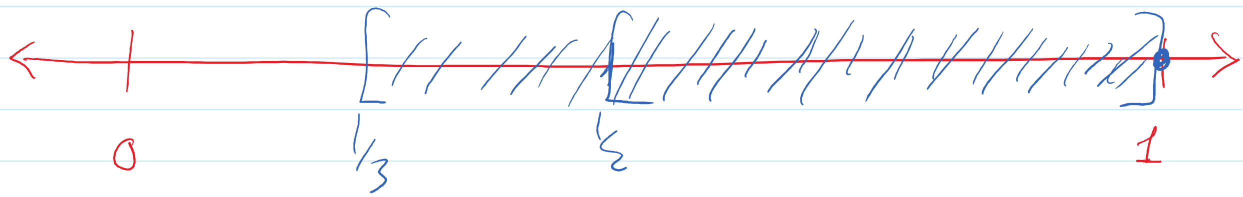
Defn: Countable Union

$$\bigcup_{i=1}^{\infty} A_i = \{x \in S \mid x \in A_i \text{ for some } i\}$$

Ex. let $S = (0, 1]$

and let $A_i = [1/i, 1]$

$A_1 = \{1\}$, $A_2 = [1/2, 1]$, $A_3 = [1/3, 1]$



$$\bigcup_{i=1}^{\infty} A_i = (0, 1]$$

Defn: Cantable Intersection

$$\bigcap_{i=1}^{\infty} A_i = \{x \in S \mid x \in A_i \ \forall i\}$$

Ex. continue from above

$$\bigcap_{i=1}^{\infty} A_i = \{1\}$$

Defn: Disjoint

We say A and B are disjoint if $AB = \emptyset$

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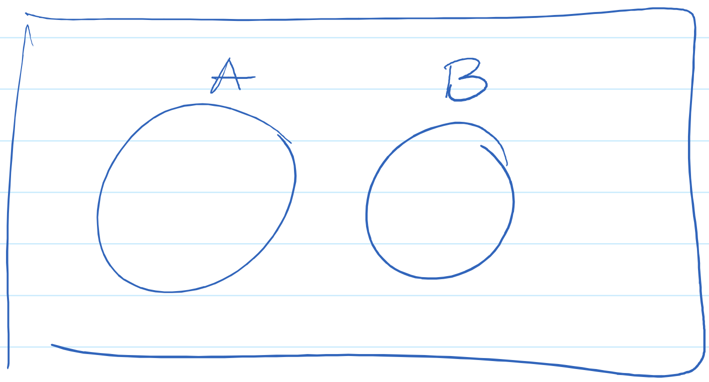


Ex. $A = \{1, 2, 3\}$ and $B = \{4, 5, 6\}$ then $AB = \emptyset$

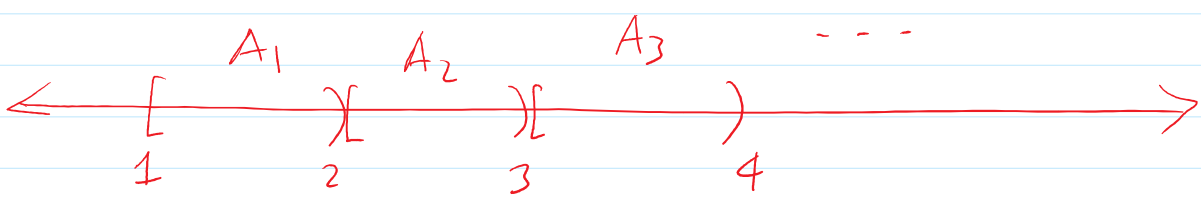
Defn: Pairwise Disjoint

A seq (A_i) is pairwise disjoint if

$$A_i A_j = \emptyset \quad \forall\, i \neq j$$

Ex. if $A_i = [i, i+1)$

then $A_i A_j = \emptyset$



Defn: Partition

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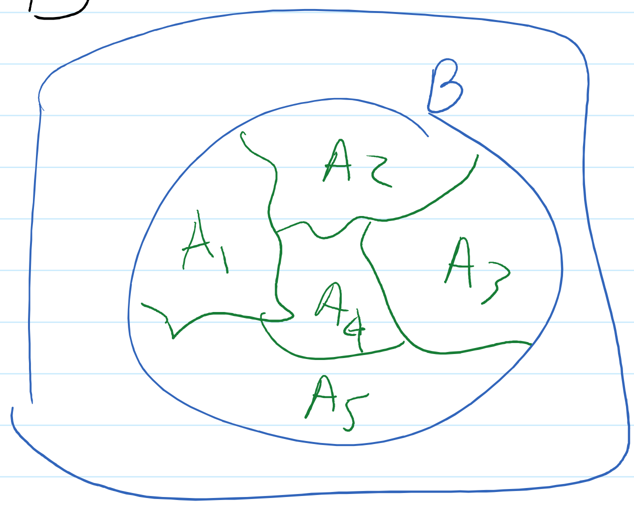
We say a seq (A_i) where $A_i \subset B$ are a partition of B

if



① the A_i are disjoint

② $\bigcup_i A_i = B$

Defn: Power Set

The power set of a set A is the collection of all subsets of A.

notation: $\mathcal{P}(A)$ or 2^A

$$2^A = \{B \mid B \subset A\}$$

Ex. $A = \{1,2\}$ then

$$2^A = \{\{1\}, \{2\}, A, \emptyset\}$$

Fact: $|2^A| = 2^{|A|}$