Defn: Set

A set is a collection of objects.

Ex. $S = \{1, 2, 3\}$

$\mathbb{N} = \{1, 2, 3, 4, \ldots\}$ "natural numbers"

$\mathbb{Q} = \{m/n : m, n \in \mathbb{Z}, n \neq 0\}$

$\mathbb{Z} = \{0, \pm 1, \pm 2, \ldots\}$

Defn: Set Membership

We say "x is in S" denoted

$$x \in S$$

if S contains x an element.

Ex. $5 \in \mathbb{N} = \{1, 2, 3, 4, 5, 6, \ldots\}$

$2/3 \in \mathbb{Q}$

$2/3 \notin \mathbb{N}$

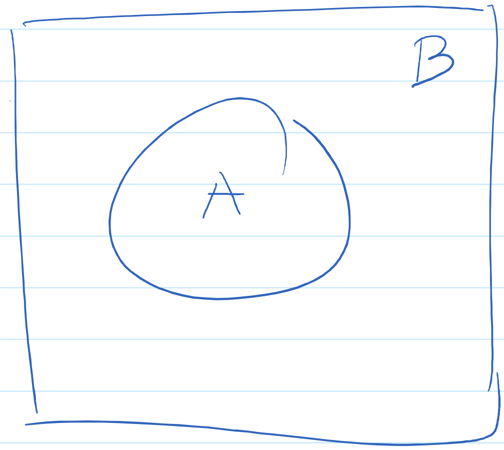
read: not in

Defn: Set Containment

We say "A is a subset of B"

denoted $A \subset B$

if $x \in A$ implies $x \in B$



$A \subset B$

$\{1,2,3\} \subset \mathbb{N}$

$\mathbb{Q} \subset \mathbb{R}$

real number

$\mathbb{N} \not\subset \{1,2,3\}$

not a subset

Defn: Set Equality

We say A and B are equal

denoted $A = B$

if $A \subset B$ and $B \subset A$.

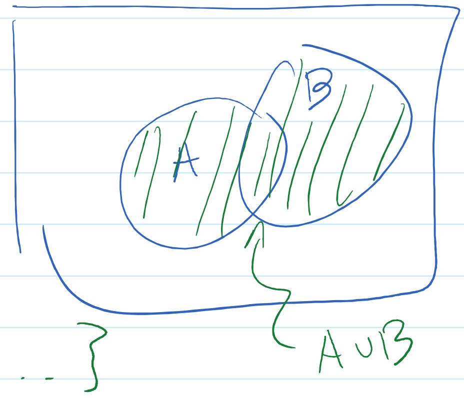
$\boxed{\text{Set Operations}}$

Defn: Union

The union of A and B denoted $A \cup B$

is defined as

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$



Ex. $A = \mathbb{N}$

$B = \{\pm 1, \pm 2, \pm 3, \dots\}$

then

$$A \cup B = \{\pm 1, \pm 2, \pm 3, \dots\}$$

Fact: $A \subset B$ then $A \cup B = B$

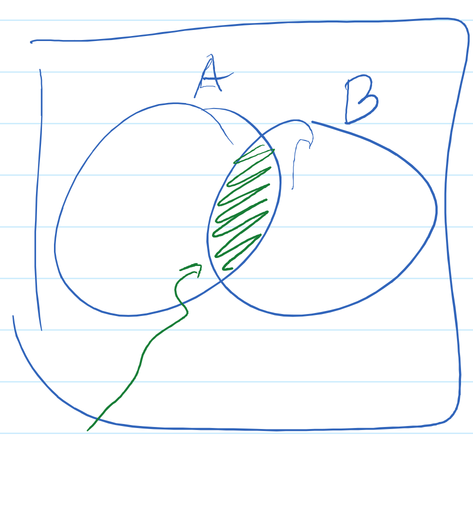
Ex. $\mathbb{Q} \subset \mathbb{R}$ so $\mathbb{Q} \cup \mathbb{R} = \mathbb{R}$

Fact: $A \cup A = A$ (Idempotency)

Defn: Intersection

We define the intersection of A and B denoted $A \cap B$ (or AB)

$$AB = \{x \mid x \in A \text{ and } B\}$$



Ex. $A = \{1, 2, 3\}$

$B = \{3, 4, 5\}$

AB

$AB = \{3\}$

Ex. $\mathbb{N}B = \emptyset$ where $B = \{-1, -2, -3, \dots\}$

empty set

Fact: $A \subset B$ then $AB = A$

Ex. $\mathbb{Q}\mathbb{R} = \mathbb{Q}$

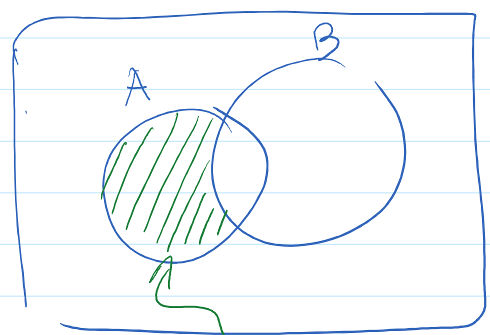
Fact: $AA = A$

Defn: Set Difference

We say ~~the~~ difference between A and B denoted

$$A \setminus B$$

is defined as



$A \setminus B$

$$A \setminus B = \{x \mid x \in A \text{ and } x \in B\}$$

Ex. $A = \{1, 2, 3\}$

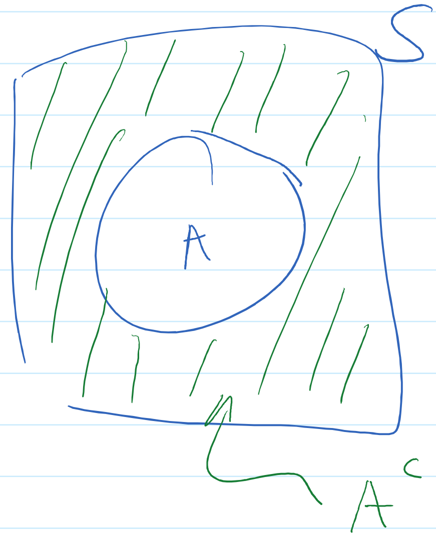
$B = \{3, 4, 5\}$

$A \setminus B = \{1, 2\}$ and $B \setminus A = \{4, 5\}$

Defn: Set Complements

Want: $A^c = \{x \mid x \notin A\}$

Need: Universe of sets S



The complement of A (against S) is

$$A^c = \{x \in S \mid x \notin A\} = S \setminus A$$

Ex. $A = \{5, 6\}$, $S = \mathbb{N}$

then $A^c = \{1, 2, 3, 4, 7, 8, \dots\}$

Basic Theorems

(1) Commutivity: $A \cup B = B \cup A$

$AB = BA$

(2) Associativity: $A \cup (B \cup C) = (A \cup B) \cup C$

$A(BC) = (AB)C$

③ Distributivity: $A(B \cup C) = AB \cup AC$

$A \cup (BC) = (A \cup B)(B \cup C)$

④ De Morgan's Laws:

① $(A \cup B)^c = A^c B^c$

② $(AB)^c = A^c \cup B^c$

Countably Infinite Set Operations

Let $A_1, A_2, A_3, \ldots$ be subsets of S

notation: $(A_i)_{i=1}^{\infty}$

Defn: Countable Union:

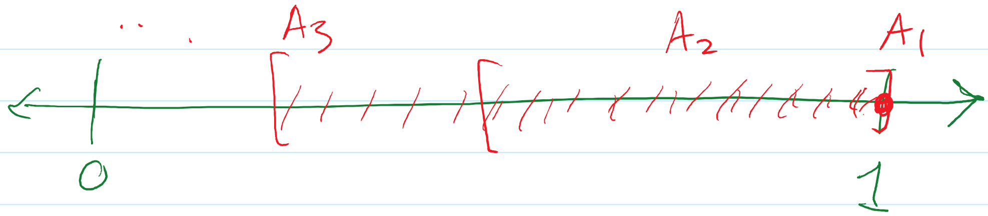
$$\bigcup_{i=1}^{\infty} A_i = \{x \in S \mid x \in A_i \text{ for some } i\}$$

Ex. Let $S = (0,1] \subset \mathbb{R}$

and $A_i = \left[\frac{1}{i}, 1\right]$

$A_1 = \{1\}, A_2 = [1/2, 1], A_3 = [1/3, 1], \ldots$

$A_1 = \{1\}$, $A_2 = [1/2, 1]$, $A_3 = [1/3, 1]$, ...



$$\bigcup_{i=1}^{\infty} A_i = (0, 1] = S$$

Defn: Countable Intersection

$$\bigcap_{i=1}^{\infty} A_i = \{x \in S \mid x \in A_i \ \forall i\}$$

Ex. continue prev.

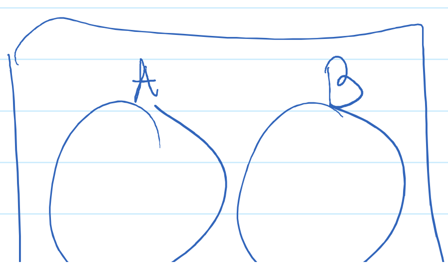
$$\bigcap_{i=1}^{\infty} A_i = \{1\}$$

Defn: Disjoint.

We say A and B are disjoint if $AB = \emptyset$.

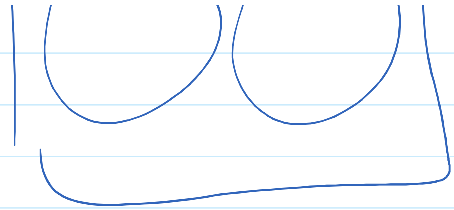
Ex. $A = \{1, 2, 3\}$

$B = \{4, 5, 6\}$



$AB = \emptyset$

$AB = \emptyset$

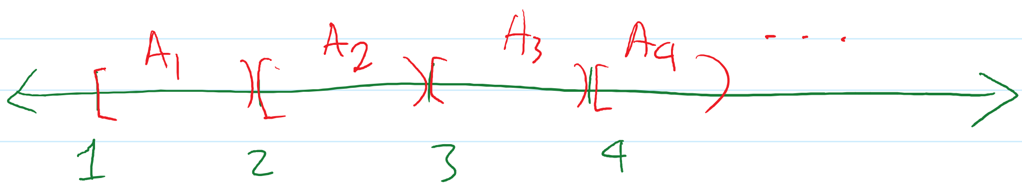


Defn: Pairwise Disjoint

If I have a seq (A_i) we say this seq is pairwise disjoint if

$$A_iA_j = \emptyset \quad \forall i \neq j.$$

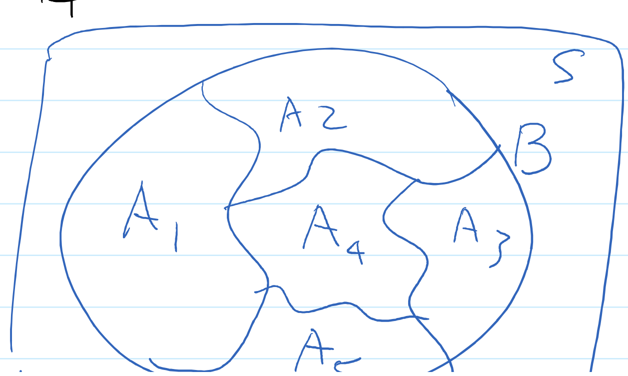
Ex. $A_i = [i, i+1)$



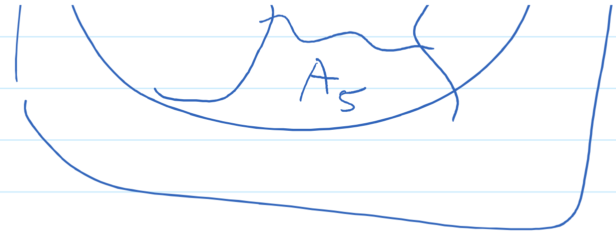
Defn: Partition

We say a seq (A_i) where $A_i \subset B$ are a partition of B if

① the (A_i) are pairwise disjoint



② $\bigcup_i A_i = B$



Defn: Power Set

The power set of a set A

denoted

$$\mathcal{P}(A) \quad \text{or} \quad 2^A$$

is defined as the set of all subsets of A

$$2^A = \{B \mid B \subset A\}$$

Ex. $A = \{1,2\}$

$$2^A = \{\{1\}, \{2\}, \{1,2\}, \emptyset\}$$

Fact: $|2^A| = 2^{|A|}$