Lecture 1: Set Notation

Defn: Set

A set is a collection of objects

Ex. $S = \{1, 2, 3\}$

$\mathbb{N} = \{1, 2, 3, 4, \ldots\}$ "natural numbers"

$\mathbb{Q} = \{ m/n : m, n \in \mathbb{N}, n \neq 0 \}$

Defn: Set Membership

We say that "x is in S" denoted

$$x \in S$$

if S contains x as an element.

Ex. $5 \in \mathbb{N}$

$2/3 \in \mathbb{Q}$

$2/3 \notin \mathbb{N}$

Defn: Containment

We say that "A is a subset of B"

denoted $A \subset B$

if $x \in A$ then $x \in B$.

Ex. $\{1,2,3\} \subset \mathbb{N}$

$\mathbb{Q} \subset \mathbb{R}$ real numbers

$\mathbb{N} \not\subset \{1,2,3\}$

Defn: Set Equality

We say A equals B, denoted

$$A = B$$

if $A \subset B$ and $B \subset A$

Set Operations

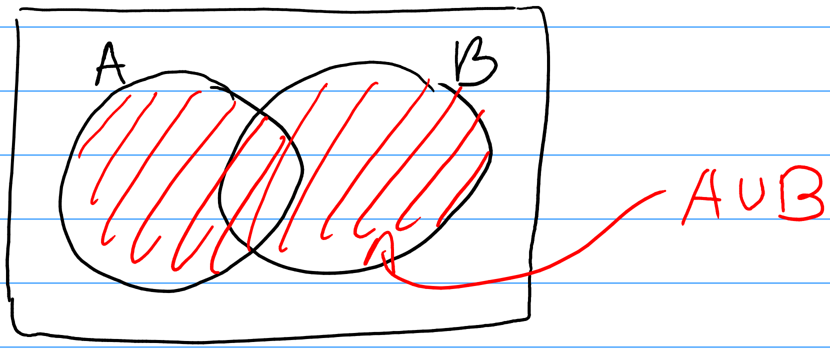
Defn: Union

The union of A and B, denoted

$$A \cup B$$

is defined as

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$



Ex. $A = \mathbb{N}$, $B = \{-1, -2, -3, \dots\}$

then

$$A \cup B = \{\pm 1, \pm 2, \pm 3, \dots\}$$

Ex. $\mathbb{Q} \cup \mathbb{R} = \mathbb{R}$ b/c $\mathbb{Q} \subset \mathbb{R}$

Fact: $A \subset B$ then $A \cup B = B$

Fact: $A \cup A = A$

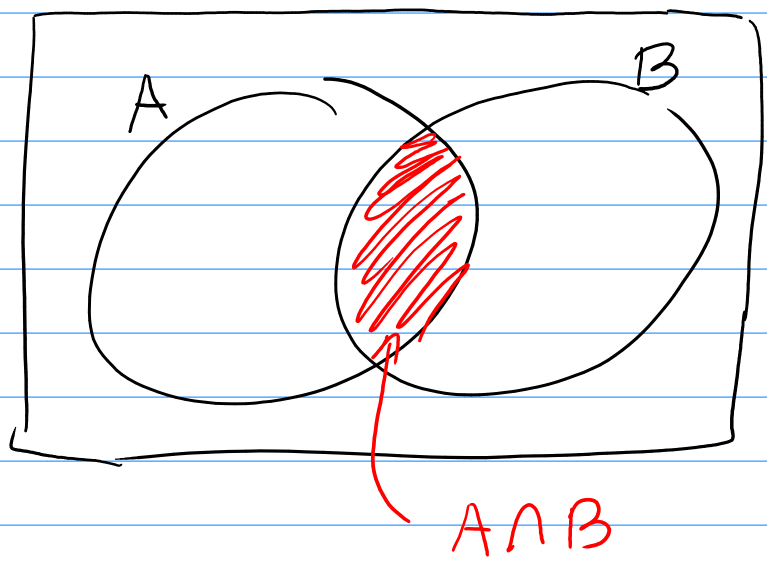
Defn: Intersection

The intersection of A and B, denoted

$$A \cap B \quad \text{or} \quad AB$$

is defined as

$$A \cap B = \{x \mid x \in A \ \underline{\text{and}} \ x \in B\}$$



Ex. $A = \mathbb{N}$, $B = \{-1, -2, -3, \ldots, \}$

then $A \cap B = \emptyset$ empty set

Ex. $\mathbb{Q}\mathbb{N} = \mathbb{N}$ b/c $\mathbb{N} \subset \mathbb{Q}$

Fact: If $A \subset B$ then $AB = A$

Fact: $AA = A$

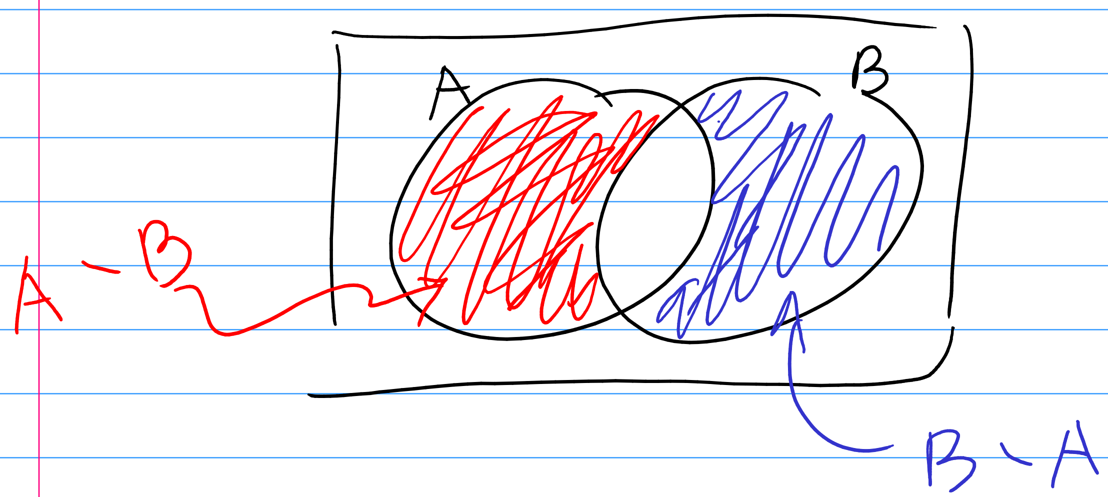
Defn: Set Difference

We say the difference between A and B denoted

$$A \setminus B$$

is defined as

$$A \setminus B = \{x \mid x \in A,\ x \notin B\}$$

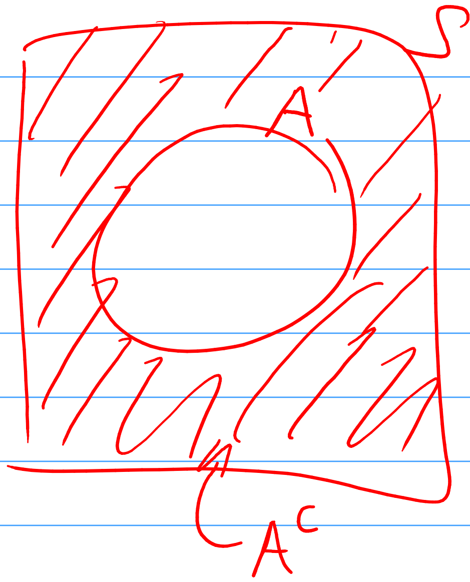


Ex. $A = \{1, 2, 3\}$

$B = \{3, 4, 5\}$

$A \setminus B = \{1, 2\}$, $B \setminus A = \{4, 5\}$

Defn: Complement



Want: $A^c = \{x \mid x \notin A\}$

Need: universe set S

$$A^c = \{x \in S \mid x \notin A\} = S \setminus A$$

Ex. $A = \{1, 2\}$, $S = \mathbb{N}$

then $A^c = \{3, 4, 5, \ldots\}$

Basic Theorems:

1. Commutivity: $A \cup B = B \cup A$
   $AB = BA$

2. Associative: $A \cup (B \cup C) = (A \cup B) \cup C$
   $A(BC) = (AB)C$

③ <u>Distributivity:</u> $A \cup (BC) = (A \cup B)(A \cup C)$

$$A(B \cup C) = (AB) \cup (AC)$$

④ <u>De Morgan's Laws</u>

Ⅰ $(A \cup B)^c = A^c B^c$

Ⅱ $(AB)^c = A^c \cup B^c$

Countably Infinite Set Operations

Let $A_1, A_2, A_3, \ldots$ be subsets of S

<u>notation:</u> $(A_i)_{i=1}^{\infty}$

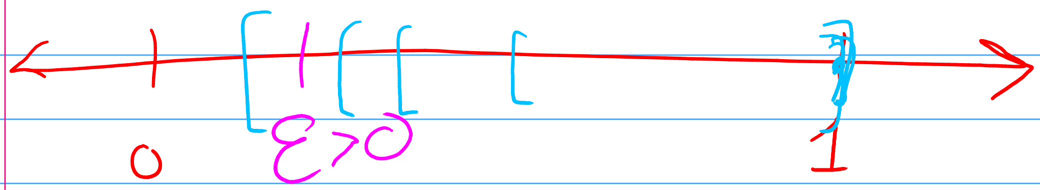
<u>Defn: Countable Union</u>

$$\bigcup_{i=1}^{\infty} A_i = \{x \in S \mid x \in A_i \text{ for some } i\}$$

<u>Ex.</u> Let $S = (0, 1]$

let $A_i = [1/i, 1]$

then $\displaystyle\bigcup_{i=1}^{\infty} A_i = (0,1] = S$



If $i > 1/\varepsilon \Rightarrow 1/i < \varepsilon$

so $\varepsilon \in A_i$

Defn: Countable Intersection

$$\bigcap_{i=1}^{\infty} A_i = \{x \in S \mid x \in A_i \text{ for all } i\}$$

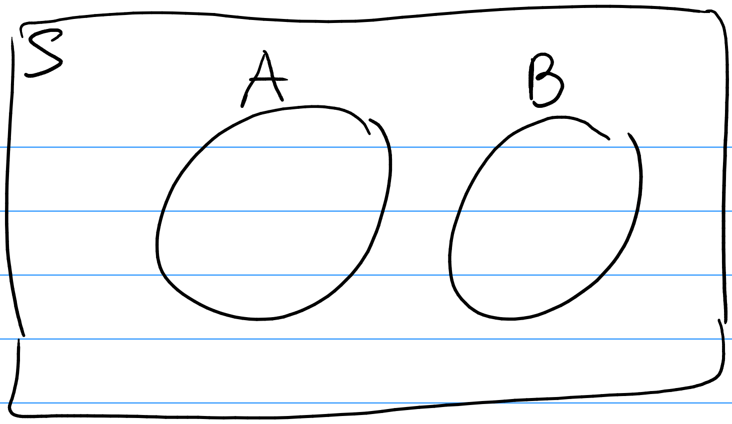
Ex. (from above)

$$\bigcap_{i=1}^{\infty} A_i = \{1\}$$

Defn: Disjoint

We say A and B are disjoint if

$$AB = \emptyset$$



$AB = \emptyset$

<u>Ex.</u> $A = \{1, 2, 3\}$
$B = \{4, 5, 6\}$
then A, B are disjoint

<u>Defn: Pairwise Disjoint</u>

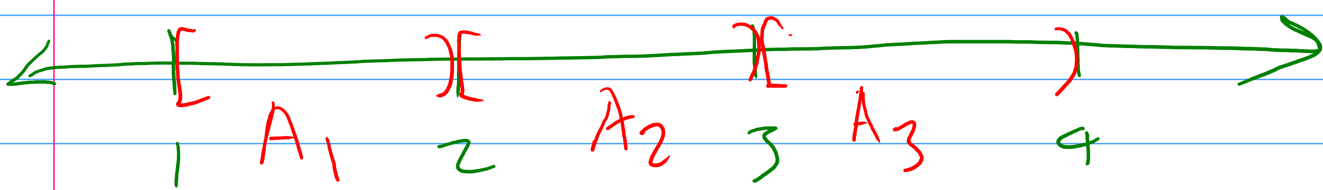
A seq (A_i) is pairwise disjoint if

$$A_i A_j = \emptyset \quad \forall\, i \neq j$$

<u>Ex.</u> If $A_i = [i, i+1)$ for $i = 1, 2, 3, \ldots$

then

$$A_i A_j = \emptyset$$



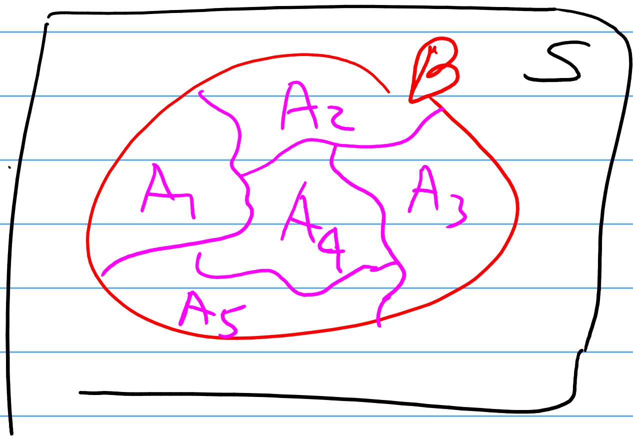
Defn: Partition:

We say a sequence (A_i) where $A_i \subset B$ partitions B

if

(I) the A_i are (pairwise) disjoint

(II) $\bigcup_i A_i = B$



Ex. $A_i = [i, i+1)$ partition $[1, \infty)$

Defn: Power Set

The power set of a set A is the collection of all subsets of A

notation: $\mathcal{P}(A)$ or 2^A

mathy:

$$2^A = \{B \mid B \subset A\}$$

Ex. $A = \{1, 2\}$ then

$$2^A = \{\{1\}, \{2\}, A, \emptyset\}$$

Fact: $|2^A| = 2^{|A|}$

$|\cdot| =$ cardinality $= \#$ of elements