Lecture 1: Basic Set Notation

Defn: Set

A set is a collection of objects

Ex. $S = \{1, 2, 3\}$

$\mathbb{N} = \{1, 2, 3, 4, 5, \ldots\}$

$\mathbb{Q} = \{ m/n$ where m, n are natural numbers, $n \neq 0 \}$

Defn: Set Membership

We say that "x is in S" denoted

$$x \in S$$

if S contains x as an element.

Ex. $5 \in \mathbb{N}$

$2/3 \in \mathbb{Q}$

$2/3 \notin \mathbb{N}$ read: not in

Defn: Containment (Subset)

We say A is a subset of B, denoted

$$A \subset B$$

if $x \in A$ implies $x \in B$.

$A \subset B$ ←
$A \subseteq B$
$A \subsetneq B$

Defn: Set Equality

We say A is equal to B if

$$A \subset B \text{ and } B \subset A.$$

We write

$$A = B.$$

Set Operations

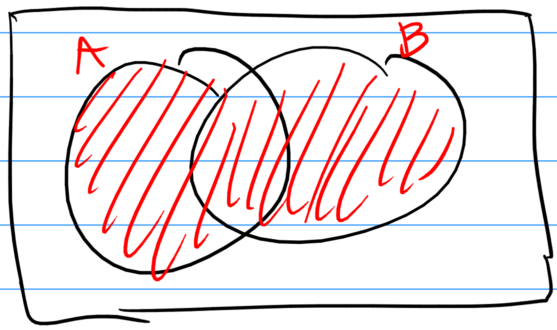
Defn: Union

The union of A and B, denoted

$$A \cup B$$

is the set

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$



Ex. $A = \mathbb{N}$, $B = \{-1, -2, -3, \dots\}$

then $A \cup B = \{\pm 1, \pm 2, \pm 3, \dots\}$

Ex. $\mathbb{Q} \cup \mathbb{R} = \mathbb{R}$ real numbers b/c $\mathbb{Q} \subset \mathbb{R}$

Fact: $A \subset B$ then $A \cup B = B$

Fact! $A \cup A = A$

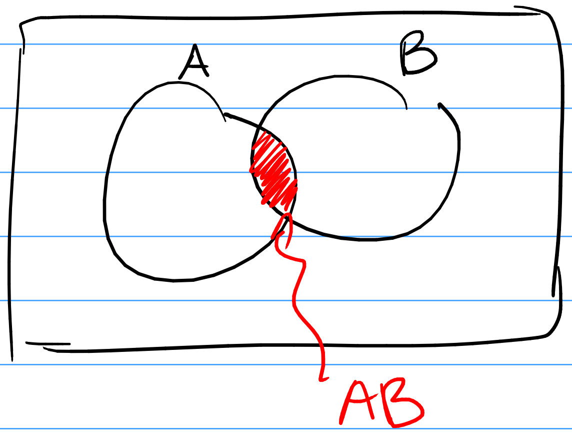
Defn: Intersection

The intersection of A and B, denoted

$$A \cap B \quad \text{or} \quad AB$$

is defined as

$$AB = \{x \mid x \in A \ \underline{\text{and}} \ x \in B\}$$



Ex. $A = \mathbb{N}$

$B = \{-1, -2, -3, \dots\}$

$AB = \emptyset$ empty set

Ex. $\mathbb{Q}\,\mathbb{N} = \mathbb{N}$ b/c $\mathbb{N} \subset \mathbb{Q}$

Fact! $A \subset B$ then $AB = A$

Fact! $AA = A$

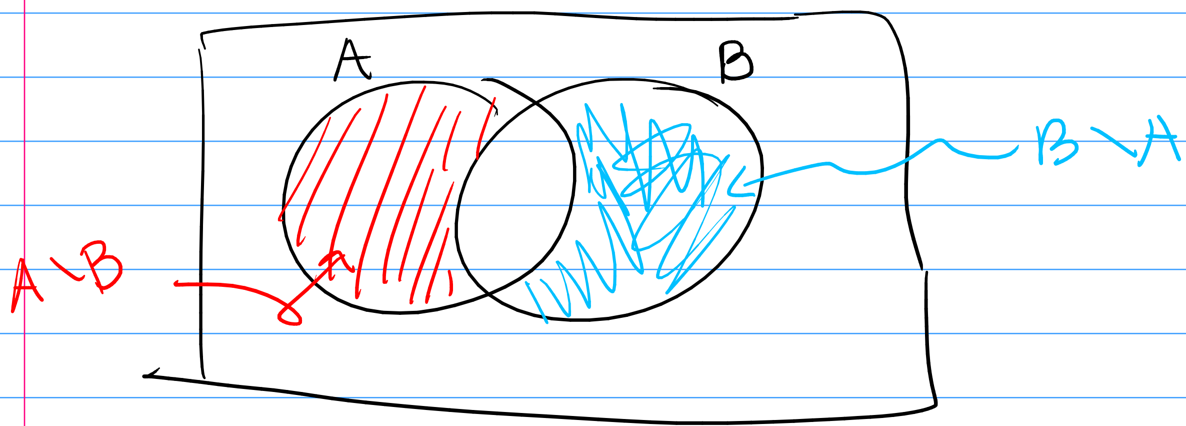
Defn! Set Difference

We say the difference btwn A and B denoted

$$A \setminus B$$

is defined as

$$A \setminus B = \{x \mid x \in A \text{ and } x \notin B\}$$



Ex. $A = \{1, 2, 3\}$
$B = \{3, 4, 5\}$

then $A \setminus B = \{1, 2\}$ and $B \setminus A = \{4, 5\}$.

Defn: Complement

Want: $A^c = \{x \mid x \notin A\}$

Need: universe of possibilities S

then $A^c = \{x \in S \mid x \notin A\} = S \setminus A$

Ex. $A = \{1, 2\}$, $S = \mathbb{N}$

then $A^c = \{3, 4, 5, \dots\}$

Basic Theorems

(1) commutivity: $A \cup B = B \cup A$

$AB = BA$

(2) Associative: $A(BC) = (AB)C$

$A \cup (B \cup C) = (A \cup B) \cup C$

(3) Distributivity: $A(B \cup C) = (AB) \cup (AC)$

$A \cup (BC) = (A \cup B)(A \cup C)$

(4) De Morgan's Laws:

(1) $(A \cup B)^c = A^c B^c$

② $(AB)^c = A^c \cup B^c$

Countably Infinite Set Operations

Let $A_1, A_2, A_3, \ldots$ be a seq of sets $A_i \subset S$

notation: $(A_i)_{i=1}^{\infty}$

Defn: Countable Union

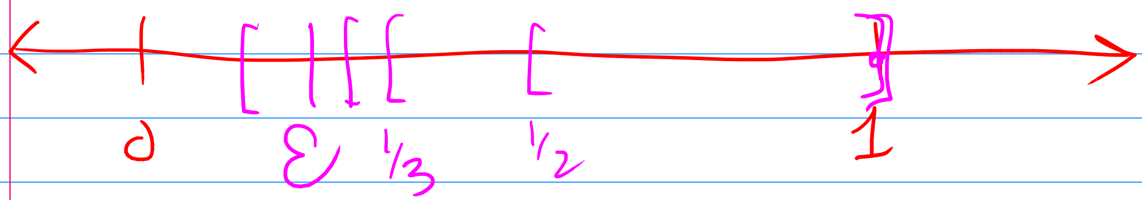
$$\bigcup_{i=1}^{\infty} A_i = \{x \in S \mid x \in A_i \text{ for some } i\}$$

Ex. $S = (0,1]$

Let $A_i = [1/i, 1]$

$A_1 = \{1\}$, $A_2 = [1/2, 1]$, $A_3 = [1/3, 1]$, ...

$$\bigcup_{i=1}^{\infty} A_i = (0,1] = S$$



If $i > 1/\varepsilon \Rightarrow 1/i < \varepsilon \Rightarrow \varepsilon \in A_i$

Countable Intersection

$$\bigcap_{i=1}^{\infty} A_i = \{x \in S \mid x \in A_i \text{ for all } i\}$$

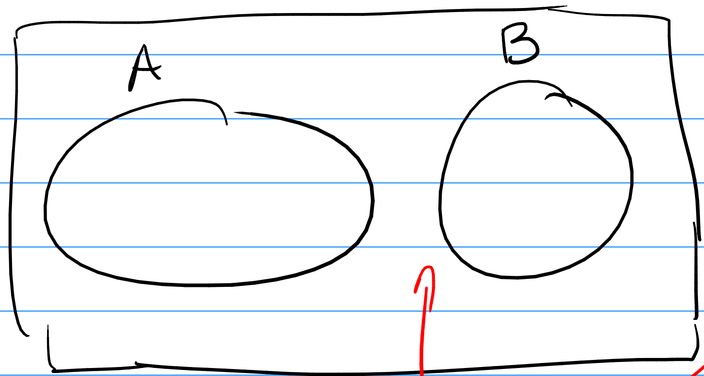
Ex. (above)

$$\bigcap_{i=1}^{\infty} A_i = \{1\}$$

Defn: **Disjoint**

We say A and B are disjoint if

$$AB = \emptyset$$



no overlap

Ex. $A = \{1, 2, 3\}$

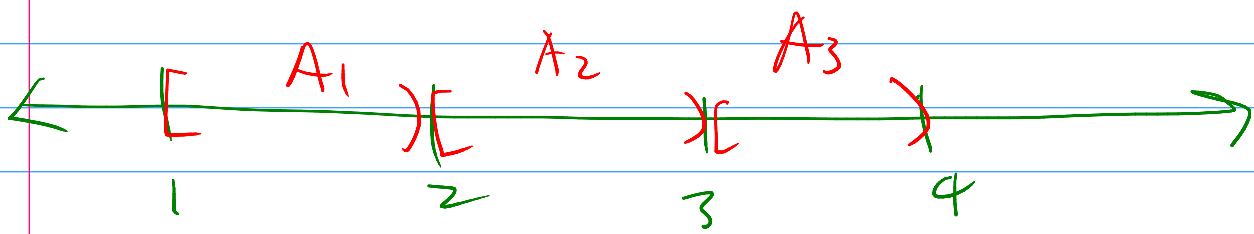
$B = \{4, 5, 6\}$ then $AB = \emptyset$

Defn: Pairwise Disjoint

A seq (A_i) is pairwise disjoint if

$$A_i A_j = \emptyset \quad \forall\, i \neq j$$

Ex. $A_i = [i, i+1)$

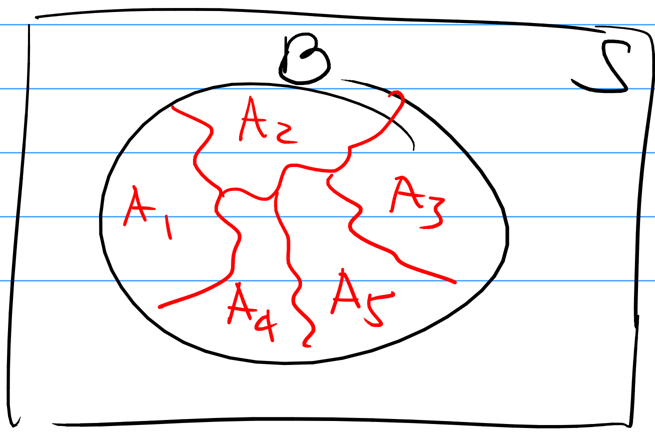


$A_i A_j = \emptyset \quad \forall\, i \neq j$ there are pairwise disjoint.

Defn: Partition

We say a seq (A_i) where $A_i \subset B$ are a partition of B if

1. the A_i are (pairwise) disjoint
2. $\bigcup_i A_i = B$



Defn: Power Set

A power set of a set A is the set of all subsets

notation: $P(A)$ or 2^A

i.e. $2^A = \{B \mid B \subset A\}$

Ex. $A = \{1, 2\}$

$2^A = \{\{1\}, \{2\}, A, \emptyset\}$

Cardinality: $|B|$ = card. = # elements

Fact: $|2^A| = 2^{|A|}$