Quiz Problem 8

Problem.
Let X_n come from a Pareto distribution so that for $\alpha, \beta > 0$,

$$f(x) = \frac{\alpha\beta^\alpha}{x^{\alpha+1}} \mathbb{1}(x \geq \beta) \text{ and } F(x) = 1 - \left(\frac{\beta}{x}\right)^\alpha.$$

Let

$$Y_N = \min_{n=1,\ldots,N} X_n.$$

We can show that the CDF of Y_N is

$$F(y) = \begin{cases} 1 - \left(\frac{\beta}{y}\right)^{\alpha N} & y \geq \beta \\ 0 & y < \beta \end{cases}$$

Show that $Y_N \xrightarrow{p} \beta$. Hint: suggested problem 7.
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