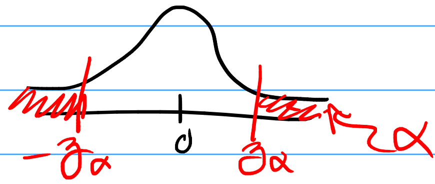
Lecture 19

Consider $X_n \overset{iid}{\sim} N(\mu, \sigma^2)$ Known



T1: rej. if $\dfrac{\bar{X} - \mu_0}{\sigma/\sqrt{N}} > z_\alpha = F_Z^{-1}(1-\alpha)$

T2: rej. if $\dfrac{\bar{X} - \mu_0}{\sigma/\sqrt{N}} < -z_\alpha$

T1': $\bar{X} > \mu_0 + \sigma/\sqrt{N}\, z_\alpha$

T2: $\bar{X} < \mu_0 - \sigma/\sqrt{N}\, z_\alpha$

Neymen-Pearsen: Simple

$H_0: \mu = \mu_0$ v. $H_a: \mu = \mu_a$

then T1 is UMP level α test

$\mu_a > \mu_0$

$H_0: \mu = \mu_0$ v. $H_a: \mu = \mu_a$

where $\mu_0 > \mu_a$

would find that T2 is the UMP level α test

Karlin-Rubin:

$H_0: \mu \le \mu_0$ v. $H_a: \mu > \mu_0$

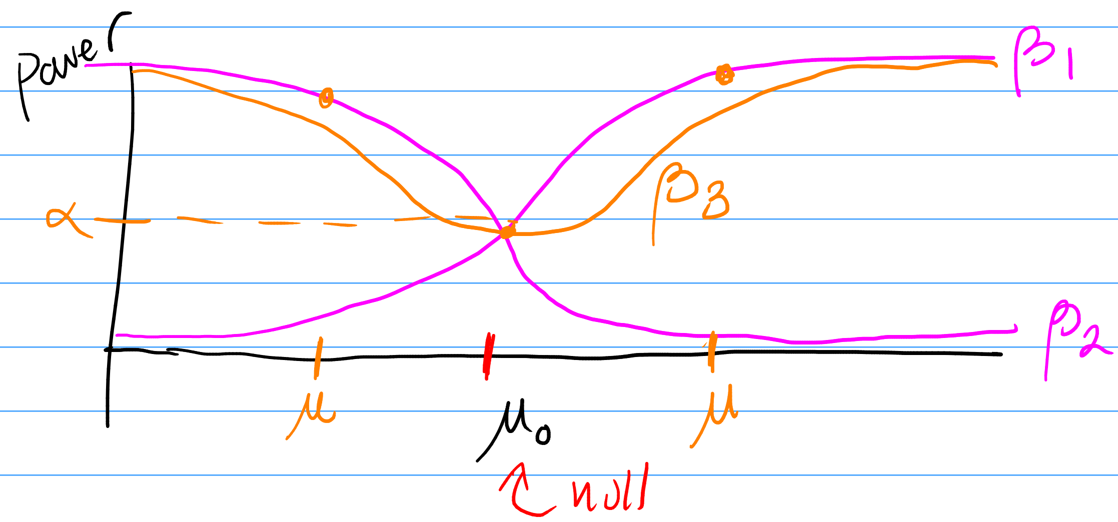
then T1 is UMP α test

$H_0: \mu \ge \mu_0$ v. $H_a: \mu < \mu_0$

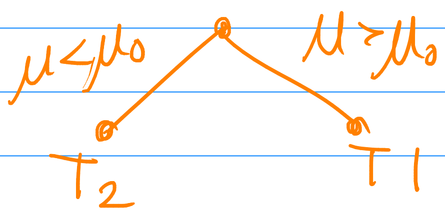
then T2 is UMP α test

Q! what about $H_0: \mu = \mu_0$ v. $H_a: \mu \ne \mu_0$.

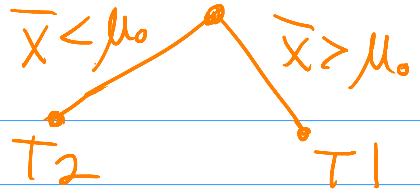
No UMP level α test.



what I would like to do



Test 3! $\dfrac{|\bar{X} - \mu_0|}{\sigma/\sqrt{N}} > z_{\alpha/2}$ ⟵ reject when this happens



Unfortunately, not uniformly best.

UMP level α unbiased test (UMPU)

Test 3 is the UMPU.

Unbiased means that β higher in Θ_a than Θ_0.