Lecture 19

Thursday, November 21, 2024 10:59 AM

P-value:

Often report results of a HT using a p-value.

Defn: A p-value is a stat. $p(\underset{\sim}{x})$ where $0 \le p(\underset{\sim}{x}) \le 1$.

idea: small $p(\underset{\sim}{x})$ gives evidence of H_a

large $p(\underset{\sim}{x})$ " H_0

Recall that a HT is just a partition of $\mathcal{X}$ into A and R:

Can define R using a p-val as

$$R = \{ \underset{\sim}{x} : p(\underset{\sim}{x}) \le \alpha \}$$

We say a p-value is valid if

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$\forall \alpha \in [0,1]$ and $\forall \theta \in \Theta_0$

$$\beta(\theta) = P_\theta(\underbrace{p(\underset{\sim}{X}) \le \alpha}_{\text{rej.}}) \le \alpha$$

i.e. If I use $p(\underset{\sim}{x})$ to defn R then the test is level α.

Typically, we actually want

$$P_\theta(p(\underset{\sim}{X}) \le \alpha) = \alpha$$

(size α test)

CDF of $P(\underset{\sim}{X})$: $F_p(\alpha) = \alpha$

i.e. $P(\underset{\sim}{X}) \sim U(0,1)$ under $H_0: \theta \in \Theta_0$.

Ex. $H_0: \theta = \theta_0$ v. $H_a: \theta \neq \theta_0$

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For some stat T

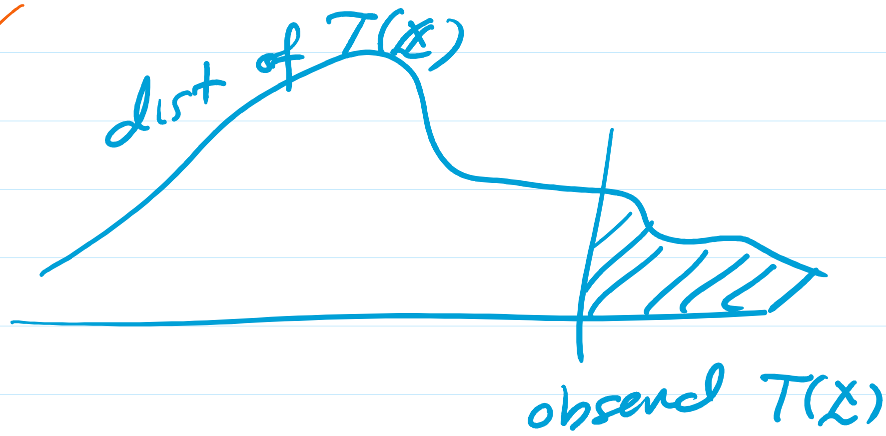
$$R = \{ \underset{\sim}{X} : T(\underset{\sim}{X}) \text{ is large} \}$$

a valid p-value is

$$p(\underset{\sim}{x}) = P_{\theta_0}\left(\overbrace{T(\underset{\sim}{X})}^{\text{random}} \geq \overbrace{T(\underset{\sim}{x})}^{\text{observed}} \right)$$

observed data

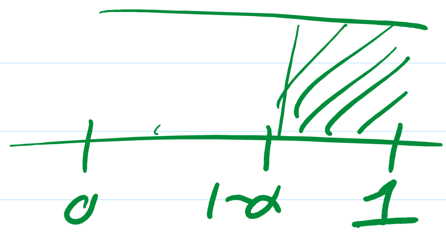
$$= 1 - F_T\left(T(\underset{\sim}{x})\right)$$



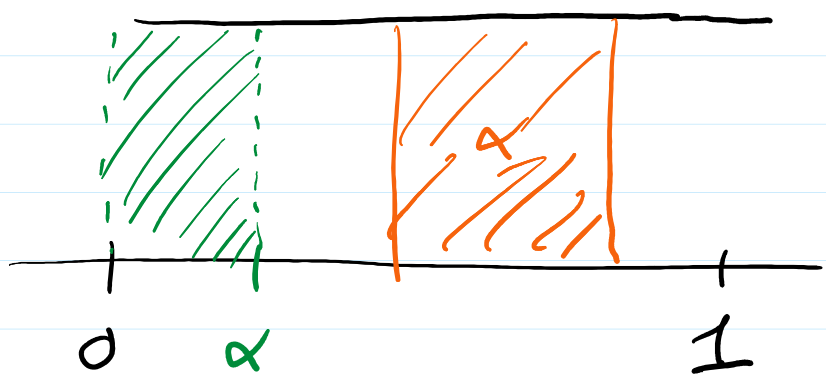
$$P_{\theta_0}\left(P(\underset{\sim}{X}) \le \alpha\right) = P_{\theta_0}\left(1 - F_T(T(\underset{\sim}{X})) \le \alpha\right)$$

$$= P_{\theta_0}\left(\underbrace{F_T(T(\underset{\sim}{X}))}_{U(0,1)} \ge 1-\alpha\right)$$

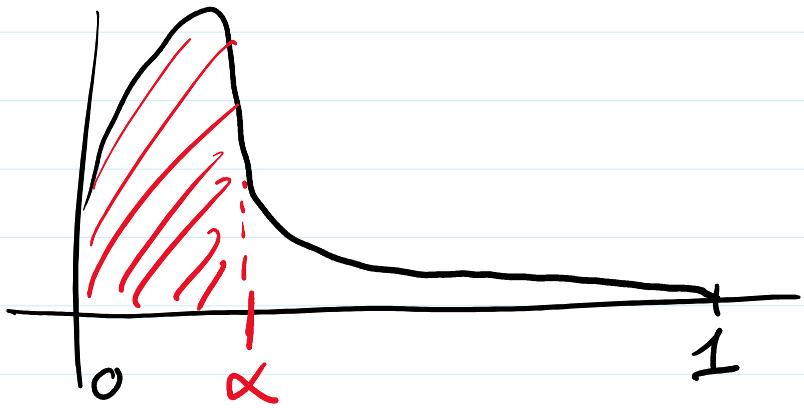
$$= \alpha$$



Punchline: under H_0 : $P \sim U(0,1)$



under H_a: typically p-val is small



Frequentist:

prob = long-run freq of occurence

Bayesian:

prob = degree of belief / information

Practically:

Frequentist: θ fixed but unknown

Bayesian: θ is "random"

Bayesian approach:

1. prior dist $\Theta \sim \pi$

2. get data

likelihood

$$\underset{\sim}{X} \mid \Theta = \theta \sim f(\underset{\sim}{x} \mid \theta)$$

3. update/combine prior and likelihood:

$$\text{posterior}: \ \pi(\theta \mid \underset{\sim}{x}) = \frac{f(\underset{\sim}{x} \mid \theta)\,\pi(\theta)}{f(\underset{\sim}{x})}$$

4. Estimate θ as

$$\hat{\theta} = E[\Theta \mid \underset{\sim}{X}]$$