Suggested Problems 7

(1) Let $X_n \stackrel{iid}{\sim} N(\theta, 1)$. Show that $(\bar{X})^2 - \frac{1}{N}$ is the UMVUE for θ^2 using the Lehmann-Scheffe theorem.

(2) Let $X_n \stackrel{iid}{\sim} \mathit{Exponential}(\lambda)$.
  (a) Find an unbiased estimator for $1/\lambda$ based on $X_{(1)}$.
  (b) Is this the UMVUE for $1/\lambda$? If not, find the UMVUE using the Lehmann-Scheffe theorem.

(3) Let $X_n \stackrel{iid}{\sim} \mathit{Bernoulli}(p)$. Find the UMVUE for $\tau(p) = p(1-p)$ using the Lehmann-Scheffe theorem.

(4) Let $X_n \stackrel{iid}{\sim} \mathit{Pois}(\lambda)$. Notice that $E[S^2] = E[\bar{X}] = \lambda$. Which estimator is better for estimating λ?

(5) Let $X_n \stackrel{iid}{\sim} \mathit{Gamma}(\alpha, \beta)$ so that

$$f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} \exp(-\beta x) \text{ for } x > 0.$$

What is the UMVUE for $\tau(\alpha, \beta) = \frac{\alpha^2}{\beta^2} + \frac{\alpha}{N\beta^2}$?

(6) Let $\hat{\theta}_1$ and $\hat{\theta}_2$ be unbiased for θ.
  (a) Show that $\hat{\theta}_3 = p\hat{\theta}_1 + (1-p)\hat{\theta}_2$ is unbiased for θ.
  (b) If $\hat{\theta}_1 \perp \hat{\theta}_2$ what value of p minimizes $\mathrm{Var}(\hat{\theta}_3)$ in terms of the variances of $\hat{\theta}_1$ and $\hat{\theta}_2$?

(7) Let $X_n \stackrel{iid}{\sim} U(0, \theta)$. Then $\hat{\theta}_1 = \frac{N+1}{N} X_{(N)}$ and $\hat{\theta}_2 = 2\bar{X}$ are unbiased for θ. Which estimator should we prefer?

(8) Let $X_n \stackrel{iid}{\sim} \mathit{Beta}(\theta, 1)$ then $-\log(X_n) \stackrel{iid}{\sim} \mathit{Exponential}(\theta)$. What is the UMVUE for θ?

(9) Let $X_n \stackrel{iid}{\sim} \mathit{Geometric}(p)$ so that $E[X_n] = 1/p$ and $\mathrm{Var}(X_n) = (1-p)/p^2$. What is the UMVUE for $(2-p)/p^2$?

(10) Let $X_1 \perp X_2$ and $E[X_1] = 5$ and $E[X_2] = 3$ and $\mathrm{Var}(X_1) = \mathrm{Var}(X_2) = \sigma^2$. What value of c makes

$$T = c(X_2^2 - X_1^2) + X_1^2$$

unbiased for σ^2?