Problem.

Let $X_1, X_2 \overset{iid}{\sim} Pois(\lambda)$ and $Y = X_1 + X_2$. Note: Y is sufficient for λ and $Y \sim Pois(2\lambda)$. Let

$$Z = \mathbb{1}(X_1 = 0).$$

Then $\mathbb{E}[Z] = P(X_1 = 0) = e^{-\lambda}$. Thus Z is an unbiased estimator for $\tau(\lambda) = e^{-\lambda}$. Define $\varphi(y)$ so that

$$\begin{aligned}
\varphi(y) &= \mathbb{E}[Z|Y = y] \\
&= P(X_1 = 0 \mid X_1 + X_2 = y) \\
&= P(X_1 = 0, X_2 = y)/P(X_1 + X_2 = y).
\end{aligned}$$

Find a formula for $\varphi(Y)$. Is $\varphi(Y)$ a better estimator than Z for $\tau(\lambda)$? What is the expectation of $\varphi(Y)$?

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