Quiz Problem 2

Let Y be a univariate random variable and and $X = (X^{(1)}, \ldots, X^{(P)})$ be a P-dimensional design random variable. Let $p(X, Y)$ be the joint distribution of these random variables so that

$$Y \mid X \sim N(X^T \beta, \sigma^2)$$

and $p(X)$ be the marginal distribution of X. Let

$$L(y, f(x)) = -\log p(x, y)$$

be a loss function and consider finding $\hat{\beta}$ as

$$\hat{\beta} = \arg\min_{\beta} \frac{1}{N} \sum_{n=1}^{N} L(y_n, x_n^T \beta).$$

Show that $\hat{\beta}$ is our typical ordinary least-squares estimate of β.