Quiz Problem 2

Let Y be a univariate random variable and and $\underline{X} = (X^{(1)}, \ldots, X^{(P)})$ be a P-dimensional design random variable. Let $p(\underline{X}, Y)$ be the joint distribution of these random variables so that

$$Y \mid \underline{X} \sim N(\underline{X}^T \beta, \sigma^2)$$

and $p(\underline{X})$ be the marginal distribution of $\underline{X}$. Let

$$L(y, f(x)) = -\log p(\underline{x}, y)$$

be our loss function. Show that the ERM estimate of $\hat{f}$ under this loss is the same as the OLS estimate of $\hat{f}$.