Quiz Problem 7

Let $X \in \mathbb{R}^{N \times P}$ be our design matrix and $Y \in \mathbb{R}^N$ be our response variables. Let $\widetilde{X}$ and $\widetilde{Y}$ be augmented versions of X and Y where

$$\widetilde{X} = \begin{bmatrix} X \\ \sqrt{\lambda} I_P \end{bmatrix}$$

adds P rows to X each with values $\sqrt{\lambda}$ and

$$\widetilde{Y} = \begin{bmatrix} Y \\ 0 \end{bmatrix}$$

adds P zeros on to the end of Y. Show that the coefficients $\hat{\beta}$ associated with regressing $\widetilde{Y}$ onto $\widetilde{X}$ is equivalent to the coefficients found from fitting a Ridge regression estimator of Y onto X. This can be interpreted as shrinking our estimate of $\hat{\beta}$ by adding hints into our data that, for many of our data points, the coefficient is zero. Hint: if

$$C_1 = \begin{bmatrix} A_1 \\ B_1 \end{bmatrix} \text{ and } C_2 = \begin{bmatrix} A_2 \\ B_2 \end{bmatrix}$$

are block matrices then

$$C_1^T C_2 = A_1^T A_2 + B_1^T B_2$$