

Problem Set 1

CSCI 688 - Statistical Learning

1. Check that

$$Q = \begin{bmatrix} 5/13 & 12/13 \\ -12/13 & 5/13 \end{bmatrix}$$

is orthogonal

2. Let

$$A = \begin{bmatrix} 9 & -2 \\ -2 & 6 \end{bmatrix}.$$

- (a) Is A symmetric?
 - (b) Determine the eigen-values and eigen-vectors of A .
 - (c) Find the spectral decomposition of A .
 - (d) Find A^{-1} using the answer to the previous part.
3. Let $I_5 = \text{diag}(1, 1, 1, 1, 1)$ be the 5×5 identity matrix.
- (a) What is the spectral decomposition of I_5 ?
 - (b) What is the singular value decomposition of I_5 ?
4. Let

$$A = \begin{bmatrix} 1 & 1 \\ 2 & -2 \\ 2 & 2 \end{bmatrix}.$$

Note that $A^T A$ has a spectral decomposition

$$A^T A = \begin{bmatrix} 9 & 1 \\ 1 & 9 \end{bmatrix} = Q \Lambda Q^T$$

where

$$Q = \begin{bmatrix} -1/\sqrt{2} & -1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \text{ and } \Lambda = \begin{bmatrix} 10 & 0 \\ 0 & 8 \end{bmatrix}.$$

What is the singular value decomposition of A ? Hint: If $A = UDV^T$ then $AV = UD$.