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Elliptic Fourier analysis of crown shapes in *Quercus petraea* trees

O. Hâruța

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Abstract. Shape is a fundamental morphological descriptor, significant in taxonomic research as well as in ecomorphology, one method of estimation being from digitally processed images. In the present study, were analysed shapes of *Q. petraea* crowns, pertaining to five different stem diameter classes, from three similar stands. Based on measurements on terrestrial digital vertical photos, crown size analysis was performed and correlations between crown and stem variables were tested. Linear regression equations between crown volumes and dbh, and crown volumes and stem volumes were derived, explaining more than half of data variability. Employment of elliptic Fourier analysis (EFA), a powerful analysis tool, permitted the extraction of the mean shape from crowns, characterized by high morphological variability. The extracted, most important, coefficients were used to reconstruct the average shape of the crowns, using Inverse Fourier Transform. A mean shape of the crown, corresponding to stand conditions in which competition is added as influential shaping factor, aside genetic program of the species, is described for each stem diameter class. Crown regions with highest shape variability, from the perspective of stage development of the trees, were determined. Accordingly, the main crown shape characteristics are: crown elongation, mass center, asymmetry with regard to the main axis, lateral regions symmetrical and asymmetrical variations.
Keywords *Quercus petraea* crown shapes, morphometry, Elliptic Fourier Transform, mean crown shape.

Author. Ovidiu Hâruța (standdyn@yahoo.com) - University of Oradea 26 Bd. Gen. Magheru, 0259412550, Oradea, Romania.
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Introduction

The architectural pattern is a general frame for tree shapes' variability, that includes genetic determinism and the influence of abiotic and biotic factors (Sachs & Novopolanski 1995). Trees display a remarkable phenotypic plasticity, which helps them to adapt under environmental conditions (Valladares & Pearcy 2000). The crown plasticity is related to the available space, being generally accepted that characteristic shape is genetically determined, and the architectural pattern is realized only when trees grow isolated (Horn 1971).

At individual level, the response to environmental influences is characterized by a high variability during age dynamics and, also, within the same dimensional category. The architecture pattern must satisfy the access to such resources - as light - under gravitational and wind generated mechanical constraints. Structural basis for optimal light interception includes leaves insertion angles, phylotaxis, branching pattern and crown shape (Valladares & Niimets 2007). Tree crown extension is an indicator of its carbon accumulation capacity, of energetic exchanges, of competition potential, and reflects in time array particular development conditions.

Crown size reflects especially the ontogenetic development, the available space for development, the influence of site conditions and the individual genetic program. The size of a tree crown has a marked effect on, and is strongly correlated with the growth of the tree and of its various parts (Hemery et al. 2005). Measurement of crown parameters and the establishment of the correlation between these characteristics are frequently aspects evaluated in forestry (Avăcăriței 2003, Korpela 2004). The differences are determined by the employed methods, means and the variable types used for evaluation.

Crown shape is the result of a complex interplay between genetic blueprint (deterministic architecture) and the response of the organism to environmental factors (Hatta et al. 1999). Thus, the form of the crown is an important factor used for the explanation of the ecological success of trees, according to environmental factors, and in term of the fitness and gaining positions within the plant community. For trees description, characterization and classification it is necessary to evaluate the general tree form, according to the relation *form = size + shape* (sensu Costa & Cesar 2001).

The morphometric description (shape and dimension analysis) is employed in the evaluation and classification of the stands, also as support for the management decisions. Whichever approach is employed description of the horizontal tree crown projections, measurements on location of phytoelements, or modern approaches, as 3D reconstruction modeling the obtained information could be used as guidlines criteria in silviculture, during different development stages of the stands. It is a measurement mode of tree shape plasticity and also, of the exploitation strategies, corresponding to each species.

Quantitative shape evaluation, as an important analysis in morphological descriptions, includes methods as contour length assessment, fractal analysis, and use of landmarks or Fourier descriptors. One of the method used in the study of tree crown shape and size is based on vertical terrestrial photography, a method employed in the quantification of horizontal and vertical crown projections (Seiler & McBee 1992, Pyysalo 2004, Ivanov & Sauerbier 2006, Rautianinen et al. 2008). It was initially developed for the analysis of the small plants and the isolated trees, the quantified aspects being, usual, the volume, the transparency and the shape of the crown. For tree growing within stands, the approaches are based on diameter measurements at different stem heights (Clark et al. 2000). Most of the papers are focused on the methodology testing the efficiency of vertical terrestrial photos from precision and information-wise points of view.

Elliptic Fourier descriptors, proposed by Kuhl and Giardina (1982), are able to separate any bi-dimensional contour, using complex procedures, such as discreet Fourier transform, image processing, contour registration, derivation of descriptors and the multivariate analysis. The most employed characteristics of the Fourier analysis consist of description of objects contour using periodic functions, the extension of functions by Fourier series, and the contour representation by Fourier coefficients.

In biological sciences, Fourier descriptors are employed as description tool for the shape of organisms, as a whole or separated on distinct parts. Quantitative characterization of irregular shapes is, most often, the first step in the analysis of the biological processes, weather they are genetic, evolutionary or functional by nature (Lestrel 1997). Elliptic Fourier coefficients are equivalents of shape descriptors, eliminating variability and translation, rotation and dimensional deformations. These permit further reconstruction of the fundamental or average shape, using Inverse Fourier transform coefficients. A set of coordinates of points within the analyzed contour are employed, expressed by a transform, named Fourier transform. Elliptic Fourier transform permits the quantification of the contour of an object using a series of sums, corresponding to weighted ellipses, described mathematically by Fourier coefficients and Fourier constants (spatial coordinates). In this way, the input data are digitalized contour coordinates, whise the output data are the coefficients describing the contour, which can be further used as ordinary variables (Lestrel 1997).

For a complete and precise description of the crown morphometry - taking into account the possibilities of stressing the differences generated by stage development - were analyzed separately the crown shape and dimension. Due to the employed measurements, it become possible to apply the classical statistics used to crown dimension and elliptic Fourier analysis employed for the first time in tree crown plasticity evaluation.

The aims of the present paper were: (i) to quantify crown plasticity at tree and population level, focusing on within population variability (trees of the same age) and between population variability (trees of different age), (ii) to perform dimensional analysis of trees using stem and crown quantitative descriptors, in order to find existing size differences between tree groups, according to stem diameter categories, and to generate simplified geometrical representations of the crown variability with age, (iii) to perform shape analysis using Elliptic Fourier Coefficients on real crown contours, in order to generate the average crown shape, realistic geometrical representations of crown models and to establish those crown regions characterized by highest shape variability during the process of aging.

Material and methods

Site location

In order to analyze crown shapes, were selected *Quercus petraea* pure stands, located in Cluj County (Romania), Forest District Cluj (Working Unit IV Făget) (Table 1). According to Cluj wheater station data, the annual average temperature is about +4.0° C on the hills and +8.0° C on valleys and depressions. The minimum annual average temperatures are recorded during January (from -2.0° C to +5.0° C), while the maxima are recorded during July (from + 15.0° C to +20.0° C); the average annual precipitations are around 710 mm/m^2.

Selected stands were even aged, located on light slopes and similar with respect to site characteristics. According to the Romanian forest typology, one of these pertain to the hilly areas, with stone oak forests, on podzol and podzol-clay illuvial soils and with mesophytic graminicolous type of flora (the sites 1, 2 and 3 from table 1)(forest type 5132, Chiriță et al. 1977); the other one is a slope stone oak forest, with graminicolous herbaceous cover, with *Luzula luzuloides* (forest type 5131 - Paşcovschi and Leandru 1958). The corresponding forest ecosystem type is 5134, stone oak forest of low and medium productivity with moder humus, vegetating on districambosoils and luvisoils, oligobasic, quasi-equilibrated in water content, with *Luzula luzuloides* (Doniță et al. 1990). All the stands were regenerated by coppice, presenting a consistency of 0.7-0.8.

Aiming to obtain a dynamic representation

Table 1 Study sites characteristics

Site no.	Latitude	Longitude	Altitude (m)	Stand	Mean dhb (cm)	Mean annual growth (m^3/ha)	Mean volume (m^3/ha)
1	46° 46′ 15.25″	23° 31′ 22.39″	480	109B	24.0	2.6	84
2	46° 46′ 08.01″	23° 30′ 15.14″	520	108A	28.3	2.8	74
3	46° 46′ 19.27″	23° 30′ 50.61″	515	111A	36.4	2.2	160

of the crown shapes and dimensions, related to the development status, care was paid for age (diameter used as a surrogate), as being in the range of 55-100 years (24-40 cm). A number of 30 trees, corresponding to a diameter at breast height (dbh) of 24 cm (site 1), 28 and 32 cm (site 2), 36 and 40 cm (site 3), were selected by random walk. For avoiding edge effects and to include only the individuals with a characteristic growth, were selected only trees located at least 100 m far from the forest edge. The canopy position was estimated as two classes, dominant and co-dominant. In order to avoid shape variability induced by the slope, only the trees growing on plain terrain were considered.

Characteristic low visibility within stands imposed a new method for extraction of the tree crown contour, by eliminating the parts occluded by neighbor trees. Thus, every tree was photographed with a Konika Minolta DiMAGE Z5 camera, using standard shooting parameters, based on a method described by Hâruța & Fodor (2010). In the field, was recorded one photograph and also was performed one drawing of each tree crown contour, from exactly the same position the photo was taken. The drawing was initially done on a transparency and then was overlaid on the photograph, by using several characteristic points of the tree crown, common for both representations. The real contour was obtained by manual vectorization, using the contour extracted from the drawing as landmark, especially for the superposition with adjacent trees areas. The result is an almost precise tracing of the shape irregularities of the crowns in 2D projection, the application of the method being facilitated in stands characterized by high degree of canopy closure. The obtained images were rectified, because during the photography session, the image plane is not parallel with the plane containing the main tree axis. Rectification permits also the transformation of image coordinates in metric coordinates, necessary for a dimensional evaluation of the photographed trees.

Using as starting point the analysis of crown contours in conjunction with their position and with respect to the stem, were quantified morphometric parameters such as: area (A), perimeter (P), crown diameters at three positioning levels (D_t at the top, D_m at the middle, D_b at the crown base), total tree height (h), height at the crown base (h_{cb}), diameter at breast height (*dbh*), maximum crown width (D_{max}). All the measurements were performed on photographs using ImageTool software (Wilcox et al. 1996). The precision was of one pixel, a real world equivalent of 3.52 cm. Height to the crown base was measured till the lowest living branch which appears to be a part of the continuous crown (Tahvanainen & Forss 2008). Stem volumes were calculated using the method proposed by Giurgiu et al. (2004) based on two entrances volume tables. For crown volumes, an algorithm was created in MathCad software (Mathsoft Inc. 1999), permitting the generation of the three dimensional object, by the rotation of crown contour, and thus the estimation of the volume. The method gives a better estimation of the crown volume as compared to the utilization of known geometric shapes, with measurement dimensions introduced by the user.

For determining which contour shape parameters define the average crown shape and the greatest variability parameters, the real contours of the crowns were extracted from photos for 150 trees from all the selected stands (Table 1), and analyzed using Shape software (Iwata & Ukay 2002). Elliptic Fourier analysis (EFA) permits the precise and complete description of crown shapes; the first step in the analysis was performed for the assessment of variability, according to the diameter classes. For each diameter class (24, 28, 32, 36, and 40 cm), 30 crown contours were processed, using the Shape software. The protocol included the contour extraction, the calculation and derivation of the normalized Elliptic Fourier coefficients, determination of the principal component which summarizes the information contained by coefficients and use of Inverse Fourier Transform, in order to obtain graphical representations of average crowns.

Crown contours were introduced as a RGB image and then transcribed in a chain code of the geometric information of the shape. Consequently, the shape was approximated by 20 harmonics, which correspond to 77 Elliptic Fourier coefficients, standardized and separated in two groups for symmetrical variations (a_n and d_n) and other two groups for asymmetrical variation (b_n and c_n), as considered with respect to the principal axis of the crown (Fig. 1).

The contour of a digitized shape can be represented as x and y coordinate sequence, corresponding to points within the contour measured clockwise and considering as starting point an arbitrary position. X coordinate of a random point p is:

$$x_p = \sum_{i-1}^{p} \Delta x_i$$

where x_i is the range between point *i* and *i-1*. Consequently, the Elliptic Fourier expansion for the coordinate sequence x is expressed as:

$$x_p = x_{cen} + \sum_{n=1}^{\infty}\left(a_n \cdot \cos\frac{2n\pi t_p}{T} + b_n \cdot \sin\frac{2n\pi t_p}{T}\right)$$

x_{cen} – coordinate of the central point
n – number of harmonics corresponding to coefficients a_n and b_n

$$a_n = \frac{T}{2n^2\pi^2}\sum_{p=1}^{k}\frac{\Delta x_p}{\Delta t_p}\left(\cos\frac{2n\pi t_p}{T} - \cos\frac{2n\pi t_{p-1}}{T}\right)$$

$$b_n = \frac{T}{2n^2\pi^2}\sum_{p=1}^{k}\frac{\Delta x_p}{\Delta t_p}\left(\sin\frac{2n\pi t_p}{T} - \sin\frac{2n\pi t_{p-1}}{T}\right)$$

In a similar way were obtained the coefficients for coordinates y (c_n and d_n). Fourier series for sines describe objects' asymmetry while those with cosines describe objects' symmetry. There is a possibility to eliminate

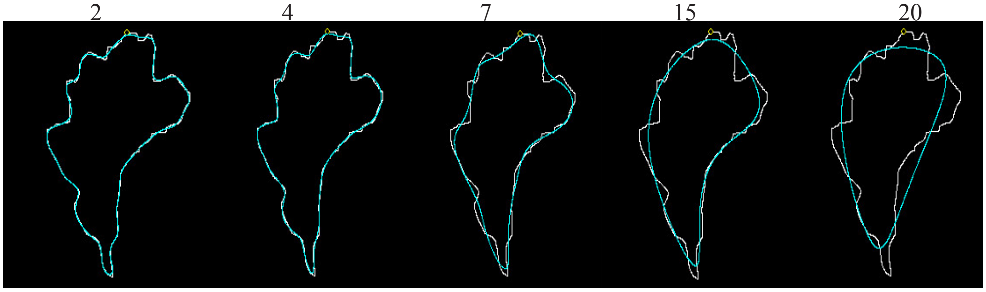

Figure1 Crown contour approximation using different numbers of harmonics

one function which leads to simplification of results and of their further interpretation. The number of Fourier coefficients obtained by analyzing a contour depends on the number of employed harmonics (every term $a_n\cos\theta$ defines a harmonic), those being fixed as function of desired precision in the description of the analyzed objects' shape irregularities.

Elliptic Fourier coefficients can be mathematically normalized for shape and rotation invariance depending on the contour starting point. During this study, standardization was applied using Shape software conforming to Kuhl and Giardina (1982) methodology:

$$\begin{bmatrix} a_n^{**} & b_n^{**} \\ c_n^{**} & d_n^{**} \end{bmatrix} = \frac{1}{E} \cdot \begin{bmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{bmatrix} \begin{bmatrix} a_n & b_n \\ c_n & d_n \end{bmatrix} \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$$

where:

$$E^* = (a_1^* + c_1^*)^{\frac{1}{2}}$$

$$\begin{bmatrix} a_1^* \\ c_1^* \end{bmatrix} = \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$$

$$\psi = \arctan\left[\frac{c_1^*}{a_1^*}\right] (0 \le \psi < 2\pi) \text{ and}$$

$$\theta = \frac{1}{2}\arctan\left[\frac{2(a_1b_1 + c_1d_1)}{(a_1^2 + c_1^2 - b_1^2 - d_1^2)}\right] (0 \le \theta < \pi)$$

Standardization is based on first harmonic ellipsis. E^* and ψ correspond to the length and direction of the main axis of first harmonic ellipsis, θ represents the position modification of the first harmonic radius vector and a_n^{**}, b_n^{**}, c_n^{**}, d_n^{**} are EF standardized coefficients.

Considering Fourier coefficients as a set of transformed variables, these can be employed in Principal Components Analysis to depict functional relationships. The basic criterion is the maximization of variables' variance which permits to find the optimal rigid axes rotation, axes being the principal components.

Statistical analysis

Basic descriptive statistics were employed: means, standard deviations, variation coefficients, as well as Tukey test for multiple comparisons and simple regression performed to underline the existing differences between and within trees from distinct stem diameter classes. For expressing the the internal variability within each diameter class, the coefficients of variation of the crown were used. It is also a useful evaluation, with the purpose of the construction of tree growth model, or for highlighting the degree of correlation between variables. The employed software was Kyplot ver. 2.0 (Yoshioka 2002) for Tukey test for multiple comparisons and SPSS ver. 14 (SPSS Inc., Chicago, Illinois, USA) for other statistical tests.

Tukey test was employed for multiple comparisons of pairwise combinations of stem diameter classes for each variable. The existence of such differences corresponds to the fact that crown or stem dimensional variables are modified from one diameter category to another.

From a practical point of view it is important to test the variables' correlation which reflects tree capacity for wood biomass accumulation or other crown constituents. For this purpose, crown volumes were regressed against dbh (Fig. 2) and crown volumes against stem volumes (Fig. 3). The calculation of Pearson correlation coefficient is a prerequisite condition necessary to perform regression in both situations - Pearson coefficient of corresponding to crown volumes versus dbh was 0.785 and crown volumes versus stem volumes was 0.779. Both results were significant at 0.01 levels of significance.

Frequencies of the crown values being not normally distributed consequently, a log transformation was applied on the data. Data normality was tested with Shapiro-Wilk test and by estimation of skewness and kurtosis of the

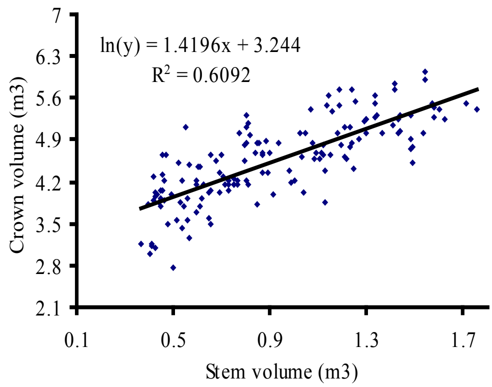

Figure 2 Linear regression of crown volumes against dbh (cumulated data for 24, 28, 32, 36 and 40 cm diameter classes)

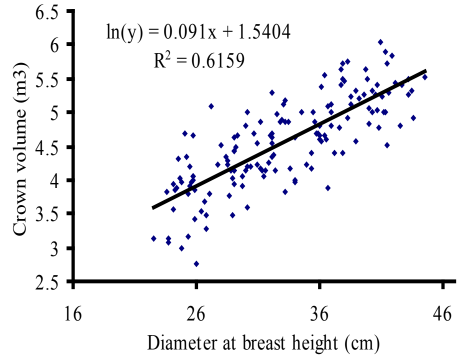

Figure 3 Linear regression of crown volumes against stem volumes (cumulated data for 24, 28, 32, 36 and 40 cm diameter classes)

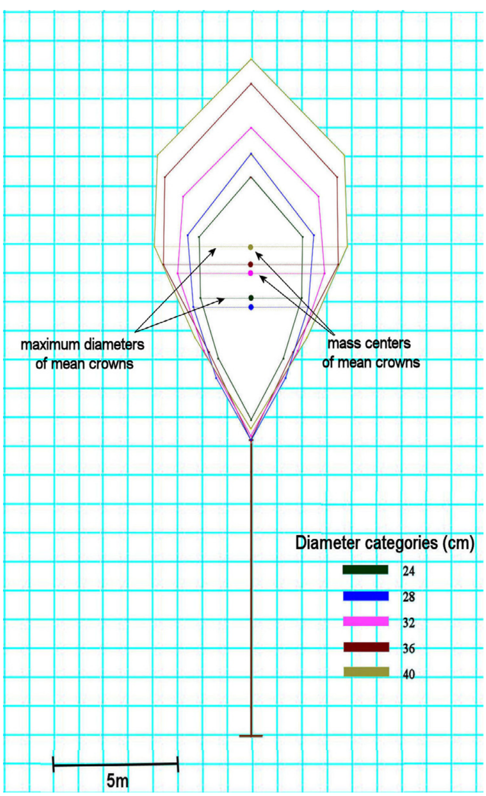

Figure 4 Geometrical representations of the crown variability with age based on mean values of morphometric variables according to diameter classes

frequency distribution. Normality null hypothesis was not rejected on log transformed data ($p < 0.391$), meanwhile the variance being modified from 6091.806 to 0.453. The ratio between skewness/standard error of skewness and kurtosis/standard error of kurtosis confirms the normalization of crown volume raw data (ratio values are -0.89 respectively -0.95). The residual plot suggests that the transformation is necessary to achieve linearity. The pattern of residuals obtained after transformation is random, suggesting that the relationship between the independent variable (stem volume) and the transformed dependent variable (logarithm of crown volume) is linear. Also the coefficient of determination value increases from 0.550 to 0.609 by data transforming. The variables dbh and stem volume present symmetrical distributions (skewness/standard error of skewness 0.33, respectively 1.52), and heavy tails (kurtosis/standard error of kurtosis -2.90, respectively -2.76). All types of data transformation applied to this variables do not improve the model resulted after regression. This is the reason why transformation was not performed in this case. Similarly to previous analysis, determination coefficient increases from 0.537

to 0.616 after log transform of crown volumes data.

A complete analysis of tree crowns needs dimension measurements being supplemented by shape evaluation. For this purpose, Elliptic Fourier Coefficient Analysis was performed employing Shape ver 1.3 software. The analysis of EF coefficients permits the extraction of the crown shape variability corresponding to each investigated stem diameter category. The high number of obtained coefficients and the difficulty raised by their interpretation necessitates the use of Principal Component Analysis. This method facilitates dimension reduction of the data set and permits efficient summarization of the information contained by the coefficients (Yoshioka et al. 2007, Andrade et al. 2008). First and second principal components of the symmetric and asymmetric coefficients concentrate the most important fraction of total variability.

Symmetrical and asymmetrical crown contour variations were separately analyzed, and the number of significant principal components summarizing maximum of variability (variance) were selected. As a final step, Inverse Fourier Transform was applied, in order to visualize the average shapes (means ± 2 standard deviations) and to permit the assessment of most variable parts of the crown, separately on symmetric (Fig. 6) and asymmetric (Fig. 7) variation. Interpretation and visualization of the principal components need the transformation of the reduced data set of Elliptic Fourier coefficients (resulted from PCA), by means of the Inverse Fourier Transform (Iwata et al. 2002, Iwata et al. 2004, Yoshioka et al. 2004). The average crown contour shapes are graphically displayed for every principal component separately, permitting the identification of the independent shape characteristics.

Results

The variables used to describe the tree sizes (means, standard deviations and variation coefficients) are depicted in Table 2. Values which indicate medium and high variability for the variation coefficients are highlighted in the table with grey color. The general trends of increasing means for analyzed variables are also shown, with one exception the height at the crown base which fluctuates among diameter categories. The highest variability, estimated through the variation coefficient was found in crown volumes and in crown diameters in the lower third of the crown. For the same variable, variation coefficients fluctuate with respect to diameter classes. Lateral crown extension, as depicted by observed means at different diameter classes, is larger in the middle third of the crown, significantly reduced to the top and fluctuant at the lower third.

The results obtained by Tukey test for multiple comparisons were employed for the matrix construction: rows correspond to variables and columns to stem diameter categories combinations. Table 3 presents the obtained values, being also included signification levels of stem and crown morphometric variables as well as the values of Tukey test. Significant and very significant values are presented on grey background: they express dimensional differences of the crowns among diameter classes.

The relationship between crown volumes and dbh is described by a linear equation which explain more than half of data variability (r^2 = 0.6159), also in the case of crown volumes - stem volumes relationship (approximately 60% of data variability explained - r^2 = 0.6092).

The graphical model of trees' increment, developed based on the means of morphometric variables (Fig. 4) show that the increment of tree heights is associated with the positioning of the crown mass center at a superior crown level. This could be considered as a consequence of a sustained growth in the upper third of the crown. Maximum height growth rate is conjunct with maximum of areas and diameters increment in the upper third of the crown, the smallest increments corresponding

Table 2 Mean values (± SD) and variation coefficients of several morphometric crown and stem variables corresponding to 24, 28, 32, 36 and 40 cm diameter classes

Variables	Statistics	Diameter classes (cm)				
		24	28	32	36	40
h (m)	Mean ± SD	19.03±1.63	19.84±1.60	20.71±1.52	22.03±1.11	22.86±1.34
	CV (%)	8.6	8.1	7.3	5.0	5.9
h_{cb} (m)	Mean ± SD	10.79±1.92	10.09±1.91	10.22±10.69	10.07±1.83	10.49±1.44
	CV (%)	17.8	18.9	16.5	18.2	13.7
D_{max} (m)	Mean ± SD	4.88±0.77	5.81±1.05	6.73±1.33	7.88±1.72	8.81±1.39
	CV (%)	15.8	18.0	19.8	21.8	15.8
A (m^2)	Mean ± SD	24.275±7.411	33.320±7.324	41.882±9.420	55.316±15.920	67.478±17.636
	CV (%)	30.5	22.0	22.5	28.8	26.1
P (m)	Mean ± SD	26.032±5.725	32.020±4.826	34.875±4.950	39.030±6.829	41.737±6.251
	CV (%)	22.0	15.1	14.2	17.5	15.0
D_t (m)	Mean ± SD	4.092±0.861	4.981±0.934	5.342±1.305	6.793±1.803	7.381±1.561
	CV (%)	21.0	18.7	24.4	26.5	21.2
D_m (m)	Mean ± SD	4.047±1.099	4.551±1.200	5.799±1.436	6.904±1.885	7.646±1.597
	CV (%)	27.2	26.4	24.8	27.3	20.9
D_b (m)	Mean ± SD	2.623±0.919	2.785±1.128	3.690±1.604	3.321±1.384	4.431±1.230
	CV (%)	35.0	40.5	43.5	41.7	27.8
V_{tr} (m^3)	Mean ± SD	0.461±0.054	0.657±0.086	0.867±0.105	1.167±0.089	1.462±0.126
	CV (%)	11.7	13.1	12.1	7.6	8.6
V_{cr} (m^3)	Mean ± SD	47.498±22.803	76.593±30.885	103.081±41.329	153.426±73.961	208.173±76.104
	CV (%)	48.0	40.3	40.0	48.2	36.6

Note: CV < 10% - indicates high homogeneity of data set; 10% < CV < 30% - indicates medium variability; CV > 30% - indicates high data variability (medium and high variability are highlighted in the table with grey color)

to the bottom third of the crown and height at the crown base. Trees extend their crowns, on average, with 43.20 m² from diameter classes of 24 to 40 cm; within smaller diameter classes, average growth rates fluctuate between 7-9 m², while within higher diameter classes it is between 12-13 m². The phenotypic crown plasticity revealed by the high size and shape variability is expressed within each diameter class. This fact is highlighted by the high values of the coefficients of variation and can be observed visually on 2D projections of the crowns. Morphometrical differences in vertical plane consist of crown variation, related to elongation and compactness, to crown diameters at different crown levels or the fragmentation degree of the contours (Fig. 5).

In practical terms, there are not two identical crowns, however classes of distinct types crown classes can be separated (characterized by high homogeneity and low variability estimates). Only visual inspection is insufficient for crown complexity, a multi-criteria analysis is needed and the employment of statistical tests, in order to underline the magnitude of differences among crown groups.

The superposition of average shapes (in black) and the corresponding standard deviations (red and blue), separated on symmetrical and asymmetrical variations, and only for components that can be functionally explained, is presented in Figures 6 and 7; the representations were generated by Inverse Fourier Transform using Shape software.

The number of significant principal components varies among diameter classes, from 5 to 7 (Tables 3 and 4). Increasing number, more contour details are depicted, but the correspondence between mathematical and visual detail is difficult to assess.

Table 3 Tukey test for multiple comparisons results for analyzed combinations of stem diameter classes

Variables	Values and significance levels - combinations of stem diameter classes (cm)									
	24 x 28	24 x 32	24 x 36	24 x 40	28 x 32	28 x 36	28 x 40	32 x 36	32 x 40	36 x 40
h (m)	N.S. 2.170	*** 4.494	*** 7.992	*** 10.225	N.S. 2.323	*** 5.821	*** 8.054	** 3.498	*** 5.731	N.S. 2.232
h_{cb} (m)	N.S. -1.542	N.S. -1.248	N.S. -1.573	N.S. -0.655	N.S. 0.294	N.S. -0.031	N.S. 0.886	N.S. -0.325	N.S. 0.592	N.S. 0.917
D_{max} (m)	N.S 2.005	*** 4.937	*** 8.184	*** 10.430	* 2.931	*** 6.178	*** 8.424	* 3.247	*** 5.493	N.S. 2.245
A (m^2)	* 2.838	*** 5.524	*** 9.739	*** 13.756	N.S. 2.686	*** 6.901	*** 10.717	*** 4.215	*** 8.031	** 3.816
P (m)	*** 4.021	*** 5.938	*** 8.729	*** 10.547	N.S. 1.917	*** 4.707	*** 6.525	* 2.790	*** 4.608	N.S. 1.818
D_t (m)	N.S. 2.566	** 3.607	*** 7.796	*** 9.491	N.S. 1.040	*** 5.229	*** 6.925	*** 4.188	*** 5.884	N.S. 1.695
D_m (m)	N.S. 1.326	*** 4.614	*** 7.523	*** 9.477	* 3.287	*** 6.196	*** 8.151	* 2.909	*** 4.863	N.S. 1.954
D_b (m)	N.S. 0.493	* 3.242	N.S. 2.122	*** 5.495	N.S. 2.749	N.S. 1.629	*** 5.002	N.S. -1.120	N.S. 2.252	** 3.373
V_{tr} (m^3)	*** 7.960	*** 16.528	*** 28.751	*** 40.758	*** 8.568	*** 20.791	*** 32.798	*** 12.223	*** 24.330	*** 12.007
V_{cr} (m^3)	N.S. 2.096	*** 4.005	*** 7.633	*** 11.578	N.S. 1.908	*** 5.536	*** 9.481	*** 3.628	*** 7.573	*** 3.945

Note: * - marginally significant, ** - significant, ***- very significant, NS - not significant

Tables 4 and 5 presents the proportions of participation of the principal components considered significant, according to the scree plot. The calculations were made for each diameter class and reflect the symmetrical and asymmetrical variations of crown shapes. The eigenvalues are numerical values corresponding to the total variation of data set. From a graphical point of view, the eigenvalues are the lengths of the major and minor axes of an ellipse, which circumscribes data swarm, re-oriented by PCA methodology.

Visual representations of the mean crown shape and the shapes that include most of the variability expressed as standard deviation (±SD) permit an easy interpretation of the significant variables of the general crown shape. The percentages correspond to eigenvalues calculated on co-variance matrix of the data and measure the variance explained by each principal component. The results presented in tables 4 and 5 depict the contributions of each principal component in the variance maximization using as input data Elliptic Fourier coefficients. The values, presented in decreasing order, show that the first two principal components explained approximately two thirds of the crown shapes variability. The calculated values for symmetrical variations are higher, as compared to asymmetrical variation within all diameter classes. The extreme values found for explained variance - 24.23 x 10^{-3} (minimum, for 40 cm diameter class) and 38.78 x 10^{-3} (maximum, for 32 cm diameter class) show that the asymmetries, with respect to main crown axis, have lesser weight than symmetrical variations for the global shape of the crowns.

According to PCA, the first 5, 6 or 7 principal components, are considered as significant for crown shape description depending on the diameter class. The interpretation of the principal components corresponding to symmetrical and asymmetrical variations is presented in Tables 4 and 5; the first two principal components summarize 60% of variability for symmetrical

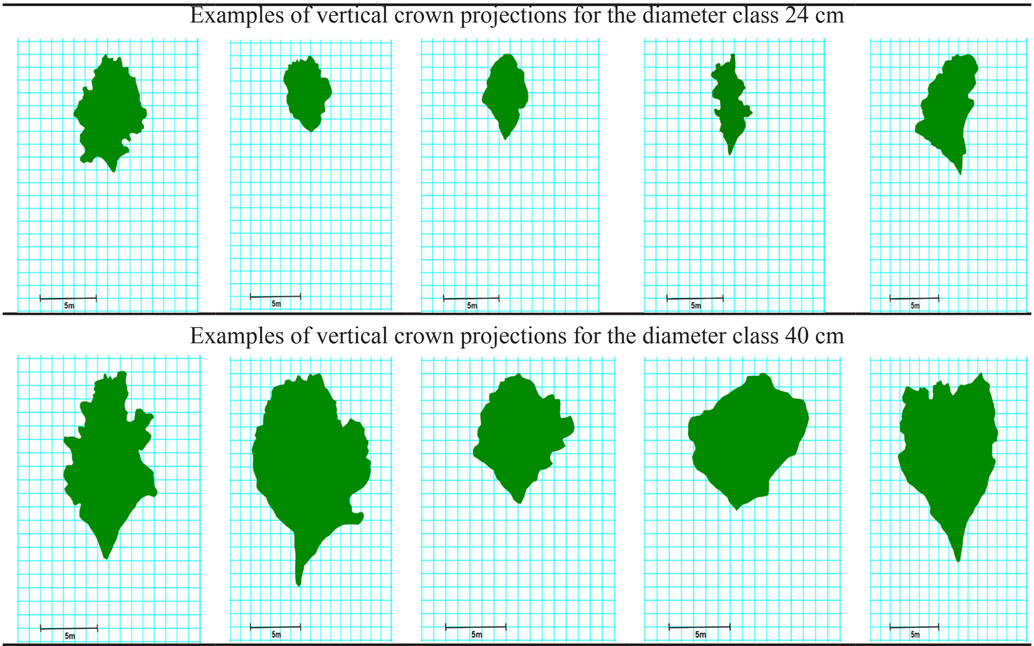

Figure 5 Examples of crown dimensional and shape variability within same diameter class and among 24 and 40 cm diameter classes

coefficients (Fig. 8) and 50% for asymmetrical coefficients (Fig. 9), for all diameter classes. It shows that crown shape is highly influenced by symmetrical variation expressed as ratio between its major and minor axes, the variation of the gravity center position, by crown asymmetry, with respect to the main axis (due to unequal lateral growth of the crown) and by the asymmetrical variations of the crown basis, lateral parts and the top, in a different manner within each diameter class. Due to high crown shape variability at approximately same age, the principal components have similar values according to analyzed diameter categories.

Graphical representations of the average crowns according to diameter classes took into consideration the average height at crown base, total tree height and maximum crown width, using sets of 30 measurement data for each diameter class (Fig. 10). According to the representation, the general shape of the crown is obovated, trees presenting a trend toward length and width allometric growth of this. The most exposed to modifications are the upper and middle third of the crown pace-wise with stem increment. Total crown height and crown areas grow in a correlated manner. The maximum height growth and crown area expansion was observed at the limit between 32-36 cm classes, where at a height growth rate of 123 cm corresponds to an area expansion with 13.44 m^2.

Height at the crown base is characterized by close values for the investigated diameter classes, minimum being observed at 36 cm diameter class (10.07 m) and maximum value at 24 cm diameter class (10.79 m).

For each diameter class, there is a specific crown development model, which reflects the competitive capacity of the trees, crown plasticity and growth characteristic is at each dimensional stem category.

The results from the morphometric measurements and the Elliptic Fourier Analysis show that for the diameter class of 24 cm, trees present a sustained height growth, com-

Table 4 Symmetrical variation – the principal components' proportion and eigenvalues (x 10^{-3}), according to the diameter classes and their interpretation

Principal components	Diameter classes (cm) - shape symmetrical variation (coefficients a_n and d_n)				
	24	28	32	36	40
PC1	50.34	57.51	44.82	54.92	38.60
PC2	17.41	12.92	22.87	15.79	22.84
PC3	9.04	7.24	10.96	9.52	10.89
PC4	6.61	6.05	7.97	5.07	8.29
PC5	4.17	4.95	3.39	3.79	6.08
PC6	2.81	2.92	2.85	-	3.31
Total %	90.38	91.59	92.86	89.09	90.01
Eigenvalues	27.942	35.401	38.780	30.438	24.234
Principal components significance					
PC1	Length/Width crown ratio				
PC2	Symmetrical variations of gravity center				
PC3	Symmetrical variations of contour convexities compared to mean shape: - 24 cm diameter class: symmetrical variation of D_b, D_m, D_t and crown base - 28 cm diameter class: symmetrical variation of D_b, D_m, D_t - 32 cm diameter class: symmetrical variation of D_b, D_t,, crown base and top - 36 cm diameter class: symmetrical variation of D_b, D_m, D_t and crown base - 40 cm diameter class: symmetrical variation of D_b, D_m, D_t and crown top				
PC4	Symmetrical variations of contour concavities compared to mean shape: - 24 cm diameter class: symmetrical variation of D_b, D_m and crown top - 28 cm diameter class: symmetrical variation of crown base and top - 32 cm diameter class: symmetrical variation of D_b, D_m, D_t and crown top - 36 cm diameter class: symmetrical variation of D_b, D_t and crown base - 40 cm diameter class: symmetrical variation of D_b, D_t and crown base				
PC5	Symmetrical variations of contour irregularities				
PC6	Hard to interpret				

peting for light which is translated in crowns' positioning. Heights variation range (between 15.3-22.5 m) exhibits greatest values, but the variation decreases at higher diameter classes. The high variation of crown basis is connected directly to the light quantity reaching this zone, trees from 28 cm diameter class crowns presented a sustained dynamic of crown basis and of the left bottom third. Height growth is active, but with lesser variability, as compared to 24 cm diameter class; for 32 cm diameter class most variable regions are the basis and left lateral. It is to observe the low variation of the crown top and right lateral region; for 36 cm diameter class, top, lateral regions and the bases variability amplify. The passage from 32 to 36 cm diameter class is realized through the most sustained growth of the height and lateral extension of the crowns; the trend consisting in the closure of spaces created by tree natural or artificial elimination in 40 cm diameter class is reflected by high variability of crown lateral regions and height, followed by a stabilization trend.

Discussion

The diverse tree architecture is the interplay of deterministic, hereditary determined growth algorithms and stochastic events determined by the environment (Champagnat et al. 1986); the assessment of its major components is the main goal of the forest ecomorphology. Tree crown architecture, including its variability, is one of the striking morphological characteris-

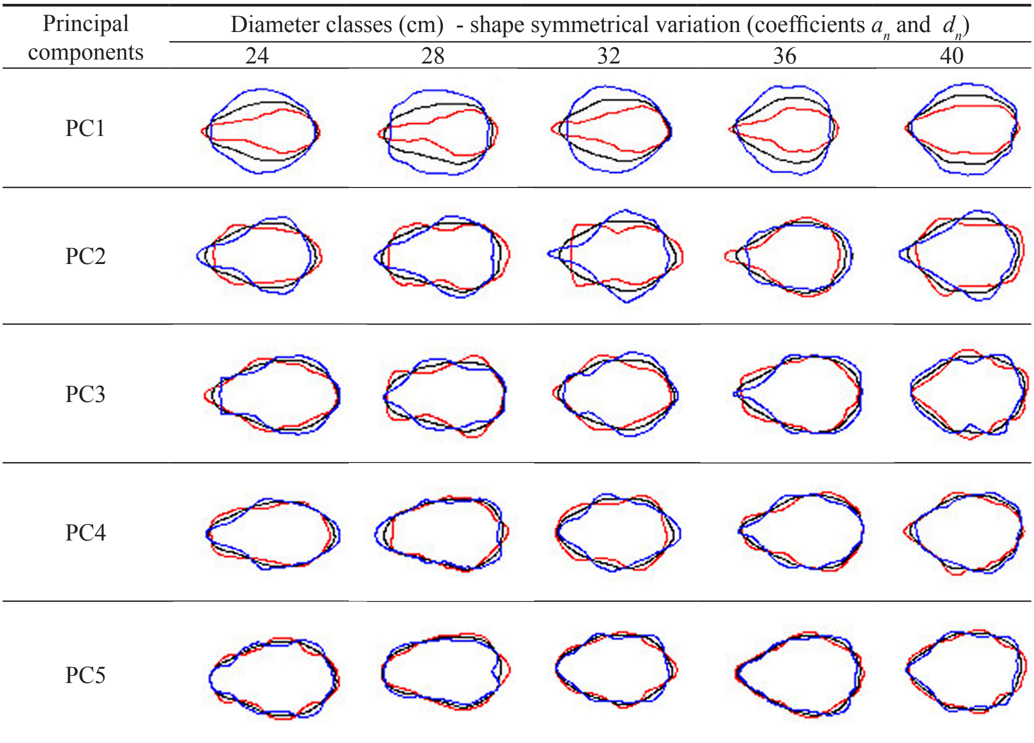

Figure 6 Mean crown shapes ±2SD obtained for symmetrical variation, corresponding to first 5 principal components, presented according to diameter classes

tics one can observe: practically, there are not two identical tree crowns within same population or species.

Within stand, the trees are difficult to be studied due to the partial crown occlusion of neighbor trees; however the assessment of crown variability in quantitative terms is important for morphometric and practical purposes. If dimensional variables are relatively easy to assess in the field, crown shape needs more sophisticated approaches in order to produce quantitative results, which to be sound also from biological point of view. The terrestrial vertical photographs taken under standardized conditions, corroborated with appropriate software offer the possibility of multiple and repeated analyses. These include measurements and analytical tools which are not supported by classical field sampling methods. Crown contours traced digitally on photos and combined with overlaid transparency drawings are good proxies for crown shape, and they also permit the generation of crown volumes, an important dimensional crown descriptor *per se*.

The results, refered to populations of stone oak, treat the crown architecture as a set of dimensional variables (crown area, perimeter, crown diameters at different crown levels, crown elongation, etc.) and dimensionless shape variables provided by Elliptic Fourier coefficients.

Dimensional information extracted from the vectorized crown contours was analysed in relationship with stem dimensions (diameters and volumes) in linear regressions. As dimensional analyses were performed within different stem diameter classes, architectural modifications with time were inferred. For instance, high-

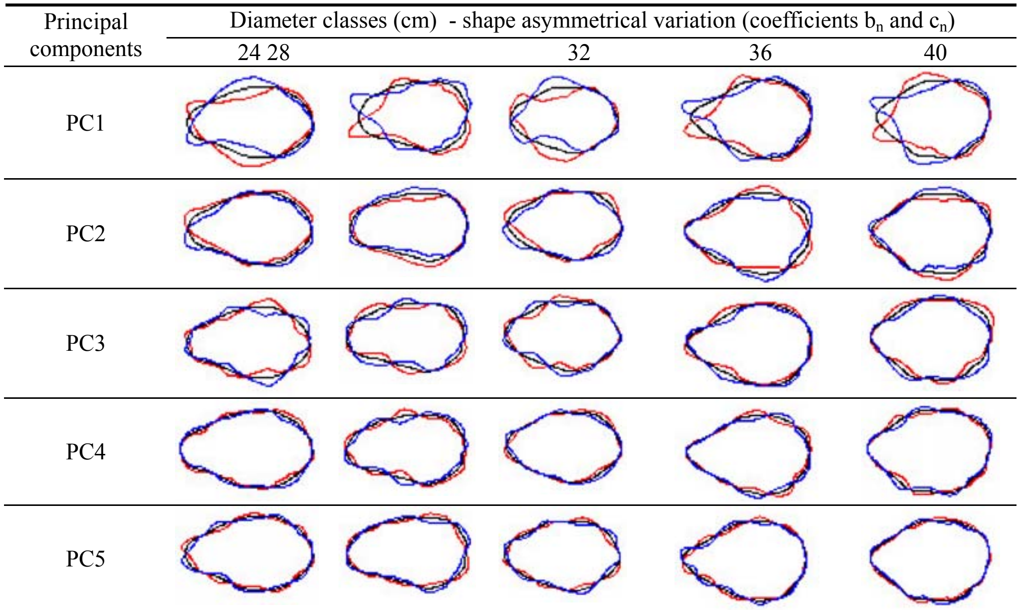

Figure 7 Mean crown shapes ±2SD obtained for asymmetrical variation, corresponding to first 5 principal components, presented according to diameter classes

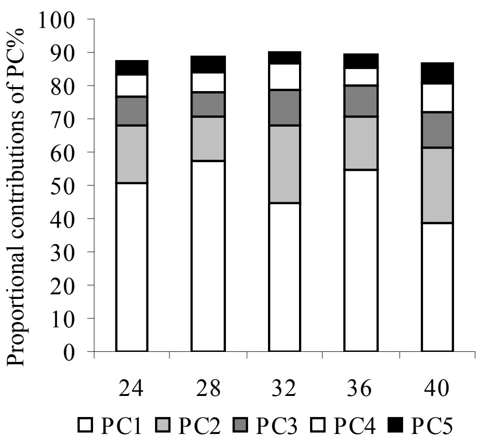

Figure 8 Proportional repartition of principal components according to diameter classes, for crown symmetrical variations (Fourier coefficients a_n and d_n)

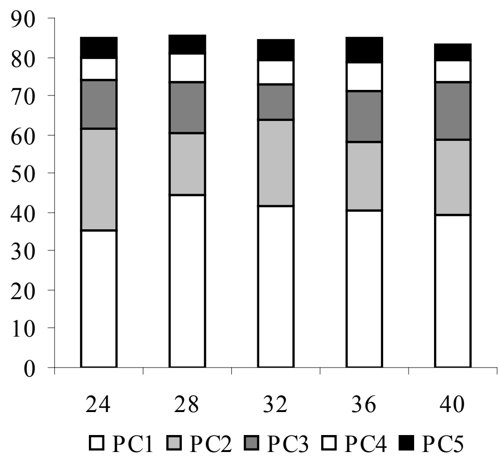

Figure 9 Proportional repartition of principal components according to diameter classes, for crown asymmetrical variations (Fourier coefficients b_n and c_n)

Tabel 5 Principal components' proportion and eigenvalues (x 10^{-3}), according to diameter classes and their interpretation for asymmetrical variation

Principal components	Diameter classes (cm) - shape asymmetrical variation (coefficients b_n and c_n)				
	24	28	32	36	40
PC1	35.04	44.27	41.41	40.65	39.26
PC2	26.48	15.89	22.19	17.44	19.46
PC3	12.27	13.08	9.22	13.27	14.60
PC4	5.82	7.42	6.41	7.27	5.82
PC5	5.54	4.74	5.32	6.02	4.09
PC6	3.63	3.31	3.64	3.47	3.42
PC7	-	2.73	-	-	2.96
Total %	88.78	91.44	88.19	88.12	89.61
Eigenvalues	16.920	20.569	19.149	12.412	14.144
Principal components significance					
PC1	Crown asymmetry related to main axis				
PC2	Asymmetrical variations of gravity center				
PC3	Asymmetrical variations of contour convexities at three crown levels: upper, middle and bottom thirds - 24 cm diameter class: asymmetrical variation of D_b, D_m and crown top - 28 cm diameter class: asymmetrical variation of D_b, D_m and crown top - 32 cm diameter class: asymmetrical variation of D_b, D_m and crown base - 36 cm diameter class: asymmetrical variation of D_b, D_t and crown base - 40 cm diameter class: asymmetrical variation of D_b, D_m, D_t and crown top				
PC4	Asymmetrical variations of contour concavities at three crown levels: upper, middle and bottom thirds - 24 cm diameter class: asymmetrical variation of D_b and D_t - 28 cm diameter class: asymmetrical variation of D_b, D_m, crown base and top - 32 cm diameter class: asymmetrical variation of D_t and crown base - 36 cm diameter class: asymmetrical variation of D_m, D_t and crown top - 40 cm diameter class: asymmetrical variation of D_b, D_m				
PC5	Asymmetrical variations of contour irregularities				
PC6	Hard to interpret				

est variability of crown volumes and crown diameters were reported for the lower third of the crown in relation to stem diameter classes, while highest general dimensional variability corresponded to the middle and top thirds of the crown in relation also to tree age. Maximum crown expansion and maximum height growth were found in the interval of 32-36 cm stem diameters; trees included in this interval can be considered effective targets for various silvicultural interventions, according to the practical purpose in view. The highest values of the coefficient of variation used for the comparisons among different variables were reported for the crown area, crown diameter at the bottom third of the crown and crown volume; apparently, these are the variables whose contribution to architectural variability are the most important. The lowest coefficient of variation corresponded to tree height (between 5 and 8.6% according to different diameter classes), a conservative variable, largely employed in empirical models.

The multiple comparisons assessed by means of Tukey test revealed the interesting fact that the majority of combinations of stem diameter classes with regard to the considered dimensional variables showed significant and very significant differences with one exception, the tree height to the first living branch

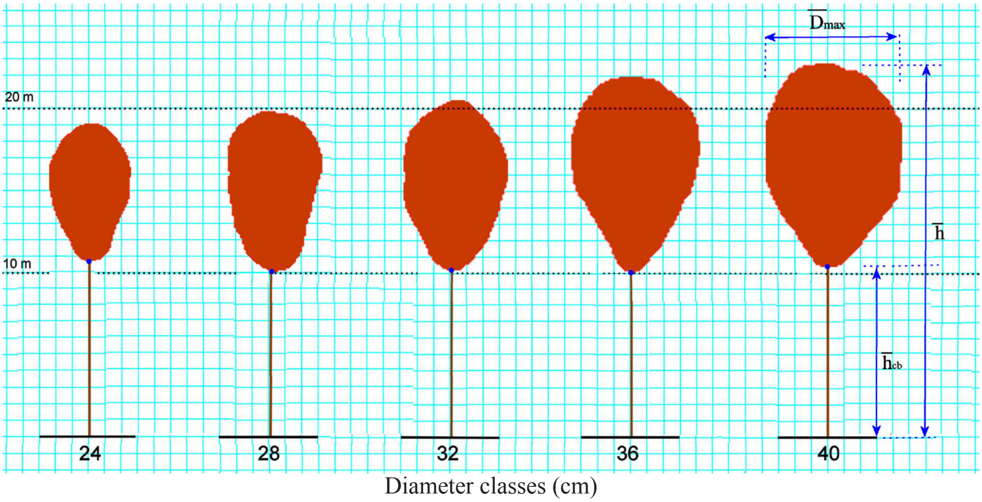

Figure 10 Graphical representation of mean crown shapes variation, according to stem diameter classes ($\overline{h}_{cor}$ – height at the crown base; $\overline{h}$ – total tree height, $\overline{D}_{max}$ - maximum crown width)

which is highly conservative across stem diameter classes (all comparisons yielded not significant differences).

The regression of crown volumes against stem diameters or stem volumes explained only a small amount of variability ($r^2 = 0.6159$ in the case of crown volume versus stem diameter and $r^2 = 0.6092$ in the case of crown volume versus stem volume), suggesting that these variables vary at a certain extent at different rates and predictions based on these models are not reliable. The statement that tree crown extension is an important dimensional parameter correlated with stem increment (Hemery et al. 2005) is only partially sustained by the presented results.

According to the results, the increment of tree heights is associated with the position of the crown mass center and depends on averages of dimensional variables. Crown mass center plays also a major role in crown mean shape, loading on the first principal component for symmetrical variation.

The graphical model, which summarizes the trends of the mean crown morphometric variables corresponding to different stem diameter classes revealed the fact that the crown growth was marked in the first place by maximum values obtained for mean crown diameters and the position of the mass gravity center. The upper third region of the crown is the most dynamic part, in conjecture with the intensive height growth and the crown area expansion. The least dynamic area corresponds to the bottom third of the crown, the same trend characterizing the height to the crown base. It is a known fact that larger crowns and slender stems (with dense wood) enhance light interception (King 1991). The upper third of the crown has the largest contribution to crown growth as already stated.

When shape is analyzed, within same stem diameter class, the crowns of stone oaks are highly variable. However, a mean shape can be extracted using different numerical representations of the shape, here the Elliptic Fourier Analysis (EFA). The method offers the possibility to assess crown shape taking into account contour details. Variability can be reduced by separating classes of similar crowns within same stem category or tree age, the classification being an useful tool for practical purposes - e.g. the tree selection according to several morphometric characteristics associated with high productivity (Hâruța 2011). The analysis of Elliptic Fourier coefficients permitted under the current study the reconstruction of the mean crown shape of stone oaks vegetating within stands, and of the dimensional descriptors with greatest determinism in the crown shape (shapes were back generated using Inverse Fourier transform). Elliptic Fourier coefficients, corroborated with PCA, led to the ordination of the original variables in a reduced space (sensu Rohlf 1990) and to the detection of small shape variations in dimension free shape analysis protocol (sensu Iwata & Ukay, 2002). This procedure separates symmetrical from asymmetrical variations and inferences can be made on the crown variables affected by the two.

From general shape analysis using Elliptic Fourier coefficients, the high mean crown shape variability (quantitatively expressed by high values of standard deviations) is explained by few variables loading on the first five principal components: for symmetrical variations the largest amount of variability is expressed by length/width crown ratio (PC1) and position of the mass center (PC2), while in asymmetry related variability, largest amount of variability corresponds to crown asymmetry related to the main axis (PC1) and asymmetrical variations of the position of the mass center of the crown (PC2). PC3 and PC4 summarizing lesser variability, explained the symmetrical and asymmetrical variations of crown contour concavities and convexities while the last morphometrically meaningful PC5 explained symmetrical and asymmetrical variations of contour irregularities, the least important to crown shape determinism.

The crown shape, as analyzed in the present study, is typical for stone oaks vegetating within stands, where asymmetries are dictated mainly by environmental factors such as the activity of defoliating insects, pathogens, intra- and interspecific competition, unbalanced development and the destruction of the apical bud in early developmental stages. Rautinianen et al. (2008) showed that crown asymmetry increases with age, an observation supported also by the present study. The reasone for this could be the enhancement of lateral growth of the crowns with age, in the modifications produced in light demands, in competitive capacity, in the need to occupy empty spaces created after tree extractions. Symmetric variations are related probably more to the growth processes than to some environmental stress. Another important observation is regarding the differences in symmetrical and asymmetrical variations among diameter classes, suggesting that the variability is expressed differently with age. Considering the modular organization of the trees, one explanation of the differential variability could be the relative autonomy of the crown regions. For instance, 32 cm stem diameter class showed symmetrical variation of crown concavities at the crown base and top together with crown diameters at the base and the top of the crown, while in 40 cm diameter class, concavity variation affected only crown top and the three crown sub-regions, where diameters were measured.

Conclusions

Dimensional and shape variability of tree crowns can be assessed from standardized vertical terrestrial photographs in conjecture with other morphometrical variables such as stem variables, which are meaningful for practical purposes. Transparency contour drawings overlaid on the photographs enhance the precision when sampling within stand conditions where visual occlusion frequently occurs. Dimensional analysis using multiple measurable variables can be performed for assessment of variability and in order to draw inferences about the variables. However, shape analysis needs more subtle approaches, provided by different methods employed in morphometrics; Elliptic Fourier coefficients and Principal Components Analysis are useful in discriminating highly variable shape descriptors and for the construction of the mean shape of the natural objects. Visual representations of the mean shape and the assessment of the variability range (expressed in standard deviation units) can be generated by Inverse Fourier Transform. Using these algorithms, mean stone oak crown was described, as well as its variability for within stand conditions. Variability affects dimensional and shape crown descriptors with different rates, according to stem diameter class. Highest shape variability was found in such descriptors, like crown center of gravity, crown length/width ratio, asymmetry from the main axis, and crown diameters, responsible for the variability of the lateral regions. Highest dimensional variability was found for crown areas and volumes, but also for diameters of the bottom third of the crowns. Nearly half of the crown volume variability is explained by stem volume or stem diameter variability, meaning that this cannot be used as a reliable predictor for practical purposes.

References

Andrade I.M., Mayo S.J., Kikup D., Van Den Berg C., 2008. Comparative morphology of populations of *Monstera* Adans. (*Araceae*) from natural forest fragments in Northeast Brazil using Elliptic Fourier Analysis of leaf outlines. Kew Bulletin 63: 193-211.

Avăcăriței D., 2001. Forma coroanei la fag – expresie a calității trunchiului şi a potențialului de creştere a arborilor [Beach crown shape – as an expression of stem quality and tree growth potential]. Anale ICAS 44(1): 151-157.

Champagnat P., Barnola P., Lavarenne S., 1986. Quelques modalités de la croissance rythmique endogène des tiges chez végétaux ligneux. Comptes rendus du coloque International sur l'Arbre. Montpellier, 9-14 Septembre 1985, Naturalia Monspeliensia, No. F hors Series: pp. 279-302.

Chiriță C., Vlad I., Păunescu C., Pătrăşcoiu N., Roşu C., Iancu I., 1977. Stațiuni forestiere [Forest sites]. Vol. II, Editura Academiei R.S.R., Bucureşti.

Clark N.A., Wynne H.R., Schmoldt L.D., Winn M., 2000. Digital terrestrial photogrammetric methods for tree stem analysis. Proceedings Integrated tools for natural resources inventories in the 21st century. Idaho, USA, pp. 353-363.

Costa I., Cesar M.R. jr., 2001. Shape analysis and classification. Theory and practice. CRC Press, Boca Raton, London, New York, Washington D.C.

Doniță N., Chiriță C., Stănescu V., 1990, Tipuri de ecosisteme forestiere din România [Forest ecosystems types from Romania], Editura Tehnică Agricolă, Bucureşti.

Giurgiu V., Decei I., Drăghiciu D., 2004. Metode şi tabele dendrometrice [Methods and yield tables]. Editura Ceres. Bucureşti.

Hâruța O., 2011. Analiza sistemică a arhitecturii coroanei la gorun şi anin negru [Systemic analysis of stone oak and black alder tree crown architecture]. Ph.D. Transilvania University of Brașov, Faculty of Silviculture and Forest Engeneering, Romania.

Hâruța O., Fodor E., 2010. The employment of the terrestrial digital photographs in the study of trees' morphometry. Analele Universității din Oradea. Fascicula Protecția Mediului XIV: 564-579.

Hatta H., Honda H., Fisher J.B., 1999. Branching principles governing the architecture of *Cornus kousa* (*Cornaceae*). Annals of Botany 84: 183-193.

Hemery G.E., Savill P.S., Pryor S.N., 2005. Application of the crown diameter-stem diameter relationship for different species of broadleaved trees. Forest Ecology and Management 215: 285-294.

Horn, H. S., 1971. The adaptive geometry of trees. Princeton University Press, N.J.

Ivanov D., Sauerbier M., 2006. Terrestrial 3-dimensional measurements of tree crowns. In: Nagel, J. (ed.). Beiträge zur Jahrestagung der Sektion Ertragskunde im DVFFA. Nordwestdeutsche Forstliche Versuchsanstalt, Göttingen. pp. 73-85.

Iwata H., Niikura S., Matsuura S., Takano Y., Ukai Y., 2004. Genetic control of root shape at different growth stages in Radish (*Raphanus sativus* L.). Breeding Science 54: 117-124.

Iwata H., Ukay Y., 2002. SHAPE: a computer program package for quantitative evaluation of biological shapes based on elliptic Fourier descriptors. The Journal of Heredity 93(5): 384-385.

King, D.A., 1991. Tree allometry, leaf size and adult tree size in old growth forests of western Oregon. Tree Physiology 9: 369-381

Korpela I., 2004. Individual tree measurements by means of digital aerial photogrammetry. Silva Fennica. (Monographs) 3: 1-93.

Kuhl F.P., Giardina C.R., 1982. Elliptic Fourier features of a closed contour. Computer Graphics and Image Processing 18: 236-258.

Lestrel P.E., 1997. Fourier Descriptors and their applications. Cambridge University Press.

Mathsoft Inc., 1999. Mathcad 2000 Reference Manual. Cambridge, MA.

Paşcovschi S., Leandru V., 1958. Tipuri de pădure din R.P.R. [Forest types from R.P.R.] Editura Agro – Silvică, Bucureşti.

Pyysalo U., 2004. Tree crown determination using terrestrial imaging for laser scanned individual tree recognition. ISPRS Congress Istanbul. Commission III papers, Vol. XXXV, part B3.

Rautiainen M., Mottus M., Stenberg P., Ervasti S., 2008. Crown envelope shape measurements and models. Silva Fennica 42(1): 19-33.

Rohlf J.F., 1990. Morphometrics. Annual Review of Ecology and Systematics 21: 299-316.

Sachs T., Novopolski A., 1995. Tree forms: architectural models do not suffice. Israel Journal of Plant Sciences 43: 203-212.

Seiler J.R., McBee P.N., 1992. A rapid technique for the evaluation of mature tree crown growth. Journal of Arboriculture 18(6): 325-328.

Tahvanainen T., Forss E., 2008. Individual tree models for the crown biomass distribution of Scots pine, Norway spruce and birch in Finland. Forest Ecology and Management 255: 455-467.

Valladares F., Niimets Ü., 2007. The architecture of plant crowns. From design rules to light capture and performance. In: Pugnaire F., Valladares F. (eds). Functional plant ecology. Taylor and Francis, New York, pp. 101-149.

Valladares F., Pearcy, R.W., 2000. The role of crown architecture for light harvesting and carbon gain in extreme light environments assessed with a realistic 3-D model. Anales Jardin Botanicos de Madrid 58(1): 3-16.

Wilcox C.D., Dove S.B., McDavid W.D., Greer D.B., 1996. Image Tool software ver. 2.0, http://ddsdx.uthscsa.edu/dig/itdesc.html.

Yoshioka K., 2002. KyPlot - a user-oriented tool for statistical data analysis and visualisation. Computational Statistics 17: 425-437.

Yoshioka Z., Iwata H., Ohsawa R., Ninomiya S., 2004. Analysis of Petal Shape Variation of Primula sieboldii by Elliptic Fourier Descriptors and Principal Component Analysis. Annals of Botany 94(5): 657-664.

Yoshioka Y., Ohashi K, Konuma A.H., Ohsawa R., Ninomiya S., 2007. Ability of bumblebees to dis-