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Division Play and Outcome Uncertainty in Sports Leagues

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Abstract

The analysis of outcome uncertainty (OU) and competitive balance (CB) has been of overwhelming importance in sports economics. Surprisingly, there is little work on the impact of the structure of play on either OU or CB. Balanced and unbalanced schedules, and division play have been used to analyze biasedness of CB measures. And the impact of the introduction of unbalanced schedules on OU has been analyzed. But the *impact* of the introduction of both unbalanced schedules and division play on OU has not been analyzed. In this paper, we assess the impacts on OU, for given choices of CB (OU/CB) of moving to division play. This includes the impact of schedule imbalance, division strength, the number of teams, and the number of divisions. We also obtain estimates of their marginal impacts on OU/CB via numerical analysis and regression. The results are compared to OU/CB from unbiased estimators, for the case of the introduction of division play in Major League Baseball. The results suggest that the approach is useful and there are policy implications.

JEL codes: L83, Z21, Z28

Keywords

sports leagues, division play, unbalanced schedules, league policy, major league baseball

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Introduction

The extensive literature on outcome uncertainty (OU) and competitive balance (CB) follows from Rottenberg's (1956) seminal work on their potential importance to fans and, thus, to profit maximizing team owners in a league. OU is typically portrayed as the standard deviation of game outcomes. CB is typically portrayed by the degree of equality of the playing strengths of teams. For our purposes, we use the standard deviation of win probability.[1]

Fort and Maxcy (2003) pointed out that there are two strands in the OU and CB literature. One assesses Rottenberg's "invariance principle," tracking movements in OU and CB over time caused by league policy or exogenous factors.[2] Another strand of the literature is about Rottenberg's "outcome uncertainty hypothesis." It analyzes the impact of OU on gate or TV viewing and, recently, explores alternative explanations for fan choice based on reference-dependent preferences.[3]

This paper belongs to the former strand in the literature, tracking changes in OU based on changes in the structure of league play—schedule imbalance, division strength, the number of teams, and the number of divisions. Comparatively little is known about the impacts of the structure of league play, itself, on OU or CB. And what there is relates almost exclusively to CB.[4]

In this paper, we analyze the relationship between the less explored OU and all these elements of the structure of league play. Sports league theory does not yet offer a detailed treatment of OU, beyond just including it as a determinant of demand. However, direct examination of the statistical definition of OU provides theoretical intuition—it depends on division strength, schedule imbalance, the number of teams in the league, and the number of divisions. Unfortunately, the partial derivatives of the statistical definitions of OU and CB with respect to the elements of the structure of league play are ambiguous with respect to even their qualitative impacts.

The intuitive theoretical relationship between OU/CB and elements of the structure of league play suggests that regression analysis can reveal the marginal effects of schedule imbalance, division strength, the number of teams, and the number of divisions on OU/CB. Since real-world episodes of altered structure of league play are few and far between, we use numerical analysis to generate simulated data. Doing so restricts our regression analysis to OU for given choices of CB, that is, OU/CB.

As a veracity check, we turn to the example of MLB. The league played a balanced schedule without divisions until 1969. After that, it introduced 2 and later 3 divisions playing unbalanced schedules. Divisions also had unequal strength and, at times, unequal numbers of teams in each division.

Using characteristics of play from the MLB example, we derive simulated data on schedule imbalance, division strength, the number of teams, and the number of divisions and OU/CB via numerical analysis. We regress those characteristics of play on our simulated OU/CB to obtain fitted values of OU/CB from the resulting coefficient estimates. We then calculate OU/CB from the actual using unbiased estimators of OU and CB for the MLB case. Comparing the regression fitted values of OU/CB to the unbiased estimates of OU/CB suggests that the regression approach using numerical analysis to generate the data has merit.

Our results should also be of interest to policy makers and policy analysts. The results suggest that if policy makers care about OU because fans do (a la Rottenberg, 1956), they would do well to recognize that their league policy choices about CB ultimately impact OU, which is the Rottenberg determinant of fan demand. And the suggested levels of these impacts from our analysis suggest that the impacts are economically significant, at the same level as recent investments in media and racial equity.

The paper proceeds as follows. Section "The Determinants of OU/CB: Theoretical Intuition" covers the determinants of OU/CB and why we must turn to numerical analysis for insight. The story of changes in the structure of league play for Major League Baseball, used to guide the rest of the analysis, is in Section "Historical Evolution of MLB Division Play." Section "Numerical Analysis: Data Generation" uses numerical analysis to generate data for analysis. In Section "Marginal Effects of Changes in the Structure of Play," we use regression to estimate the marginal effects of the elements of the structure of play on OU/CB. In Section "Values and Unbiased Estimators: OU/CB for MLB," OU/CB is calculated from unbiased estimators on what MLB data are available. These are compared to the fitted values of OU/CB from the regression coefficient estimates. Summary and conclusions round out the paper in Section "Summary and Conclusions."

The Determinants of OU/CB: Theoretical Intuition

Following Fort and Lee (2020), we restrict our attention to within-season outcomes so that playing strength is fixed during a season. We also assume there is no home field advantage. Let $p_{ij},\ i, j = 1, \ldots, N$ be the win probability for team i playing team j, assumed to represent the relative playing strengths of the two teams. N is the number of teams. In this setting, Fort and Lee (2020) express league-wide CB starting with the squared average deviation from the mean playing strength:

$$\sigma^2_{\mathrm{CB}} = \frac{2}{N(N-1)} \sum_{i=1}^{N-1} \sum_{j>i}^{N} (p_{ij} - 0.5)^2 \tag{1}$$

The standard deviation has intuitive scale, so we use the standard deviation of winning percentages, $\sigma_{\mathrm{CB}} = \sqrt{\sigma^2_{\mathrm{CB}}}$ in (1). As σ_{CB} increases, the distribution of winning percentages includes more and more strong and weak teams. That is, as σ_{CB} increases, CB itself decreases.

In the same setting, Fort and Lee (2020) began the expression of OU with the variance of game outcomes by the squared average deviation from the mean win probability[5]:

$$\sigma^2_{\mathrm{OU}} = \frac{2}{N(N-1)}\frac{1}{K}\sum_{i=1}^{N-1}\sum_{j>i}^{N} K_{ij}(p_{ij}-0.5)^2 \tag{2}$$

In (2), K_{ij}, $i, j = 1, \ldots, N$ is the number of games in a season between teams i and j and K is the average number of K_{ij} over j and we assume all teams has the same number of games.

Again, the standard deviation has intuitive scale, so we use the standard deviation of game outcomes, $\sigma_{\mathrm{OU}} = \sqrt{\sigma^2_{\mathrm{OU}}}$ in (2). As σ_{OU} increases, the difference between weaker and stronger teams increases and games become more rather than less predictable. That is, as σ_{OU} increases, OU itself decreases.

Equations (1) and (2) give league-wide CB and OU, respectively. To compare league-wide CB with CB after the leagues have been split into divisions, we also offer a new definition of within-division competitive balance, $\mathrm{CB_W}$. The definition utilizes the average of the within-division variation in win probability across all division play. For example, with 2 divisions, (1) becomes:

$$\sigma^2_{\mathrm{CB_W}} = \frac{1}{2}\left[\frac{2}{N_1(N_1-1)}\sum_{i=1}^{N_1-1}\sum_{j>i}^{N_1}(p_{ij}-0.5)^2 + \frac{2}{N_2(N_2-1)}\sum_{i=N_1+1}^{N-1}\sum_{j>i}^{N}(p_{ij}-0.5)^2\right] \tag{3}$$

In (3), N_1 and N_2 are the number of teams in division 1 and 2, respectively.[6] It is appropriate to think of the first term in braces on the right-hand side of (3) is σ^2_{CB} of one division and the second term is σ^2_{CB} of the other division.

As with the measurement convention for league-wide CB and OU, we use the standard deviation $\sigma_{\mathrm{CB_W}} = \sqrt{\sigma^2_{\mathrm{CB_W}}}$ in (4) for $\mathrm{CB_W}$. As $\sigma_{\mathrm{CB_W}}$ increases, the distribution of within-division winning percentages includes more and more strong and weak teams. That is, as $\sigma_{\mathrm{CB_W}}$ increases, $\mathrm{CB_W}$ itself decreases.

We will also utilize "plug-in" estimators of (1) through (3) in the empirical work that follows.[7] Fort and Lee (2020) show that the plug-in estimator for σ_{CB} in (1) is biased and provide the "bias corrected standard deviation" (BCSD) plug-in estimators:

$$\mathrm{BCSD}(\hat{\sigma}_{\mathrm{CB}}) = \sqrt{\max(0,\, \hat{\sigma}^2_{\mathrm{CB}})} \tag{4}$$

$\hat{\sigma}_{\mathrm{CB}}$ is the plug-in result from (1). Fort and Lee (2020) show that the plug-in estimator for σ_{OU} in (2) also is biased and derives the BCSD plug-in estimator:

$$\mathrm{BCSD}(\hat{\sigma}_{\mathrm{OU}}) = \sqrt{\max(0,\, \hat{\sigma}^2_{\mathrm{OU}})} \tag{5}$$

$\hat{\sigma}_{\mathrm{OU}}$ is the plug-in result from (2).

Similarly (without proof, since it is just a partition of CB), the "bias corrected standard deviation" plug-in estimator for $\mathrm{CB_W}$ is

$$\mathrm{BCSD}(\hat{\sigma}_{\mathrm{CB_W}}) = \sqrt{\max(0, \hat{\sigma}^2_{\mathrm{CB_W}})} \tag{6}$$

where $\hat{\sigma}^2_{\mathrm{CB_W}}$ is the plug-in result from (3).

As described in a subsequent section of the paper, our analysis is restricted to OU *for given CB*, or OU/CB, and $\mathrm{CB_W}$ *for given CB*, or $\mathrm{CB_W}$/CB. From (1) and (2):

$$\begin{aligned}\frac{\sigma_{\mathrm{OU}}}{\sigma_{\mathrm{CB}}} &= \sqrt{\frac{(2/(N(N-1)))(1/K)\sum_{i=1}^{N-1}\sum_{j>i}^{N} K_{ij}(p_{ij}-0.5)^2}{(2/(N(N-1)))\sum_{i=1}^{N-1}\sum_{j>i}^{N}(p_{ij}-0.5)^2}}\\ &= \sqrt{\frac{(1/K)\sum_{i=1}^{N-1}\sum_{j>i}^{N} K_{ij}(p_{ij}-0.5)^2}{\sum_{i=1}^{N-1}\sum_{j>i}^{N}(p_{ij}-0.5)^2}}\end{aligned} \tag{7}$$

And from (1) and (3):

$$\frac{\sigma_{\mathrm{CB_W}}}{\sigma_{\mathrm{CB}}} = \sqrt{\frac{(1/2)\left[(2/(N_1(N_1-1)))\sum_{i=1}^{N_1-1}\sum_{j>i}^{N_1}(p_{ij}-0.5)^2 + (2/(N_2(N_2-1)))\sum_{i=N_1+1}^{N-1}\sum_{j>i}^{N}(p_{ij}-0.5)^2\right]}{(2/(N(N-1)))\sum_{i=1}^{N-1}\sum_{j>i}^{N}(p_{ij}-0.5)^2}} \tag{8}$$

The theory of sports leagues does not include the structure of league play, as we discuss it here, but (7) and (8) offer some intuitive theoretical insights. The ratios in (7) and (8) depend on schedule imbalance, division strength, the number of teams, and the number of divisions.

Schedule imbalance is incorporated into K_{ij}/K, and grouping of teams in the divisions with N_1 and N_2 teams each. Division strengths group different subsets of teams (and their respective p_{ij}), depending on which teams are in which division. Changes in the number of teams, N, would alter both the composition of the two divisions (N_1 and N_2 teams each) and the limits of the summations. Finally, the number of divisions allows further variation in both division strength and schedule imbalance, as well as determining the number of partitions in (3).

It would be helpful to the investigation of impacts of schedule imbalance, division strength, number of teams, and number of divisions on OU/CB if the partial derivatives of (7) or (8) with respect to these factors were sign determinant. However, to the best of our ability, this ends up not the case. For example, for given K, as K_{ij} increases, schedule imbalance increases. But try as we might, we could not determine the sign of $\partial(\sigma_{\mathrm{OU}}/\sigma_{\mathrm{CB}})/\partial K_{ij}$, that is, the change in OU given CB with respect to an increase in schedule imbalance. Nor could we determine the sign of the other partial derivatives of OU/CB with respect to division strength, number of teams, or number of divisions.

Absent guidance from the theory of sports leagues in its current state, or from the definition of OU/CB, we use the theoretical intuition from (7) and (8) to identify the following relationship:

$$\frac{\sigma_{\mathrm{OU}}}{\sigma_{\mathrm{CB}}} = f(\mathrm{SI}, \mathrm{DS}(\mathrm{SI}), N, D) \tag{9}$$

SI is schedule imbalance, DS is division strength which also depends on SI, N is the number of teams, and D is the number of divisions. Individual divisions can be denoted $D_i, \quad i = 1, \ldots D$. And we have no predictions even about the signs of the impact of the independent variables on OU/CB.

Further, even across leagues, there have not been very many changes in the structure of play. There is no actual real-world league data to use to calculate marginal effects of the changes in the structure of play. This leads us to employ numerical analysis to generate simulated data on OU/CB and its theoretically intuitive determinants in order to estimate those marginal effects.

We use the following real-world case of MLB to inform the numerical analysis and then as the source of a veracity check on our subsequent estimates of independent variables in (9) on OU/CB.

Historical Evolution of MLB Division Play

MLB is the example throughout the paper, and before turning to the numerical analysis, we document the changes in CB, schedule imbalance, division strengths, number of teams, and number of divisions that occurred in that league. This is useful just for the understanding of our example, but also instructive in the design of the numerical analysis in the next section.

Divisions were *not* created with the first MLB expansion, from 8 to 10 teams each in 1961 for the American League (AL) and 1962 for the National League (NL). For the second expansion, from 10 to 12 teams in both the AL and NL in 1969, division play was introduced. East and West Divisions of 6 teams each were created in both leagues.

Expansion occurred again for the 1977 season in the AL, but both leagues maintained the same East and West division structure. For the third expansion, after an inter-league shuffle and division realignment, the 1993 season saw East, Central, and West divisions in both the AL and NL. With another inter-league shuffle, and further realignment, the leagues settled on 5 teams in each of their 3 divisions from 2013 onward.

Thus, MLB history included the following regarding division play. No divisions prior to 1969, 2 divisions from 1969 to 1992, and 3 divisions from 1993 onward. The number of teams in the league, and in each division, also changed over time. Facts about the intuitive determinants of OU/CB are covered in the remainder of the section—CB, schedule imbalance, and division strength.

Table 1 contains calculations from the unbiased estimator BCSD($\hat{\sigma}_{CB}$) in (4) over the decades when both the AL and NL existed. The historical decade averages are in the interval (0.052, 0.117) and the largest observed values were never greater than 0.145 for the NL 1919. The averages for 2 and 3 divisions, for the period 10 years before and 10 years after, lie in the interval (0.047, 0.065). The largest value is in the NL in with 2 divisions (0.112, 1962). This helps guide our numerical analysis in the next section.

Schedule imbalance information is in Table 2.[8] Prior to division play, both the AL and NL played balanced schedules. When both leagues had 2 divisions, there were brief periods when the degree of schedule imbalance was greater in the NL than that in the AL. Then, with 3 divisions, the AL appears to remain "Unbalanced" while the NL schedule appears "Most Unbalanced," historically speaking. This information on actual degrees of schedule imbalance is also used in the work to follow.

Table 3 shows actual MLB division strengths based on average finishing place (rank) for 2 and 3 divisions. In both the AL and NL, two things are apparent. *Within-division* strengths are typically unbalanced as witness the averages well over 5 in all periods for all divisions, and *between-division* strengths vary. With 2 divisions, the between-division differences are less than one finishing place. With 3 divisions, the between-division differences in the AL are about 1 finishing place but reach 1.6 finishing places in the NL. Since it is difficult to precisely pin down

Table 1. Estimates of MLB σ_{CB}, 1901–2019.

	AL			NL		
Decades	Average	Max	Year	Average	Max	Year
1901–1909	0.087	0.111	1904	0.117	0.145	1909
1910–1919	0.094	0.126	1915	0.079	0.108	1912
1920–1929	0.081	0.109	1927	0.077	0.113	1928
1930–1939	0.098	0.133	1932	0.080	0.120	1935
1940–1949	0.077	0.109	1949	0.086	0.124	1942
1950–1959	0.086	0.133	1954	0.066	0.103	1952
1960–1969	0.064	0.088	1961	0.072	0.112	1962
1970–1979	0.063	0.089	1977	0.057	0.076	1976
1980–1989	0.054	0.068	1980	0.049	0.072	1985
1990–1999	0.052	0.069	1999	0.055	0.083	1998
2000–2009	0.068	0.095	2002	0.053	0.069	2004
2010–2019	0.062	0.117	2018	0.056	0.076	2015
2 Divisions 1969 ± 10 Years						
1959–1968	0.062	0.088	1961	0.065	0.112	1962
1969–1978	0.063	0.089	1977	0.061	0.093	1969
3 Divisions 1994 ± 10 Years						
1984–1993	0.047	0.063	1988	0.053	0.081	1993
1994–2003	0.065	0.095	2002	0.057	0.083	1998

Note: Calculated using BCSD($\hat{\sigma}_{CB}$) in (4).

Table 2. Schedule Imbalance, MLB, Division Play Only.

AL					NL				
Seasons	*N*, *D*	Div.	Non-D.	Ratio	Seasons	*N*, *D*	Div.	Non-D.	Ratio
1969–1976	12,2	18	12	1.50	1969–1992	12,2	18	12	1.50
1977–1978	14,2	15	10–11	1.43	1993–1997	14,2 and 14,3	12–13	10–12	1.14
1979–1996	14,2 and 14,3	13	12	1.08	1998–2000	16,3	13	7–9	1.62
1997–2000	14,3	12–13	9–11	1.25	2001–2012	16,3	16–19	5–9	2.50
2001–2012	14,3	18–19	6–9	2.47	2013-on	15,3	19	6–7	2.92
2013-on	15,3	19	6–7	2.92					

Note: Summarized from daily "Scores and Standings", by season, at Baseball-Reference.com.
N = number of teams, *D* = number of divisions. 2 divisions start 1969 and 3 divisions start 1994 .

Table 3. Average Division Strength, MLB Episodes.

	AL			NL		
	East	Central	West	East	Central	West
2-Divisions						
1959–1968	—	—	—	—	—	—
1969–1978						
Average division strength	7.1	—	6.3	6.1	—	6.9
#Dominant seasons	6	—	1	3	—	6
#Equal strength seasons		3			1	
3 Divisions						
1984–1993						
Average division strength	7.8	—	7.2	6.9	—	6.3
#Dominant seasons	7	—	3	5	—	4
#Equal strength seasons		0		1	1	
1994–2003						
Average division strength	7.8	6.8	7.9	8.2	7.3	8.9
#Dominant seasons	3	2	5	3	0	7
#Equal strength seasons		0			1	

Note: Calculated from annual season place (rank) at Baseball-Reference.com.

relative division strength, the analysis to follow characterizes division strengths as "Equal," "Unequal," and "Most Unequal."

Numerical Analysis: Data Generation

The relationship in (9) lends itself naturally to regression analysis to empirically analyze the impact of SI, DS, *N*, and *D* on OU/CB. However, as previously noted, even across leagues, there have not been very many changes in the structure of play. So, the data from real-world league choices are sparse.

This leads us to employ numerical analysis to generate simulated data on OU/CB and its theoretically intuitive determinants in (9). Then, regression is used to determine the signs and marginal effects of the determinants of OU/CB. The numerical analysis technique for league-wide CB in (1) is in Lee et al. (2019a, 2019b). The extension to OU in (2) is in Fort and Lee (2020) and the extension to CB_W is new in this paper.

In a nutshell,[9] define relative strengths $S_1 > S_2, \ldots, > S_N$ and the relationship between these strengths, say, $S_1 - S_2 = S_2 - S_3 = \ldots = S_{N-1} - S_N$. Choose values $\sigma_{CB} = \bar{\sigma}_{CB}$ and $N = \bar{N}$ in (1) and solve for:

$$\bar{p}_{ij} = \frac{\exp(S_i)}{\exp(S_i) + \exp(S_j)} \tag{10}$$

Where S_i satisfy:

$$S_1 - S_2 = S_2 - S_3 = \ldots = S_{N-1} - S_N \tag{11}$$

And $\bar{p}_{ij}$ satisfy:

$$\sqrt{\frac{2}{\bar{N}(\bar{N}-1)} \sum_{i=1}^{\bar{N}-1} \sum_{j>i}^{\bar{N}} (p_{ij} - 0.5)^2} = \bar{\sigma}_{CB} \tag{12}$$

Expressions (10) through (12) represent a system of equations, whose solution are $\bar{p}_{ij} = p_{ij}(\bar{\sigma}_{CB}, \bar{N}), \quad i, j = 1, \ldots \bar{N}$. For the sake of generating data, different p_{ij} can be had by varying $(\bar{\sigma}_{CB}, \bar{N})$.

In turn, obtaining $\bar{p}_{ij} = p_{ij}(\bar{\sigma}_{CB}, \bar{N})$ allows us to calculate either σ_{OU}, from (2), or σ_{CB_W} from (3). As observed earlier, this is the reason our analysis is restricted to OU/CB or CB_W/CB. Literally, both OU and CB_W are calculated *for a given league-wide CB* (i.e. $\sigma_{CB} = \bar{\sigma}_{CB}$).[10]

For example, to analyze the impact of SI on OU/CB *without division play* use $\bar{p}_{ij}$, $i = 1, \ldots, \bar{N}$ and vary K_{ij} in (2).[11] Schedule imbalance with division play is a bit more complicated. For a season of G games, let K_1 be the number of games against each division opponent, assumed the same for all division opponents. Let K_2 be the number of games against each non-division opponent, assumed the same for all non-division opponents. We only consider $K_1 \geq K_2$ so that schedule imbalance increases as the number of games played against each division opponent increases. And the calculation of OU depends on how changing K_1, in turn, changes K_{ij} in (2). Thus, simple ratio $(K_1 / K_2) \geq 1$, is one index of SI.

When it comes to estimating (9), we aggregate to the ratio of the number of games against all division opponents out of the total number of games for SI. The number of games against division opponents is calculated as $G_1 = K_1[(N / D) - 1]$. While the number of games played against the opponents in each of the other divisions is

$G_2 = (D - 1)K_2(N / D)$. And, of course, $G = G_1 + G_2$. For SI, we use a proportional index of schedule imbalance, G_1 / G where $((1 / D) - (1 / N)) \leq (G_1 / G) \leq 1$.[12]

Schedule imbalance specifications for our numerical analysis are in Table 4. The "Balanced" schedule with 2 divisions would be the same as if there were no divisions. Each team plays each other 18 times in a single season when $N = 10$. That would be $G_1 = 18[(10 / 2) - 1] = 72$ games against division opponents and $G_2 = (2 - 1)18(10 / 2) = 90$ games against opponents in the other division, for a total $G = 162$. As a result, $G_1 / G = 72 / 162 = 0.44$. MLB teams have played 162 games since 1961 in the AL and since 1962 in the NL so this situation mimics MLB quite closely. In the numerical analysis, the number of games in a season also approximates the 162 games in an MLB season.[13] Relative to Table 3, the degrees of schedule imbalance are not exactly those observed for MLB, but the point is variation to facilitate estimation.

Division strength is the relative strength of one division compared to another. Approximately equal division strengths would have $DS_1 \cong DS_2 \cong \ldots \cong DS_D$. Unequal strengths have $DS_1 > DS_2 > \ldots > DS_D$ (remember, smaller rank numbers are higher strengths). The "Most" unequal version also has $DS_1 > DS_2 > \ldots > DS_D$ but with the starkest contrast, putting the bottom N_D teams in D_D, the next strongest N_{D-1} teams in D_{D-1}, and so on up to the N_1 strongest teams in D_1.

The case of 2 and 3 divisions, and three division strengths is characterized in Table 5. So, one way to look at strength is just the averages across divisions, also shown in Table 5. For example, the "Most Unequal" result, for 3 divisions would have the strongest division with the 5 best teams and $DS_1 = 3.0$, down to the weakest division with the 5 worst teams $DS_3 = 15.6$. This is one way to envision division strength. In Table 5, showing the rankings of division teams, the chosen division strengths in the numerical analysis mimic those observed for MLB in Table 2.[14]

Having chosen D and the DS_i, $i = 1, \ldots, D$, again using $\bar{p}_{ij} = p_{ij}(\bar{\sigma}_{CB}, \bar{N})$, the natural characterization of the impact of *DS* is CB_W, restricted to be *relative to league-wide CB*, that is, $\sigma_{CB_W} / \sigma_{CB}$. For given league-wide σ_{CB}, an increase in σ_{CB_W} implies division strength becomes more equal. For the chosen range of $0.050 \leq \bar{\sigma}_{CB} \leq 0.250$, consistent with actual MLB results in Table 1, and for chosen $\bar{N}$, Table 6 compares σ_{CB_W} and $\sigma_{CB_W} / \sigma_{CB}$ across the 3 division strength characterizations, for 2 and 3 divisions.

For, example, for the choice of $\bar{\sigma}_{CB} = 0.05$ and $\bar{N} = 10$, the resulting $\bar{p}_{ij}$ from (1), are grouped according to a division strength specification to calculate σ_{CB_W} in (3) with $\bar{N}_1 = \bar{N}_2 = 5$. For 2 divisions, $\sigma_{CB_W} = (0.071, 0.058, 0.035)$ and $\sigma_{CB_W} / \sigma_{CB} = (1.060, 0.866, 0.522)$.[15] The conclusion is that within-division CB is better than league-wide CB, that is, $\sigma_{CB_W} / \sigma_{CB}$ is *smallest* (0.522), when division strength is "Most" unequal. Further, inspection across Table 5 shows this same pattern, for all chosen $\sigma_{CB} = \bar{\sigma}_{CB}$ and for both 2 and 3 divisions.

To generate the data set we use to estimate (9), $\sigma_{OU} / \sigma_{CB}$, G_1 / G, $\sigma_{CB_W} / \sigma_{CB}$, $\bar{N}$, and $\bar{D}$ were derived using the following variations. We use 5 different choices of $\bar{\sigma}_{CB}$ and 6 different $(\bar{N}, \bar{D}) = (10, 2), (12, 2), (20, 2), (12, 3), (15, 3), (20, 4)$. Each

Table 4. Schedule Balance Specifications.

	N	K_1	K_2	G	K_1/K_2	G_1	G_1 / G	N	K_1	K_2	G	K_1/K_2	G_1	G_1 / G
Schedules				2 Divisions							3 Divisions			
Balanced	10	18	18	162	1.00	72	0.44	15	12	12	168	1.00	48	0.29
	10	20	16	160	1.25	80	0.50	15	16	10	164	1.60	64	0.39
	10	24	14	166	1.71	96	0.58	15	20	8	160	2.50	80	0.50
	10	26	12	164	2.17	104	0.63	15	26	6	164	4.33	104	0.63
	10	28	10	162	2.80	112	0.69	15	30	4	160	7.50	120	0.75
Most unbalanced	10	30	8	160	3.75	120	0.75	15	36	2	164	18.00	144	0.88
Balanced	12	15	15	165	1.00	75	0.45	12	15	15	165	1.00	45	0.27
	12	18	12	162	1.50	90	0.56	12	19	13	161	1.46	57	0.35
	12	20	10	160	2.00	100	0.63	12	24	11	160	2.18	72	0.45
	12	23	8	163	2.88	115	0.71	12	28	10	164	2.80	84	0.51
	12	25	6	161	4.17	125	0.78	12	32	8	160	4.00	96	0.60
Most unbalanced	12	28	4	164	7.00	140	0.85	12	36	7	164	5.14	108	0.66
Balanced	20	9	9	171	1.00	81	0.47				4 Divisions			
	20	10	8	170	1.25	90	0.53	20	9	9	171	1.00	36	0.21
	20	12	6	168	2.00	108	0.64	20	12	8	168	1.50	48	0.29
	20	14	4	166	3.50	126	0.76	20	16	7	169	2.29	64	0.38
	20	15	3	165	5.00	135	0.82	20	18	6	162	3.00	72	0.44
Most unbalanced	20	16	2	164	8.00	144	0.88	20	26	4	164	6.50	104	0.63
								20	30	3	165	10.00	120	0.73

Notes: N = number of teams, K_1 = games against each division team, K_2 = games against each non-division team, G = total number of games per team, G_1 = total number of games against division opponents.

Table 5. Variation in Division Strengths, 2 and 3 Divisions.

Division Strength	Division Strength Assignments (Final Standings of Teams); Average Rank	
	2 Divisions $N_i = 5$	3 Divisions $N_i = 5$
Equal; $D_1 \cong D_2 \cong D_3$:		
D_1	$(S_1, S_4, S_5, S_8, S_9)$; 5.4	$(S_1, S_6, S_7, S_{12}, S_{13})$; 7.8
D_2	$(S_2, S_3, S_6, S_7, S_{10})$; 5.6	$(S_2, S_5, S_8, S_{11}, S_{14})$; 8.2
D_3	—	$(S_3, S_4, S_9, S_{10}, S_{15})$; 8.2
Unequal; $D_3 > D_2 > D_1$:		
D_1	$(S_1, S_2, S_3, S_6, S_7)$; 3.8	$(S_1, S_2, S_7, S_8, S_9)$; 5.4
D_2	$(S_4, S_5, S_8, S_9, S_{10})$; 7.2	$(S_3, S_4, S_{10}, S_{11}, S_{12})$; 8.0
D_3	—	$(S_5, S_6, S_{13}, S_{14}, S_{15})$; 10.6
Most; $D_3 > D_2 > D_1$:		
D_1	$(S_1, S_2, S_3, S_4, S_5)$; 3.0	$(S_1, S_2, S_3, S_4, S_5)$; 3.0
D_2	$(S_6, S_7, S_8, S_9, S_{10})$; 8.0	$(S_6, S_7, S_8, S_9, S_{10})$; 8.0
D_3	—	$(S_{11}, S_{12}, S_{13}, S_{14}, S_{15})$; 15.6

Table 6. Division Strengths, 2 and 3 Divisions.

		Division Strength					
		Equal		Unequal		Most	
Divisions $N_i = 5$	$\bar{\sigma}_{CB}$	σ_{CB_W}	$\sigma_{CB_W} / \sigma_{CB}$	σ_{CB_W}	$\sigma_{CB_W} / \sigma_{CB}$	σ_{CB_W}	$\sigma_{CB_W} / \sigma_{CB}$
2	0.050	0.071	1.060	0.058	0.866	0.035	0.522
	0.100	0.142	1.060	0.117	0.873	0.073	0.545
	0.150	0.214	1.059	0.180	0.891	0.116	0.574
	0.200	0.287	1.059	0.250	0.923	0.168	0.620
	0.250	0.364	1.058	0.328	0.953	0.242	0.703
3	0.050	0.074	1.088	0.064	0.941	0.024	0.353
	0.100	0.148	1.080	0.130	0.949	0.051	0.372
	0.150	0.222	1.078	0.199	0.966	0.082	0.398
	0.200	0.298	1.080	0.272	0.986	0.123	0.446
	0.250	0.379	1.077	0.348	0.989	0.190	0.540

Notes: $\bar{\sigma}_{CB}$ is chosen as described in the text, p. 10. σ_{CB_W} = within-division CB_W. N_i = teams in each division. Calculated from BCSD($\hat{\sigma}_{CB}$) in (4) and BCSD($\hat{\sigma}_{CB\,W}$) in (6).

$(\bar{N}, \bar{D})$ is considered in 6 different SI cases and 3 different DS cases. Thus, the data set contains $5 \times 6 \times 6 \times 3 = 540$ observations.

Marginal Effects of Changes in the Structure of Play

Using the data set from the numerical analysis, it is possible to estimate the marginal effects of SI, DS, N, and D on OU/CB[16]. The regression specification of the general intuitive theoretical relationship in (9), using the data from the last section, is:

$$\begin{aligned}\left(\frac{\sigma_{\mathrm{OU}}}{\sigma_{\mathrm{CB}}}\right)_i &= \beta_0 + \beta_2\left(\frac{G_1}{G}\right)_i + \beta_3 d_i\left(\frac{G_1}{G}\right)_i + \beta_1\left(\frac{\sigma_{\mathrm{CB_W}}}{\sigma_{\mathrm{CB}}}\right)_i \\ &\quad + \beta_4\left(\frac{\sigma_{\mathrm{CB_W}}}{\sigma_{\mathrm{CB}}}\right)_i\left(\frac{G_1}{G}\right)_i + \beta_5 \ln N_i + \beta_5 \ln \mathrm{D}_i + \varepsilon_i\end{aligned} \tag{13}$$

for the $i = 1, \ldots 540$ observations generated in the numerical analysis of the last section. The dependent variable $\sigma_{\mathrm{OU}} / \sigma_{\mathrm{CB}}$ is OU/CB. G_1 / G is the measure of SI. G_1 / G increases as schedules become more and more unbalanced (the number of games against each division team increases). $\sigma_{\mathrm{CB_W}} / \sigma_{\mathrm{CB}}$ is relative DS, decreasing as division strength becomes more and more unequal. The dummy variable d_i separates the two situations for $\sigma_{\mathrm{CB_W}} / \sigma_{\mathrm{CB}}$; $d_i = 1$ if $(\sigma_{\mathrm{CB_W}} / \sigma_{\mathrm{CB}}) < 1$ and $d_i = 0$ if $(\sigma_{\mathrm{CB_W}} / \sigma_{\mathrm{CB}}) \geq 1$. We also interact $\sigma_{\mathrm{CB_W}} / \sigma_{\mathrm{CB}}$ and G_1 / G to take account of possible interdependence. The number of teams and divisions, N and D, respectively, are entered in log form.[17]

Given the specification in (13), the marginal effect of $(\sigma_{\mathrm{CB_W}} / \sigma_{\mathrm{CB}})$ on $(\sigma_{\mathrm{OU}} / \sigma_{\mathrm{CB}})$, is $\partial(\sigma_{\mathrm{OU}} / \sigma_{\mathrm{CB}}) / \partial(\sigma_{\mathrm{CB_W}} / \sigma_{\mathrm{CB}}) = \beta_1 + \beta_4(G_1 / G)$. With $(\sigma_{\mathrm{CB_W}} / \sigma_{\mathrm{CB}}) < 1$ for $d_i = 1$, the marginal effect of G_1 / G on $\sigma_{\mathrm{OU}} / \sigma_{\mathrm{CB}}$ is $(\partial(\sigma_{\mathrm{OU}} / \sigma_{\mathrm{CB}}) / \partial(G_1 / G)) = \beta_2 + \beta_3 + \beta_4(\sigma_{\mathrm{CB_W}} / \sigma_{\mathrm{CB}})_i$. On the other hand, with $(\sigma_{\mathrm{CB_W}} / \sigma_{\mathrm{CB}}) \geq 1$ for $d_i = 0$, the marginal effect of (G_1 / G) on $(\sigma_{\mathrm{OU}} / \sigma_{\mathrm{CB}})$ is $(\partial(\sigma_{\mathrm{OU}} / \sigma_{\mathrm{CB}}) / \partial(G_1 / G)) = \beta_2 + \beta_4(\sigma_{\mathrm{CB_W}} / \sigma_{\mathrm{CB}})_i$.

Estimation results for (13) are in Table 7. Model 1 excludes the number of teams and the number of divisions. Model 2 adds the number of teams and the number of

Table 7. OLS Estimation Results, OU/CB, Expression (13).

Variables	Model 1	Model 2	Model 3
Constant	1.315* (115.37)	1.345* (82.03)	1.340* (99.58)
$\frac{\sigma_{\mathrm{CB_W}}}{\sigma_{\mathrm{CB}}}$	−0.309* (−22.32)	−0.314* (−22.73)	−0.314* (−22.78)
$\frac{G_1}{G}$	−1.147* (−50.76)	−1.164* (−49.54)	−1.166* (−50.56)
$d_i\left(\frac{G_1}{G}\right)$	0.038* (7.53)	0.042* (7.57)	0.043* (8.19)
$\left(\frac{\sigma_{\mathrm{CB_W}}}{\sigma_{\mathrm{CB}}}\right)\left(\frac{G_1}{G}\right)$	1.138* (44.46)	1.145* (44.74)	1.146* (45.02)
$\ln(N)$		−0.002 (−0.47)	
$\ln(D)$		−0.015* (−2.69)	−0.016* (−3.41)
NOBS	630		
R^2	0.9315	0.9330	0.9330
Adjusted R^2	0.9310	0.9322	0.9323

Notes: t-Values in parentheses.
*Statistically significant, 95 percent level of confidence.

divisions. Statistical significance and model fit statistics are of dubious value since the data were all created with in-depth analysis and researcher-imposed relationships. Interestingly, both N and D are negatively signed, but only the number of divisions is statistically significant. The rest of the discussion in this paper is based on Model 3 that drops N with nearly no impact on coefficient estimates and a bit of an increase in adjusted R^2.

The estimated behavior of the marginal effects is portrayed in Figure 1. Since the marginal effect of G_1 / G depends on $\sigma_{CB_W} / \sigma_{CB}$, and vice versa, representative values were chosen for the presentation of marginal effects. In the top panel, when $d_i = 0$, division strength approximately is equal. When $d_i = 1$, division strengths are unequal. The top panel of Figure 1 shows that marginal effect of G_1 / G is negative in the range of $d_i = 1$ $((\sigma_{CB_W} / \sigma_{CB}) < 1)$ and positive in the range of $d_i = 0$ $((\sigma_{CB_W} / \sigma_{CB}) \geq 1)$. In the bottom panel, as schedule imbalance increases toward more games with division opponents, the marginal effect of division strength increases.

The top panel of Figure 2 shows the relationship between the fitted value of OU/CB, denoted, $(\sigma_{OU} / \sigma_{CB})^*$, using the estimated coefficients from (13), chosen $\sigma_{CB_W} / \sigma_{CB}$ representing cases where $d_i = 0$ and $d_i = 1$, and a range of schedule imbalance. As a reminder, OU/CB increases as $(\sigma_{OU} / \sigma_{CB})^*$ decreases. The marginal effect of schedule imbalance is the slope of the relationship, that varies with $\sigma_{CB_W} / \sigma_{CB}$. For equal division strength, $d_i = 0$, the top panel shows increased schedule imbalance leads to negligible decrease in OU/CB (increase in $(\sigma_{OU} / \sigma_{CB})^*$). When division strengths are unequal, $d_i = 1$, increased schedule imbalance increases OU/CB (decrease in $(\sigma_{OU} / \sigma_{CB})^*$). The more unequal is the division strength, for example, moving from $\sigma_{CB_W} / \sigma_{CB} = 0.75$ to $\sigma_{CB_W} / \sigma_{CB} = 0.45$, increased schedule imbalance increases OU/CB at a greater rate.

In the bottom panel of Figure 2, when schedules are pretty much balanced ($G_1 / G = 0.30$), there is negligible increase in OU/CB (decreases in $(\sigma_{OU} / \sigma_{CB})^*$) when division strength moves toward more and more inequality, that is, as $\sigma_{CB_W} / \sigma_{CB}$ decreases along the x-axis. As schedules become more imbalanced (e.g., G_1 / G increases from 0.55 to 0.75), the rate of increase in OU/CB becomes more and more pronounced.

Values and Unbiased Estimators: OU/CB for MLB

In this section, we perform a veracity check of the results in Table 7 and Figures 1 and 2 on actual MLB episode data. First, we generate fitted OU/CB, denoted $(\sigma_{OU} / \sigma_{CB})^*_{MLB}$, using the estimated coefficients from (13) for actual MLB G_1 / G, $\sigma_{CB_W} / \sigma_{CB}$, N, and D. Then we generate OU/CB obtained from unbiased estimators applied to the same actual MLB episode data, denoted $(\sigma_{OU} / \sigma_{CB})_{UE}$ (the suffix for "unbiased estimators"). The veracity check is to compare $(\sigma_{OU} / \sigma_{CB})^*_{MLB}$ to $(\sigma_{OU} / \sigma_{CB})_{UE}$. The $(\sigma_{OU} / \sigma_{CB})^*_{MLB}$ and $(\sigma_{OU} / \sigma_{CB})_{UE}$ values for the comparison were obtained as follows.

First, for fitted results from Table 7, Model 3, that is, $(\sigma_{OU} / \sigma_{CB})^*_{MLB}$. These values are derived from the actual MLB data in Tables 2 and 3, denoted $(G_1 / G)_{MLB}$ and

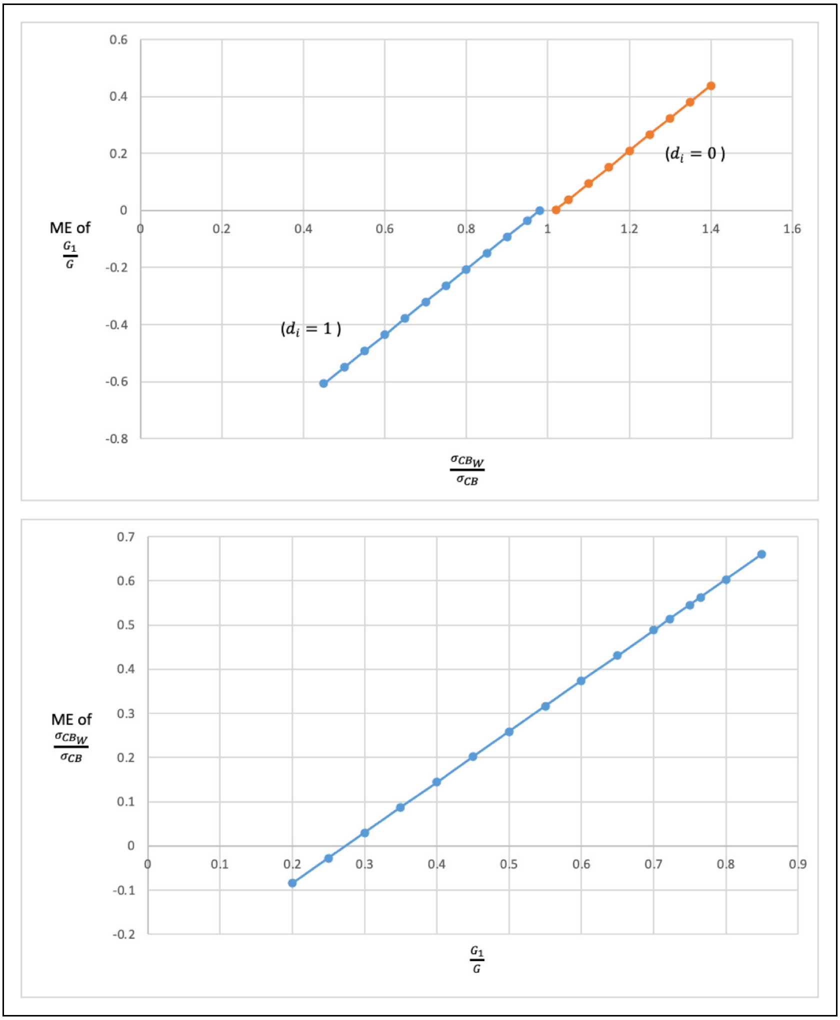

Figure 1. Marginal effects of schedule imbalance and division strength, 2 divisions. *Notes:* Calculated for the range of independent variables from estimated coefficients in Table 7, Model 3, and the definitions of marginal effects in the text.

$(\sigma_{CB_W} / \sigma_{CB})_{MLB}$, respectively. The actual N and D values are also from the actual MLB episodes.

The derivation of $(G_1 / G)_{MLB}$ is shown for an illustrative example, the AL with 3 divisions. Table 3 shows the AL, 1984–1993, with $N = 14$, $D = 2$, with $K_1 = 13$, so $(G_1 / G)_{MLB} = 0.481$. Referring to Table 6 gives the label "0.481 (Balanced – Unbalanced)" entered in Table 8.

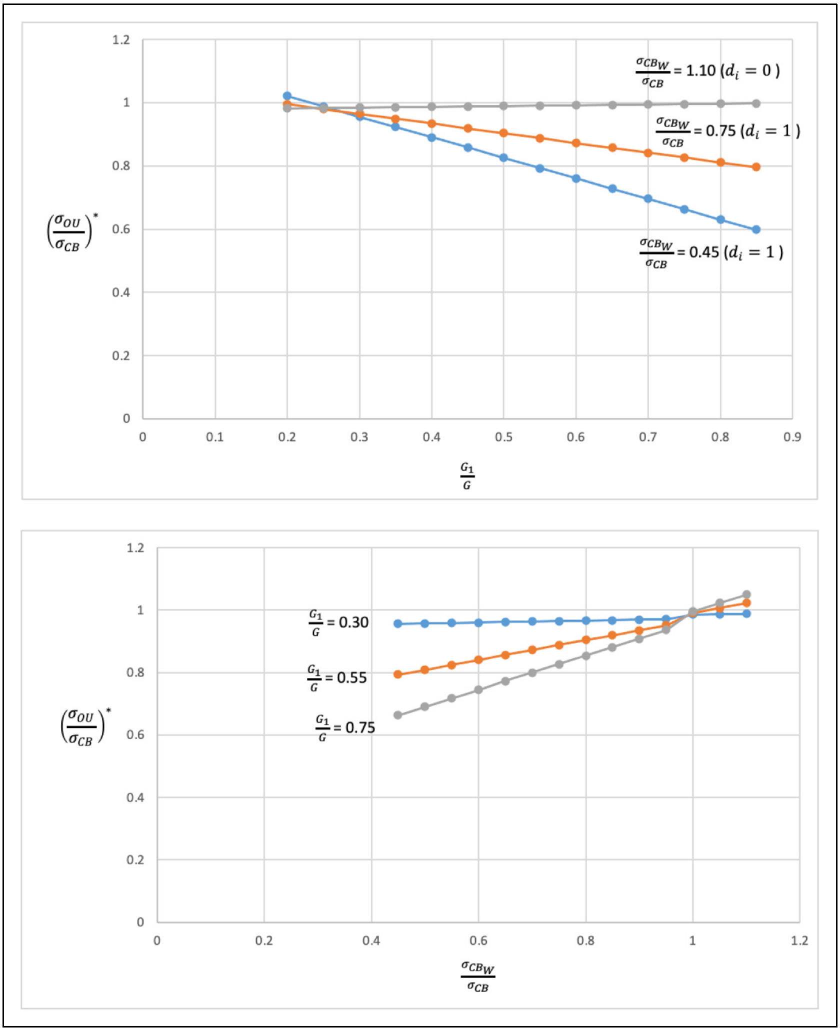

Figure 2. Schedule imbalance and division strength impacts on OU/CB, 3 divisions. *Note:* See Figure 1.

For some episodes, K_1 changes within the period and we turn to weighted averages to calculate $(G_1 / G)_{\text{MLB}}$. We continue with the illustrative example of the AL with 3 divisions. Table 3 shows the following: for 1994–1996, $K_1 = 13$; for 1997–2000, $K_1 = 12 \sim 13$; for 2001–2003, $K_1 = 18 \sim 19$. So, for 1994–1996, $(G_1 / G)_{1994-1996} = 0.321$. Using the midpoints for K_1, in the other two periods, $(G_1 / G)_{1997-2000} = 0.361$ and $(G_1 / G)_{2001-2003} = 0.500$. The weighted average for

Table 8. $(G_1 / G)_{MLB}$ and $(\sigma_{CB_W} / \sigma_{CB})_{MLB}$, MLB Episodes.

MLB Episode	Schedule Imbalance	Division Strength
	$(G_1 / G)_{MLB}$	$(\sigma_{CB_W} / \sigma_{CB})_{MLB}$
2 Divisions		
AL		
1959–1968	—	—
1969–1978	0.556 (Unbalanced)	0.764 (Unequal-Most)
NL		
1959–1968	—	—
1969–1978	0.556 (Unbalanced)	0.946 (Equal)
3 Divisions		
AL		
1984–1993	0.481 (Balanced – Unbalanced)	0.850 (Unequal)
1994–2003	0.391 (Balanced – Unbalanced)	0.769 (Unequal-Most)
NL		
1984–1993	0.548 (Unbalanced)	1.013 (Equal)
1994–2003	0.395 (Balanced – Unbalanced)	0.624 (Most)

Note: Calculated as described in the text.

the entire period, 1994–2003, is $(G_1 / G)_{MLB} = 0.3 \times 0.321 + 0.4 \times 0.361 + 0.3 \times 0.500 = 0.391$. Again, referring to Table 6 gives the "0.391 (Balanced – Unbalanced)" entry in Table 8. The entries for $(G_1 / G)_{MLB}$ for the NL were derived in the same way.

For $(\sigma_{CB_W} / \sigma_{CB})_{MLB}$ in the AL and NL, we use the unbiased plug-in estimators for σ_{CB} and σ_{CB_W} in (4) and (6), respectively. These unbiased estimates were calculated using MLB data for the appropriate episode. For example, also in Table 8, and continuing the AL example for 3 divisions, over the period 1984–1993, $(\sigma_{CB_W} / \sigma_{CB})_{MLB} = 0.850$. From Table 6, this is "Unequal" division strength and, hence, the "0.850 (Unequal)" label in Table 8. The rest of the entries in Table 8 were obtained in the same way for the specific episodes.

The estimated coefficients in Table 7 and the results for $(G_1 / G)_{MLB}$ and $(\sigma_{CB_W} / \sigma_{CB})_{MLB}$ in Table 8 allow the calculation of the fitted model OU/CB, $(\sigma_{OU} / \sigma_{CB})^*_{MLB}$. Generically, for $d_i = 0$, $(\widehat{\sigma_{OU}} / \sigma_{CB})_{MLB} = 1.340 - 1.166(G_1 / G)_{MLB} - 0.314 (\sigma_{CB_W} / \sigma_{CB})_{MLB} + 1.146(G_1 / G)_{MLB}(\sigma_{CB_W} / \sigma_{CB})_{MLB} - 0.016 \ln(D)$. If $d_i = 0$, add another $0.043(G_1 / G)_{MLB}$. Again, for our AL example with 3 divisions, for 1984–1993, $(\sigma_{OU} / \sigma_{CB})^*_{MLB} = 0.996$ in Table 9. The rest of the fitted results in Table 9 were obtained in the same fashion.

For $(\sigma_{OU} / \sigma_{CB})_{UE}$, we used the unbiased plug-in estimators BCSD$(\hat{\sigma}_{CB})$ and BCSD$(\hat{\sigma}_{OU})$ in (4) and (5), respectively.[18] Figure 3 charts the unbiased estimators BCSD$(\hat{\sigma}_{CB})$ and BCSD$(\hat{\sigma}_{OU})$ for the AL and NL since the appearance of the former, 1901–2019. It is easy to see that CB and OU are the same thing prior to 1969, with balanced schedules in both leagues. Their unbiased estimates overlap each other lock step to 1968 and then deviate from each other from 1969 onward.

Table 9. Veracity Comparison.

	Fitted Model $(\sigma_{OU} / \sigma_{CB})^*_{MLB}$		Unbiased Estimators $(\sigma_{OU} / \sigma_{CB})_{UE}$	
	AL	NL	AL	NL
2 Divisions				
1959–1968	1.000	1.000	1.000	0.999
1969–1978	0.957	1.016	0.963	1.024
Difference	−0.043	0.016	−0.037	0.025
% Difference	−4.3%	1.6%	−3.7%	2.5%
3 Divisions				
1984–1993	0.996	1.014	1.005	1.011
1994–2003	0.996	0.975	0.975	0.984
Difference	−0.000*	−0.039	−0.030	−0.027
% Difference	−0.0%*	−3.8%	−3.0%	−2.7%

Notes: $(\sigma_{OU} / \sigma_{CB})^*_{MLB}$ from results in Tables 7 and 8. $(\sigma_{OU} / \sigma_{CB})_{UE}$ Calculated using plug-in estimators in (4) and (5).
*Negative beyond the depicted three decimal places.

Calculated results for $(\sigma_{OU} / \sigma_{CB})_{UE}$ over MLB episodes are in Table 9. For our continuing example of the AL in 1984–1993 with 3 divisions, $(\sigma_{OU} / \sigma_{CB})_{UE} = 1.005$ in Table 9. The rest of the $(\sigma_{OU} / \sigma_{CB})_{UE}$ results in Table 9 were calculated in the same way.

Table 9, then, contains the elements for our proposed veracity check. For the move to 2 divisions starting in 1969, in the AL, Table 9 shows a 4.3 percent increase in OU/CB, (a decrease in $(\sigma_{OU} / \sigma_{CB})^*_{MLB}$) for the fitted model and a 3.7 percent increase in OU/CB (a decrease in $(\sigma_{OU} / \sigma_{CB})_{UE}$) for the unbiased estimates.[19] In the NL, it's a 1.6 percent *decrease* by the fitted model and a 2.5 percent *decrease* in OU/CB by the unbiased estimators. The fitted model results and unbiased estimators are in qualitative agreement each time and predict both levels and changes in OU/CB that are quite close in the magnitude.

For the move to 3 divisions starting in 1994, in the AL, Table 9 shows a very slight increase in OU/CB (a very slight decrease in $(\sigma_{OU} / \sigma_{CB})^*_{MLB}$) for the fitted model and a 3.0 percent increase in OU/CB (a decrease in $(\sigma_{OU} / \sigma_{CB})_{UE}$) for the unbiased estimates. In the NL, it's a 3.8 percent increase by the fitted model and a 2.7 percent increase in OU/CB by the unbiased estimators. As with the 2-division case, the fitted model results and unbiased estimators are in qualitative agreement each time, predicting changes in OU/CB that are quite close to each other.

The largest absolute difference between the fitted model results and unbiased estimators is for the AL, 1994–2003, but $|(\sigma_{OU} / \sigma_{CB})_{UE}| > |(\sigma_{OU} / \sigma_{CB})^*_{MLB}|$ by only 3.8 percentage points. The directions of the changes are the same in all cases. There is no detectable pattern in the differences in Table 9, by either the unbiased estimators or the fitted model. The fitted model and unbiased estimators appear to be highly congruent.

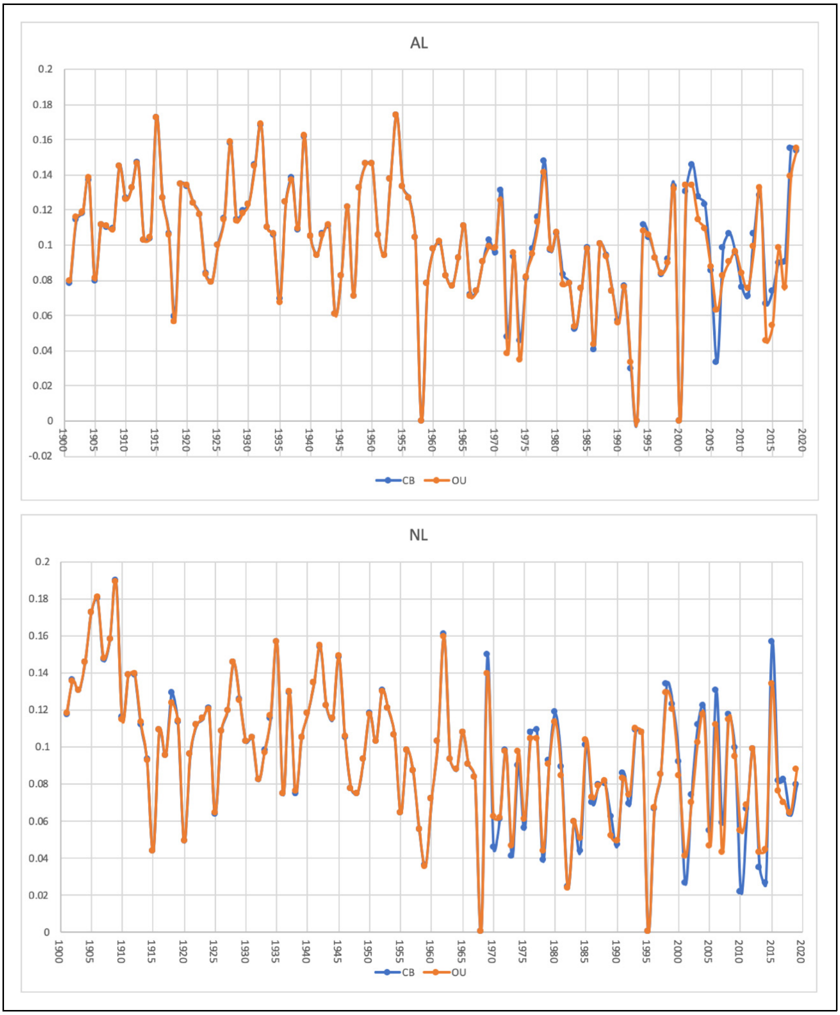

Figure 3. Unbiased estimators of CB and OU, AL and NL, 1901–2019. *Note:* Calculated using BCSD($\hat{\sigma}_{CB}$) in (4) and BCSD($\hat{\sigma}_{OU}$) in (5).

We offer two final observations. First, the changes in the structure of league play summarized in Table 9 appear economically significant. Revenues in MLB for 2019 (the last stable year before Covid) were about $11 billion. So, a 1 percent change is about $110 million. This is close to the $120 million MLB put together to launch MLBAM over 4 years starting in 2000, and more than the $77 million it ended up taking to get MLBAM off the ground (Popper, 2015). Or one might compare to the $150 million maximum it recently committed to increase black representation in baseball (Brown, 2021).

Suppose unit elasticity of MLB revenue with respect to OU/CB.[20] Then a 1 percent increase in OU/CB would increase MLB revenues by the above \$110 million. From Table 9, either league has enjoyed up to a 4.3 percent increase in OU/CB. Unitary elastic revenues in MLB would generate $4.3 \times 110 = \$473$ million, easily important compared to its MLBAM investment. On the other hand, from Table 9, the NL might have lost $2.5 \times 110 = \$275$ million in the move to 2 divisions. Of course, elastic revenue response to OU/CB would make these numbers larger, and inelastic revenue response would make them lower.

Our second observation is that while we know of no tests of statistical significance there are plausible speculations on the differences between unbiased estimators and fitted model results in Table 9. First, the specification in the numerical analysis may have missed the mark for the relative degree of within-division strength imbalance. While the numerical analysis is designed with variation in between-division strengths, it is not designed with within-division unequal strength for all divisions. Second, it could be that the regression in (13) suffers specification errors. A third is that the estimated regression results from (13) used the true values of the data while the unbiased estimators of OU and CB in MLB are random variables.

Finally, in actual MLB data, CB may change due to many other factors than division strengths or schedule. And since OU is *for a given CB* is the analysis here, it may be possible that other factors that impact CB are at work. For example, there could simply have been a great difference in the team characteristics for the NL expansion in 1993. The impact of team location (via team moves and expansions) found in other work on baseball may well be in operation here as well (Lee & Fort, 2005).

This also helps explain the differences in Table 9, for MLB, compared to the differences found in Fort and Lee (2020) for the Scottish Professional Football league move to unbalanced schedules. There simply was no expansion or team movement to contaminate the numerical analysis of the Scottish league. We expect that the results here for MLB will be distinguished from any other league that does not have team movement and league expansion.

Summary and Conclusions

Theoretical intuition suggests that OU depends on the structure of play—schedule imbalance, division strength, number of teams, and number of divisions. Lacking formal sports league theory as a guide, and with actual league examples few and far between, numerical analysis is used to generate data on these elements of the structure of play. One of the limitations of the numerical analysis is to model OU for a given CB, or OU/CB. Regression analysis using these data produces the marginal effects of schedule imbalance, division strength, and the number of divisions on OU/CB.

As a veracity check, fitted values of OU/CB from the regression estimates and actual MLB episodes were compared to unbiased estimator results for OU/CB for the same MLB episodes. The predicted *directions* of OU/CB change are identical for the fitted model results and the unbiased estimator results. The *magnitude* of those changes is also quite close for both the unbiased estimator and fitted model results. This is encouraging for our numerical analysis and regression approach.

In the move to 2 divisions, both approaches show OU/CB increased in the AL, but the opposite is true in the NL. In the move to 3 divisions, OU/CB increased in both leagues. The absolute value of the changes predicted by the two approaches differ by no more than 3.8 percentage points. The closeness of the comparison suggests the model provides important insights for league policy makers and outside analysts regarding both the neglected issue of league policy impacts on OU and the sources of those impacts.

The results are important for both league designers and outside observers of league choices concerning OU, including sports economists. For league designers bent on altering CB, the results in this paper show that policy alterations can affect both OU and CB. And if fans care about OU, then designers would do well to include these impacts in their considerations. Some designs intended to influence CB can end up influencing OU, and vice versa.

The policy value of our analysis is reinforced by examples. First, unlike the soccer case in Fort and Lee (2020), that considered unbalanced schedules intentionally designed by the league to increase OU/CB, the within-division CB for MLB may be worse than the overall league-wide CB. In such a situation, schedule imbalance with more division games and fewer non-division games may decrease OU/CB.

Second, suppose MLB owners are considering a greater degree of schedule imbalance for the sake of increased "balance". Our results suggest that in its current 3-divsion format, a policy that increases schedule imbalance will only increase OU/CB unequal division strength, and more so the more unequal is division strength. OU relative to CB *will not increase* much if divisions strengths are approximately equal with 3 divisions. Presumably, if fans care about OU as well as CB, such an offset should count against any increase in CB that might occur. League designers must pay attention to both schedule strength and the degree of schedule imbalance.

Third, consider the current MLB competitive balance tax, referred to as the "luxury" tax. Suppose the idea arises among league members to decrease the threshold and increase the tax rate, that is, to drive more larger-revenue owners away from talent, reducing talent price to smaller-revenue owners. Unless all smaller-revenue teams are in the same division, this should lead to more equal division strength, across all divisions in the league. With 3 divisions as MLB is currently comprised, for any degree of schedule imbalance, moving from unequal division strength toward more equal division strength, will always *decrease* OU/CB.

For outside analysts, consider the further observation regarding the competitive balance tax. The move to 3 divisions occurred for the 1994 season, nearly adjacent to the first version of the tax. Our results show that OU/CB increased for that episode. But was this due to the tax, or the corresponding increase in the inequality of division strength after moving to 3 divisions? Since that first version of the tax was only paid by the Yankees, it does not necessarily follow that the tax design "worked" to increase "balance". Instead, our results suggest that the increase could have occurred because both OU and CB changed due to the concurrent change in number of divisions.

Finally, by way of example, the results suggest that league policy makers proceed with caution with expansion and alterations in the number of divisions. Typically, OU/CB increases, but not universally. The move to 2 divisions decreased OU/CB in the NL, quite probably because that league ended up with nearly equal division strength, but also possibly because of individual characteristics of the expansion teams. In any event, the underlying characteristics of any form of chosen division play can run the risk of decreasing OU/CB.

As for suggestions for future work, a natural next step is to further extend the model to an assessment of CB, rather than analyzing OU/CB. A more complete numerical analysis would add the determinants of CB, rather than determining relative win probabilities for a given level of CB. Another added dimension would be the introduction of interleague play (in MLB, the first season of interleague play was 1997). These added dimensions are beyond the aims in this paper, but given our results, are suggested as a move toward a more general understanding of the actual determinants of OU and CB.

We close by noting that the previous focus on CB, to the near exclusion of OU, ignores at least three important sports league issues. First, OU and CB are not the same thing unless *teams in a league play a balanced schedule*. OU increases as the probability that one team beats another move toward 0.5. In general, CB increases with decreases in the variation in playing talent across the teams in a league. A league with higher CB would be expected to have games with high OU as well. However, some of our findings reveal that OU and CB can move in opposite directions, creating a tradeoff between the two for league policy makers and analysts of league outcomes.

Second, Rottenberg (1956) originally based what is now called the "Outcome Uncertainty Hypothesis" on OU, not CB (Fort, 2005). He did go on to suggest implications of fan preferences for OU on the management of CB from the perspective of league decision makers. But the foundation was OU, not CB. Analyzing the impacts of league choices on OU, directly, puts analysis in accord with the OU foundations of this fundamental sports economics hypothesis.

Third, it is well-established that CB depends on the variation in marginal revenue across teams in a league. Thus, CB could change due to exogenous changes in the distribution of income or population across cities, expansion in the number of teams, relocation of existing teams, or player market regulations. But little is even known about how these changes impact OU/CB analyzed in this paper.

In any event, the results in this paper suggest that future theoretical and empirical work on sports league outcomes would do well go pay attention to OU as well as CB.

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Notes

1. CB also has been used to describe playoff access, playoff success, and ultimately dynasties across seasons.
2. The invariance principle holds that the distribution of playing talent in a league is invariant with respect to rules governing the rights to values created by that talent. For the literature on the invariance principle, see the recent review by Fort et al. (2016), and the update in Fort (2022).
3. The outcome uncertainty hypothesis is that, while fans prefer their home team win, they also prefer both that their team wins but also tightly contested games. On the outcome uncertainty hypothesis literature, and recent reference-dependence alternatives, see the review in Johnson and Fort (2022).
4. Lenten (2015) analyzed the impact of unbalanced schedules on CB. Lenten (2008) added the impact of division play on CB. Lee et al. (2019a) showed that previous measures of CB were biased, designed an unbiased estimator of CB with balanced schedules, and compared their measure to previous measures. Lee et al. (2019b) designed an unbiased estimator of CB with unbalanced schedules and repeated the comparison with past measures. But they do not examine the impact of unbalanced schedules on OU. Fort and Lee (2020) is the only paper on OU. They analyze the behavior of OU with schedule imbalance but without division play.
5. The general case of time-varying playing talent during a season due to roster changes (contract sales, contract trades, and injuries) is given in Fort and Lee (2020). They also provide the mathematical details of the derivation of (1) and (2) used in this paper.
6. The "between-division" standard deviation can also be derived. However, a higher CB_W implies a lower between-division CB with a fixed league-wide CB. So, for our analysis, between-division standard deviation is redundant.
7. The "plug-in" estimator is obtained by plugging the observed values of N, K, and K_{ij} and the observed winning percentage of team i against team j for p_{ij}, into (1) and (2).
8. We ignore strike interruptions of 1981 and 1994–1995 since games appeared to follow planned schedule imbalance even though full seasons of games were not played.
9. This is just descriptive of numerical analysis approach. For the full method up to unbalanced schedules with no divisions, see Lee et al. (2019a, 2019b) and Fort and Lee (2020).
10. Analyzing OU/CB also is in keeping with Rottenberg's (1956) observation that one would expect OU to be higher with higher CB. See Fort and Lee (2020) for a more extensive discussion.

11. This is the case analyzed in Fort and Lee (2020).
12. To see the bounds, $\frac{G_1}{G} = \frac{K_1[(N/D)-1]}{K_1[(N/D)-1]+(D-1)K_2(N/D)}$ which is a minimum of $\frac{1}{D} - \frac{1}{N}$ when $K_1 = K_2$ and a maximum of 1 when $K_2 = 0$.
13. Technically, the MLB season length was 154 games until the move to 162 games in 1961 for the AL and 1962 for the NL.
14. The AL and NL each had 10 teams prior to 2 divisions. Then, there were 6 teams in each NL division. There were 6 teams in each AL division until 1977 when 2 teams were added for 7 teams in each division. The NL continued with 6 teams per division until 1993 when two teams were added, and each division then had 7 teams. Moving to 3 divisions, the AL had 2 divisions of 5 teams and one with 4 teams for the entire period. The NL started with the same as the AL, but then went to 2 divisions of 5 teams and one of 6 teams from 1998 to the end of the period.
15. $\frac{\sigma_{CB_W}}{\sigma_{CB}} = 1.060$ is an example of approximately equal schedule strength. Thus, "equal" represents that CB_W is worse than or the same as league-wide CB.
16. We could attempt to gain insight into the qualitative impacts just by inspecting a cross-tab of OU/CB and the independent variables in (13). However, the impact of *SI* or *DS* cannot be compared across the 2-division and 3-division cases because the number of teams also changes, and we cannot set the same degree of SI or DS for both 2-division and 3-divisions. So, we allow both *N* and *D* to vary in the regression specification in (13).
17. We also estimated (13) with *N* and *D* in a linear form with similar results.
18. While not the point of the work in this paper, we did examine Noll-Scully "RSD" values for the two MLB Episodes. Since RSD is a standard deviation measure like those used here, higher RSD values go with lower CB. RSD predicts the same direction change as $BCSD(\hat{\sigma}_{CB})$ except in one case, the AL move to 2 divisions. RSD also predicted larger changes than did $BCSD(\hat{\sigma}_{CB})$ for the NL, but not the AL. It appears that the bias in RSD is in magnitude, not direction, relative to $BCSD(\hat{\sigma}_{CB})$.
19. Since we are comparing variances, a natural to test the differences for the unbiased estimates in Table 9 would be an *F*-test. However, without explicit knowledge of the distribution of the variance, that would be a bit of a shot in the dark. A discussion of economic significance appears later.
20. We know of no revenue elasticity estimates in the literature useful at the game level. All the works we could find estimating revenues were at the team level, or the league-level aggregate. The literature testing the outcome uncertainty hypothesis at the game level appears to all be demand estimation, not revenue estimation (Johnson & Fort, 2022).

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