Chinese Logic: An Introduction

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Introduction

As late as 1898, logic was seen by the Chinese as "an entirely alien area of intellectual inquiry": the sole Chinese-language textbook on logic was labeled by Liang Qichao (梁启超)—at that time a foremost authority on Western knowledge—as "impossible to classify" (无可归类), alongside museum guides and cookbooks (Kurtz, 2011: 4-5). This same textbook had previously been categorized by Huang Qingcheng (黄庆澄) as a book on 'dialects' (方言). The Chinese word for logic (*luójí* 逻辑) itself is merely a transliteration from the English—the entire Chinese lexicon had no word resembling it (Lu, 2009: 98). Hence it never occurred even to specialists that this esoteric discipline might have close affinities with the roots of Chinese philosophy, from the *I Ching* (易经) to the ancient Chinese dialecticians (辩者), as well as the famous paradoxes of Buddhism.

With the advent of computers, "there is now more research effort in logics for computer science than there ever was in traditional logics" (Marek & Nerode, 1994: 281). This has led to a proliferation of logical methods, including modal logic, temporal logic, epistemic logic, and fuzzy logic. Further, such new logical systems permit multiple truth values, semantic patterns based on games, and even logical contradictions. In light of these possibilities, research in 'Chinese logic' aims to reinterpret the history of Chinese thought by means of such tools.

This essay consists of three parts: the mathematics of the *I Ching*, the debates within the School of Names, and the paradoxes of Buddhism. The first section will, through examining the binary arithmetic of the *I Ching*, provide an introduction to basic logical notation. The second section will explore Gongsun Long's famous *bái mǎ fēi mǎ* (白马非马) paradox, as well as the logical system of the Mohist school. The third section will explain the seven-valued logic of the Buddhist monk Nāgārjuna by way of paraconsistent logic.

1 The *I Ching* (易经) & Binary Arithmetic

The *I Ching* is one of the oldest books in history. Throughout the world, there is no other text quite like it. Its original function was for divination, giving advice for future actions; yet, after centuries of commentary, it has taken on a fundamental role in Chinese culture. In part, this is because its commentaries became (apocryphally) associated with Confucius, thereby establishing it as a classic.

Its survival of the 'burning of books and burying of scholars' (焚书坑儒) in 213-210BC has magnified the *I Ching*'s importance. Historically, the Zhou dynasty was marked by hundreds of years of war and dissension. Finally, Qin Shi Huang united the nation in 221BC, to become China's first emperor. According to the standard account, in order to unify thought and political opinion, Emperor Qin Shi Huang ordered that all books not about medicine, farming, or divination be burned. And so, the vast majority of ancient Chinese knowledge has been lost to history. Yet, since the *I Ching* was about divination, it avoided sharing the same fate. In a sense, then, the *I Ching* has come to represent the collective wisdom of ancient China—it embodies their entire philosophical cosmology.

Confucius's interest in the *I Ching* is well known. In verse 7.16 of the *Analects*, he says: "If some years were added to my life, I would give fifty to the study of the *Yi* [*I Ching*], and then I might come to be without great faults." Curiously, this appears at odds with the rest of his philosophy. After all, the *Analects* elsewhere says: "The subjects on which the Master did not talk, were—extraordinary things, feats of strength, disorder, and spiritual beings." (7.21). That is, Confucius had no interest in oracles. Hence we can conclude that for Confucius, the main content of the *I Ching* was not divination, but philosophy.

The core tenet of the *I Ching* is deeply metaphysical, namely: the complementarity of Yin (阴) and Yang (阳). Yin represents negativity, femininity, winter, coldness and wetness. Yang represents positivity, masculinity, dryness, and warmth. Accordingly, the *gua* (卦) or fundamental components of the *I Ching*'s hexagrams, are two lines: '⚋' for Yin, and '⚊' for Yang.

The trigrams, made up of three lines, have 8 combinations ($2^3 = 8$), and so are called the *bagua* (八卦), where *bā* (八) means 8. The *bagua* and its associated meanings are: ☰ (乾/天: the Creative/Sky), ☱ (兑/泽: the Joyous/Marsh), ☲ (离/火: the Clinging/Fire), ☳ (震/雷: the Arousing/Thunder), ☴ (巽/风: the Gentle/Wind), ☵ (坎/水: the Abysmal/Water), ☶ (艮/山: Keeping Still/Mountain), ☷ (坤/地: the Receptive/Earth). The *I Ching*'s commentaries revolve around 64 hexagrams of six lines ($2^6 = 64$ combinations). There are multiple ways of ordering the hexagrams: the most well-known is the King Wen (文王) sequence, but the most important for our purposes is the Fu Xi (伏羲) sequence.

In the 17th century, the mathematician Gottfried Wilhelm Leibniz attempted to develop a system of arithmetic using only the numbers 0 and 1, called binary arithmetic. Binary arithmetic is in base 2: its key point is that any integer can be uniquely represented as a sum of powers of two. For example, $7 = 4+2+1 = 1{\times}(2^2)+1{\times}(2^1)+1{\times}(2^0)$, and since each of the coefficients is 1, therefore the binary representation of 7 is (111). Conversely, $5 = 4+1 = 1\times(2^2)+0\times(2^1)+1\times(2^0)$, where the middle coefficient is 0, so that 5 in binary is (101). For larger numbers, we simply include larger powers of two: $2^3 = 8$, $2^4 = 16$, etc.

Leibniz corresponded with various Christian missionaries in China, and had received a poster containing the Fu Xi sequence. To his astonishment, by letting ⚋ $= 0$ and ⚊ $= 1$, the Fu Xi sequence of 64 hexagrams exactly corresponds with the binary numbers from 0 to 63! Using the trigrams as a simplified example, from top to bottom we read: ☱ $= (110) = 1\times(2^2)+1\times(2^1)+0\times(2^0) = 4+2 = 6$, ☵ $= (010) = 0\times(2^2)+1\times(2^1)+0\times(2^0) = 2$, and so on. Thus, according to the Fu Xi and binary sequence, the *bagua* are ordered as: ☷, ☶, ☵, ☴, ☳, ☲, ☱, ☰.

Further, since we can treat the trigrams as numbers, we can also perform on them arithmetic operations such as addition and multiplication. To do this involves modular arithmetic, which for pedagogical purposes is occasionally called ‘clock arithmetic’. Its main feature is that it is cyclical: after arriving at the base number (‘mod n’, in our case: mod 2), we start up once again at zero. So in mod 2 arithmetic, $1 + 1 = 0$: we only use the numbers 0 and 1. In the same way, a 12-hour clock only involves the numbers 1 to 12, and so is ‘mod 12’; hence, 15:00 is the same as 3:00, and so on. Therefore, the mod 2 addition of the *I Ching*’s trigrams can be represented by the following table:

$+_2$	1	0
1	0	1
0	1	0

$\veebar$	T	F
T	F	T
F	T	F

Note that this is equivalent to the ‘$\veebar$’ (exclusive or) operation in Boolean logic. (Boolean logic simply uses 0 for ‘false’ and 1 for ‘true’.) This logical point of view comes most in handy for defining multiplication, since binary multiplication is equivalent to the logical ‘$\wedge$’ (and) operation (Schöter, 1998: 6):

$\times_2$	1	0
1	1	0
0	0	0

$\wedge$	T	F
T	T	F
F	F	F

The advantage of logic over modular arithmetic is that we can define complements ($\neg$). For example, Fire (☲/101) and Water (☵/010) are complementary, and so are Sky (☰/111) and Earth (☷/000). The use of logic is actually quite helpful in analyzing the trigrams’ associated meanings. Using the slightly different terminology of lattice theory (Schöter, 1998: 9):

1. The Creative [乾/☰] is the union ($\veebar$) of complements.
2. The Joyous [兑/☱] is the union ($\veebar$) of the Arousing [震/☳] and Abyss [坎/☵].
3. Fire [火/☲] is the union ($\veebar$) of the Arousing [震/☳] and Stillness [艮/☶].
4. The Gentle [巽/☴] is the union ($\veebar$) of the Abyss [坎/☵] and Stillness [艮/☶].
5. Arousing [震/☳] is the intersection ($\wedge$) of the Joyous [兑/☱] and Fire [火/☲].
6. Abyss [坎/☵] is the intersection ($\wedge$) of the Joyous [兑/☱] and Gentle [巽/☴].
7. Stillness [艮/☶] is the intersection ($\wedge$) of Fire [火/☲] and the Gentle [巽/☴].
8. The Receptive [坤/☷] is the intersection ($\wedge$) of complements.

In a beautiful essay, Goldenberg (1975) uses a branch of mathematics called group theory to unify the above points. A group is an algebraic structure with two operations (e.g. addition and multiplication). It turns out that the *I Ching*’s hexagrams satisfy many of the conditions for a group, which are as follows. 1) *Closure*: any operation between two hexagrams produces a new hexagram. 2) *Associativity*: in arithmetic operations, the order of the hexagrams does not matter, e.g. (☵+☶)+☱=☵+(☶+☱)=☲. 3) *Identity Element*: there exists a hexagram (the identity element) such that an operation with it and any other hexagram produces that same hexagram, e.g. ☷+☱=☱, as well as ☰×☱=☱. 4) *Inverse*: for every hexagram, there exists another hexagram, such that an operation combining them produces the identity element; here, for the addition operation, every hexagram is its own inverse, e.g. ☶+☶=☷. Note, however, that there does not exist a multiplicative inverse. Further, addition and multiplication both satisfy the property that $a \cdot b = b \cdot a$, so that the hexagrams are commutative. So while the hexagrams' lack of a multiplicative inverse precludes them from being a group, since they satisfy the remaining properties they are thus a 'commutative ring'.

Goldenberg (1975: 163) goes on to define the "Fundamental Theorem of the Algebra of the *I Ching*":

> For any hexagram-pair there exists a third, unique, mediating hexagram which transforms either member of the pair into the other under addition.

That is, for any two hexagrams there is a hidden third hexagram linking them. It's tempting to speculate that this may give a key to interpreting the *I Ching*—which is precisely what Goldenberg promised to do in the sequel. Sadly, however, this was never published. In any case, recent avant-garde research in the mathematics of the *I Ching* makes use of lattice theory, the mathematics of crystals, to identify elaborate networks among the hexagrams (Schöter, 2004), and it will be very interesting to observe where things go from here.

As a matter of fact, modern computers all rely on binary arithmetic. A 'bit' just means a choice between 0 or 1; a byte contains 8 bits, and so on. So in a sense, we can say that the *I Ching* played a hand in the birth of modern computing.

2 Mohist Logic & Analogical Reasoning

The basic units of logic are propositions, where each has a truth value, true (T) or false (F). The basic logical operations are: $\neg$ (not), $\wedge$ (and), $\vee$ (inclusive or), $\veebar$ (exclusive or), $\rightarrow$ (implies). '$\neg$' reverses a proposition's truth value. A compound proposition $p \wedge q$ is true when both p and q are true, otherwise it is false. $p \vee q$ is true if one or both of p and q is true. '$\veebar$' slightly differs from '$\vee$', in that if p and q are both true, then $p \veebar q$ is false. As for '$\rightarrow$', only when a false proposition p implies a true proposition q is $p \rightarrow q$ false; otherwise, $p \rightarrow q$ is always true. If both $p \rightarrow q$ and $q \rightarrow p$ are the case, then we can write $p \Leftrightarrow q$, which behaves much like '='. To better understand these operations, we can use truth tables.

p	q	$\neg p$	$p \wedge q$	$p \vee q$	$p \veebar q$	$p \rightarrow q$
T	T	F	T	T	F	T
T	F	F	F	T	T	T
F	T	T	F	T	T	F
F	F	T	F	F	F	T

From a modern perspective, Chinese logic often appears rather puzzling. For instance, rather than '$p \Leftrightarrow q$', Chinese philosophers used the far more complicated formula '$(p \rightarrow q) \wedge (\neg p \rightarrow \neg q)$' (Boh, 1972: 752). Yet, by using a truth table we can see that these formulas are equivalent.

p	q	$p \Leftrightarrow q$	$(p \to q) \land (\neg p \to \neg q)$
T	T	**T**	**T**
T	F	**F**	**F**
F	T	**F**	**F**
F	F	**T**	**T**

The basic laws of classical logic are the law of identity ($p \Leftrightarrow p$), the law of non-contradiction ($p \veebar \neg p$), and the law of the excluded middle ($\neg\neg p \Leftrightarrow p$). The most useful is the law of non-contradiction, as many logical proofs proceed by assuming that a proposition is false, and then deriving a contradiction, from which we conclude that the proposition is true. The law of non-contradiction dates back to Aristotle, who defined a contradiction as a proposition "both affirming and negating a given subject-predicate statement at the same time and in the same context" (Cheng, 1965: 201).

Curiously, in Chinese logic the law of non-contradiction is defined in a rather different manner. According to the Legalist philosopher Han Fei Zi (韩非子), a proposition is contradictory if it entails two incompatible *relations* (ibid.). In logical notation, let $a\mathrm{R}b$ and $c\mathrm{R}'d$ be propositions expressing two incompatible relations. We can express this incompatibility by $\neg(a\mathrm{R}b \land c\mathrm{R}'d)$. When p entails both relations, we get: $p \to a\mathrm{R}b \land c\mathrm{R}'d$. Yet, supposing $\neg(a\mathrm{R}b \land c\mathrm{R}'d)$, then from Han Fei Zi's notion of contradiction it follows that p cannot be maintained.

The reason for this roundabout way of thinking is due in part to the grammar of classical Chinese. "A is B" (A是B) is in classical Chinese written "A，B也"—that is, rather than 'is' as an infix, it is instead a postfix. The usage of 'not' (非/$\neg$) and 'implies' (所以/$\to$) were, and still are, the same as in logical notation, i.e. infixes. However, classical Chinese lacked a word for conjunction: it had neither 'inclusive or' ('或者'/$\vee$) nor 'exclusive or'/XOR ('还是'/$\veebar$) that could be used as logical operators. Therefore, rather than '$p \vee q$', classical Chinese philosophers would instead write '$\neg p \to q$' (Boh, 1972: 751). Logically, the two are equivalent:

p	q	$p \vee q$	$(\neg p) \to q$
T	T	**T**	**T**
T	F	**T**	**T**
F	T	**T**	**T**
F	F	**F**	**F**

In the history of Chinese philosophy, the school of thought that focused most on the logical structure of the Chinese language was the 'School of Names' (名家) in the Warring States period. 'School of Names' is largely used for convenience: in reality, the philosophers under this category offered no unified ideas or doctrine. For our purposes, we can roughly define the 'School of Names' dialecticians by their love of debate (辩) and paradox (悖论).

2.1 白马非马: A White Horse is Not a Horse

Around 300BC, a philosopher named Gongsun Long (公孙龙) wrote a sentence that would be written into the annals of history: “A white horse is not a horse” (白马非马). Obviously, it makes no sense. On this basis, many Western scholars simply dismiss Chinese philosophy as absurd. Yet, from a logical point of view, Gongsun Long’s statement does in fact have meaning.

The simplest explanation is, it all depends on the copula—what the meaning of the word ‘is’ is. (For simplicity’s sake, we’ll focus on ‘is’ rather than ‘not’.) If the word ‘is’ indicates strict identity, namely ‘equals’, then we get the following: 白马 = 马(a white horse equals a horse), but also 黑马 = 马(a black horse equals a horse). From this is follows that 白马 = 马 = 黑马 → 白马 = 黑马 — a white horse equals a black horse. So if ‘is’ means ‘equals’, then ‘a white horse is not equal to a horse’ makes sense. In reality, we ought to say: a white horse ‘belongs to’ the category of horse, rather than being equal to it.

Using set theoretic notation, we write: 白马 ⊂ 马, which we read: white horses are a subset of horses. Black horses are likewise a subset of horses: 黑马 ⊂ 马. However, 白马 ∩ 黑马 = ∅ — there are no horses that are both white and black, so that the intersection of white horses and black horses is the empty set (∅). From this we can conclude: 白马 ≠ 黑马. Compared to the equivalence relation (=), set notation (⊂) is more rigorous for expressing what we mean. Set theory therefore lets us identify the shift in meaning that produces this paradox.

Of course, the above is not the only explanation. In fact, attempts to rigorously explain 白马非马 date back at least to Greniewski & Wojtasiewicz (1956). (Their analysis is also based on set theory, so we omit it.) From there on, researchers began to experiment with progressively more complicated logical methods. Two approaches worth noting are Rieman’s (1981) axiomatic system, as well as Vierheller’s (1993) distinction between object-language and meta-language.

These detailed logical expositions present an excellent opportunity for computational philosophy. First, some background. In medieval times, Saint Anselm produced a ‘proof’ of the existence of God, known as the ontological argument. Put roughly: if we can think the concept of God, then God must necessarily exist, since existence (as more ‘perfect’ than inexistence) is part of the concept of God. Oppenheimer & Zalta (2011) translated this into an actual logical proof, so that regardless of whether you accept Anselm’s premises, the argument is valid. They then went a step further, using a tool called automated theorem proving.

Theorem provers are a type of software that can verify software or logical proofs. In essence, after the user inputs a proof, the theorem prover tells them whether or not it contains any mistakes, and sometimes can even suggest more simplified (and more elegant) versions of a proof. Theorem provers have already been used to formalize philosophical arguments by Spinoza, Leibniz, and Plato. However, this method has never been used to formalize Chinese philosophy.

“A formal proof is a proof in which every logical inference has been checked all the way back to the fundamental axioms of mathematics” (Hales, 2008: 1371). Since we must spell out *every* presupposition, a formal philosophical proof can help us to exactly identify the universe of discourse behind each formal approach.

From there we're in a better position to identify which premises would have been accepted by Gongsun Long. Further, perhaps someday a 'computational Daoism' or 'computational Confucianism' could reveal new logical structures that have eluded commentators for millennia.

2.2 Types (类) vs. Categories (范畴)

Another discourse by Gongsun Long, titled "On Understanding Change" (通变论), makes the claim: "A chicken has three legs" (鸡足三). He elaborates:

> In speaking of fowl's foot, it is one; in counting a fowl's feet there are two; two and one—therefore three. In speaking of ox-ram's foot, it is one; in counting the feet there are four; four and one—therefore five.

That is, by a flattening of ontological hierarchies we get the abstract concept of 'leg' plus the two physical legs, totaling three. By an intentional mix-up of concrete and abstract, Gongsun Long once again hits on a charming sophism.

Continuing where Gongsun Long left off, the Mohists (墨家) developed this into a mature theory. At that time, the majority of philosophical debates revolved around the concept of 'names' (名), for instance the Confucian 'rectification of names' (正名) — hence the term 'School of Names'. Organizing names unavoidably involves the use of categories (类). Yet, by modern standards, the Chinese notion of *lei* (类) behaves rather oddly. As opposed to a class or a set, we cannot say that X and Y 'belong to' a *lei*, but rather that they are 'of the same type' (同类), or else 'of different types' (不类) (Yuan, 2005: 197, n. 6). Yuan (2005) compares the Mohist notion of *lei* to Aristotle's notion of categories (范畴), concluding that: "The terms in a categorical proposition which are located in the relations of a hierarchical system of classes can be taken as...a 'tree' of genus and species," whereas the 'names' (名) of Chinese logic "are located within changeable contexts [and] can be taken as...a 'net' of associations" (2005: 183).

However, *lei* can lead to various seemingly self-contradictory statements, as in the 'Lesser Qu' (小取) chapter of the *Mozi* (《墨子》):

1. His parents are people. But serving his parents is not serving people.
2. His younger brother is a handsome man. Loving his younger brother is not loving a handsome man.
3. The robber is a person. Killing the robber is not killing a person.

To account for the second case, we can say that the brother is loved *qua* brother, not *qua* 'handsome man'. Yuan (2006) interprets this in terms of 'possible worlds' using modal logic. Modal logic is characterized by its focus on necessity ($\Box$) and possibility ($\Diamond$). The key here is that if a proposition is true in at least one possible world, then it is possible; if it is true in all possible worlds, then it is necessary. In Yuan's view, "Chinese logic is about 'indicating' [指] a world. A valid Chinese logical argument is valid in a specifically indicated possible world" (2006: 149). She views 'possible worlds' in terms of different *lei*, and concludes that modal logic is more suitable than classical logic for Chinese logic. Regrettably, her analysis is rather shallow, yet she makes a noteworthy distinction between the 'intension' and 'extension' of terms in Chinese logic (2005: 187-8). In order to get to the root of the problem, however, we must go deeper.

Since the relation 'of the same type' (同类) is not the same as 'equals' nor 'belongs', researchers in Chinese logic have turned to mathematics to seek out other types of equivalence relation. Cikoski (1975) makes use of group theory (as seen above) to demonstrate that the analogical reasoning (类推逻辑) of the School of Names is in no way inferior to propositional logic. He claims that in ancient Chinese logic, the usage of the word '是' ($\approx$ 'to be') is equivalent to the group theoretic notion of homomorphism, so that '非' ('not') signifies non-existence of a homomorphic mapping (1975: 339-42). From this very rigorous standpoint, 白马非马 is true. Further, to clarify the meaning of *lei* (类), he suggests that we ought to make use of an extremely deep branch of mathematics known as category theory (范畴论). He concludes: "To the mid-twentieth century algebraist, analogy and syllogism are but two sides of one coin" (1975: 352).

Lucas (2005: 364, n. 5) concurs with this opinion, noting how category theory has already been successfully used to explain Aristotelian term logic. The name 'category theory' is adopted from the philosophies of Aristotle, Kant, and Peirce, but is given a new mathematical definition. It is the most abstract theory in mathematics, used to connect widely disparate mathematical systems, such as discrete and continuous mathematics. It has found applications in logic, computer science, quantum physics, and even economics. To integrate the widely varying approaches to Chinese logic, a categorical point of view seems essential. Yet, no such research has been done. Nevertheless, research in Chinese logic certainly does not lack avant-garde formal tools, as we will see in the next section.

2.3 Game Semantics & Mohist Debate Logic

A recent research programme in logic lends itself quite well to the study of the Mohist dialecticians. From the point of view of game semantics, the Western concept of truth is a 'zero-agent' notion, just as proof is a 'single-agent' notion (van Benthem, 2006: 2). Naturally, this raises the question of what a multi-agent truth process would look like. In fact, many applications in computer science are of precisely this form, namely: "verification, argumentation, communication, or general interaction" (ibid.). Game semantics draws upon the concepts of game theory in economics, which is a theory of the behavior of rational agents (e.g. competing businesses); here, a 'game' is simply a competitive situation. In the same way, for game semantics, truth arises from an interactive process.

The two logicians Catarina Dutilh Novaes and Sara Uckelman make use of this method to explain the logic of medieval debates. Inspired in part by this approach, Liu (刘奋荣), Seligman and van Benthem (2011) apply game semantics to Mohist debate logic. In brief, their paper uses a graphical notation to represent a form of logical game known as 'bisimulation'. Bisimulation denotes a system where two agents each try to model the other's behavior. Two debaters attempt to categorize an object: the first claims 'it is X', the second claims 'it is not X, but Y'. Each turn, the first debater points out a property of the object corresponding to 'X', then the second points out an aspect not corresponding to 'X', and so on. The authors cite a simplified example from the *Mozi* (2011: 66-7):

> 之马之目眇则谓之马眇，之马之目大而不谓之马大。
> If this horse's eyes are blind, we say this horse is blind; though this horse's eyes are big, we do not say that it is a big horse.
>
> 之牛之毛黄则谓之牛黄，之牛之毛众而不谓之牛众。
> If these oxen's hairs are yellow, we say these oxen are yellow; though these oxen's hairs are many, we do not say that these oxen are many.

While the above is far too simplistic, the following explanation from the *Mozi* helps to make sense of a famous Mohist paradox: "A whelp [狗] is a dog [犬], but to kill a whelp is not to kill a dog" (ibid., 69):

> The things that something is called are different. In the case where they are the same, one man calling it 'whelp' and the other man a 'dog', or where they are different one calling it an 'ox' and the other a 'horse', and neither winning, is failure to engage in *bian* [辩]. In '*bian*', one says it is this and the other that it is not, and the one who fits the facts is the winner.

In a logical game, "when the categorisation structure of one player has a consistent and complete extension" then that player has a winning strategy (2011: 78, fn. 32). If this game has a finite number of rounds, then by Zermelo's theorem in game theory it follows that one of the players either has a winning strategy or can force a draw. If player 2 ('it is not X, but Y') has a winning strategy, then the bisimulation is broken. If player 1 ('it is X') has a winning strategy, then there exists a congruence relation between X and Y—the two are the same, as with 'whelp' and 'dog'.

Although bisimulation games may not sound like 'logic' proper, they are in fact an important tool in theoretical computer science, notably distributed computing. In order to apply bisimulation to Mohist debate logic, we simply treat debaters as state transition systems. Perhaps the most interesting aspect of Liu, Seligman and van Benthem's approach is that it can interpret such abstract concepts in terms of everyday situations. In the next section, we will see that the 'contradictions' of everyday life may not be so separate from logic after all.

3 Buddhist Logic & The Catuṣkoṭi

The notion of paradox has existed for millennia, dating back at least to the Greek 'Liar paradox'. Epiminides' original formulation runs as follows: "This sentence is false." If it is true, it's false; if it's false, it's true. Hence we have a self-referential contradiction: its truth value is neither true nor false.

Classical two-valued logic has only two states, namely true or false. If a proposition is not true, it is false, and any proposition must be one or the other—the law of non-contradiction. However, there also exist multi-valued logics.

As a practical example, modern airline companies often overbook their flights because it is very likely that someone will cancel their ticket or not show up. The airline has calculated that it makes more money providing refunds for overbooked flights than it does leaving empty seats on its flights. The problem is that "the set of promises made by the airline is inconsistent with the physical fact of the plane's capacity" (Parikh, 2002: 194), which therefore requires that the airline's database use a *paraconsistent logic* that can tolerate such inconsistencies.

In Buddhism, one such multi-valued logic is well-known as the catuṣkoṭi (四句破). Its most famous proponent is Nāgārjuna, who in his *Mūlamadhyakamakārikā* (or: *Fundamental Verses of the Middle Way*) writes:

All things (dharma) exist: affirmation of being, negation of non-being
All things (dharma) do not exist: affirmation of non-being, negation of being
All things (dharma) both exist and do not exist: both affirmation and negation
All things (...) neither exist nor do not exist: neither affirmation nor negation

This form of logic has 4 truth values: 1) 'exists' (p); 2) 'does not exist' ($\neg p$); 3) 'both exists and does not exist' ($p \wedge \neg p$); and 4) 'neither exists nor does not exist' $[\neg(p \vee \neg p)]$. Thanks to the very original work of Priest (2008), we now know that it is possible to interpret this in terms of modern logic. (Regrettably, to go into detail here would take us too far afield.)

The Chinese monk Xuanzang's journey to India to retrieve ancient Buddhist scriptures is known to every child throughout China, as this was the inspiration behind the classic novel *Journey to the West*. Less known, however, is that among these scripts were several texts on the Indian 'science of reasons (*hetuvidyā* / 因明). In 647AD, Xuanzang undertook to translate Śaṅkarasvāmin's *Introduction to Logic* (《因明入正理论》), thereby introducing into China the tradition of Buddhist logic (Zamorski, 2014: 151).

In Chinese philosophy, however, contradiction and multi-valued logics were not a foreign idea. Zhuangzi once wrote (in Graham, 1972: 99):

> 有有也者，有无也者，有未始有无也者，有未始有夫未始有无也者
> There is there-being, there is there-not-being, there is there-not-yet-having-begun-to-be there-not-being, there is there not yet having begun to be there-not-yet-having-begun-to-be there-not-being'.

Here, Zhuangzi uses a 4-valued logic; yet, through the influence of Buddhism, Chinese philosophers proceeded to develop systems based on 8-valued logics.

To explain the 6 new truth-values (besides 'true' and 'false'), Butzenburger (1993: 315) proposes that these arise from three different forms of contradiction:

1. A proposition p is a *paradox* when it is true if any only if it is false (that is, bidirectional implication): $[\varphi(p) = T] \Leftrightarrow [\varphi(p) = F]$;
2. A proposition p is an *antinomy* when it is false if it is true (i.e. one-directional implication): $\varphi(p) \Rightarrow [\varphi(p) = F]$;
3. A proposition p is a *problem* when by positing it multiple times, it becomes false: $[\varphi(p) \wedge \varphi(p)] = F$.

As an example of a ‘problem’, consider the overused notion of ‘paradigm shift’. Nearly every day we hear about how some technology or other is causing a ’paradigm shift’ in some industry. For all we know, some of these claims may be true. But they can’t all be true, otherwise the concept would lose all meaning. Logically, we have a system where although any claim $\varphi(s)$ of a paradigm shift may be true, it is also the case that $\varphi(s_1) \wedge \varphi(s_2) \wedge \cdots \wedge \varphi(s_n)$ becomes false as n approaches infinity. More succinctly: $\bigwedge_{i \in I} s_i \Leftrightarrow F$ as $|I| \to \infty$.

In modern logic, a clever way to get around this problem is known as ‘linear logic’, in which each proposition may be used only once. That is, a logical proof can ‘run out’ of propositions. Truth is not a fixed essence, but a scarce resource. For a simple example, a story tells of a Buddhist monk who on returning to his hometown said: “I am not the same person as the child who grew up here.” From the perspective of linear logic, this is not mere rhetoric, but observes that all the propositions from his childhood have been used up.

This is the modern solution; the ancient Chinese philosophers’ solution was entirely different. Butzenburger spells the latter out in detail by linking the above three types of paradox (respectively) to the three forms of change expounded in the *Xici* (系辞) chapter of the *I Ching* commentaries (ibid., 336):

1. 变 (*biàn*): there is a cyclic change from s_1 to s_2 and reverse;
2. 易 (*yì*): state s_2 is succeeding state s_1, i.e. state s_1 changes into state s_2;
3. 同 (*tóng*): both states are pervading each other & occurring simultaneously.

Butzenburger suggests that in order to deal with the problem of change, Chinese philosophers proceeded from a 2-valued logic (是非) to a 4-valued logic ($2^2 = 4$), by “combining the set {⚋,⚊} with itself” (1993: 322). The values are ‘wholly p’, ‘partially p, partially $\neg p$’, ‘partially $\neg p$, partially p’, and ‘wholly $\neg p$’, which we can represent in Yin-Yang notation as:

	⚊	⚋
⚊	⚌	⚍
⚋	⚎	⚏

	T	F
T	TT	FT
F	TF	FF

So with the introduction of Buddhist logic into China, Chinese philosophers discovered to their surprise that “the Buddhist catuṣkoṭi is structurally isomorphic to the Chinese system with 2^2 values” (1993: 324).

But then, of course, the problem arises of transitions among these states. So, the chart was again combined with itself to yield an 8-valued ($2^3 = 8$) logic corresponding exactly to the *I Ching*’s bagua (八卦). While in principle the number of reiterations could be infinite, the authors of the *I Ching* contented themselves with the power set $(2^3)^2 = 2^6 = 64$ of the above 4-valued system, that is, its total number of subsets (1993: 323).

Thus, Chinese philosophers preferred to go beyond the catuṣkoṭi’s ‘tetralemma’ (四歧式) in favor of a reduced form of the *I Ching*’s octolemma. One notable example is the *Treatise of the Five Teachings* (《华严五教章》) by the Chinese Buddhist Fazang (法藏), which uses the octolemma “to exhaustively categorize extant theories on causation, i.e. all attempts on solving the problem of transition" (1993: 325). In order to reduce the combinatorial possibilities, Fazang introduces several principles from Buddhism in the form of logical propositions.

Fazang begins with the metatheoretical assumption that any Buddhist theory "can be expressed as a complex proposition which contains either an affirmative or a negative version of each of the following one-place predicates: (P_1) empty; (P_2) efficacious; (P_3) dependent" (ibid.). That is:

$$(P_1 \veebar \neg P_1) \wedge (P_2 \veebar \neg P_2) \wedge (P_3 \veebar \neg P_3) \qquad \{\mathrm{A}\}$$

This by itself would yield a system with $2^3 = 8$ values, but then Fazang goes on to introduce a Buddhist axiom, namely (ibid.):

> an object is causally connected to another either by being its condition, i.e. by being efficacious with regard to that object; or by being conditionally caused by that other object, i.e. by not being independent.

Put in propositional form, this leads to a new restriction for {A}:

$$P_2 \vee P_3 \qquad \{\mathrm{B}\}$$

This rules out two cases, namely 100 [☳/TFF] and 000 [☷/FFF]. By disposing of these we "arrive at a hexapartite version of our original octolemma" (ibid.). The 'Lesser Qu' (小取) chapter of the *Mozi* contains a similar 7-valued logic.

We noted earlier that the *I Ching*'s 64-hexagram system is in some sense a compromise, as a true representation of the transitions between changing states is capable of infinite reiterations, yielding an infinite-valued logic. Today, in fact, such an infinite-valued logic exists—known as 'fuzzy logic'. Fittingly, it is now a leading subject of research in China.

Lastly, Butzenburger concludes his article with a discussion of the 'metatheoretical' role of Buddhist logic. It is commonly accepted among Buddhist philosophers that no intellectual system is adequate for describing reality. Thus, from the Buddhist point of view, "logic must be able to metatheoretically cope with the necessary transitions from one system to another without ever being confined to one single and fixed system" (1993: 337). He concludes that the use of paradox is a formal tool for reinforcing such transitions *ad infinitum*.

Conclusion

A logical system may be understood purely in the abstract, but perhaps the main contribution of Chinese logic is to show that we can truly *live* these logics. We have seen that in some sense the differences between Chinese philosophical schools of thought can be attributed to the different logical systems in which they take place. Thus by understanding these systems we can better understand our own universe of discourse. Further, through the use of more avant-garde tools in logic we may be able to construct hybrid philosophical systems, or perhaps a unique system of 'Chinese logic' to complement that of the West.

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