Algorithmic Psychoanalysis:
Lacan's Logic of Suspicion

Graham Joncas

Introduction

Lacan once remarked on the Cartesian cogito that etymologically, "the French verb *penser* (to think)...means nothing other than *peser* (to weigh)" (1961: 14). Lacan's mix of bad puns, abuse of notation, cryptic aphorisms and immense erudition has created a new style of writing, and even of thinking. A new language, in short, lacking any method for non-experts to weigh its words.

Of all things, Lacan's earliest papers take the form of math puzzles—a lucid (albeit horrendously verbose) derivation, then reframing of the problem as metaphor. One such paper—"The Number Thirteen and the Logical Form of Suspicion"—has largely been forgotten. This paper aims to recast the puzzle using discrete mathematics, and show how it bears upon Lacan's later ideas.

What I like about this mathematical allegory is that even if one believes Lacan is a charlatan, here one need not immerse oneself in psychoanalysis, but only *think*.

I hope that Lacanians will find working through my derivation a challenging exercise, and that mathematicians will be piqued at the idea of treating a math problem as a philosophical 'text'. I for one would be glad to see more such texts.

1 Lacan's Algorithm

We are given 12 identical pieces, and told one of them is 'bad'—either lighter or heavier than the rest, we're not sure which. Having only a scale with two plates, and no way to gauge numerical weight, we must find the bad piece in 3 weighings.

If we knew whether the bad piece was lighter or heavier, the problem would be easy: just split the pieces into two groups of 6, then split the 'bad' half into two groups of 3, then simply weigh two of the bad three. But we don't.

Here, we'll overview Lacan's account for 12 pieces, and then in the next section we'll consider n pieces, and try to explain *why* Lacan's algorithm works.

Lacan begins by placing on the scales two groups of 4. Suppose they balance. Then the bad piece is in the remaining 4, so we can just weigh any 2 of the 4. If those balance, the 2 left-out pieces are bad; if they don't, the 2 pieces on the scale are bad. So weigh one of the bad 2 against a good piece: if they balance, the other piece is bad; if they don't, then the piece on the scale is bad. Simple.

Note how this was equivalent to the sub-problem of finding a bad piece out of 4 pieces, in 2 weighings. The sub-problem is embedded in the larger problem.

If the two groups of 4 don't balance, we use the method of *tripartite rotation*.

The scales don't balance, so one is heavier (H), one lighter (L). So, we select 3 pieces from H, L, and the remainder (R), and rotate them: $H \to L \to R \to H$.

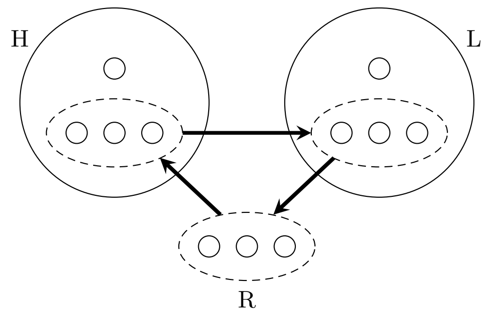

Fig. 1. Tripartite rotation

Case 1: Scales balance – the bad piece is in the 3 moved to R, and too light.
Case 2: Balance shifts – the bad piece is in the 3 moved to L, and too heavy.
Case 3: Unbalance doesn't change – the bad piece is in the 2 unmoved pieces.

In cases 1 and 2, just weigh 2 of the bad pieces: if they're equal, the remainder is bad; if not, we know the bad piece is the lighter (case 1) or heavier piece (case 2). For case 3, just pick one and weigh it against a good piece. And we're done.

Lacan then considers the case of 13 pieces: 4 on each scale, 5 remainders. It's clear that if the scales don't balance, the problem is the same as with 12 pieces when the scale didn't balance—the remainders are all good, whether 5 or 4.

Here, when the scales balance, we have a new problem. Recall how we could treat 4 pieces as a separate problem. So let's examine the 5-piece sub-problem.

Start with 2 pieces on each scale and 1 remainder. If we're lucky, the scales balance and the remainder is bad. If not, we have 4 pieces, but we know the 4-piece case takes two weighings, so the 5-piece case must take three weighings.

It's the same even for 1 piece on each scale and 3 remainders. If we're unlucky, the scales balance, giving a new sub-problem with 3 pieces—the smallest solvable version of Lacan's problem. Weigh any 2. If they balance, the remainder is bad. If not, weigh a piece on the scale against the good piece. Total: three weighings.

So both the 3-piece and 4-piece cases take two weighings, 5 pieces takes three weighings, so it would seem that 13 pieces must take four weighings. Nope.

Actually, for 13 pieces, the 5 remainders aren't truly a separate sub-problem. There's a difference: we have 8 good pieces. For 3 or 4 pieces this doesn't matter, but for 5 pieces, Lacan can introduce a new trick: the *'by-three-and-one' position.*

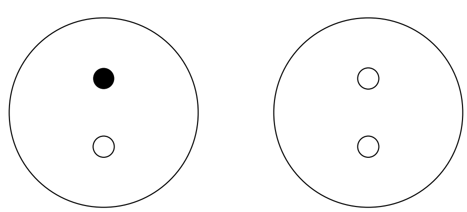

Fig. 2. The 'by-three-and-one' position

Here, we have 2 pieces in each plate, with one of the 4 a good piece, and 2 remainders. If the scales balance, just weigh one remainder against a good piece and we're done. If they don't balance, here's the trick: we can do the smallest possible tripartite rotation, $H \to L \to R \to H$, where R is a good piece.[1]

Case 1: Scales balance – R is the bad piece.
Case 2: Balance shifts – L is the bad piece.
Case 3: No change – the unmoved piece is bad.

Thus, the 5 remainders take two weighings, and the 13-piece case takes three.

In this case, treating the 5 remainders as a sub-problem was the wrong way to go, making it seem impossible to solve in 3 weighings. More pieces means more ways to divide between scales and remainder, increasing the risk of such pitfalls.

Thus, Lacan's task is to find a general algorithm for any number of pieces, including a uniform way to divide them. The algorithm must minimize the maximum amount of weighings—i.e. find the minimum, assuming we don't get lucky.

The problem also raises some new questions. The main one is: for a given number of pieces, how many weighings are needed? As in the solutions outlined above, Lacan answers this question, but fails to explain *why* his solution works.

Hence, the next section will diverge from Lacan's exposition, using discrete mathematics to give an algorithm for n pieces. This will help us see how Lacan's problem relates to the logic of suspicion, which we will outline in the final section.

2 The Mathematics of Discretion

Lacan's solution starts by dividing the pieces into three groups: two groups go on the scales, the third is the remainder. Call the total number of pieces n, the number of pieces on each scale k, and the number of pieces in the remainder r. This gives the simple identity: $n = 2k + r$, which implies $r = n - 2k$.

If n is divisible by three, everything is easy: $k = r = \frac{n}{3}$. If it isn't, then r is either one more or one less than $\frac{n}{3}$. This is easy to understand intuitively, but to express this in math we'll need a special notation.

The 'floor' function $\lfloor x \rfloor$ can be thought of as rounding x down to the greatest integer less than or equal to x. So unlike the usual rounding, $\lfloor 0.01 \rfloor = \lfloor 0.99 \rfloor = 0$.[2]

Intuitively, we can define $k = \left\lfloor \frac{n}{3} \right\rfloor$, dividing n into thirds and putting any extras in the remainder. However, the following formula is easier to deal with:

$$k = \left\lfloor \frac{n+1}{3} \right\rfloor$$

Recall how we could divide 5 pieces in two different ways: either $k = \left\lfloor \frac{5}{3} \right\rfloor = 1$ and $r = 3$, or else $k = \left\lfloor \frac{5+1}{3} \right\rfloor = 2$ and $r = 1$. So adding the $+1$ means we have smaller remainders for n of the form $3k - 1$. It's essentially a stylistic choice.

This means that now we can define r purely in terms of n:

$$r = n - 2k = n - 2\left\lfloor \frac{n+1}{3} \right\rfloor$$

This gives $k_{12} = 4$ and $r_{12} = 4$ as well as $k_{13} = 4$ and $r_{13} = 5$, just as in Lacan's examples. It also gives $k_{14} = 5$ and $r_{14} = 4$. Now recall how our main trouble with 13 pieces was dealing with the 5 remainders. Things went smoothly for 12 pieces where $r_{12} = k_{12}$, but harder for $r_{13} = k_{13} + 1$, which brought a new sub-problem that we could only solve using 'by-three-and-one'.

For 14 pieces, we have $r_{14} = k_{14} - 1$. If the scales balance, it's the same as for 12 pieces: $r_{14} = r_{12} = 4$. If they don't balance, we can use tripartite rotation. For 12 and 13 pieces, we rotated $k_{13} - 1 = 3$ pieces in k and r. Yet, notice that $k_{13} = 4$, and also $k_4 = 1$. So in fact $k_{13} - 1$ is a special case of a larger formula.

Imagine if we rotate $k - 1$ pieces when n is large. If the scales stay the same (i.e. one of the 2 unmoved pieces is bad), we can finish in one more weighing, even if we have plenty left. It's a waste—far better to divide the weighings evenly among the three outcomes (same, balance, switch). Thus, by rotating $k - \lfloor \frac{k+1}{3} \rfloor$ pieces, we're dividing n' into thirds: $2k' = 2\lfloor \frac{k+1}{3} \rfloor$ pieces get rotated, leaving r'.[3]

So for 14 pieces we rotate $5 - \lfloor \frac{5+1}{3} \rfloor = 3$. If the scales stay the same, the bad piece is in the 4 unmoved pieces. If they don't balance, it's in one of the groups of 3, and we know whether it's light or heavy. That's two weighings so far.

Recall: using 'by-three-and-one' for 5 pieces took two weighings. We can't do better for $k = 4$. So 14 pieces require a worst-case minimum of 4 weighings. Thus, the significance of the number 13 for Lacan is that it's the maximum number that we can solve in 3 weighings. We'll write this as $\max_3 = m_3 = 13$. Now we can write everything we know about the puzzle in the form of a table.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
k	0	1	1	1	2	2	2	3	3	3	4	4	4	5	5	5	6	6	6	7
r	1	0	1	2	1	2	3	2	3	4	3	4	5	4	5	6	5	6	7	6
m	1	–	2	2	3	3	3	3	3	3	3	3	3	4	4	4	4	4	4	4

Table 1. Note how r has the cyclical form $(k-1,\ k,\ k+1)$.

To see the pattern for r, recall way back when we defined k using $\lfloor x \rceil$. We did so because we want to divide the pieces into 3 groups, but the remainder may have one piece more or less than k, so we have to round. That is, r is either $k - 1$, k, or $k + 1$. Plugging different types of n into $r = n - 2k$, this is very easy to see:

$$\begin{aligned} n &= 3k - 1 \;\rightarrow\; r = 3k - 1 - 2k = k - 1 \\ n &= 3k \;\;\;\;\;\;\;\;\rightarrow\; r = 3k \;\;\;\;\;\;- 2k = k \\ n &= 3k + 1 \;\rightarrow\; r = 3k + 1 - 2k = k + 1 \end{aligned}$$

Furthermore, as n increases, we can see that this pattern is cyclical:

$$\begin{aligned} n + 1 &= 3k - 1 + 1 \rightarrow n + 1 = 3k \\ n + 1 &= 3k \;\;\;\;\;\;\;+ 1 \rightarrow n + 1 = 3k + 1 \\ n + 1 &= 3k + 1 + 1 \rightarrow n + 1 = 3k + 3 - 1 = 3(k + 1) - 1 \end{aligned}$$

This implies a corresponding cycle for r. The crucial point here is that the only time k goes up by one is when we add 1 to n of the form $3k+1$. We can also show that any $m_w = 3k + 1$. The proof is by contradiction, and not very informative, so we'll relegate it to a footnote.[4] This implies that $k_{m_{w+1}} = k_{m_w} + 1$.

Another interesting pattern is that for any m_w, its k is the previous m_{w-1}. For $m_3 = 13$, $k_{13} = 4 = m_2$, and $k_4 = 1 = m_1$, and even $k_1 = 0 = m_0$. This also applies for $m_w + 1$, where $k_{m_w+1} = m_{w-1} + 1$, down to m_2, which is unsolvable. The proof gives some insight into the puzzle, so we'll go over the more jazzy bits.

Key to the proof is that Lacan's puzzle is composed of two embedded puzzles: the first, when we don't know whether the bad piece is lighter or heavier; the second, when we do. After tripartite rotation, the former becomes the latter.

In this simpler sub-puzzle, our algorithm is to divide n into thirds of size k (putting any remainders in r) and discard the 'good' two-thirds, repeating until $k = 1$. It's intuitive, then, that the number of weighings for n pieces is the number of times we can divide n by 3, plus an extra if there are any remainders.

Thus, $\hat{m}_w$ occurs when n is perfectly divisible by 3 for w rounds. That is:

$$\hat{m}_w = 3^w$$

Hence, we can tabulate this sub-puzzle as follows:

	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	...	27	28
k	1	1	1	2	2	2	3	3	3	4	4	4	5	5	5	6	6	6	7	...	9	9
r	0	1	2	1	2	3	2	3	4	3	4	5	4	5	6	5	6	7	6	...	9	10
m	1	1	2	2	2	2	2	2	3	3	3	3	3	3	3	3	3	3	3	...	3	4

Table 2. Like before, r follows the cycle $(k-1,\ k,\ k+1)$.

From here, the proof gets pretty dense, so we'll put it in a footnote.[5] The gist of it is to compare m_w and m_w+1 after the tripartite rotation step. If the latter still takes one more weighing, then the former must contain a $\hat{m}_w$ in the sub-puzzle. We can use the formula for $\hat{m}_w$ to prove by contradiction that $k = 3k' + 1$, and these arguments apply to k'', k''', all the way down to $k^* = 0 = m_0$. All these k's are actually m_w's, and we can modify the formula for $\hat{m}_w$ into a new formula for m_w. Finally, this formula lets us prove by induction that $k_{m_{w+1}} = m_w$.

Let's recap: any m_w is expressible as $3k+1$, and for m_{w+1}, we get $k_{m_{w+1}} = m_w$. Putting these together gives a recurrence relation of the following form:

$$m_{w+1} = 3m_w + 1$$

We'll see how important this is once we try some examples (where $m_0 = 0$).

$$\begin{aligned} m_1 &= 3m_0 + 1 = & 3^0 &= 1 \\ m_2 &= 3m_1 + 1 = & 3^1 + 3^0 &= 4 \\ m_3 &= 3m_2 + 1 = & 3^2 + 3^1 + 3^0 &= 13 \end{aligned}$$

In other words, we can express m_w far more elegantly in summation notation:

$$m_w = \sum_{k=0}^{w-1} 3^k$$

Likewise, for m_4 we need only add 3^3 and m_3 to get 41, then $m_5 = 3^4 + m_4 = 121$, and so on for any number of weighings we'd like.

With this formula in hand, Lacan invokes the biblical Judgment Day, in which men's sins are weighed on a divine scale. Suppose all souls are good except for one, and that the number of souls being weighed is a trillion or less. Our formula shows that $m_{26} = 1,270,932,914,164$. Lacan concludes with a touch of black humour that the ceremony "would take effect, with room, in 26 stages, and therefore...would have no reason to prolong itself for too long a time" (1946: 14).

Lacan introduces one more trick for the case $n = 40$. In fact, it's just a larger version of 'by-three-and-one', but it's worth elaborating because his analysis on page 10 is near-incomprehensible without going through the derivation.

We begin by setting $k = \left\lfloor \frac{40+1}{3} \right\rfloor = 13$, with $r = 14$. If the scales don't balance, we do a tripartite rotation with $13 - \left\lfloor \frac{13+1}{3} \right\rfloor = 9$ pieces. If the scale balances or switches, then we know whether the piece is heavier or lighter, and need only weigh 3 against 3, and then weigh any 2 pieces. If, after the tripartite rotation, the scales stay the same, then we again weigh 3 against 3 (this time with $r' = 2$), and then any 2. Following these steps, the maximum number of weighings is 4.

The main problem arises if, weighing 13 against 13, the scale balances, leaving 14 pieces to weigh in 3 moves. As we know, $m_3 = 13$, so how is this possible?

Lacan's trick is to do both tripartite rotation and by-three-and-one at once.

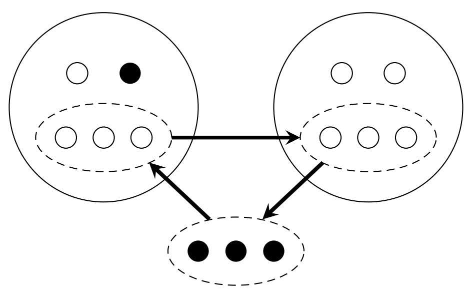

Fig. 3. Tripartite rotation plus 'by-three-and-one' position

We place 9 unknown pieces on the scale: 5 on one, 4 plus 1 good piece on the other, and 5 remainders. If the scales balance, then we must weigh the 5 remainders in two weighings—the same problem as with 13 pieces, which we know is solvable. If the scales don't balance, then we do a tripartite rotation with 3 pieces. If the scale balances or switches, we just weigh any 2. If the scale stays the same, then we just do one last 1-piece tripartite rotation on the by-three-and-one pieces and then we'll know the bad piece. Three weighings.

In general, it's clear that given scales that don't balance, tripartite rotation is optimal. By letting us know that the bad piece is light or heavy, tripartite rotation *changes the problem itself*, which 'by-three-and-one' can't do. Yet, it's not as clear what is optimal if the scales do balance. In fact, both methods usually take the same number of weighings, except—as we saw—when $n = m_w + 1$. Here, 'by-three-in-one' wins out, and so is optimal when the scales balance.

To conclude, Lacan's general algorithm for $n \geq 3$ pieces runs as follows:

1) On each scale, put $k = \lfloor \frac{n+1}{3} \rfloor$ pieces, leaving $r = n - 2k = n - 2\lfloor \frac{n+1}{3} \rfloor$.

If the scales don't balance, use tripartite rotation:

a) Rotate $k - \lfloor \frac{k+1}{3} \rfloor$ pieces from $H \to L \to R \to H$, where R is good pieces.

If the scales balance, then the bad piece is in R, and too light.
If the balance shifts, then the bad piece is in L, and too heavy.
If there's no change, the bad piece is in the r' unmoved pieces.

b) Divide n into $k = \lfloor \frac{n+1}{3} \rfloor$. Discard the good pieces, repeat until $n = 1$.

If the initial scales do balance, use 'by-three-and-one':

i) If $n = 3$ or 4, weigh any 2; if $n = 2$, weigh 1 and a good piece; if $n = 1$, stop.
If $n \geq 5$, weigh k against $k - 1$ plus a good piece, the rest as remainders.

If the scales balance, repeat (i).
If the scales don't balance, do (a).[6]

The puzzle's philosophical import is still far from clear, so let's focus on this now.

3 The Logic of Suspicion

Lacan explains the 'logic of suspicion' as follows (1946: 13). In suspicion,

> the norm with which [a deviation] relates itself is not a specified nor a specifying rule, but only a relation of individual to individual within the collection—a reference not to the species, but to uniformity.

That is to say, deviant individuals are defined extensionally, not intensionally. 'Deviance' cannot be defined by internal criteria, but only differentially, through examples and comparisons to a norm.

We can think of tripartite rotation as converting extensional to intensional difference: once our scales don't balance, we leverage the effects of this 'deviance' to isolate its precise type. As we saw, this qualitatively changes the puzzle from one of suspicion to one of rooting out. Conversely, for 'by-three-and-one' we need only *suspect* that one piece is bad: even if we don't know for sure there's a bad piece, repeatedly doing 'by-three-and-one' ensures we don't miss it.

Lacan stresses in particular that "[t]his 'by-three-and-one' position is the original form of the logic of suspicion" (1946: 9). He notes that although tripartite rotation only has two steps, 'by-three-and-one' involves three "proofs" (ibid., 13):

1. Setting aside at least one good piece in a prior weighing
2. Weighing the bad pieces along with the good piece
3. Doing tripartite rotation to find the bad piece

It's typical to frame discrimination as irrational, hating someone who is different. Lacan claims just the opposite: discrimination is algorithmic—a logic whose sole arbitrary 'seed value' is its choice of an individual as paradigmatic 'good piece'.

In the logic of suspicion, difference is purely extensional: discrimination is not tied to any specific *quality* of the deviant. This algorithm does not even require *knowledge* of a deviation beforehand (as in Lacan's examples), but may simply be used to confirm that all the pieces correspond to a pre-selected 'good' one.

Note that despite this rhetoric of good and bad pieces, 'discrimination' is meant here in the most general, non-pejorative sense. Yet, once the magnitude of this difference is determined relative to the standard, this logic becomes merely iterative, repeating the same step over and over until the deviant is singled out.

If Lacan's aim was to theorize discrimination, we would rightly fault him for not generalizing his model for $n \geq 2$ 'bad' pieces. Yet, as Lacan reinterpreted this puzzle in the light of his later oeuvre, the 'unrealistic' assumption of foreknowing the presence of exactly one bad piece takes on a far deeper meaning.

We'll first review extant readings of Lacan's puzzle. As the English-language literature is so small (and its overlap so large), we'll get by with quoting its main points. Our first source is Clark's (2014: 136) annotated bibliography of Lacan:

> Lacan's...problem...involves the isolation of a single member of a group through a process of reasoning that discovers the pure difference of the item—whether it is heavier or lighter than the other items—and he claims that the numerical combinations leading to this discovery manifest the "logical form of suspicion" and that they lead to "the absolute idea of difference, the root of the form of suspicion." The essay concludes with [a] defense of the ancient faith in mathematical analysis as a means of investigating the "generative function of phenomena."

The second, most original take is Bousseyroux (2012: 2-3), who links the problem's triadic nature to Lacan's later interest in the tripartite Borromean ring:

> [T]he operations of weighing [t]hat he calls the *by-three-and-one* position and...*tripartite rotation*...resonate fairly well with the Borromean Lacan of the 1976 'Preface': there is no possible weighing, on the two-pan scale of truth and the real, of the urgency present in the initial request [for analysis] and which is to be satisfied at the end without the introduction of the position of *by-three-and-one* into the analytic operation, which is an excellent way of qualifying the position of the symptom, as the fourth ring in the Borromean knotting that orients the analysis towards the real.

To unpack this a bit, Lacan divides reality into 3 'orders': Imaginary, Symbolic, and Real (RSI), where Imaginary corresponds to internal (mental) phenomena, Symbolic to external phenomena (objects), Real to the fundamentally unknowable noumenon. This way, Lacan can bring the theories of psychoanalysis to bear on the Imaginary, semiotics on the Symbolic, and structuralism on the Real.

The Borromean ring is a topological object composed of three rings (cf. RSI), so that if one is cut, the others break free. Lacan often presents it as a three-circled Venn diagram, labelling each intersection. In this metaphor, the 'symptom' acts as a fourth ring that keeps the others together even if one is cut.

At the beginning of psychoanalysis, the analyst knows something is wrong, but doesn't know the precise nature of the analysand's 'deviance'. The 'scales' of truth and the real are usually correlated, but what the analyst is really looking for is some word or sign—a Freudian slip, a revealing association—that evinces far more of the underlying Real than its mere superficial truth.

The analyst weighs the analysand's words, performing 'by-three-and-one' until hitting upon an unbalance. The analyst then aims to isolate the type and magnitude of this deviance: tripartite rotation. The knowledge of this magnitude (the symptom or *sinthome*) acts as a fourth term that changes the nature of the analysis, from seeking a symptom to rooting out the Real.

Last, Forrester (1990: 183-6) lucidly overviews Lacan's puzzle, emphasizing its continuity with Lacan's other early essay "Logical Time" (ibid., 187-8):

> These two papers are the primary sources for what Lacan called his 'subjective logic': not the logic that a subject obeys, which is objective logic, nor the logic that a subject employs, which is subjectivised logic, but the logic which the concept of a subject, in relation to other subjects, and in relation to an individuating propensity, inherent in the rules of the game, requires. In ["#13"], it is the logic governing the use one makes of the others in determining which one is in deficit, the logic through which suspicion finally and permanently falls on the element that has not moved, that has not been displaced amidst the general tripartition.

Another crucial point is how Freud stressed the herd-like conformity of groups, their "effacement of difference," whereas Lacan "demonstrate[s] how difference emerge[s] out of the rules that specif[y] the constitution of the group" (ibid., 183).

Lacan doesn't mention this puzzle in any other writings. The closest he gets is a curiously-worded metaphor to describe his concept of objet petit *a*.

Consider the platitude that desire is unfulfillable. Lacan doesn't try to 'justify' this, but rather, condenses it into an *object*, denoted *a*. This lets him fuse the psychoanalytic theory of phantasy with French phenomenology's focus on the subject–object relation. The typical gloss of object *a* is lucid enough: 'the object-cause of desire'—desire's asymptote, the 'little other' as particularity (versus the Big Other *A* as universality). What's difficult is the many associations in which object *a* is embedded, e.g. as the 3-way overlap of Lacan's RSI schema. If anything, reading object *a* through Lacan's puzzle only complicates things further.

In a lecture on Plato's *Symposium* (1960-1: 128-9), Lacan parallels object *a* to the Greek term *agalma* (ἄγαλμα). Just as Alcibiades tries to seduce the famously-ugly Socrates whose knowledge he desires, likewise an *agalma* is a beautiful offering to the gods placed in a statue of a satyr (Cake, 2009: 226-7).

It's become almost banal to characterize love as "tak[ing] the other as a subject and not at all purely and simply as our object" (Lacan, 1960-1: 126). This even takes on a moral tone: to view another as object devalues them, makes them 'replaceable' as opposed to a singularity.

To this, Lacan replies: how is it any better to consider the other as a subject? After all, 'subject' for Lacan has a very strict meaning: a speaking being, a being inducted into language. Yet, language is imposed from without — beings are *subjected* to what Lacan calls the symbolic order by no choice of their own: "I is another" (Rimbaud). In this view, taking the other as subject is even worse than as object. It's almost a narcissism, a mere guarantee that this linguistic being "can respond to our combination by his own combinations" (ibid., 127).

Lacan aims not to tear down this platitude, but instead show it's not the whole story. It must be that "in the subject there is a part where it speaks for itself, this thing from which nevertheless the subject remains suspended" — "something which is the aim of desire as such, that which accentuates one object among all as being without equivalence to the others" (ibid.). Object *a*.

Lacan states that "*agalma*...is the major point of analytic experience" (1960-1: 129). Most curious of all is how Lacan consistently—with no further explanation—refers to *agalma* as a 'weight':

> In a word, if [an] object impassions you it is because within, hidden in it there is the object of desire, *agalma* (the **weight**, the thing that makes it interesting to know where this famous object is, to know its function and to know where it operates just as much in inter- as in intrasubjectivity) and in so far as this privileged object of desire, is something which, for each person, culminates at this frontier, at this limiting point which I have taught you to consider as the metonomy of the unconscious discourse where it plays a role that I tried to formalise...in the phantasy.

Lacan identifies the object *a* with the partial object of Melanie Klein: a pre-individual object, non-complete but not incomplete, as in the fragmentary, underdeveloped perception of infants.[7] A partial object is its own synecdoche.

In Klein, the partial object takes on the role of 'good object' or 'bad object' for the infant comprehending it. In the 'dialectic' of psychoanalysis, claims Lacan, this partial object operates as a "primordial given" (1960-1: 128). Hence, just as Lacan's puzzle begins with the knowledge of a unique 'bad' piece (ibid.):

> [T]his object, *agalma*, little *a*, object of desire, when we search for it according to the Kleinian method, is there from the beginning before any development of the dialectic, it is already there as object of desire. The **weight**, the intercentral kernel of the good or the bad object...is this [partial] object that Melanie Klein situates somewhere in this origin, this beginning of beginnings...

What the lover seeks is this "supreme point where the subject is abolished in the phantasy, his *agalmata*" (ibid., 138). Any love-relation involves three parties: lover, loved, and *agalma*. "[I]t is only through the Other and for the Other that Alcibiades, like each and every person, wants to make his love known" (ibid., 129).

Lacan stresses over and over again how the object *a* as asymptote is approached triadically. The example that most resonates with our puzzle is his conclusion that "our subjectivity is something we entirely construct in plurality, in the pluralism of these levels of identification which we will call the Ego-Ideal

[$I(a)$, in Lacan's algebraic notation], the Ideal Ego [$i(a)$], which we will also call the desiring Ego" (ibid., 128)—and, of course, the object a (ibid., 138).

All these parallels open up far more questions than they answer. It's unrealistic to suppose that this forgotten text of Lacan's holds a 'key' to object a—an *agalma* of the *agalma*, so to speak. If this parallel is more than rhetorical, however, it holds open the tantalizing possibility of, someday, an algorithmic psychoanalysis.

Lacan's algorithm: a finite version of the infinite pursuit of object a.

Conclusion

In continental philosophy, a statement is judged not by whether it is true or false, but by how much it expands one's universe of discourse. Lacan intends his models to bring new nuances to light, not merely to obtain in a given setting.

Underemphasized in Lacanian scholarship is the use of recreational mathematics to generate new ideas. Such problems are singled out for certain elegant properties, which lend themselves all the more to metaphorical 'mis-recognition'.

Lacan's paper was part of a project of "a *collective logic* [to] complete classical logic" (1945: 18)—where "the relations of the individual to the collection must be defined before a class is constituted, that is, before the individual is specified" (1946: 2). One can only imagine what he could have done with the full repertoire of today's formal methods, from multiagent logics to game semantics.

By framing his problem in math, Lacan opens it to further generalizations and re-mappings. We can imagine a variant of Lacan's puzzle with ≥ 2 'bad' pieces, or a probabilistic version with a faulty scale. We can ask new questions, such as the algorithmic complexity of Lacan's solution. The puzzle may even turn out to be congruent to a well-studied problem in another branch of math.[8]

Mathematics is filled with beautiful theories with no applications. In the midst of such a scandal, even toy models like Lacan's can open up brand new metaphorizations, bringing tomes of theorems to bear on new conceptual territories.

References

1. Bousseyroux, M. (2012). "The Response in Urgent Cases." *7th Meeting of IF-SPFLF*. Retrieved from `www.champlacanien.net/public/docu/2/rdv2012pre9.pdf`
2. Cake, A. (2009). "Lacan's Psychoanalysis and Plato's Symposium: Desire and the (In)Efficacy of the Signifying Order." *Analecta Hermeneutica* 1, pp. 224-39
3. Clark, M. (2014). *Jacques Lacan (Volume I) (RLE: Lacan): An Annotated Bibliography*. New York: Routledge, p. 136: C48
4. Forrester, J. (1990). *The Seductions of Psychoanalysis: Freud, Lacan and Derrida*. Cambridge: Cambridge University Press.
5. Kirshner, L. (2005). "Rethinking Desire: The *Objet petit a* in Lacanian Theory." *Journal of the American Psychoanalytical Association* 53(1), pp. 83-102
6. Lacan, J.; Fink, B. & Silver, M. (trans.). (1988 [1945]). "Logical Time and the Assertion of Anticipated Certainty: A New Sophism." *Newsletter of the Freudian Field* 2(2), pp. 4-22

7. Lacan, J.; Gross, S. (trans.). (1946). "The Number Thirteen and the Logical Form of Suspicion." *Cahiers d'art*, pp. 389-93
8. Lacan, J.; Gallagher, C. (trans.). (1960-1). *Seminar 8: Transference*. Retrieved from `www.lacaninireland.com/web/translations/seminars`.
9. Lacan, J.; Gallagher, C. (trans.). (1961-2). *Seminar 9: Identification*. Retrieved from `www.lacaninireland.com/web/translations/seminars`

Notes

[1]This ordering is purely a convention: $L \to H \to R \to L$ works just as well.

[2]Analogously, the 'ceiling' function $\lceil x \rceil$ rounds *up*, so $\lceil 0.01 \rceil = \lceil 0.99 \rceil = 1$. We can also define $k = \left\lceil \frac{n-1}{3} \right\rceil$, whose equivalence with $\left\lfloor \frac{n+1}{3} \right\rfloor$ follows from the property $\left\lfloor \frac{n}{m} \right\rfloor = \left\lceil \frac{n-m+1}{m} \right\rceil = \left\lceil \frac{n+1}{m} \right\rceil - 1$. Here I stick with the floor version, as it's a bit more intuitive.

[3]After an unbalanced weighing, r will always have at least $k - \left\lfloor \frac{k+1}{3} \right\rfloor$ pieces. First, note that at the very least, $r = k - 1$. So we want to show that $k - \left\lfloor \frac{k+1}{3} \right\rfloor \leq k - 1 \leq r$. But this amounts to showing that $\left\lfloor \frac{k+1}{3} \right\rfloor \geq 1$, which must be true because if $k = 1$ then we've already solved the puzzle, and this inequality holds for any other $k > 1$.

[4]First, suppose $m_w = 3k - 1$, so that $r_{m_w} = k - 1$. Then $m_w + 1 = 3k$, and $r_{m_w+1} = k$. So the worst case for m_w is weighing the two k's. But $m_w + 1$ also has two k's, so should take the same number of weighings as m_w — a contradiction.

Second, suppose $m_w = 3k$, so that $r_{m_w} = k$. Then $m_w + 1 = 3k + 1$, and $r_{m_w+1} = k + 1$. For $m_w + 1$, the worst case is 'by-three-and-one' with $k + 1$.

i) If $k = 3k' - 1$, then $\left\lfloor \frac{k+1}{3} \right\rfloor = \left\lfloor \frac{k+2}{3} \right\rfloor = k'$. Weigh k' plus a good piece against $k' + 1$ pieces. (It shouldn't matter how we divide.) This leaves remainders $r_{m_w} = k - 2$ and $r_{m_w+1} = k - 1$. The worst case is weighing the two $k' + 1$'s, which is the same for both m_w and m_{w+1} — a contradiction.

ii) If $k = 3k'$, then $\left\lfloor \frac{k+1}{3} \right\rfloor = \left\lfloor \frac{k+2}{3} \right\rfloor = k'$. Divide as before, so $r_{m_w} = k - 1$ and $r_{m_w+1} = k$. The worst case is still the $k' + 1$'s — the same contradiction.

iii) If $k = 3k' + 1$, then $\left\lfloor \frac{k+1}{3} \right\rfloor = k'$ but $\left\lfloor \frac{k+2}{3} \right\rfloor = k' + 1$. For m_w, put $k' - 1$ plus a good piece on one scale, k' on the other, leaving $k' + 2$ remainders. For m_{w+1}, weigh k' plus a good piece against $k' + 1$, leaving $k' + 1$ remainders. The worst case for m_w is 'by-three-and-one' with $k' + 2$ pieces, which is higher than $k' + 1$ pieces for m_{w+1} — another contradiction.

Each case contradicts m_w as maximum for w. By elimination, $m_w = 3k + 1$.

[5]The first step of our proof is to establish by contradiction that $k = 3k' + 1$.

Suppose $k = 3k'$. Then $k' = \frac{1}{3}k$, $t_k = 2k'$ and $t_{k+1} = 2k' + 1$. For both, k' is the same, but the tripartite rotations differ. Rotating gives us the simpler sub-puzzle. Since m_w and $m_w + 1$ differ by one weighing, it must be that $2k' = \hat{m}_w$ for some w. But: $\hat{m}_w = 3^w$, and no power of 3 is even — a contradiction.

Suppose $k = 3k' - 1$. Then $k' = \frac{1}{3}k$, $t_k = 2k' - 1$ and $t_{k+1} = 2k'$. So $2k' - 1 = 3^{w-2} \to k' = \frac{3^{w-2}+1}{2}$. Thus $k = 3k' - 1 \to k = \frac{3^{w-2}+3}{2} - 1 = \frac{3^{w-2}+1}{2}$, and $m_w = \frac{3^w+3}{2} + 1 = \frac{3^w+1}{2} + 2$. So we've got a formula. Plugging in values for w, we get: $m_0 = 3$, $m_1 = 4$, $m_3 = 7$, $m_4 = 16$ — all wrong. Thus $k \neq 3k' - 1$.

By elimination, $k = 3k' + 1$. Thus, for both m_w and m_{w+1}, $t = 2k' + 1$, so the difference must come from k' vs. $k' + 1$. This implies that $k' = m_w$ for some w, thus $k' = 3k'' + 1$. Further, it can't be that $2k' + 1 = 3^w + 1$, which would mean 3^w is even. Likewise, if $2k' + 1 = 3^w - 1$, then $2k' + 2 = 3^w \rightarrow 2(k' + 1) = 3^w$, the same contradiction. Therefore, $2k' + 1 = 3^{w-2} \rightarrow m_w = \frac{3^w - 1}{2}$.

Using our formula for k' in m_w+1 gives the same proportions as before ($t' = 2k''+1$, etc.), and all the same arguments apply — $k'' = 3k''' + 1$, and so on. So k'' is a max, and our formulas hold all the way down to $k^* = \frac{3^0 - 1}{2} = 0 = m_0$.

We can encapsulate this in an induction proof. We've seen that $m_w = \frac{3^w - 1}{2}$ and that $\forall k$ in m_w, $k = 3k' + 1$. Now let's prove the claim that $k_{m_{w+1}} = m_w$.

Base case: $m_0 = \frac{3^0 - 1}{2} = 0$, then $3(0) + 1 = 1 = \frac{3^1 - 1}{2} = m_1$

Induction: $m_w = \frac{3^w - 1}{2} \rightarrow 3(\frac{3^w - 1}{2}) + 1 = \frac{3^{w+1} - 3}{2} + \frac{2}{2} = \frac{3^{w+1} - 1}{2} = m_{w+1}$

[6]This step also adds the restriction of not rotating the good piece.

[7]Kirshner (2004: 88) gives a very accessible introduction to the concept of object a:

> Unlike the transitional object, which I believe Lacan considered a sign of the child's entry into the symbolic order as a separate subject, the *objet petit a* cannot be concretized as an actual thing. [I]t has been compared to the part object fantasy of Kleinian theory in that it represents an imaginary link between the infantile body and the mother—the breast, for example. However, it is neither a concrete feature of her anatomy nor a specific memory, but the fantasy of a loss established retrospectively, after the child has been 'subjectified'.

[8]A numerical analogue of Lacan's puzzle is to have n variables, all of which have value 1 except one with a value of either 0 or 2. The problem in this light is to isolate the 'bad' variable using only the addition and subtraction operators, where we can add k variables from one group ('on one scale') and subtract k variables from another.