Élie Ayache's *The Medium of Contingency*

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Introduction

Ayache's project is to outline the ontology of quantitative finance as a discipline. That is, he wants to find what distinguishes it as a *genre*, distinct from economics or even stocks and bonds—what most of us associate with 'finance'. Quantitative finance, dealing with derivatives, is a whole new level of abstraction. So Ayache has to show that economic and social concerns are *exogenous* (external) to derivative prices: the underlying asset can simply be treated as a stochastic process. His issue with probability is that it is epistemological—a shorthand for when we don't *know* the true mechanism. Taleb's notion of black swans as radically unforeseeable (unknowable) events is simply an extension of this. Conversely, market-makers—those groups of people yelling at each other in old movies about Wall Street—don't need probability to do their jobs. Ayache's aim is thus to introduce into theory the *practice* of derivatives trading—from within, rather than outside, the market. And it's reasonable to think that delineating the ontology of this immensely rich field will yield insights applicable elsewhere in philosophy.

This is not a didactic book. People coming from philosophy will not learn about finance, nor about how derivatives work. Ayache *reinterprets* these, assuming familiarity with the standard view. Even Pierre Menard—Ayache's claim to fame—is only given a few perfunctory mentions here. People coming from finance will not learn anything about philosophy, since Ayache assumes a graduate-level knowledge of it. Further, Ayache's comments on Taleb's *Antifragile* are limited to one page. The only conceivable reason to even skim this book is that you'd like to see just how abstract the philosophy of finance can get.

I got interested in Ayache because I write philosophy of economics. I wanted to learn what quantitative finance is all about, so several years ago I read through all his articles in *Wilmott Magazine*, gradually learning how to make sense of sentences like "Only in a diffusion framework is the one-touch option...replicable by a continuum of vanilla butterflies" (2006: 19). I've made it through all of Ayache's published essays. Now I've read this entire book, and I deserve a goddamn medal. I read it so that you don't have to.

Much of Ayache's reception so far has been quite silly. I recently came across an article (Ferraro, 2016) that cited Ayache's concept of 'contingency' as an inspiration behind a game based on sumo wrestling. (You can't make this stuff up.) Frank Ruda (2013), an otherwise respectable philosopher, wrote a nonsensical article comparing him to Stalin![1] Philosophy grad students occasionally mention

his work to give their papers a more 'empirical' feel (which is comparable in silliness to the sumo wrestling), especially Ayache's clever reading of Borges' short story on Pierre Menard—from which these graduate students draw sweeping conclusions about capitalism and high-frequency trading.

Ayache expects the reader to have already read *The Blank Swan*, which itself is not understandable without reading Meillassoux's *After Finitude*. Thus, for most readers, decreasing returns will have long set in. My goal here is to summarize the main arguments and/or good ideas of each chapter, divested of the pages and pages of empty verbosity accompanying them. I try to avoid technical jargon from finance and philosophy except as needed to explain the arguments, though I do provide requisite background knowledge that Ayache has omitted. So first, let's cover the most important concepts that the reader may find unfamiliar.

1 Basic Concepts

Derivatives "were invented in order to see what difference it makes today that a difference should occur later" (Ayache, 2013). Less prosaically, a derivative is a contract giving the right to buy or sell an asset—the *underlying*. For instance, a *call option* gives the right to buy an asset at a pre-specified price (the *strike price*); the idea is that if the asset's market price goes up, you can buy the asset at the strike price from the person who sold you the option, and then sell the asset at the market price, pocketing the difference. A *put option* gives the right to sell the asset at some strike price, so if the price goes down, you can buy the asset at the market price and sell it at the strike price. Insurance, in fact, is a kind of put option, and many other contractual relations that we take for granted can be construed (and valued) as options. Options, derivatives, and contingent claims are all the same thing. Note that a derivative is written on an underlying (which is what makes them 'derivative'). More complex derivatives (*exotic options*) can be written on other derivatives, or specify any kind of payoff conditions one likes.

Volatility is how much a variable wiggles as it goes along its trend. It's the same as *variance* in statistics. In finance, there are two ways to measure volatility. The first is to collect observations of the asset's price movements over time, and measure *historical volatility* using statistics. The second uses the Black-Scholes-Merton or **BSM formula** for pricing options. Originally, traders would plug all of the variables (like volatility) into the formula to get the proper price for an option. However, since at any time we can check the prices of other options selling on the market, that means we can just input the price into the BSM formula to get the *implied volatility*. Another nice thing about this method is that if a trader is using a specific model or algorithm, they can use prices on the market to infer if any changes have occurred in the variables have occurred (say, a jump in volatility). If market prices consistently differ from what your model says they should be, then you can use these prices to **recalibrate** your model, adjusting parameters such as volatility so that current market prices make sense.

The **Volatility Smile** is a curious finding related to implied volatility. Historical volatility is a fixed parameter—it ought to be the same for any option

written on the same underlying. But in fact, the implied volatility tends to go up the more an option's strike price differs from the market price.[2] The *smile problem* is a major outstanding problem in finance, and much work has been done trying to develop a better formula than BSM that avoids it, but even more complex models have higher-order 'smile problems' of their own. One way of interpreting the volatility smile is as the market's way of accounting for black swans: these options with a high difference between strike and market price tend to only pay off if an unlikely event should occur, so in effect people betting on black swans are forced to pay a premium. In this book, Ayache says little about the volatility smile directly, as that was the main focus in *The Blank Swan*. Yet, it remains the foundational problem behind his analysis. (See also p. 393. n. 23.)

Taleb defines **Black Swans** by three properties: 1) being an outlier; 2) having an extreme impact; and 3) having people explain it afterwards in a way that makes it seem predictable. Ayache's interpretation admits only the last category: retro-narrative (2010: 8). Taleb tends to frame black swans in epistemological terms: we don't *know* the correct statistical distribution, so instead of a bell curve (which ignores unlikely events), we ought to use fat-tailed distributions. Conversely, Ayache takes an ontological approach, arguing that the problem is with probability itself. For Ayache, a true black swan event creates the possibilities that will have led to it (Bergson). It thus has much in common with Badiou's definition of the event as a set that contains itself: it is non-wellfounded, having no foundation or narrative other than itself.

Contingency is the idea that something could have been different. Ayache adopts most of his thinking on contingency from the philosopher Quentin Meillassoux. In a very well-written argument, Meillassoux argues that contingency precedes possibility: that is, we can think that something could have been different without necessarily knowing a set of states so that if A was not the case, then B would be. Ayache spends a great deal of time expounding on this concept, as much of the appeal of Ayache's work to philosophers seems to be that he can relate such abstract speculations to reality so as to reveal insights that couldn't be got otherwise. Yet, unlike its central role in *The Blank Swan*, the notion of contingency contributes very little to Ayache's arguments in this book, with the exception of a clever reading of the Efficient Market Hypothesis in ch. 8.

The **Efficient Market Hypothesis** (EMH) states that all available information is expressed in price. This usually takes the form of a back-induction: if someone could make a consistent profit by some investment, other people already would have done so until the opportunity was gone. That is, you're not very likely to find a $20 bill on the street because someone else will have already picked it up. The EMH is not strictly true: its weaker form would rule out the existence of technical trading (quants who look for patterns in price data) and its stronger form would rule out the existence of fundamental trading (analysts who invest based on accounting and financial statements). Still, it is a very useful heuristic; in financial theory, it manifests itself in the '=' sign that lets us say anything at all. While Ayache purports to be critical of financial theory, he leans heavily on EMH-type reasoning, so he's not critical in this sense.

Immanence vs. **Transcendence** is a distinction that arises quite often in continental philosophy, and is likely unfamiliar to anyone outside of it. *Transcendence* is roughly the idea of a 'beyond' to human thought, such as a God that we can never personally know. Ayache therefore associates transcendence with epistemology, which is just the viewpoint in finance that he wants to avoid. Historically, many philosophers relied on transcendent ideas to advance their arguments, so it's now fashionable among continentals to emphasize immanence. *Immanence* is associated with 'within' or 'inherent'-ness, like the set of instructions codified in DNA to create a human being. Ayache wants to develop an immanent theory of the market, i.e. one that does not depend on anything outside the market itself. So while Taleb's 'black swan' is transcendent—we can never know it—Ayache's '*blank* swan' is a reinterpretation of it in an immanent way, so that black swans are inherent to the probabilistic worldview itself.

2 Overview

The **Introduction** summarizes Ayache's position by introducing all of the book's ideas at once. There's no way the reader can follow everything that's happening here, so it's mainly a chance to get used to Ayache's writing style.[3] What's worse, he frames the entire introduction through the motif of 'matter' (claiming that the market is 'material', and so on), which is purely empty rhetoric.

Ayache is interested in the genesis of the market for derivatives—not in the historical sense, but rather in the sense of necessary and sufficient conditions for the concepts of quantitative finance to coalesce. Ayache's concept of the market (or, occasionally, arché-market) is a purely abstract one—simple, not complex. "The market never starts, we are already in the middle of it" (2). For now, the most useful definition of what Ayache means by 'market' is a perpetual givenness of prices. At any time, we can find out an offer or asking price for a given option.

Price is different from value: Ayache wants to distance himself from economic or social concerns, such as someone's willingness to pay for an option (how much they *value* it), or how much a firm's accounting statements say an option is *worth*. "[A]gainst value as given by the valuation models, I argue for the immanence of the market and, consequently, that price is all there is" (5). From this abstract perspective, Ayache wants to argue for "a flattening of the hierarchy of underlying and derivative" (4). In a word, Ayache wants to show that the idea of derivatives 'deriving' from the underlying is misleading.

Valuation implies that derivative prices arise from deeper states of the world, which collectively act as a kind of 'random generator' that financial modelers can never truly know, but must try to approximate. This is precisely the viewpoint that Ayache wants to overcome. For him, the 'givenness' of prices introduces an *ontological* element, as opposed to the epistemological usage of probability. If we could reduce all financial concepts to price, and price is always 'given', then we will have constructed a suitable ontology of markets—all without probability. This, in a nutshell, is his aim in this book.

Ayache asks the reader to "picture the trading floor" (8), the kind of place Ayache used to work every day buying and selling options. The reason he knows that a non-probabilistic theory of markets is possible is that this is precisely the approach he took as a market-maker. For Ayache, concerns about probabilities are a step *out* of the market; he wants a theory that occurs in the middle of it. Now he complements this with his experience as a technology provider, where things that ordinary traders take for granted (like recalibration) become central—even though they're never even mentioned in the mathematics of derivative pricing. This tension between formalism and practice becomes more stark as the book progresses. Ayache's goal is to frame recalibration not as signifying an error in one's financial model, but instead as the essence of the market.[4]

Part I gives Ayache's main arguments, often multiple times using different words.

Chapter 1 summarizes Ayache's philosophical ideas. For Taleb, black swans tend to be overlooked due to backward narratives by 'experts'; for Ayache, probability *itself* is a backward narrative.[5] From the fact that something could have been different (contingency) we infer a set of possible states. A "radically-emergent event" thus occurs outside of probability (19). Such an event "changes the whole universe of possibilities" (20), and Ayache claims that this incommensurability rules out typical statistical notions such as expectations and Bayes' theorem. Instead, he points to quantum mechanics as a "meta-probabilistic calculus" (21), which inspires his own approach.[6]

In the context of finance, "history becomes quantitative" (46): the impact of any event is expressed solely in the dimension of price. Fluctuations in an asset's price are summarized as its *volatility*. The BSM formula, we recall, has two degrees of freedom: price and volatility. So while its canonical use is to input a volatility to price a derivative, it can also be inverted to infer the volatility *implied* by the price at which a derivative is trading (implied volatility). For Ayache, a suitable theory of markets must be similarly 'inverted'.

Ayache introduces a motif that recurs throughout the book: the insurance company.[7] Insurance companies' actuarial models claim to be able to predict the date of a particular person's death. They do this by accumulating massive amounts of health and mortality data, minutely classified by demographic characteristics. However, for Ayache this is not actual prediction, but instead merely an accounting identity (128-9): the company charges rates so that, conditional on its past business, it will turn a profit. To call this a 'prediction' is to confuse *ex-post* accounting (looking *backward*) with an *ex-ante* hypothesis of the unknown (looking *forward*). Ayache identifies this with frequentism in statistics, which infers probabilities from past data, but thereby opens itself to the black swan problem of whether the distribution might change. Note, however, that since this accounting equation is based on real data, it is in a sense *immanent* rather than transcendent. Lastly, Ayache suggests that viewing markets in terms of Brownian motion offers a way around frequentist views.

Chapter 2 summarizes how Ayache's ideas relate to finance. While recalibration is typically thought of as a tedious chore external to financial theory,

Ayache takes it as foundational. Many traders try to overcome the smile problem in BSM models by making volatility stochastic; yet, such models run into higher-order smile problems of their own, which even more complex models try to fix by adding jumps, and so on. Instead, Ayache advocates regime-switching models as a meta-model that integrates recalibration into the model itself.[8]

By blurring the difference between calibration and recalibration, regime-switching models draw not from 'states of the world', but rather from "*difference without underlying states*" (43). The notion of 'state' is Ayache's main bugbear throughout the book. In financial theory, it is embodied in the concept of Arrow-Debreu securities, a tool used in proofs of general equilibrium. The essential idea is to partition the world into states for everything, with securities betting on all such states.[9] Yet, Ayache notes that prices computed based on these states of the world will themselves become states of the world, which gives rise to "a perpetual breaching or non-closure of the ontological circle" (31).[10] For similar reasons, in *The Blank Swan* (2010: 166) Ayache concludes that since derivatives can be written on derivatives, financial markets virtually embody a Cantorian transfinite of derivatives written on derivatives written on derivatives...

In the standard view, such meta-derivatives are redundant and could never trade. A derivative is *redundant* if a combination of underlying securities can be made that behaves exactly the same way (e.g. the same payoff), i.e. if it can be *replicated* by the underlying. The standard method of expressing derivatives as functions of underlying prices makes them all redundant (42). Even in more complex models, derivatives should not trade because in so doing they become redundant (32). Yet, for Ayache, an asset trading on a market must by definition vary from any pre-assigned value (73), otherwise no-one would buy or sell it. Hence any derivative that actually *trades* must diverge from the function used to replicate it. Therefore, no derivative trading in a market can be redundant. This *principle of non-redundancy* thus establishes the non-closure of markets.

Chapter 3 spells out one of the most interesting arguments in the book: interpreting implied volatility through Brownian motion, which runs as follows. Derivative prices follow a random walk expressible as Brownian motion. Brownian motion is fractal, i.e. scale-invariant. Therefore all information about a derivative's random walk is contained in a single instant. This is what is being taken advantage of with implied volatility, i.e. when traders invert the Black-Scholes option-pricing formula to infer volatility from an option's current market price. Add this to his claim that derivatives intrinsically engender the creation of more and more complex derivatives (up to infinity) in a way similar to Cantor diagonalization, and we see that markets contain a 'double infinity'. This is why Ayache originally wanted to call his book *On the Infinity of Markets* (xiii).

Also of note in this chapter is Ayache's reading of Taleb's *Antifragile* on p. 55. He sums up Taleb's argument as: "The contrary of fragility (1) is not robustness (0), but antifragility (–1)." Taleb's advice to live in such a way as to benefit from volatility is to "adopt a static buy and hold strategy" rather than the dynamic hedging that has been the major accomplishment of finance. It amounts to "an apology for the asymmetric payoffs (also known as derivatives) that one can

find and purchase for free, outside the market" – that is, in life. Ayache's main criticism is that in *The Black Swan*, nothing at any scale was safe from a radical event, whereas antifragility nevertheless relies on scale. (Muscles get stronger from wear and tear, but not from getting hit by a bus.) What's really needed, suggests Ayache, is to think a form of antifragility that does not involve scale.

Chapter 4 is mostly spent reiterating Ayache's position, and its main new material is Shafer & Vovk's game theoretic interpretation of probability.[11] As everyone knows, probabilities are valued between 0 and 1, so that the values for all possibilities in a given scenario sum to 1. This is a mathematical system, axiomatically grounded in measure theory, similar to real analysis as a rigorous foundation for calculus. Curiously, however, the proposition that "an event whose probability is very close to 1 will happen" (Cournot's principle) is *external* to the formalism itself: it is an *interpretation* meant to bridge it with reality (389, n. 8). Shafer & Vovk's radical move is to *abandon* Cournot's principle.

Rather than values contingent on 'states of the world', Shafer & Vovk interpret probabilities as prices in a game against Reality. Probabilities are abstract, but money isn't: in the event of a black swan shifting the universe of possibilities, "we know what it means to lose money but we don't know what it means to lose probability" (94). For Shafer & Vovk, a probability of 1 means that there exists a winning strategy in the game, i.e. we can make an infinite amount of money—or else that Reality yields, and the event is realized. Here, probabilities do not depend on transcendent states, so Ayache characterizes game theoretic probability as 'immanent': all that matters is the trading strategy. His main complaint, however, is that the formalism still takes place in time (120): we must 'wait and see' how our strategy performs. Instead, Ayache believes that using the fractal property of Brownian motion, we should be able to "condense the whole market in a single point and a single place" (96).

Part II gives the philosophical foundations for the rest of the book.

Chapter 5 was published previously in *Collapse VIII* as "A Formal Deduction of the Market," extended while it should have been shortened. Ayache's thesis is that the law of large numbers takes the market from pure formalism into actual existence (108). The *law of large numbers* states that as the number of trials (say, rolls of a die) approaches infinity, the outcome approaches the true probability ($\frac{1}{6}$ for each side). Here, Ayache compares two ways to derive it.

The first is from Kolmogorov's axiomatic foundation of probability—a purely abstract account, where intuitive ideas such as "experiment, trial, event, or realization" are simply an *interpretation* (104). His main tool is measure theory, which helps both to eliminate impossible events (sets of measure zero) and to frame the law of large numbers as a limit (105). Kolmogorov's key problem is to represent the concept of independent trials: independence + repetition, both ideas being external to measure theory. He overcame this by the notion of a *random variable*: a single outcome, conceived of as variable (111). From there, the rest is just measure theory.

Another derivation by Richard von Mises actually defines probability *by* the law of large numbers (106). After postulating "a limiting frequency in an infinite sequence of trials," he defines a probability "*as* this limiting frequency" (107). His main mathematical problem thus becomes preventing probability from depending on a *specific* sequence, and also ruling out sequences that repeat themselves (109-10). His solution is an *axiom of randomness*, where in the infinite sequence behind the law of large numbers, it's not possible to pick a sub-sequence with a different limiting frequency (110). He gives an existence proof for such infinite sequences (called *collectives*), and shows that—out of all infinite sequences—quite a lot are collectives. The problem is that the very idea of *constructing* collectives is contradictory: if we could, it could be predicted, and so would not be random.

For Ayache, neither the ontology presupposed "by the von Mises's axiom of randomness [n]or by the un-identification in Kolmogorov" (119) is satisfactory. Kolmogorov's abstract framework requires us to postulate Cournot's principle for it to apply to reality (113), and von Mises's axiom re-introduces many of the problems inherent in the notion of 'random generator' (114).

Instead, Ayache advocates a 'non-chronological' modification of Shafer & Vovk's ontology that avoids both Cournot's principle (as we saw earlier) and also any notion of frequency akin to the results of a random generator (127). Ayache proposes that "time is not money, because money is more fundamental than time" (132). That is, in Shafer & Vovk's alternate to Cournot's principle, a probability-1 event "doesn't take place in time, but in money." Thinking in terms of price allows a "playing without expecting" (136), setting it completely apart from other interpretations of probability (frequentist, expectational, etc.). We "no longer wait for the event to happen" (139); our strategy is entirely 'local'.

Ayache conceptually links this to the radically 'local' nature of Brownian motion. His argument seems to be that we can index Brownian motion 'in money' rather than in time, and from there we can use the mathematics of derivatives (the stochastic integral) to shrink this fractal Brownian process to a single instant and "make the law of large numbers happen on the spot" (140). This in turn gives a causal priority to 'local' derivative pricing: "One should not infer Brownian volatility from the history of prices of the underlying asset, but from the instantaneously given derivative price"—that is, implied volatility (142). That is, Ayache seems to claim that this 'local' character of markets—at any time we are 'given' a price from which we can infer volatility—*creates* its infinitely fractal Brownian nature in a 'Big Bang'. What he wants to argue is that the market is formally deduced, even instantiated by, the theory. Yet, this Big Bang is short-lived as it starts to conflict with the principle of non-redundancy (145):

> Once we have the market constituted and all grades of derivatives trading,...it becomes a later elaboration to see that the derivatives prices do not accord with the hypothesis of their underlying following Brownian motion. As a matter of fact, since all the derivatives of all orders are given in the Big Bang, trading at prices that cannot be redundant, the conclusion follows that the underlying price could follow no stochastic process at all for the reason that any kind of process, once assumed, will

> make some derivative redundant. [...] Brownian motion is just needed to ignite the market and to constitute its meaning, and then it disappears.

This line of argument is quite representative of Ayache's style throughout the book: dense, long-winded expositions connected by ad-hoc quasi-aphorisms.

Chapter 6 recasts what was argued in ch. 5 through the terms ex-post vs. ex-ante. He situates possibility as "the ex-ante reformulation of an ex-post phenomenon" (147), whereas contingency is ex-post due to the blind fact(icity) that "a contingent thing...*could have been* different" (148). A purely ex-ante stance vis-à-vis the event is "an illusion" (146), as a "genuine event *can only happen*," i.e. cannot be conceived beforehand. This distinction is linked to Ayache's metaphor of the insurance company: "Counting is ex-ante whereas accounting is ex-post" (155). The main takeaway is a more concise synopsis of his argument in ch. 5, which I'll quote in full since I don't fully understand it myself (152):

> The derivative price is given, together with the idea of the infinitely fractal randomness of the underlying process which enables the convergence of the ex-post accounting equation on the spot and calls for the price of that derivative as a component of that accounting equation. We say it is an idea because Brownian motion cannot ultimately be given as a process. There are no stochastic processes that may be given in the market (either Brownian motion or any of its cognates) and their idea should never materialize because only derivative prices are given and the mere thought of an underlying stochastic process being given would make some derivative redundant and would prevent its price from being independently given. Even though the idea of Brownian motion is needed to trigger the whole argument and to produce the market as a consequence, it has to disappear almost as instantaneously because the whole place is created at once: derivatives of all grades of complexity are written at once and their prices are given at once, non-redundantly.

Ayache's 'inverted' view of the market replaces the notion of random generator with the givenness of market prices (from which we may imply volatility, etc.). The bare bones of his argument is that his principle of non-redundancy conflicts with the idea of representing derivatives by any underlying stochastic process whatsoever. Hence, says Ayache, we should toss out probability.

Part III is the most abstruse section of the book, as in *The Blank Swan*.

Chapter 7 links two main ideas: that the 'written' character of derivatives contracts lets options encompass contingency in a way not contained within the formalism, and that tree diagrams, each of whose branches representing a possible state, are unable to represent this contingency. Tree diagrams are helpful in thinking about finance because we are concerned with two states: either the price goes up or down, creating two 'branches', and from the next node it either goes up or down, and so on. In fact, the BSM formula can be viewed as a limit as the number of branches in the tree approaches infinity. However, no matter

how we modify a tree, it cannot encompass a state where the tree itself becomes invalid—a shift in probabilities due to a black swan.

Conversely, a derivative contract behaves similar to a tree in that it specifies a payoff if some state A should happen, and otherwise nothing. Yet, its difference from the abstract tree model is that it is *written* on paper (or else as a file on a computer, etc.). This *material* quality of the paper opens it to contingent events "outside the range that has been coded as *content* inside the paper" (166). The contract could be lost in a fire, or the bank could default, and so on. Ayache views this link between derivatives, writing, and contingency as foundational: "My whole argument is that the ontological property of writing, which is that we discover its provisions only after the fact, is the same ontological property as the market" (175; see also 177).

Ayache then parallels this with EMH-type reasoning to argue that markets are prone to irreducible randomness that can only be mediated by price as "the translator in time of future contingency" (168), and not by probability. Due to the EMH, "present price is ex-post, in a sense" (170); "a residue" or "almost a *trace* of the future event." He describes the ontology of price as follows (164):

> Price is not a value (a number); it is more archaic than value in the sense that an observable in quantum mechanics is more 'archaic' than the particular eigenvalue or in the sense that writing, in Derrida, is more archaic than state or being. The numerical value of price should never be abstracted away from price.

Ayache's aim is to divest finance of any reliance on epistemology. "Writing or not writing is an ontological question,...not an epistemological one" (175). As a result, many arguments in this chapter overextend his framework outside finance—for example, he (absurdly) argues that buying an orange from a supermarket should not be thought of in terms of supply and demand, but instead in terms of derivatives (161). Last, he presages that the formalism of derivatives avoids problems associated with temporality because it "doesn't cross the path of epistemology, or even ontology" (175).

Chapter 8 opens by paralleling Badiou's notion of the 'multiple' with Ayache's reading of price as "a multiple within which multiple states are not identified,...which would therefore be one...in a sense that does not contradict multiplicity" (185). He then interprets the EMH in terms of contingency, as follows. 1) In finance, if we knew what the price would be in the future, it would be that price *now*. 2) So, to be a price now implies that we *don't* know the future price: it could be different. 3) In this sense, we can say contingency precedes identity.

Ayache goes on to suggest that, rather than thinking differentially, we can avoid being caught up in temporality by thinking *integrally*, as in integrals in calculus (197). "Time does not exist; all that exists are the different ways in which we look at writing" (197). Instead, he frames money as 'material time' or 'time turned physical' (202). "Money is the number (the *numéraire*) of writing, and it alone permits the materialization of the time of contingency and of the passage of the event" (203). He then defines the market as "the continual event"

(205), and parallels the irreversibility of losing money with the irreversibility of the event (207). "The market, this infectious real that is opposed to the possible, takes place before the event in this new scale of time which is money" (206). So rather than predicting the event, one must performatively *write* the event from within (211). "Writing has no aim except to explore the other face of the event, to write it in advance, outside of time" (201).

Chapter 9 extends the ideas of ch. 8, which he recapitulates as follows (213):

> The market is the special case where money has replaced time and the number of money has replaced the number of probability (which was inactive in time), thus placing us in the middle of the event.

While this chapter is even more abstract than the last one, a primary goal is to explain how it is that alternate interpretations of finance theory arise to thought. His answer is that it depends on whether one's philosophical orientation is based on transcendence (other theories) or immanence (his theory) [215]:

> The exchange of contingency is impossible in transcendence and necessary in immanence. Given contingency as absolute, the exchange can be defined either as immanence or as transcendence, depending on whether we decide to start from contingency and draw the conclusion of necessity (that is to say, deduce matter: speculative materialism) or that of impossibility (the black hole of possibility before contingency) – depending on whether we decide to venture, with the event, into place or into time, into the synchronicity of money or into the impropriety of time.

Therefore: "[t]he market is this inversion of metaphysics" from transcendence to immanence (216). Ayache's repertoire of concepts thus oppose standard concepts as follows: "Money is the diversion of time and price is the diversion of probability (the contingent claim is the diversion of the possible state)" [216].

Chapter 10 is more of the same, but is at least short. The most useful excerpt is a somewhat more 'concrete' explanation of money as replacing time (243):

> The possible state is only a state, as we have said, in virtue of the transition that makes it proceed from possible to actual, and which operates in time. But money replaces time, and is concerned with the transition between the price of the contingent claim (or the place that takes place 'before' and which is the market) and its final strike. Just as time is supposed to realize possibility in a false role-play with reality (a representation), money *realizes* contingency in the true sense of reality. It realizes contingency in the sense that that which translates the contingent strike at expiry—money—precisely does not make a *state* out of it (for the state is that which lends itself to a false realization, that of possibility) but a mark. The state $(A, B, ...)$ that is written on the contingent claim (if A, pay $x(A)$, if B, pay $x(B)$, ...) is a 'semblance of a state'. It is the mark of contingency and not of identification.

Part IV is the most technical part of the book, in which Ayache argues, step by step, that all state variables in finance are reducible to prices. Readers from finance: read Part IV, skip the rest. Philosophers: skip Part IV.

Chapter 11 critiques the mathematical foundations of quantitative finance, such as the fundamental theory of asset pricing, martingales, and Girsanov's theorem. Readers familiar with finance should start here. I admit that much of it is above my level, so I draw heavily from Johnson (2015).

First, let's recall the EMH. In an efficient market, the expected value of a price tomorrow is its price today; otherwise, people would have traded based on that information and the price would already be tomorrow's price. (So the EMH can be thought of as a *no-arbitrage* rule.) In algebra, we can represent this state of affairs as $\mathbf{E}_{\mathbb{Q}}[X_T] = X_0$, where X_0 is today's price, X_T is tomorrow's price, and $\mathbf{E}_{\mathbb{Q}}$ is our 'expectation' operator, defined using measure theory. The $\mathbb{Q}$ is what is known as a risk-neutral measure or *martingale*, which is defined precisely so that this equality exists (Johnson, 2015: 53). It is called *risk-neutral* because it does not take into account how risk-averse the investor is: normally, investors prefer a less risky asset with a small payoff to a risky asset with a large payoff, even if the *expected payoffs* are the same. (Conversely, in markets such as gambling and lotteries, investors are risk-seeking.) With a risk-neutral measure, expected values are defined simply as equal to the expected payoff. This makes martingales very handy for derivative pricing, as we completely avoid having to measure risk aversion; we can deduce it purely from market prices.

The first fundamental theorem of asset pricing is that if a market satisfies no-arbitrage, then a martingale measure exists, and vice versa. The second fundamental theorem is that if an arbitrage-free market is complete, then the martingale measure is unique (there is only one)—and also vice versa. Since both theorems work in both directions, they say nothing about causality. Yet, Ayache claims that "what seems to underlie both arguments and their reciprocity is the *assumption of definite states*" (252).

Ayache's aim is thus to show that the state variables in finance can all be replaced with *price*, and use that to reinterpret these theorems. For instance, volatility is representable as the price of a variance swap, a kind of derivative whose underlying is just the volatility of some asset (252). Similarly, the state of whether a corporation will or will not default is contained in the price of a credit default swap (CDS), a derivative that pays off if default occurs. Hence, "[a]ny complication and further enrichment of the 'process' of the underlying should be thought through the richness of prices of derivatives of increasing complexity, not through the complication of its stochastic process" (264).

Ayache points out that "the simultaneous existence of two prices (of the underlying and of the option) implies that the first is stochastic (255); otherwise, the derivative would always be redundant. He goes on to argue against mathematical representations of derivatives as lotteries (258-9)—a standard mathematical simplification—since these, in one way or another, imply redundancy. Ayache thus reinterprets lotteries in terms of a "pre-probabilistic randomness" of prices: prices are random due to the EMH, and "its being a price is what

fixes the value of the lottery that it is because it is random" (260). Price is a better state variable than lotteries because it is both itself stochastic and also "its stochastic nature is *represented* by something of the same nature" (255).

Ayache next wants to show that the framework of martingale measures avoids both subjective probability *and* objective probability. He stresses: "the market is not a subjective probability problem, in which agents are asked to assign prices to derivatives in an internally coherent way" according to their preferences over lotteries (264). Ayache claims that valuation of a lottery is not a *price* in his sense of the term, since this view attributes any price change to some state change in the underlying—exactly what Ayache is trying to avoid. Instead, Ayache claims that "[t]he market has always been an evaluator in this scenario (verifying non-arbitrage, selecting the martingale measure); it has never been a generator (a creator) of prices" (265). Similarly, Ayache sidesteps objective probability by saying that "The derivatives market...has always already selected the martingale measure" (265), which by definition is equivalent to the objective measure (264).

Lastly, Ayache foreshadows that in his later arguments he will analyze martingales with the same reasoning with which he treated Brownian motion (266):

> Once the framework is in place in which no derivative is redundant and the state space is all made up of independent prices of all grades of derivatives (no longer valuations), we will see that the first picture in which we spoke of martingale measure had by the market, or what the market *believes* the future volatility should be, and so on, doesn't make sense. The whole chain is a single point which communicates directly with the event (the continual event).

Chapter 12 extends Ayache's principle of non-redundancy. He links it to the notion of incomplete markets, which he claims is a "contradiction in terms" (400, n. 1). One definition is that a market is incomplete when "the contingencies of the world or the economy...exceed the number of traded assets" (276). Hence it is "indissolubly...linked with the metaphysics of background states of the economy" (280), and Ayache has to rule it out. Another reason this is important is that if a market is arbitrage-free and complete, then there is a unique risk-neutral measure (as we saw earlier), but if an arbitrage-free market is *incomplete*, uniqueness fails—and so, derivatives lack a unique price.

But recall Ayache's claim that derivatives written on derivatives make up a transfinite number of (virtual) derivatives, all of which in principle admit a price: hence, in Ayache's view, a market can be *ontologically* incomplete, whereas the standard definition of incomplete markets is impossible. The market is independent of the economy, but as a "total that is never totalized" (280).

Chapter 13 is easily the book's strangest.[12] Here, Ayache argues that derivatives markets don't exist. Ayache helpfully summarizes the chapter on p. 301:

> We didn't need probability when we argued that price could move up or down (the binomial [tree]) and that we could trade variable size at any price, [which is] equivalent to the writing of contingent payoffs, still without probability.

Recall from ch. 7 that Brownian motion can be viewed as a continuous limit of the discrete binomial tree. Further, if the market's stochastic process follows either of these, the market is complete (283). Ayache favors Brownian motion because "it comes before probability because of the EMH, and...*for this reason* it has the martingale representation property and all contingent payoffs are replicable" (284). That is, Ayache interprets the EMH as implying that "[r]andomness in the market precedes probability" (283); he also cites a study by Vovk showing that "probability emerges second to trading" (267).

Ayache then centers on the distinction between contingent payoffs and contingent claims. This is, in fact, one of the major problems that led him to write this book.[13] As we know, a contingent claim is simply another word for 'derivative'. A *contingent payoff*, by contrast, can be thought of as a replication of a derivative using only the underlying. While examining the formalism of derivative pricing, Ayache noticed that such papers only mention the market for the underlying asset—and not that derivatives trade in a market of their own. So in such treatments, any 'derivatives' being dealt with are only *contingent payoffs*. Ayache (2014: 2) states that this was the major discovery of BSM: that in a market defined by Brownian motion, we can buy or sell and dynamically readjust the underlying "in order to end up with any payoff profile as a function of S, or $f(S)$, at any time horizon"—i.e., we can use the BSM formula to manufacture any contingent payoff we want.

To treat a contingent claim purely within the formalism, we would have to assume that "derivatives independently exist as written contracts...that may admit of an independent price" (Ayache, 2014). But the formalism doesn't do this. Ayache draws an odd conclusion from this: since contingent claims do not exist within the formalism, by fiat they cannot be redundant. And since they're not redundant, they must therefore admit of a market (299). "Redundancy is such a negation of the market that it needs something stronger than negation, which is non-existence" (299). It's precisely because contingent claims do not exist that they must exist.

Chapter 14 deals with the fact that Brownian motion is the limit of the binomial tree model as the number of branches approaches infinity. This convergence poses a problem for Ayache, as it requires the use of probability "in order to make sense of the limit" (301). To get around this problem, Shafer & Vovk used non-standard analysis (304), which Ayache views as a compromise.

Historically, the concept of a continuous-time stochastic process originated with Louis Bachelier's treatment of markets as a (Brownian) random walk (300). In fact, the EMH implies that Brownian motion must be random at all scales; if it were predictable at some scale, people would have traded based on it until it was likewise random (302). Thus, points out Ayache, we can view price as ontologically prior to Brownian motion (302). "The price is random because it is a price, and because it is a price, it is its own present value" (306): that is, price is likewise prior to expectations (of present value). Further, since "price is not random for reasons that are external to price," therefore "[t]he market is the

self-contained theory of its own randomness and of its own valuation" (306.) In a sense, the market is its own theory—inseparable from empirical 'reality' (309).

Ayache wants to invalidate objections either that Brownian motion isn't the case, or that it is the case but its volatility is not constant (310). His first answer is that "Brownian motion with constant volatility is *formally* given and only formally given" (310), and that "[t]he only interpretation of the formalism is implied volatility" (311). His second answer is that the timelessness of the formalism, coupled with the fact that prices are instantaneously given in the market, makes the question of whether volatility is constant or not in (temporal) *reality* merely 'philosophical', in the derogatory sense. Both within the formalism and when we apply the formalism (e.g. implying volatility) the question doesn't arise; time only arises in a probabilistic mindset (312). "[I]t is what is formal in the formalism (as such written, detached from physical time or physical reality, detached even from epistemology and ontology) that engenders the market" (313).

Recall how BSM admits two kinds of volatility: historical volatility (by recording price movements in the underlying over time) and implied volatility (instantaneous). For Ayache, "BSM is the expression of the intimate link between Brownian motion and the couple formed by underlying and derivative" (316), because in a market with Brownian price dynamics, the fractal nature of Brownian motion makes these two orders of discourse—temporal and atemporal—the same. This fractality is "why volatility is the only thing at work in the relationship between the price of the underlying and of the option" (316). Therefore (317):

> The form that Brownian motion is, is the same form that the market is, when it is understood that the *form of the market* is the relation between underlying and derivative. Because of this equivalence of form, the instant option price is the instant estimator of volatility. Either the option price exists to translate the form of formal Brownian motion, or it doesn't exist and it jumps into existence because of the martingale representation property, because the contingent payoff gets written as a substitute for the withholding of time. The form of BSM that needs generalizing is the one that says that the options market doesn't exist. Generalizing it amounts to saying that this market exists.

Chapter 15 is ostensibly meant to philosophically tie the book's arguments together. In reality, much of the chapter is self-praise, where Ayache baldly compares himself to David Hilbert (333), Paul Cohen (341), and Einstein (351).

Ayache criticizes sociologists' view of markets as 'performative', where models act as self-fulfilling prophesies: "Models of quantitative finance are not performed or enacted, as sociologists of finance hold; they join the archive" (322). In this sense (as he goes on at length) the market is a 'book'. For Ayache, the market "cannot be formalized by theory, but only by a book" (331), and "[t]here is a book of the market insomuch as there is no theory" (336). As well as the present book (330), by 'book' Ayache also has in mind a derivative pricing technology.

Since Ayache's ideas have (rather inexplicably) found favor among artists, on pp. 339-40 he spends a section drawing a very strained parallel of derivatives

and contemporary art. In his view, "[t]he ontology of contemporary art should borrow...from the market of contingent claims...with regard to chronology and to the relation with representation and external thought or framing" (340). Ayache relates an anecdote where he "was told the purpose of a museum of contemporary art is nothing else, any longer, than...presenting, as of today, what future generations will read as the history of contemporary art" (340); compare the EMH with regard to price. Thus, "[t]he reason why the market of contingent claims has no end is the same reason why contemporary art has no end" (339).

Ayache then extends on the theme of prices as a kind of double lottery, that is, "that the underlying price is both producing the randomness and is the price of the endogenous lottery" (341) as we saw in ch. 11. He now confusingly (and irritatingly) refers to this as 'trading force'. In a thought experiment trying to describe the genesis of derivatives, he argues that if we start out with an asset trading 'with force' and then add derivatives afterwards, "inter-temporal arbitrage will force non-arbitrage relations between the two price processes prior to expiry," thereby "robbing the derivative of its trading force and its originality" (342-3). Ayache concludes that "to respect the order of thought of the formalism,...the only way to have derivatives formally and to have them trade with force is to create their market out of the void (the genesis)" [343].

Ayache notes that in his qualitative view of price, "the only contingencies are up and down prices movement of a traded asset, [so] by definition the contingencies are spanned by a traded contingent claim" (349), hence the market is complete by fiat. "The whole language of martingales and incomplete markets comes from the probabilistic framework" (344) because "there is no martingale measure [for the market] to select," just as "there is no contingent claim" (344-5). "*Derivatives are valued at once as prices*," bypassing the "intermediary step of valuation by non-arbitrage in the martingale measure" (336).

Ayache closes by noting how the present book follows from his last (350):

> In *The Blank Swan*, I said that the whole book was trying to find the meaning of implied volatility. This is the same void as the blank interval between contingent payoff and contingent claim. [N]othing exists in the blank and that matter has to be created anew. Hence, the book.

The **Denouement** outlines the stakes involved in choosing between Ayache's interpretation vs. the standard view in finance. As we've seen earlier (353):

> The two incompatible presentations are the following: either states of the world and their probabilities are considered first, or the trading assets and their prices are considered first. Either randomness is considered first – ...of which it is only one possible and *later observation* that prices of an underlying trading asset are coincident with its states (and along with this initial randomness general lotteries are considered, whose payoffs are defined in all future states of the world and whose values are studied in the present state) – or the market is the only thing that is considered, [as] the only source of randomness and the only horizon. Either the problem

> is one of valuation of lotteries, or it is one of trading, in which case there should be no valuation but only transactions and prices.

The standard view in finance is to say that BSM is wrong—volatility is not constant but stochastic, and so we must use more complex models, which in turn engender even more complex models, ad infinitum. Conversely, for Ayache "BSM is not a model and, because it is not a model, no model can surpass it" (371). Rather, BSM mediates between two separate orders of discourse (360):

> How the contingent payoff becomes a contingent claim whose price is available directly without there having been the intermediary step of valuation, is by inverting the BSM formula. This, we think, is the meaning of implied volatility.

Implied volatility is entirely separate from the σ^2 written on paper. "Implied volatility can be stochastic...without entailing that we should revise the model and make its volatility stochastic" (361). Rather, its purpose is to let "traders recognize the alternative road...in which there are no stochastic processes, no probability, no valuation and, consequently, no principle of non-arbitrage" (361).

Ayache characterizes his own view of Brownian motion as 'qualitative': the qualities of 'up vs. down' of a binomial tree, as we know by now, converge to Brownian motion as "the continuous-time limit of this binomial walk" (356). Conversely, innovations such as trinomial trees "differ in nature, not in degree or number of states" from binomial trees by introducing states of the world, giving rise to the 'quantitative' standard view (356). Brownian motion thereby becomes just another stochastic process, thus "an infinity of equivalent martingale measures is admissible" (357), and the market becomes incomplete.

Purely within the formalism, we can only treat contingent payoffs, whereas the fact that the formalism was written for the purpose of actually *trading* derivatives is relegated to the status of tacit knowledge (357). Ayache offers a new interpretation of the formalism, and "[a]n interpretation of the formalism is always a step outside the formalism" (360). Rather than a transcendental view, establishing the uniqueness of the martingale measure in a deus ex machina choice, Ayache offers a theoretical edifice that forces contingent claims into the formalism by means of the void, without collapsing them into valuations (361-2):

> A formalism of the market of contingent claims therefore requires recalibration processes, not stochastic processes. It is a meta-formalism vis-à-vis the formalism of stochastic processes because what is being recalibrated is a stochastic process. This is how the *usage* of BSM (and of its cognates, such as stochastic volatility models or jump-diffusion models, and so on) can become part of a superior formalism.

The **Conclusion** concisely summarizes the arguments made throughout the book, and links them to the technology developed by Ayache's company, ITO33.

As Ayache's arguments are all confined within financial theory, it's easy to be skeptical that they at all relate to the actual practice of trading. In fact, many of

them spring from his viewpoint as technology provider. To objections of "Who cares?", Ayache notes that financial algorithms crucially rely on concepts such as random generators and states of the world, while excluding recalibration (365). Algorithms therefore reify the probabilistic interpretation of markets.

Ayache tells a story in one of his lectures that helps to underscore the point.[14] When Ayache used to work as a market-maker, at one point there was a guy who would call in to buy out-of-the-money puts—a derivative that gives you the right to sell the underlying at a specified price (the strike), but where the strike price is lower than the current market price.[15] He wanted to buy puts that were so out-of-the-money (the strike price was so low) that they only costed one cent.

For such a derivative to actually make a profit was extremely unlikely—so much so that the market-makers had no statistical data about how out-of-the-money an option would be worth that little. So instead, they inverted the BSM formula, plugged in the other variables, and then made a plausible-sounding guess. In essence, they *created probability.* A human can do that, but an algorithm can't. Ayache conjectured that guys like him are responsible for the fact that people who buy options with very low or very high implied volatility have to pay a premium—that is, the volatility smile. Because eventually, a black swan event *will* happen, and guys like him will make a ton of money. (Selling a worthless stock for a few cents is profitable with enough shares, and this guy bought as many as they would sell him.) For Ayache, cases like this reveal the market's essentially human character—one entirely lost to algorithmic and high-frequency trading.

Conclusion

To understand the statement "Time is money", who should we ask: a taxi driver, who has lived and breathed its truth every day, or some greasy graduate student who has read lots and lots of Deleuze? Like it or not, Ayache is our taxi driver, and the difference between him and the legions of humanities professors pontificating about capitalism is that Ayache actually has something to say. An unreadable book by Ayache is worth ten thousand 'scholarly' humanities papers.

If this book was a third or even half its size, stripped down to its fundamentals, it would be a classic. Instead, Ayache writes as if he is coming up with his ideas as he writes, rather than communicating results so that others can understand. In five or six years, Ayache will no doubt write an even thicker book that presumes the reader has gone through both *The Blank Swan* **and** *The Medium of Contingency.* Let's just say, I think I'll hold out for the audiobook version.

Griping aside, Ayache is an excellent speaker, and those hoping for more insight into Ayache's ideas are encouraged to listen to his online lectures. For those foolish enough to read this book: it is not a book to be read with pleasure, but rather a quintessential subway book—to be read in small increments in time otherwise lost. This review having taken care of the arguments, the casual reader is free to focus on the philosophical bits. Lastly, the reader need not pay full price, as they should have little trouble finding a (very lightly) used copy.

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Notes

[1]For the record, Ruda's account is worth very little because he does not understand the difference between prices and derivatives. (Yes, really.) He writes (2013: 75) "Prices are contingent claims that produce a difference"—which is wrong because contingent claims (another name for derivatives) and prices are quite separate things, and because neither derivatives nor prices (as such) 'produce' a difference.

[2]An option is *out-of-the-money* if by exercising the option now, you would lose money. For a call, this is when you have the right to buy an option at a higher strike price than its market price; for a put, it's when you have the right to sell an option at a lower strike price than its market price. An option is *at-the-money* if by exercising it now you would break even—that is, the strike price is equal to the market price. (This is the same for calls and puts.) An option is *in-the-money* if by exercising it now, you would make a profit. For a call, this is when the market price exceeds the strike price; for a put, this is when the strike exceeds the market price. In general, we refer to which of these states an option is in as the **Moneyness** of an option.

[3]Ayache tells me that originally the introduction and conclusion were one text, later split in half—hence the overlap in style and concepts (e.g. 'trading force', martingales). This gloss of it is very gentle so as to avoid scaring readers away at the outset, but by the end of this review the reader will have a far better idea of what Ayache is getting at.

[4]In a short film about his ideas entitled 'Contingent Claim' (2012), Ayache gives a far more concrete explanation of what he means by markets as a 'medium' of contingency: "Markets are here to actually trade events before they happen" (30:10–14), and "I argue that [option traders] are producing the bridge, the material bridge or the material medium which is the market, with which to connect the future events that may happen, to the stuff that is trading" (34:08–25). The definition in the book is far more obscure.

[5]In a very dense argument in ch. 9 of *The Blank Swan*, Ayache argues that both possibility *and* necessity are established via retro-narrative

[6]Ayache's interpretation of quantum theory draws mainly from Michel Bitbol.

[7]Ayache gives a more detailed exposition of actuarial 'single-case statistics' in his *Wilmott* articles of February, March, and October 2011.

[8]In an interview (2007: 262), Ayache explains regime-switching models as follows:

> The key observation regarding the regime-switching model is that it is self-similar under its own stochasticization. It incorporates its own meta-model. Recalibration will make the regime-switching model stochastic like any other model. However, since stochastic variables are expressed in regimes, regimes of regimes are also regimes. So all we have are regimes; we don't have to model recalibration a priori; recalibration and expansion of the state space occur whenever they occur and they put new names on the regimes. Since the regimes have no predefined names, it is not clear that the new, richer stochastic model and its larger state space were not with us all along, only we didn't distinctly perceive them. The model is open to change and upgrading, because at no point do we lock ourselves in the myth of perfect calibration and its correlate, perfect hedging. Rather, we propose optimal hedges and we measure the standard deviation of the P&L. This provides just the right leeway for the model to be robust under successive recalibrations.

Ayache also includes a very helpful appendix; see especially the caption to fig. A.3.

[9]This Arrow-Debreu world in which *everything* has a market seems very strange, but like with many mathematical ideas, it is far easier to consider infinite markets than extremely big finite markets. Note that in the complete market of Arrow & Debreu, all that matters is the underlying, and all derivatives are redundant.

[10]This argument is due to Philippe Henrotte, Ayache's colleague at ITO33.

[11]Ayache's exposition spills into ch. 5, but here I'll treat it all at once.

[12]Of note in ch. 13 is a passage (297-8) likely intended as a reply to Jon Roffe. Roffe's Ph.D thesis draws heavily from *The Blank Swan*, and his deepest criticism of Ayache is that he doesn't address the entire market. While at first glance this may appear to be low-hanging fruit, Roffe has found favor among readers who think Ayache is too much of a capitalist, as he focuses too much on actual markets and not enough on Deleuze.

[13]Ayache puts his argument most clearly in a lengthy abstract for a conference paper (Ayache, 2014), which I'll be following quite closely in this section.

[14]This is from Ayache's 2011 DOCH lectures. He mentions the story on p. 333 of *The Blank Swan*, but focuses on another aspect of it.

[15]See note 2 above for an explanation of the term 'out-of-the-money'.