1. Starkville Police Department requires different numbers of police officers on different days of the week, as shown in the following table:

Day	Number of police officers required
Monday	20
Tuesday	16
Wednesday	18
Thursday	20
Friday	15
Saturday	17
Sunday	12

Typically, police officers work a shift consisting of five consecutive days (e.g., Monday-Friday, Tuesday-Saturday, etc.). However, suppose that the police department can force officers to work one day of overtime each week. For example, an officer whose regular shift is Monday to Friday can also be required to work on Saturday. Each officer is paid $50 a day for each of the first five days worked during a week and $62 for the overtime day (if any).

A. Formulate an LP whose solution will enable the police department to minimize the cost of meeting its weekly work requirements.
B. Suppose the police department had 25 full-time officers and was not allowed to hire or fire any officers. Show how you could modify your formulation from part A to schedule the officers in order to maximize the number of weekend days off received by the officers.

Part A: Formulate an LP whose solution will enable the police department to minimize the cost of meeting its weekly work requirements.

Variables

- Let x_1 = Number of officers who start work on Sunday and work 5 days, x_2 = Number of officers who start work on Monday and work 5 days... x_7 = Number of officers who start work on Saturday and work 5 days.
- Also let o1 = Number of officers who start work on Sunday and work 6 days... o7 = Number of officers who start work on Saturday and work 6 days.

Then the appropriate LP is

$\min z = 250(x_1+x_2+...x_7) + 312(o1+o2+...o7)$
s.t. $x_1+x_4+x_5+x_6+x_7+o1+o3+o4+o5+o6+o7 \geq 12$ (Sunday)
$x_1+x_2+x_5+x_6+x_7+o1+o2+o4+o5+o6+o7 \geq 20$ (Monday)
$x_1+x_2+x_3+x_6+x_7+o1+o2+o3+o5+o6+o7 \geq 16$ (Tuesday)
$x_1+x_2+x_3+x_4+x_7+o1+o2+o3+o4+o6+o7 \geq 18$ (Wednesday)
$x_1+x_2+x_3+x_4+x_5+o1+o2+o3+o4+o5+o7 \geq 20$ (Thursday)
$x_2+x_3+x_4+x_5+x_6+o1+o2+o3+o4+o5+o6 \geq 15$ (Friday)
$x_3+x_4+x_5+x_6+x_7+o2+o3+o4+o5+o6+o7 \geq 17$ (Saturday)
All variables nonnegative

```
var x1 integer >= 0; #start on Sunday and work 5 days
var x2 integer >= 0; #start on Mon and work 5 days
var x3 integer >= 0;
var x4 integer >= 0;
var x5 integer >= 0;
var x6 integer >= 0;
var x7 integer >= 0; #start on Sat and work 5 days
var o1 integer >= 0; #start on Sunday and work 6 days
var o2 integer >= 0; #start on Monday and work 6 days
var o3 integer >= 0;
var o4 integer >= 0;
var o5 integer >= 0;
var o6 integer >= 0;
var o7 integer >= 0; #start on Mon and work 6 days

minimize Cost: 250*(x1+x2+x3+x4+x5+x6+x7) +
312*(o1+o2+o3+o4+o5+o6+o7);

s.t. SundayDemand: x1+x4+x5+x6+x7+o1+o3+o4+o5+o6+o7 >= 12;
s.t. MondayDemand: x1+x2+x5+x6+x7+o1+o2+o4+o5+o6+o7 >= 20;
s.t. TuesdayDemand: x1+x2+x3+x6+x7+o1+o2+o3+o5+o6+o7 >= 16;
s.t. WednesdayDemand: x1+x2+x3+x4+x7+o1+o2+o3+o4+o6+o7 >= 18;
s.t. ThursdayDemand: x1+x2+x3+x4+x5+o1+o2+o3+o4+o5+o7 >= 20;
s.t. FridayDemand: x2+x3+x4+x5+x6+o1+o2+o3+o4+o5+o6 >= 15;
s.t. SaturdayDemand: x3+x4+x5+x6+x7+o2+o3+o4+o5+o6+o7 >= 17;
```

```
reset;
model 4c-police.mod;
option solver cplex;
solve;
display x1,x2,x3,x4,x5,x6,x7,o1,o2,o3,o4,o5,o6,o7;

ampl: include 4c-police.run;
CPLEX 20.1.0.0: optimal integer solution; objective 6056
13 MIP simplex iterations
0 branch-and-bound nodes
x1 = 4
x2 = 0
x3 = 1
x4 = 0
x5 = 2
x6 = 1
x7 = 0
o1 = 0
o2 = 8
o3 = 0
o4 = 3
o5 = 0
o6 = 0
o7 = 2
```

Part B: Suppose the police department had 25 full-time officers and was not allowed to hire or fire any officers. Show how you could modify your formulation from part A to schedule the officers in order to maximize the number of weekend days off received by the officers.

- Add the constraints: $x_1+x_2+x_3+x_4+x_5+x_6+x_7 + o_1+o_2+o_3+o_4+o_5+o_6+o_7 = 25$
- change the objective function to max $z = x_1+2x_2+x_3 + o1 + o2$

```
var x1 integer >= 0; #Sunday-Thur (1)
var x2 integer >= 0; #Mon-Fri (2)
var x3 integer >= 0; #Tue-Sat (1)
var x4 integer >= 0; #Wed to Sun (0)
var x5 integer >= 0; #Thur to Mon (0)
var x6 integer >= 0; #Fri to Tue (0)
var x7 integer >= 0; #Sat to Wed (0)
var o1 integer >= 0; #Sun-Fri (1)
var o2 integer >= 0; #Mon-Sat (1)
var o3 integer >= 0; #Tue-Sun (0)
var o4 integer >= 0; #Wed-Mon (0)
var o5 integer >= 0; #Thu-Tue (0)
var o6 integer >= 0; #Fri-Wed (0)
var o7 integer >= 0; #Sat-Thu (0)
```

```
maximize Cost: x1+2*x2+x3 +o1+o2;

s.t. SundayDemand: x1+x4+x5+x6+x7+o1+o3+o4+o5+o6+o7 >= 12;
s.t. MondayDemand: x1+x2+x5+x6+x7+o1+o2+o4+o5+o6+o7 >= 20;
s.t. TuesdayDemand: x1+x2+x3+x6+x7+o1+o2+o3+o5+o6+o7 >= 16;
s.t. WednesdayDemand: x1+x2+x3+x4+x7+o1+o2+o3+o4+o6+o7 >= 18;
s.t. ThursdayDemand: x1+x2+x3+x4+x5+o1+o2+o3+o4+o5+o7 >= 20;
s.t. FridayDemand: x2+x3+x4+x5+x6+o1+o2+o3+o4+o5+o6 >= 15;
s.t. SaturdayDemand: x3+x4+x5+x6+x7+o2+o3+o4+o5+o6+o7 >= 17;
s.t. CannotHirOrFire: x1+x2+x3+x4+x5+x6+x7 = 25;
s.t. CannotHirOrFireOT: o1+o2+o3+o4+o5+o6+o7 = 25;

reset;
model 4c-police-modified.mod;
option solver cplex;
solve;
display x1,x2,x3,x4,x5,x6,x7,o1,o2,o3,o4,o5,o6,o7;

ampl: include 4c-police-modifed.run;
CPLEX 20.1.0.0: optimal integer solution; objective 21
2 MIP simplex iterations
0 branch-and-bound nodes
x1 = 8
x2 = 0
x3 = 5
x4 = 0
x5 = 4
x6 = 0
x7 = 0
o1 = 0
o2 = 8
o3 = 0
o4 = 0
o5 = 0
o6 = 0
o7 = 0
```