

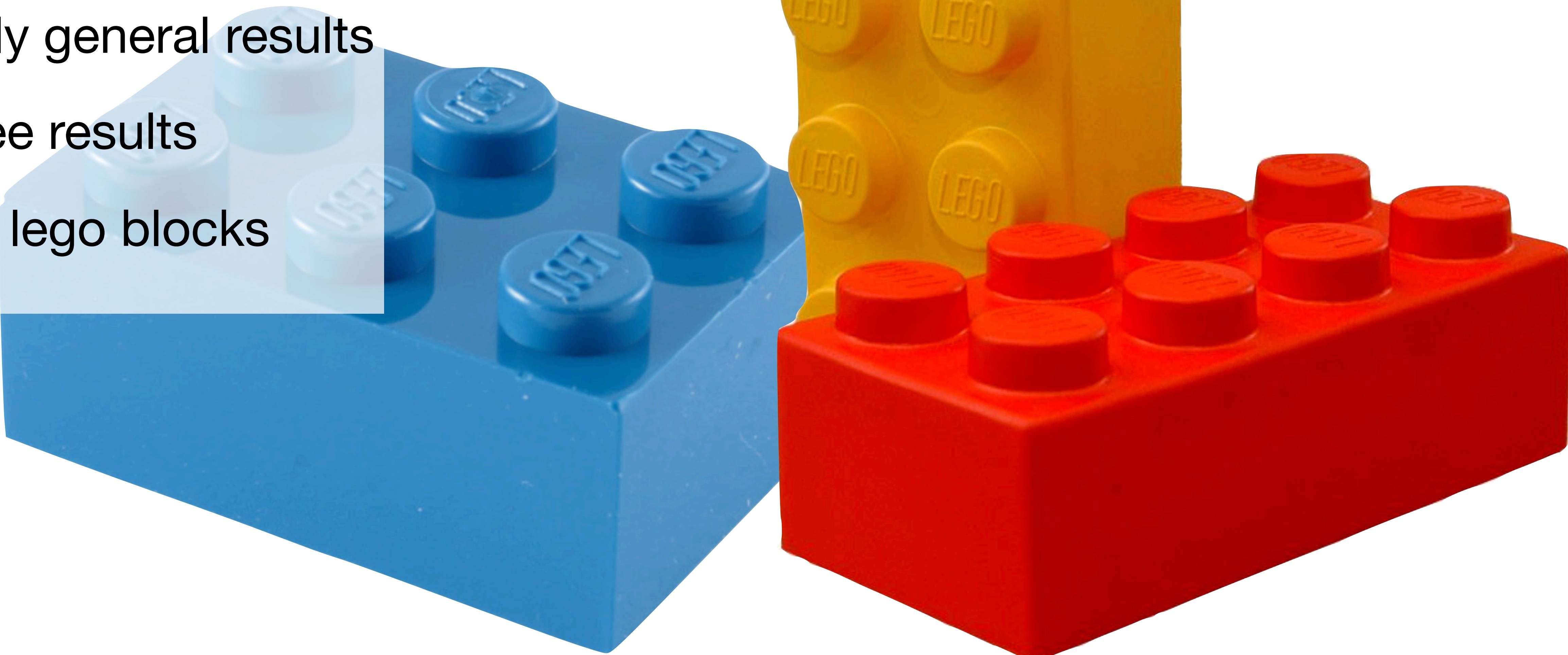
Probability Axioms

Three Axioms

Imply general results

Three results

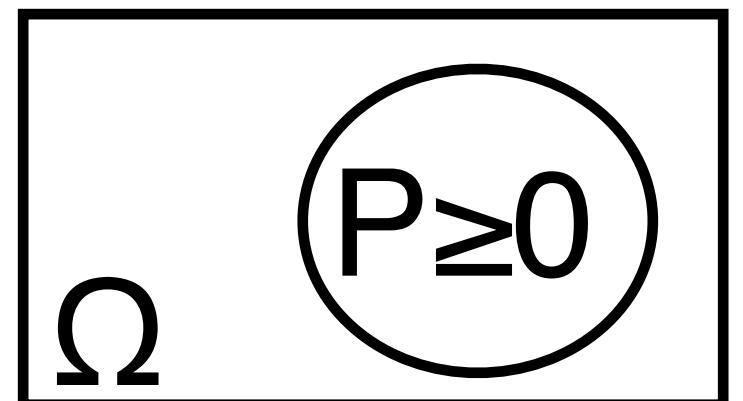
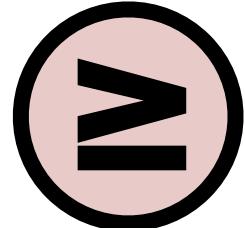
Use lego blocks



Three Axioms

Non-negativity

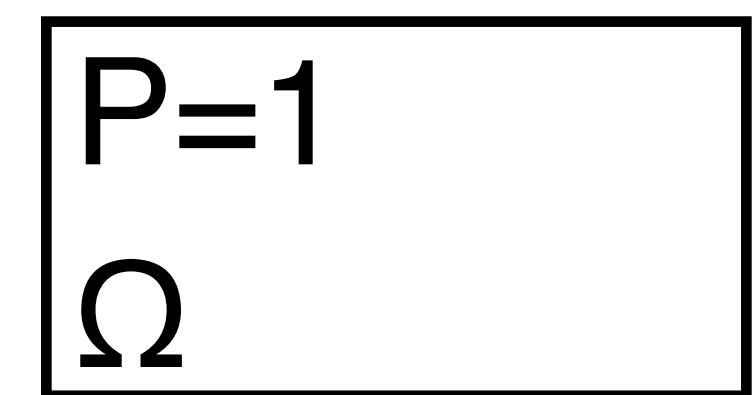
$$P(A) \geq 0$$



Short for $\forall A \ P(A) \geq 0$

Unitarity

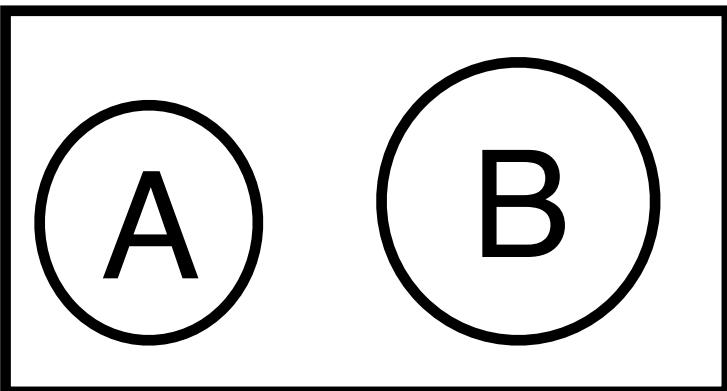
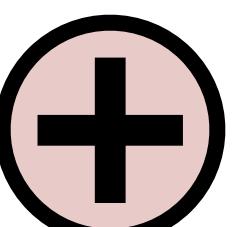
$$P(\Omega) = 1$$



Addition rule

A, B disjoint \rightarrow

$$P(A \cup B) = P(A) + P(B)$$



A_1, A_2, \dots disjoint

$$\rightarrow P(A_1 \cup A_2 \cup \dots) = P(A_1) + P(A_2) + \dots$$

Countable unions only

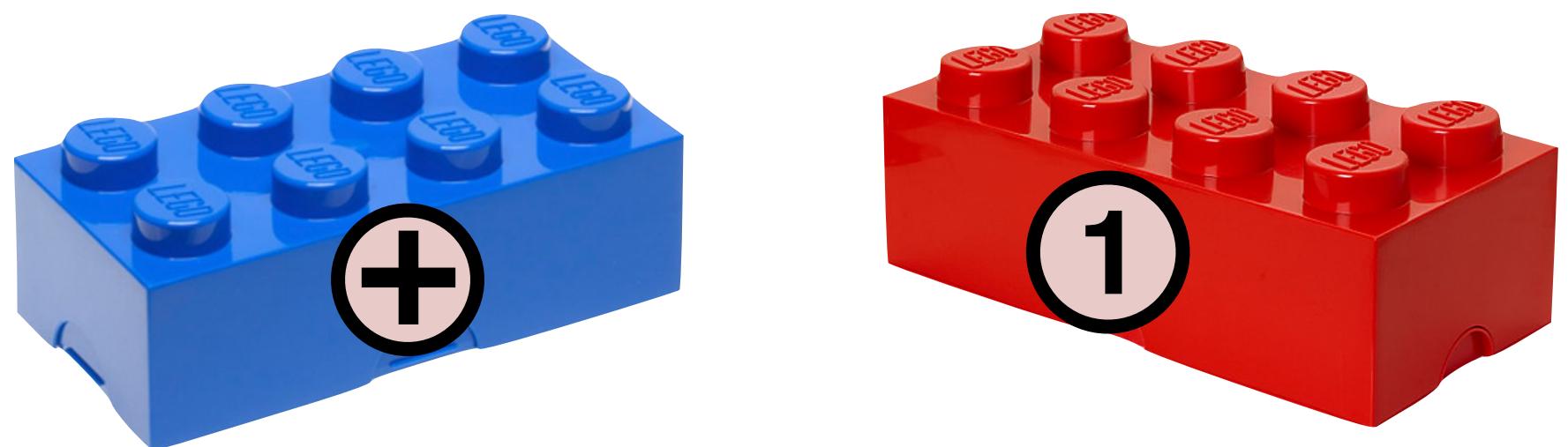
Uncountable later

“Building” Results

Use lego blocks

construct three simple results

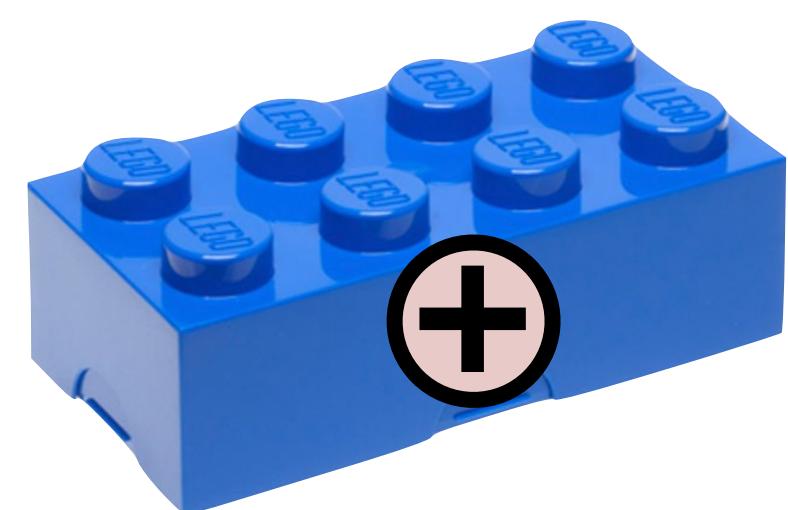
Complement rule



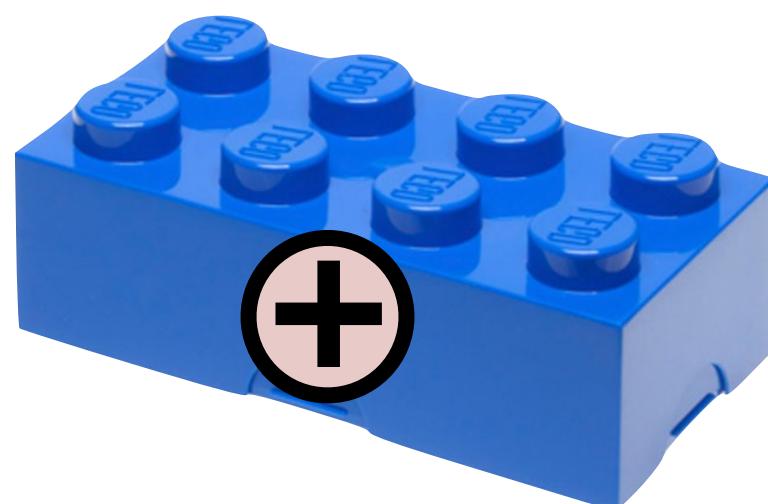
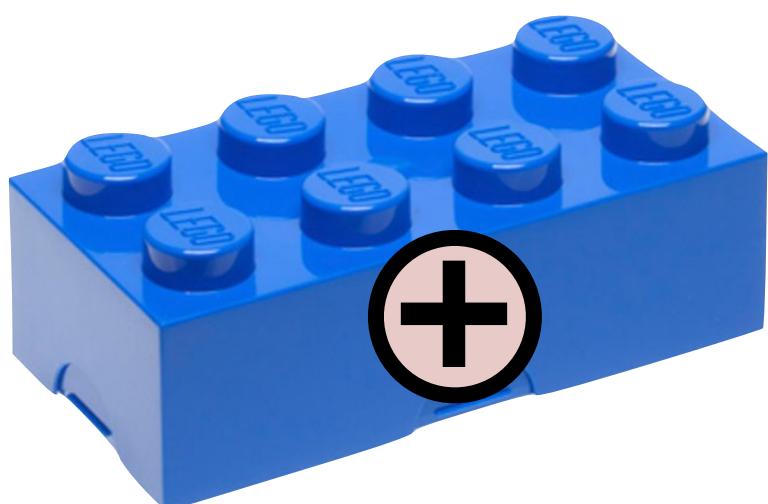
Subtraction

nested sets

general sets



Inclusion-exclusion



Complement Rule

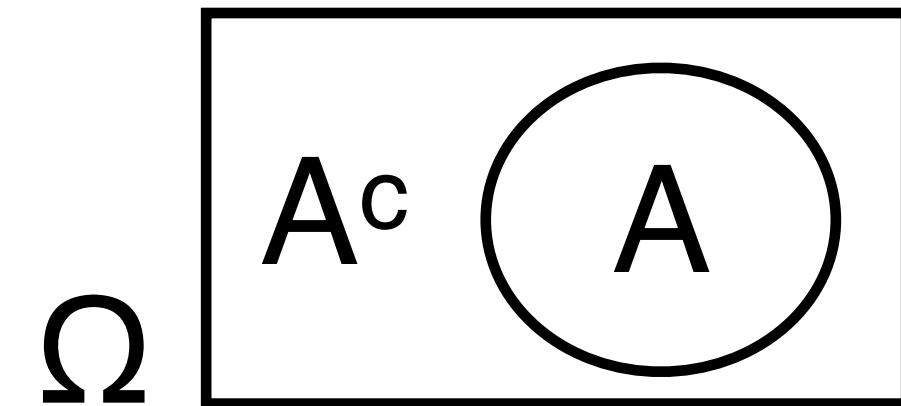
Complement rule for counting

$$|A^c| = |\Omega| - |A|$$

Same for



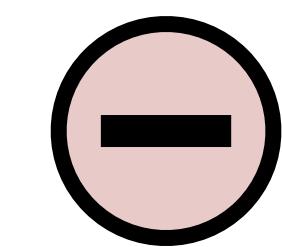
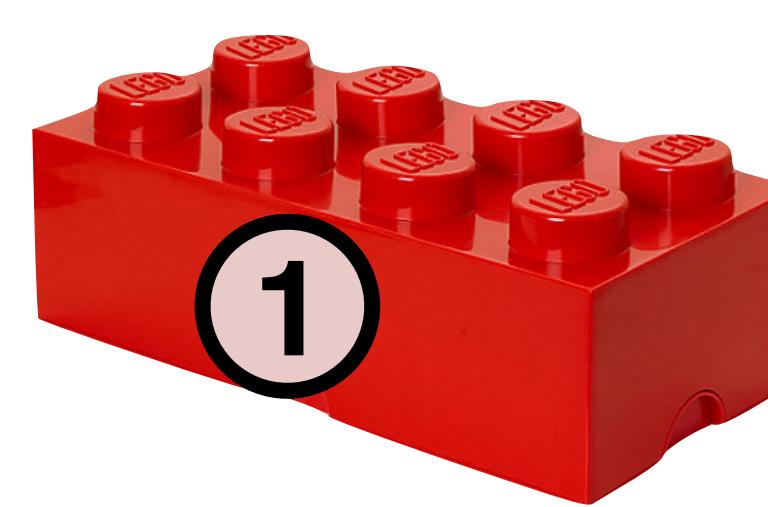
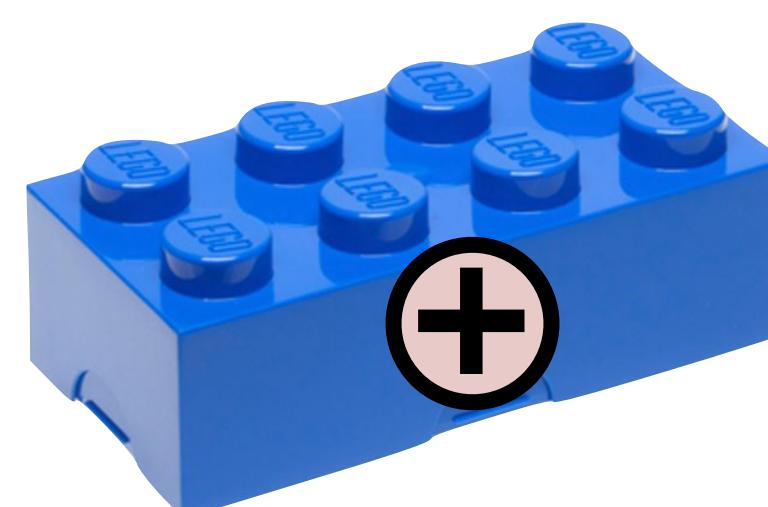
$$A \cup A^c = \Omega$$



$$P(A) + P(A^c) = P(A \cup A^c) = P(\Omega) = 1$$

$$P(A^c) = 1 - P(A)$$

Complement rule for probability



Subtraction Rule - Nested Sets

Complement rule

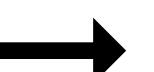
$$A \subseteq \Omega$$

$$P(A^c) = 1 - P(A)$$

$$P(\Omega - A) = P(\Omega) - P(A)$$

Generalize

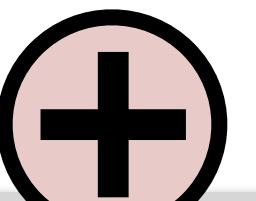
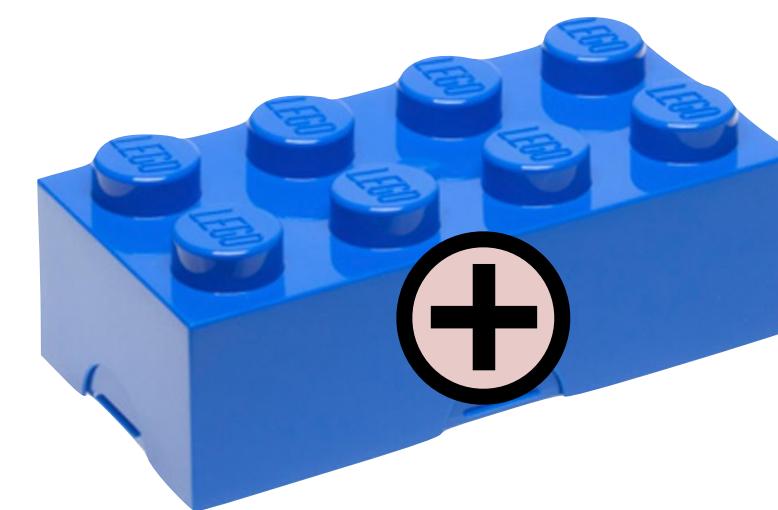
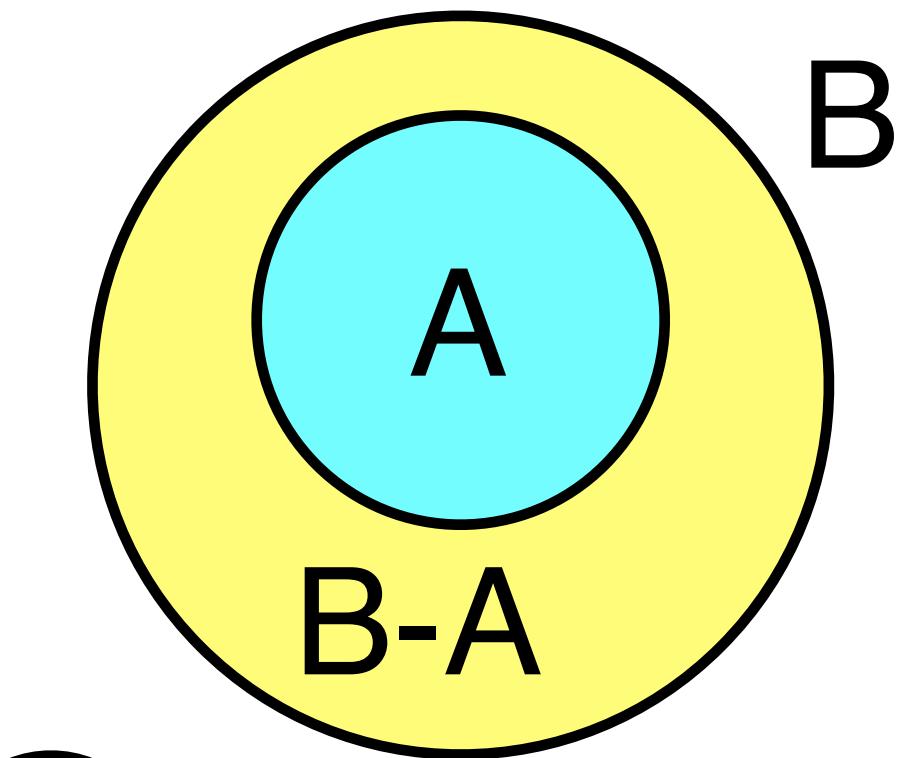
$$A \subseteq B$$



$$P(B - A) = P(B) - P(A)$$

$$A \subseteq B \rightarrow$$

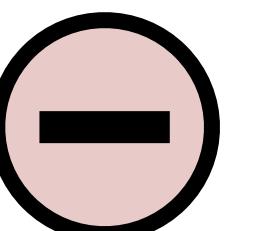
$$B = A \cup (B - A)$$



$$P(B) = P(A \cup (B - A)) = P(A) + P(B - A)$$

$$P(B - A) = P(B) - P(A)$$

Subtraction rule for nested sets



Subtraction Rule - General Sets

Nested

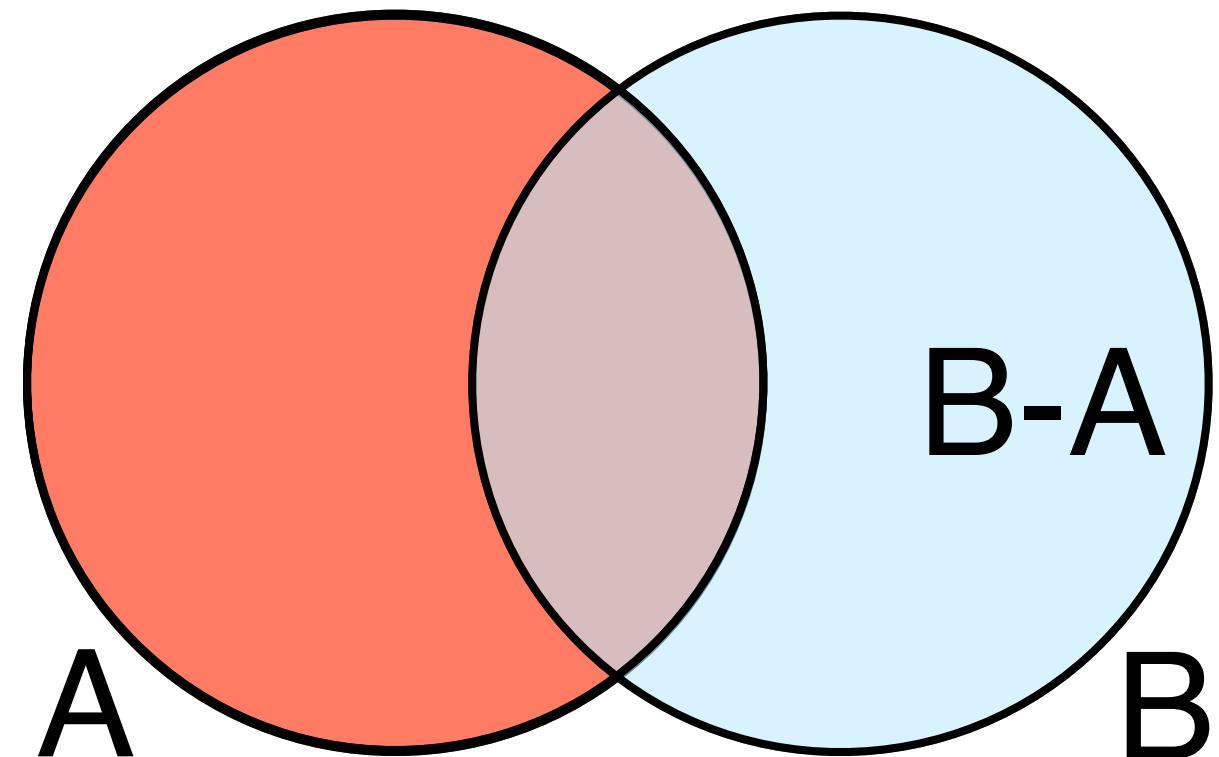
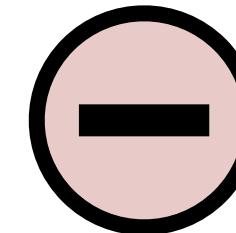
$$A \subseteq B$$

$$P(B-A) = P(B) - P(A)$$

General

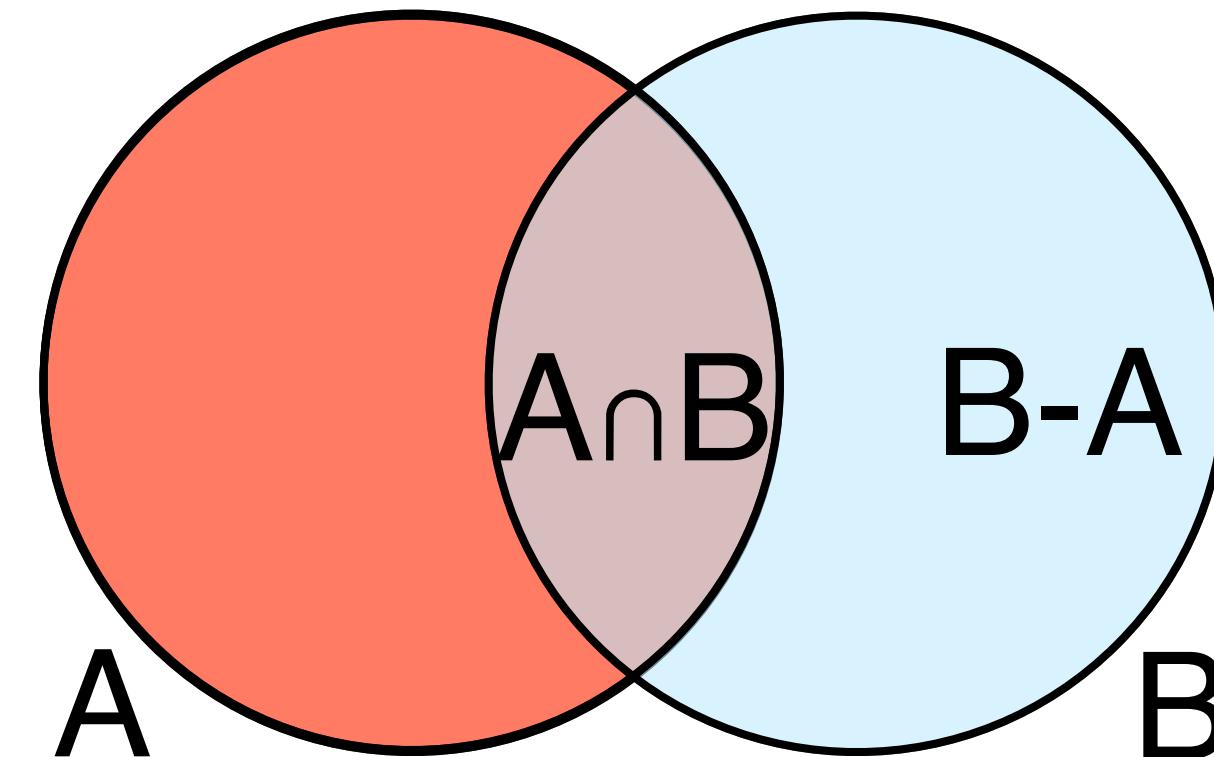
$$\forall A, B$$

$$P(B-A) = P(B) - P(A \cap B)$$

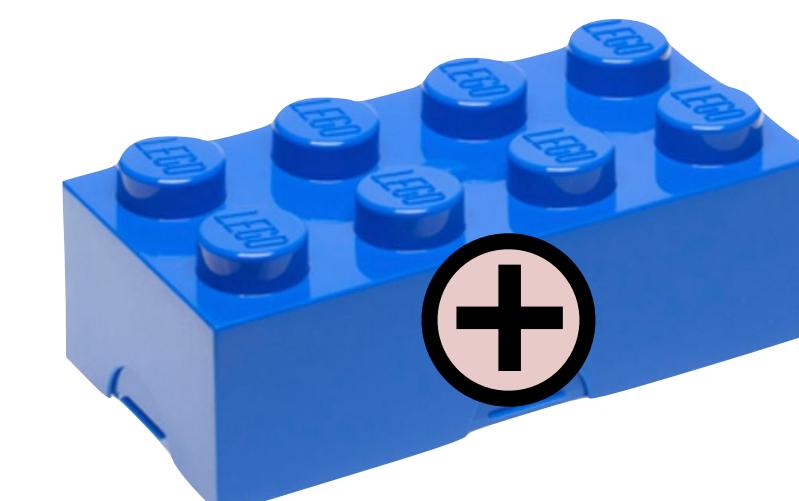


$$B - A = B - (A \cap B)$$

$$A \cap B \subseteq B$$



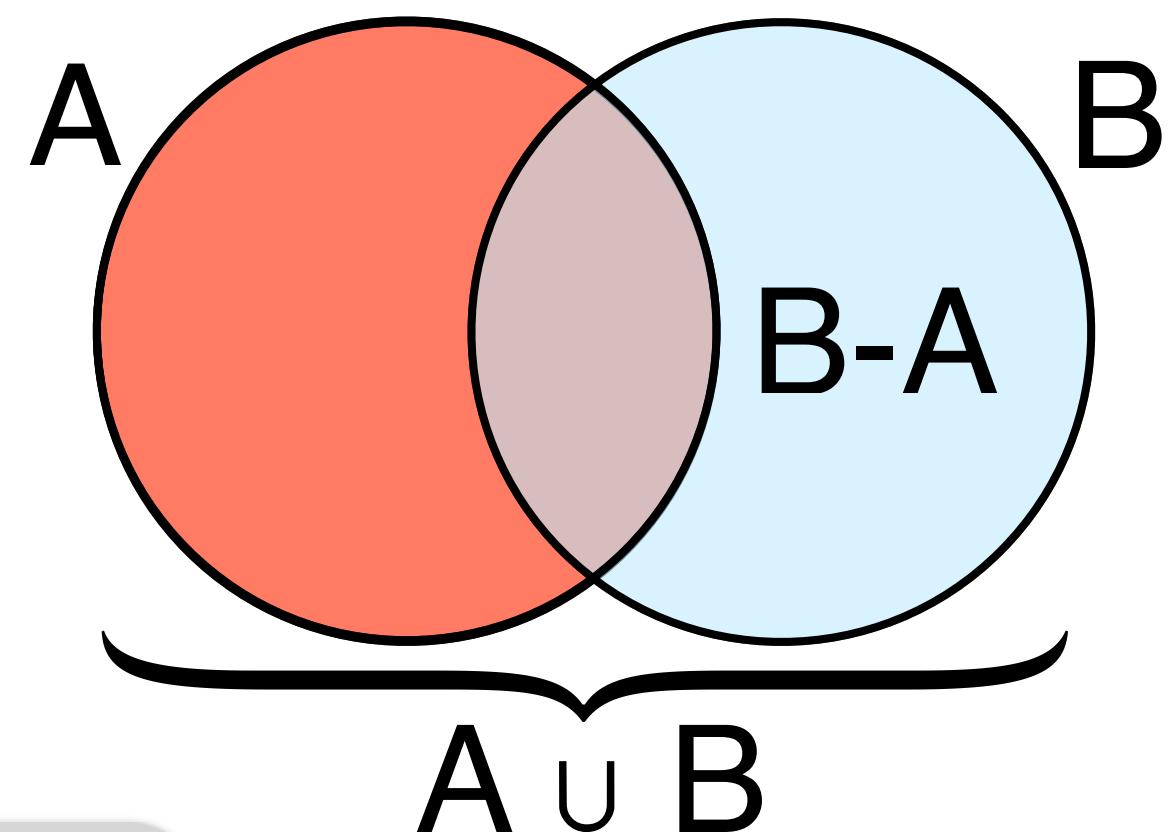
$$P(B - A) = P(B - (A \cap B)) = P(B) - P(A \cap B)$$



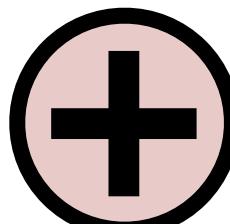
Inclusion Exclusion

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$A \cup B = A \cup B - A$$



$$P(A \cup B) = P(A \cup B - A)$$



$$= P(A) + P(B - A)$$

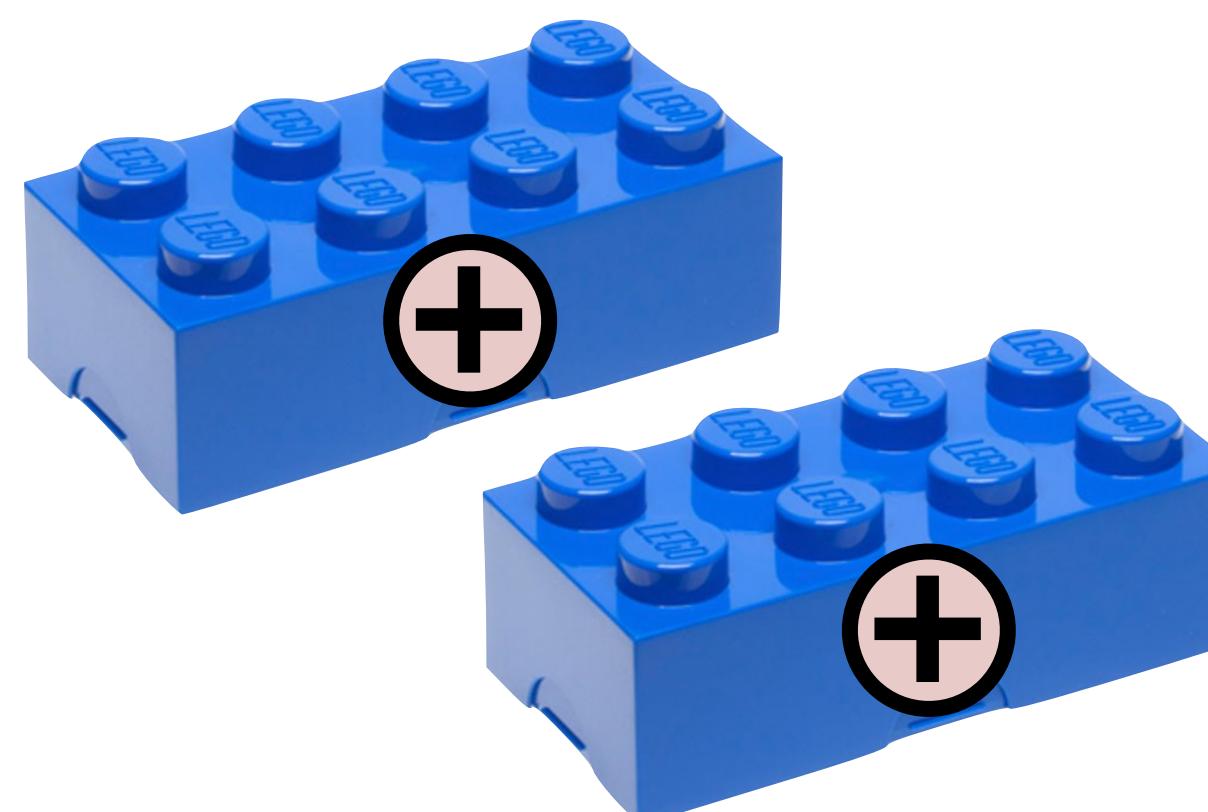
$$= P(A) + P(B) - P(A \cap B)$$

General subtraction

$$P(B - A) = P(B) - P(A \cap B)$$



X 2



More Sets

Two sets

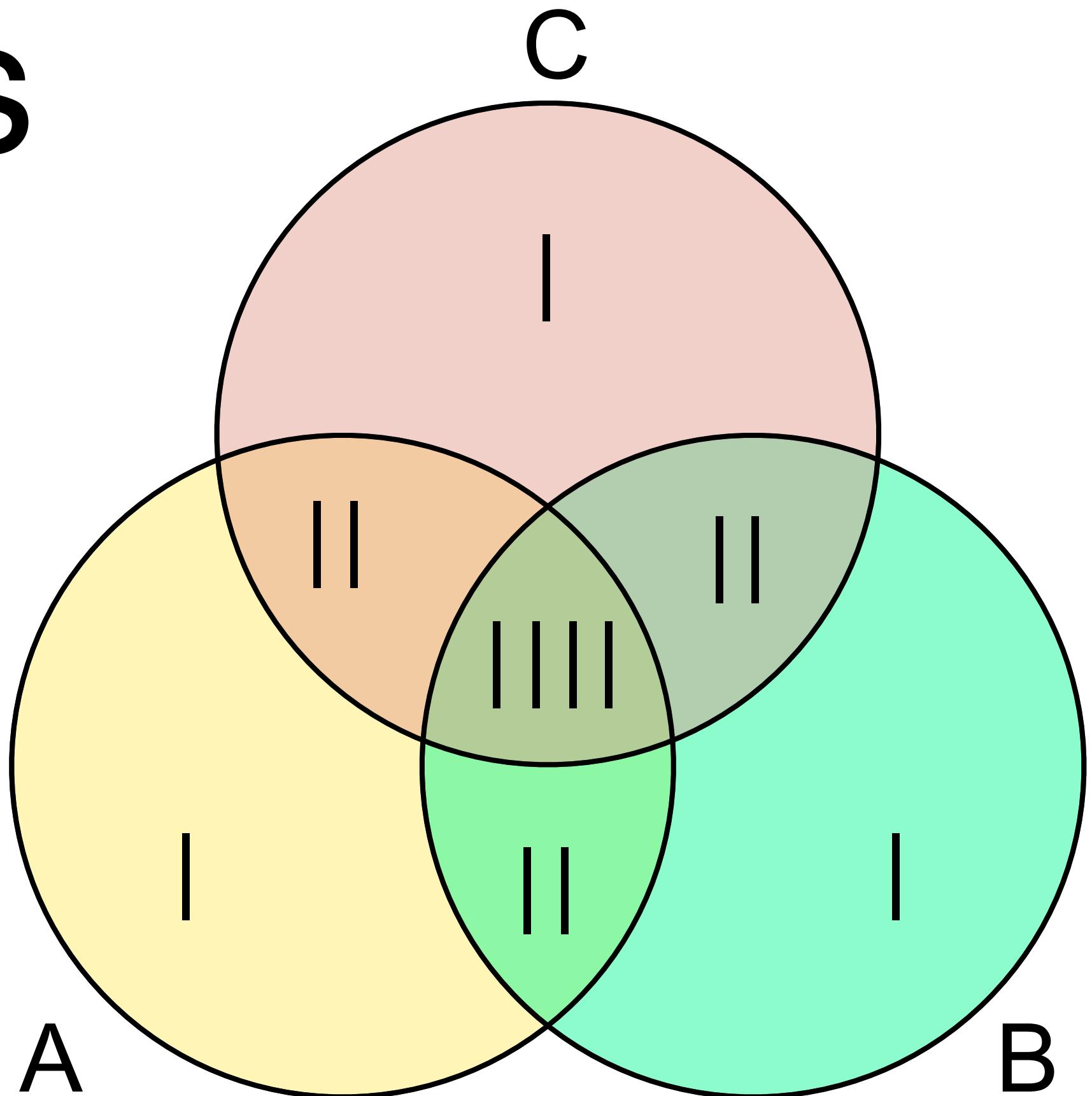
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Three sets

$$\begin{aligned} P(A \cup B \cup C) &= P(A) + P(B) + P(C) \\ &\quad - P(A \cap B) - P(A \cap C) - P(B \cap C) \\ &\quad + P(A \cap B \cap C) \end{aligned}$$

n sets

$$P\left(\bigcup_{i=1}^n A_i\right) = \sum_{1 \leq i \leq n} P(A_i) - \sum_{1 \leq i < j \leq n} P(A_i \cap A_j) + \dots + (-1)^{n-1} P\left(\bigcap_{i=1}^n A_i\right)$$



Probability Axioms

Three Axioms

Imply general results

Three results

Use lego blocks

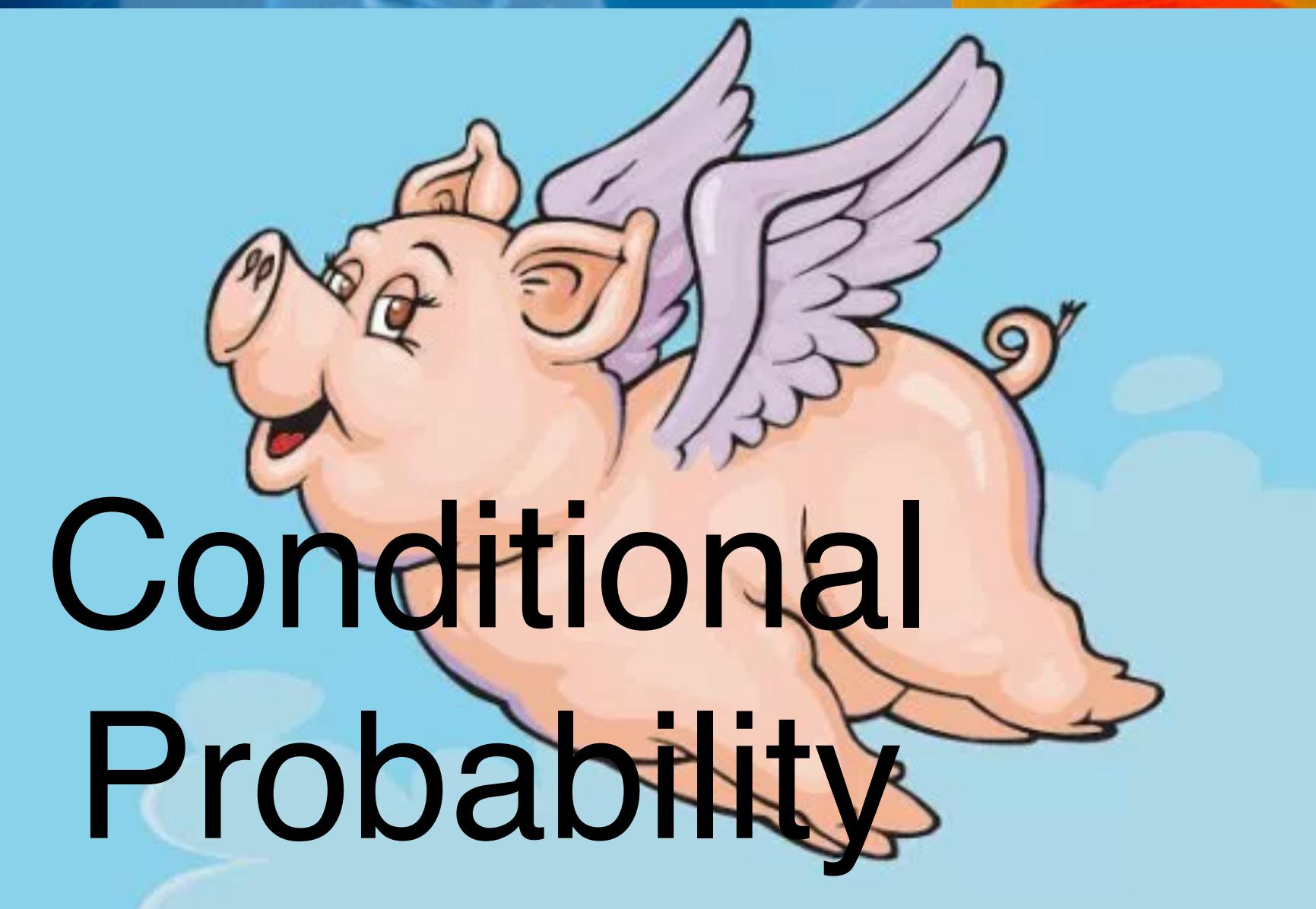


Axioms

Three Axioms

Some implications

\$1.4M question



Conditional
Probability

