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```

#### Basic 1

1

```
1.1 Increase Stack Size
//stack resize (linux)
#include <sys/resource.h>
void increase_stack_size() +
  const rlim_t ks = 64*1024*1024;
   struct rlimit rl;
   int res=getrlimit(RLIMIT_STACK, &rl);
   if(res==0){
     if(rl.rlim_cur<ks){</pre>
      rl.rlim_cur=ks;
      res=setrlimit(RLIMIT_STACK, &rl);
} } }
1.2 Misc
編譯參數: -std=c++14 -Wall -Wshadow (-fsanitize=
     undefined)
//check special cases for example (n==1)
//check size arrays
#include <random>
mt19937 gen(0x5EED);
int randint(int lb, int ub)
{ return uniform_int_distribution<int>(lb, ub)(gen); }
#define SECs ((double)clock() / CLOCKS_PER_SEC)
struct KeyHasher {
  size_t operator()(const Key& k) const {
    return k.first + k.second * 100000;
};
typedef unordered_map<Key,int,KeyHasher> map_t;
 __builtin_popcountll //換成二進位有幾個1
1.3 check
for ((i=0;;i++))
     echo "$i"
     python3 gen.py > input
     ./ac < input > ac.out
     ./wa < input > wa.out
    diff ac.out wa.out || break
done
1.4 python-related
int(eval(num.replace("/","//")))
 from fractions import Fraction
from decimal import Decimal, getcontext
getcontext().prec = 250 # set precision
itwo = Decimal(0.5)
two = Decimal(2)
N = 200
def angle(cosT):
   """given cos(theta) in decimal return theta"""
   for i in range(N):
    cosT = ((cosT + 1) / two) ** itwo
  sinT = (1 - cosT * cosT) ** itwo
return sinT * (2 ** N)
pi = angle(Decimal(-1))
2
     flow
 2.1 ISAP
```

```
struct Maxflow {
 static const int MAXV = 20010;
 static const int INF = 1000000;
 struct Edge {
    int v, c, r;
   Edge(int _v, int _c, int _r):
      v(_v), c(_c), r(_r) {}
```

```
int s, t;
vector<Edge> G[MAXV*2];
  int iter[MAXV*2], d[MAXV*2], gap[MAXV*2], tot;
  void init(int x) {
    tot = x+2;
    s = x+1, t = x+2;
for(int i = 0; i <= tot; i++) {
   G[i].clear();</pre>
       iter[i] = d[i] = gap[i] = 0;
  void addEdge(int u, int v, int c) {
   G[u].push_back(Edge(v, c, SZ(G[v]) ));
    G[v].push_back(Edge(u, 0, SZ(G[u]) - 1));
  int dfs(int p, int flow) {
    if(p == t) return flow;
    for(int &i = iter[p]; i < SZ(G[p]); i++) {</pre>
       Edge &e = G[p][i]
       if(e.c > 0 \&\& d[p] == d[e.v]+1) {
         int f = dfs(e.v, min(flow, e.c));
         if(f) {
           e.c -= f;
           G[e.v][e.r].c += f;
           return f;
    if( (--gap[d[p]]) == 0) d[s] = tot;
    else {
       d[p]++;
       iter[p] = 0;
       ++gap[d[p]];
    return 0;
  int solve() {
    int res = 0;
    gap[0] = tot;
    for(res = 0; d[s] < tot; res += dfs(s, INF));
    return res;
  void reset() {
    for(int i=0;i<=tot;i++) {</pre>
       iter[i]=d[i]=gap[i]=0;
} } flow;
```

### 2.2 MinCostFlow

```
struct MinCostMaxFlow{
typedef int Tcost;
  static const int MAXV = 20010;
static const int INFf = 1000000;
  static const Tcost INFc = 1e9;
  struct Edge{
    int v, cap;
    Tcost w;
    int rev
    Edge(){}
    Edge(int t2, int t3, Tcost t4, int t5)
    : v(t2), cap(t3), w(t4), rev(t5) {}
 };
  int V, s, t;
 vector<Edge> g[MAXV];
 void init(int n, int _s, int _t){
    V = n; s = _s; t = _t;
    for(int i = 0; i <= V; i++) g[i].clear();</pre>
  void addEdge(int a, int b, int cap, Tcost w){
    g[a].push_back(Edge(b, cap, w, (int)g[b].size()));
    g[b].push_back(Edge(a, 0, -w, (int)g[a].size()-1));
  Tcost d[MAXV];
  int id[MAXV], mom[MAXV];
  bool inqu[MAXV];
  queue<int> q:
 pair<int,Tcost> solve(){
    int mxf = 0; Tcost mnc = 0;
    while(1){
      fill(d, d+1+V, INFc);
      fill(inqu, inqu+1+V, 0);
fill(mom, mom+1+V, -1);
      mom[s] = s;
      d[s] = 0;
      q.push(s); inqu[s] = 1;
```

```
while(q.size()){
          int u = q.front(); q.pop();
          inqu[u] = 0;
          for(int i = 0; i < (int) g[u].size(); i++){</pre>
            Edge &e = g[u][i];
            int v = e.v;
            if(e.cap > 0 \& d[v] > d[u]+e.w){
               d[v] = d[u] + e.w;
               mom[v] = u;
               id[v] = i
               if(!inqu[v]) q.push(v), inqu[v] = 1;
            }
          }
        if(mom[t] == -1) break;
        int df = INFf;
        for(int u = t; u != s; u = mom[u])
  df = min(df, g[mom[u]][id[u]].cap);
for(int u = t; u != s; u = mom[u]){
          Edge &e = g[mom[u]][id[u]];
          e.cap
          g[e.v][e.rev].cap += df;
       mxf += df;
       mnc += df*d[t];
     return {mxf,mnc};
} flow;
2.3 Dinic
const int MXN = 10000;
```

```
struct Dinic{
  struct Edge{ int v,f,re; };
  int n,s,t,level[MXN];
  vector<Edge> E[MXN];
  void init(int _n, int _s, int _t){
    n = _n; s = _s; t = _t;
for (int i=0; i<n; i++) E[i].clear();</pre>
  void add_edge(int u, int v, int f){
    E[u].PB({v,f,SZ(E[v])});
     E[v].PB({u,0,SZ(E[u])-1});
  bool BFS(){
    for (int i=0; i<n; i++) level[i] = -1;</pre>
     queue<int> que;
     que.push(s)
     level[s] = 0;
    while (!que.empty()){
       int u = que.front(); que.pop();
       for (auto it : E[u]){
  if (it.f > 0 && level[it.v] == -1){
           level[it.v] = level[u]+1;
           que.push(it.v);
      }
    return level[t] != -1;
  int DFS(int u, int nf){
     if (u == t) return nf;
     int res = 0;
     for (auto &it : E[u]){
       if (it.f > 0 && level[it.v] == level[u]+1){
         int tf = DFS(it.v, min(nf,it.f));
         res += tf; nf -= tf; it.f -= tf;
         E[it.v][it.re].f += tf;
         if (nf == 0) return res;
      }
    if (!res) level[u] = -1;
    return res;
  int flow(int res=0){
     while ( BFS() )
       res += DFS(s,2147483647);
     return res;
}flow;
```

### 2.4 Kuhn Munkres 最大完美二分匹配

```
struct KM{ // max weight, for min negate the weights
   static const int MXN = 2001; // 1-based
   static const ll INF = 0x3f3f3f3f;
  int n, mx[MXN], my[MXN], pa[MXN];
ll g[MXN][MXN], lx[MXN], ly[MXN], sy[MXN];
bool vx[MXN], vy[MXN];
void init(int _n) {
     n = _n;
for(int i=1; i<=n; i++) fill(g[i], g[i]+n+1, 0);</pre>
   void addEdge(int x, int y, ll w) \{g[x][y] = w;\}
   void augment(int y) {
     for(int x, z; y; y = z)
  x=pa[y], z=mx[x], my[y]=x, mx[x]=y;
   void bfs(int st) {
     for(int i=1; i<=n; ++i) sy[i]=INF, vx[i]=vy[i]=0;</pre>
     queue<int> q; q.push(st);
     for(;;) {
        while(q.size()) {
           int x=q.front(); q.pop(); vx[x]=1;
for(int y=1; y<=n; ++y) if(!vy[y]){
    ll t = lx[x]+ly[y]-g[x][y];
              if(t==0){
                 pa[y]=x
                 if(!my[y]){augment(y); return;}
                 vy[y]=1, q.push(my[y]);
              }else if(sy[y]>t) pa[y]=x,sy[y]=t;
        11 cut = INF;
        for(int y=1; y<=n; ++y)</pre>
           if(!vy[y]&&cut>sy[y]) cut=sy[y];
         for(int j=1; j<=n; ++j){
  if(vx[j]) lx[j] -= cut;</pre>
           if(vy[j])_ly[j] += cut;
           else sy[j] -= cut;
         for(int y=1; y<=n; ++y) if(!vy[y]&&sy[y]==0){
           if(!my[y]){augment(y); return;}
           vy[y]=1, q.push(my[y]);
     }
   ll solve(){
     fill(mx, mx+n+1, 0); fill(my, my+n+1, 0); fill(ly, ly+n+1, 0); fill(lx, lx+n+1, -INF);
     for(int x=1; x<=n; ++x) for(int y=1; y<=n; ++y)</pre>
        lx[x] = max(lx[x], g[x][y]);
      for(int x=1; x<=n; ++x) bfs(x);</pre>
     11 \text{ ans} = 0;
     for(int y=1; y<=n; ++y) ans += g[my[y]][y];
     return ans;
}graph;
```

### 2.5 Directed MST

```
/* Edmond's algoirthm for Directed MST
* runs in O(VE) */
const int MAXV = 10010;
const int MAXE = 10010;
const int INF = 2147483647;
struct Edge{
  int u, v, c;
  Edge(int x=0, int y=0, int z=0) : u(x), v(y), c(z){}
int V, E, root;
Edge edges[MAXE];
inline int newV(){ return ++ V; }
inline void addEdge(int u, int v, int c)
{ edges[\pm\pmE] = Edge(u, v, c); }
bool con[MAXV];
int mnInW[MAXV], prv[MAXV], cyc[MAXV], vis[MAXV];
inline int DMST(){
  fill(con, con+V+1, (int r1 = 0, r2 = 0;
  while(1){
    fill(mnInW, mnInW+V+1, INF);
    fill(prv, prv+V+1, -1);
```

```
REP(i, 1, E){
    int u=edges[i].u, v=edges[i].v, c=edges[i].c;
     if(u != v && v != root && c < mnInW[v])
      mnInW[v] = c, prv[v] = u;
  fill(vis, vis+V+1, -1);
  fill(cyc, cyc+V+1, -1);
  r1 = 0;
  bool jf = 0;
REP(i, 1, V){
  if(con[i]) continue;
  if(prv[i] == -1 && i != root) return -1;
    if(prv[i] > 0) r1 += mnInW[i];
    int s;
    for(s = i; s != -1 && vis[s] == -1; s = prv[s])
       vis[s] = i;
     if(s > 0 \& vis[s] == i){
        // get a cycle
       jf = 1; int v = s;
       do{
         cyc[v] = s, con[v] = 1;
         r2 += mnInW[v]; v = prv[v];
       }while(v != s);
       con[s] = 0;
  if(!jf) break ;
REP(i, 1, E){
    int &u = edges[i].u;
    int &v = edges[i].v;
    if(cyc[v] > 0) edges[i].c -= mnInW[edges[i].v];
if(cyc[u] > 0) edges[i].u = cyc[edges[i].u];
    if(cyc[v] > 0) edges[i].v = cyc[edges[i].v];
    if(u == v) edges[i--] = edges[E--];
} }
return r1+r2;
     SW min-cut (不限 S-T 的 min-cut)
```

```
// global min cut
struct SW{ // O(V^3)
  static const int MXN = 514;
  int n,vst[MXN],del[MXN];
  int edge[MXN][MXN],wei[MXN];
  void init(int _n){
    n = _n; FZ(edge); FZ(del);
  void addEdge(int u, int v, int w){
    edge[u][v] += w; edge[v][u] += w;
  void search(int &s, int &t){
    FZ(vst); FZ(wei);
     s = t = -1
     while (true){
       int mx=-1, cur=0;
for (int i=0; i<n; i++)
  if (!del[i] && !vst[i] && mx<wei[i])</pre>
           cur = i, mx = wei[i];
       if (mx == -1) break;
      vst[cur] = 1;
       s = t; t = cur;
       for (int i=0; i<n; i++)
         if (!vst[i] && !del[i]) wei[i] += edge[cur][i];
    }
  int solve(){
     int res = 2147483647;
     for (int i=0,x,y; i<n-1; i++){</pre>
       search(x,y);
       res = min(res,wei[y]);
       del[y] = 1;
for (int j=0; j<n; j++)</pre>
         edge[x][j] = (edge[j][x] += edge[y][j]);
    return res;
}graph;
```

### 2.7 Max flow with lower/upper bound

```
// flow use ISAP
// Max flow with lower/upper bound on edges
```

```
// source = 1 , sink = n
int in[ N ] , out[ N ];
int l[ M ] , r[ M ] , a[ M ] , b[ M ];//0-base,a下界,b
int solve(){
  flow.init(n); //n為點的數量,m為邊的數量,點是1-
         base
  for( int i = 0 ; i < m ; i ++ ){
  in[ r[ i ] ] += a[ i ];
  out[ l[ i ] ] += a[ i ];
  flow.addEdge( l[ i ] , r[ i ] , b[ i ] - a[ i ] );
  // flow from l[i] to r[i] must in [a[ i ], b[ i ]]</pre>
   int nd = 0;
  int nu = v,
for( int i = 1 ; i <= n ; i ++ ){
   if( in[ i ] < out[ i ] ){
     flow.addEdge( i , flow.t , out[ i ] - in[ i ] );
     nd += out[ i ] - in[ i ];
}</pre>
      if( out[ i ] < in[ i ] )</pre>
         flow.addEdge( flow.s , i , in[ i ] - out[ i ] );
   // original sink to source
  flow.addEdge( n , 1 , INF );
if( flow.maxflow() != nd )
      // no solution
      return -1;
   int ans = flow.G[ 1 ].back().c; // source to sink
   flow.G[1].back().c = flow.G[n].back().c = 0;
   // take out super source and super sink
   for( size_t i = 0 ; i < flow.G[ flow.s ].size() ; i</pre>
          ++ ){
      flow.G[ flow.s ][ i ].c = 0;
Edge &e = flow.G[ flow.s ][ i ];
      flow.G[ e.v ][ e.r ].c = \overline{0};
   for( size_t i = 0 ; i < flow.G[ flow.t ].size() ; i</pre>
     ++ ){
flow.G[ flow.t ][ i ].c = 0;
Edge &e = flow.G[ flow.t ][ i ];
flow.G[ e.v ][ e.r ].c = 0;
  flow.addEdge( flow.s , 1 , INF );
flow.addEdge( n , flow.t , INF );
   flow.reset();
   return ans + flow.maxflow();
```

### 2.8 HLPPA (稠密圖 flow)

```
template <int MAXN, class T = int>
struct HLPP {
  const T INF = numeric_limits<T>::max();
  struct Edge {
    int to, rev; T f;
  int n, s, t;
  vector<Edge> adj[MAXN];
  deque<int> lst[MAXN];
  vector<int> gap[MAXN];
  int ptr[MAXN];
  T ef[MAXN]:
  int h[MAXN], cnt[MAXN], work, hst=0/*highest*/;
  void init(int _n, int _s, int _t) {
    n=_n+1; s = _s; t = _t;
for(int i=0;i<n;i++) adj[i].clear();</pre>
  void addEdge(int u,int v,T f,bool isDir = true){
  adj[u].push_back({v,adj[v].size(),f});
    adj[v].push_back({u,adj[u].size()-1,isDir?0:f});
  void updHeight(int v, int nh) {
    work++;
    if(h[v] != n) cnt[h[v]]--;
    h[v] = nh;
    if(nh == n) return;
    cnt[nh]++, hst = nh; gap[nh].push_back(v);
    if(ef[v]>0) lst[nh].push_back(v), ptr[nh]++;
  void globalRelabel() {
    work = 0;
    fill(h, h+n, n);
```

```
fill(cnt, cnt+n, 0);
for(int i=0; i<=hst; i++)
     lst[i].clear(), gap[i].clear(), ptr[i] = 0;
queue<int> q({t}); h[t] = 0;
     while(!q.empty()) {
        int v = q.front(); q.pop();
       for(auto &e : adj[v])
  if(h[e.to] == n && adj[e.to][e.rev].f > 0)
            q.push(e.to), updHeight(e.to, h[v] + 1);
       hst = h[v];
   void push(int v, Edge &e) {
     if(ef[e.to] == 0)
        lst[h[e.to]].push_back(e.to), ptr[h[e.to]]++;
     T df = min(ef[v], e.f);
     e.f -= df, adj[e.to][e.rev].f += df;
     ef[v] -= df, ef[e.to] += df;
   void discharge(int v) {
     int nh = n;
     for(auto &e : adj[v]) {
        if(e.f > 0) {
          if(h[v] == h[e.to] + 1) {
            push(v, e);
            if(ef[v] <= 0) return;</pre>
          else nh = min(nh, h[e.to] + 1);
     } }
     if(cnt[h[v]] > 1) updHeight(v, nh);
     else {
        for(int i = h[v]; i < n; i++) {</pre>
          for(auto j : gap[i]) updHeight(j, n);
          gap[i].clear(), ptr[i] = 0;
     } }
   T solve() {
     fill(ef, ef+n, 0);
ef[s] = INF, ef[t] = -INF;
     globalRelabel();
     for(auto &e : adj[s]) push(s, e);
for(; hst >= 0; hst--) {
       while(!lst[hst].empty()) {
          int v=lst[hst].back(); lst[hst].pop_back();
          discharge(v);
if(work > 4 * n) globalRelabel();
     } }
     return ef[t] + INF;
} };
```

#### 2.9 Flow Method

```
Maximize c^T x subject to Ax \leq b, x \geq 0; with the corresponding symmetric dual problem, Minimize b^T y subject to A^T y \geq c, y \geq 0.

Maximize c^T x subject to Ax \leq b; with the corresponding asymmetric dual problem, Minimize b^T y subject to A^T y = c, y \geq 0.

Minimum vertex cover on bipartite graph = Maximum matching on bipartite graph = Max flow with source to one side, other side to sink
```

To reconstruct the minimum vertex cover, dfs from each unmatched vertex on the left side and with unused edges only. Equivalently, dfs from source with unused edges only and without visiting sink. Then, a vertex is chosen

iff. it is on the left side  $\mbox{and}$  without visited or on the right side  $\mbox{and}$  visited through dfs.

Maximum density subgraph (  $\sum W_e + \sum W_v$  ) / |V|

```
Binary search on answer:
For a fixed D, construct a Max flow model as follow:
Let S be Sum of all weight( or inf)

1. from source to each node with cap = S

2. For each (u,v,w) in E, (u->v,cap=w), (v->u,cap=w)

3. For each node v, from v to sink with cap = S + 2 * D

- deg[v] - 2 * (W of v)

where deg[v] = \sum weight of edge associated with v
```

If maxflow < S \* IVI, D is an answer.

```
Requiring subgraph: all vertex can be reached from source with edge whose cap > 0.
```

### 3 Math

#### 3.1 FFT

```
// const int MAXN = 262144;
// (must be 2^k)
// before any usage, run pre_fft() first
typedef long double ld;
typedef complex<ld> cplx; //real() ,imag()
const ld PI = acosl(-1);
const cplx I(0, 1)
cplx omega[MAXN+1];
void pre_fft(){
  for(int i=0; i<=MAXN; i++)
  omega[i] = exp(i * 2 * PI / MAXN * I);</pre>
void fft(int n, cplx a[], bool inv=false){
  int basic = MAXN / n;
  int theta = basic;
  for (int m = n; m >= 2; m >>= 1) {
    int mh = m >> 1;
for (int i = 0; i < mh; i++) {</pre>
      cplx w = omega[inv ? MAXN-(i*theta%MAXN)
                             : i*theta%MAXN];
       for (int j = i; j < n; j += m) {
         int k = j + mh;
         cplx x = a[j] - a[k];
         a[j] += a[k];
         a[k] = w * x;
    theta = (theta * 2) % MAXN;
  int i = 0;
  for (int j = 1; j < n - 1; j++) {
for (int k = n >> 1; k > (i ^= k); k >>= 1);
    if (j < i) swap(a[i], a[j]);
  if(inv) for (i = 0; i < n; i++) a[i] /= n;
cplx arr[MAXN+1];
inline void mul(int _n,ll a[],int _m,ll b[],ll ans[])
  int n=1, sum=_n+_m-1;
  while(n<sum)</pre>
    n<<=1;
  for(int i=0;i<n;i++)</pre>
    double x=(i<_n?a[i]:0), y=(i<_m?b[i]:0);
    arr[i]=complex<double>(x+y,x-y);
  fft(n,arr);
  for(int i=0;i<n;i++)</pre>
    arr[i]=arr[i]*arr[i];
  fft(n,arr,true);
  for(int i=0;i<sum;i++)</pre>
    ans[i]=(long long int)(arr[i].real()/4+0.5);
```

#### 3.2 NTT

```
// Remember coefficient are mod P
  p=a*2^n+1
        2^n
  n
                                     root
                                а
        65536
                    65537
                                     3 */
   20
       1048576
                    7340033
// (must be 2^k)
template<LL P, LL root, int MAXN>
struct NTT{
 static LL bigmod(LL a, LL b) {
    LL res = 1;
    for (LL bs = a; b; b >>= 1, bs = (bs * bs) % P)
      if(b&1) res=(res*bs)%P;
    return res:
 static LL inv(LL a, LL b) {
    if(a==1)return 1;
```

```
return (((LL)(a-inv(b%a,a))*b+1)/a)%b;
   LL omega[MAXN+1];
  NTT() {
     omega[0] = 1;
     LL r = bigmod(root, (P-1)/MAXN);
     for (int i=1; i<=MAXN; i++)
        omega[i] = (omega[i-1]*r)%P;
   // n must be 2^k
   void tran(int n, LL a[], bool inv_ntt=false){
     int basic = MAXN / n , theta = basic;
for (int m = n; m >= 2; m >>= 1) {
        or (int m = i, ...
int mh = m >> 1;
int i = 0; i < mh; i++) {
           LL w = omega[i*theta%MAXN];
           for (int j = i; j < n; j += m) {
  int k = j + mh;
  LL x = a[j] - a[k];</pre>
              if (x < 0) x += P;
              a[j] += a[k];
              if (a[j] > P) a[j] -= P;
             a[k] = (w * x) \% P;
        theta = (theta * 2) % MAXN;
     int i = 0;
     for (int j = 1; j < n - 1; j++) {
  for (int k = n >> 1; k > (i ^= k); k >>= 1);
  if (j < i) swap(a[i], a[j]);</pre>
     if (inv_ntt) {
        LL ni = inv(n,P);
        reverse( a+1 , a+n );

for (i = 0; i < n; i++)

a[i] = (a[i] * ni) % P;
  }
};
const LL P=2013265921,root=31;
const int MAXN=4194304;
NTT<P, root, MAXN> ntt;
```

## 3.3 Fast Walsh Transform

```
/* xor convolution:
 * x = (x0,x1) , y = (y0,y1)
* z = (x0y0 + x1y1 , x0y1 + x1y0)
 * x' = ( x0+x1 , x0-x1 ) , y' = ( y0+y1 , y0-y1 )

* z' = ( ( x0+x1 )( y0+y1 ) , ( x0-x1 )( y0-y1 ) )

* z = (1/2) * z''
  * or convolution:
  * x = (x0, x0+x1), inv = (x0, x1-x0) w/o final div
  * and convolution:
  * x = (x0+x1, x1), inv = (x0-x1, x1) w/o final div */
typedef long long LL;
const int MAXN = (1 << 20) + 10;
const LL MOD = 1e9+7;
inline LL inv( LL x )
   return mypow( x , MOD-2 );
inline void fwt( LL x[ MAXN ] , int N , bool inv=0 ) {
   for( int d = 1 ; d < N ; d <<= 1 ) {</pre>
      int d2 = d << 1;
      for( int s = 0 ; s < N ; s += d2 )
        for(int i = s , j = s+d ; i < s+d ; i++, j++){
    LL ta = x[i], tb = x[j];
    x[i] = ta+tb;
          x[j] = ta-tb;
if( x[i] >= MOD ) x[i] -= MOD;
           if(x[j] < 0) x[j] += MOD;
        }
   if( inv )
     for( int i = 0 ; i < N ; i++ ) {
    x[ i ] *= inv( N );
    x[ i ] %= MOD;</pre>
}
```

### 3.4 Poly operator

```
struct PolyOp {
#define FOR(i, c) for (int i = 0; i < (c); ++i)
NTT<P, root, MAXN> ntt;
  static int nxt2k(int x) {
     int i = 1; for (; i < x; i <<= 1); return i;
  void Mul(int n, LL a[], int m, LL b[], LL c[]) {
   static LL aa[MAXN], bb[MAXN];
     int N = nxt2k(n+m)
    copy(a, a+n, aa); fill(aa+n, aa+N, 0); copy(b, b+m, bb); fill(bb+m, bb+N, 0); ntt.tran(N, aa); ntt.tran(N, bb); FOR(i, N) c[i] = aa[i] * bb[i] % P;
     ntt.tran(N, c, 1);
  void Inv(int n, LL a[], LL b[]) {
     // ab = aa^{-1} = 1 \mod x^{(n/2)}
     // (b - a^-1)^2 = 0 mod x^n
     // bb - a^{-2} + 2 ba^{-1} = 0
     // bba - a^{-1} + 2b = 0
     // bba + 2b = a^{-1}
     static LL tmp[MAXN];
     if (n == 1) {b[0] = ntt.inv(a[0], P); return;}
     Inv((n+1)/2, a, b);
     int N = nxt2k(n*2);
     copy(a, a+n, tmp);
fill(tmp+n, tmp+N, 0);
     fill(b+n, b+N, 0);
     ntt.tran(N, tmp); ntt.tran(N, b);
     FOR(i, N) {
       LL t1 = (2 - b[i] * tmp[i]) % P;
if (t1 < 0) t1 += P;
        b[i] = b[i] * t1 % P;
     ntt.tran(N, b, 1);
     fill(b+n, b+N, 0);
  void Div(int n, LL a[], int m, LL b[], LL d[], LL r
        []) {
     // Ra = Rb * Rd mod x^{n-m+1}
     // Rd = Ra * Rb^-1 mod
     static LL aa[MAXN], bb[MAXN], ta[MAXN], tb[MAXN];
     if (n < m) {copy(a, a+n, r); fill(r+n, r+m, 0);</pre>
           return;}
     // d: n-1 - (m-1) = n-m (n-m+1 terms)
     copy(a, a+n, aa); copy(b, b+m, bb);
reverse(aa, aa+n); reverse(bb, bb+m);
     Inv(n-m+1, bb, tb)
     Mul(n-m+1, ta, n-m+1, tb, d);
     fill(d+n-m+1, d+n, 0); reverse(d, d+n-m+1);
         r: m-1 - 1 = m-2 (m-1 terms)
     Mul(m, b, n-m+1, d, ta);
FOR(i, n) { r[i] = a[i] - ta[i]; if (r[i] < 0) r[i]
  void dx(int n, LL a[], LL b[]) { REP(i, 1, n-1) b[i -1] = i * a[i] % P; }
   void Sx(int n, LL a[], LL b[]) {
     b[0] = 0;
     FOR(i, n) b[i+1] = a[i] * ntt.inv(i+1, P) % P;
  void Ln(int n, LL a[], LL b[]) {
  // Integral a' a^-1 dx
  static LL a1[MAXN], a2[MAXN], b1[MAXN];
     int N = nxt2k(n*2);
     dx(n, a, a1); Inv(n, a, a2);
Mul(n-1, a1, n, a2, b1);
Sx(n+n-1-1, b1, b);
     fill(b+n, b+N, 0);
  void Exp(int n, LL a[], LL b[]) {
     // Newton method to solve g(a(x)) = \ln b(x) - a(x)
     // b' = b - g(b(x)) / g'(b(x))
// b' = b (1 - lnb + a)
static LL lnb[MAXN], c[MAXN], tmp[MAXN];
assert(a[0] == 0); // dont know exp(a[0]) mod P
     if (n == 1) {b[0] = 1; return;}
Exp((n+1)/2, a, b);
fill(b+(n+1)/2, b+n, 0);
```

```
Ln(n, b, lnb);
    fill(c, c+n, 0); c[0] = 1;
    FOR(i, n) {
      c[i] += a[i] - lnb[i];
       if(c[i] < 0) c[i] += P
       if (c[i] >= P) c[i] -= P;
    Mul(n, b, n, c, tmp);
    copy(tmp, tmp+n, b);
} polyop;
3.5 O(1)mul
LL mul(LL x,LL y,LL mod){
  LL ret=x*y-(LL)((long double)x/mod*y)*mod;
  return ret<0?ret+mod:ret;</pre>
3.6 BigInt
struct Bigint{
  static const int LEN = 60;
  static const int BIGMOD = 10000;
  int s
  int vl, v[LEN];
  // vector<int> v;
  Bigint() : s(1) \{ vl = 0; \}
  Bigint(long long a) {
    s = 1; vl = 0;
if (a < 0) \{ s = -1; a = -a; \}
    while (a) {
      push_back(a % BIGMOD);
      a \neq BIGMOD;
  Bigint(string str) {
    s = 1; vl = 0;
    int stPos = 0, num = 0;
    if (!str.empty() && str[0] == '-') {
      stPos = 1;
      s = -1:
    for (int i=SZ(str)-1, q=1; i>=stPos; i--) {
  num += (str[i] - '0') * q;
  if ((q *= 10) >= BIGMOD) {
        push_back(num);
        num = 0; q = 1;
    if (num) push_back(num);
    n();
  int len() const {
    return vl;
          return SZ(v);
  bool empty() const { return len() == 0; }
  void push_back(int x) {
    v[vl++] = x;
           v.PB(x);
  void pop_back() {
    vl--;
// v.pop_back();
  int back() const {
    return v[vl-1];
          return v.back();
  void n() {
    while (!empty() && !back()) pop_back();
  void resize(int nl) {
    vl = nl;
    fill(v, v+vl, 0)
    //
           v.resize(nl);
           fill(ALL(v), 0);
  }
  void print() const {
    if (empty()) { putchar('0'); return; }
if (s == -1) putchar('-');
```

```
printf("%d", back());
  for (int i=len()-2; i>=0; i--) printf("%.4d",v[i]);
friend std::ostream& operator << (std::ostream& out,</pre>
     const Bigint &a) {
  if (a.empty()) { out << "0"; return out; }</pre>
  if (a.s == -1) out << "-";
  out << a.back();
  for (int i=a.len()-2; i>=0; i--) {
    char str[10];
snprintf(str, 5, "%.4d", a.v[i]);
    out << str;
  return out;
int cp3(const Bigint &b)const {
  if (s != b.s) return s - b.s;
if (s == -1) return -(-*this).cp3(-b);
  if (len() != b.len()) return len()-b.len();//int
  for (int i=len()-1; i>=0; i--)
    if (v[i]!=b.v[i]) return v[i]-b.v[i];
  return 0;
bool operator<(const Bigint &b)const</pre>
  { return cp3(b)<0; }
bool operator<=(const Bigint &b)const</pre>
  { return cp3(b)<=0; ]
bool operator==(const Bigint &b)const
  { return cp3(b)==0;
bool operator!=(const Bigint &b)const
  { return cp3(b)!=0; }
bool operator>(const Bigint &b)const
  { return cp3(b)>0; }
bool operator>=(const Bigint &b)const
  { return cp3(b)>=0; }
Bigint operator - () const {
  Bigint r = (*this);
  r.s = -r.s;
  return r;
Bigint operator + (const Bigint &b) const {
  if (s == -1) return -(-(*this)+(-b));
if (b.s == -1) return (*this)-(-b);
  Bigint r;
  int nl = max(len(), b.len());
  r.resize(nl + 1);
for (int i=0; i<nl; i++) {
    if (i < len()) r.v[i] += v[i];
if (i < b.len()) r.v[i] += b.v[i];</pre>
    if(r.v[i] >= BIGMOD) {
       r.v[i+1] += r.v[i] / BIGMOD;
       r.v[i] %= BIGMOD;
    }
  }
  r.n();
  return r;
Bigint operator - (const Bigint &b) const {
  if (s == -1) return -(-(*this)-(-b));
  if (b.s == -1) return (*this)+(-b);
  if ((*this) < b) return -(b-(*this));</pre>
  Bigint r;
  r.resize(len());
  for (int i=0; i<len(); i++) {</pre>
    r.v[i] += v[i];
    if (i < b.len()) r.v[i] -= b.v[i];</pre>
    if (r.v[i] < 0) {
       r.v[i] += BIGMOD;
       r.v[i+1]--;
    }
  }
  r.n();
Bigint operator * (const Bigint &b) {
  Bigint r;
  r.resize(len() + b.len() + 1);
  r.s = s * b.s;
  for (int i=0; i<len(); i++) {</pre>
    for (int j=0; j<b.len(); j++) {
  r.v[i+j] += v[i] * b.v[j];</pre>
       if(r.v[i+j] >= BIGMOD) {
```

```
r.v[i+j+1] += r.v[i+j] / BIGMOD;
          r.v[i+j] %= BIGMOD;
      }
    }
    r.n();
    return r;
  Bigint operator / (const Bigint &b) {
    Bigint r
    r.resize(max(1, len()-b.len()+1));
    int oriS = s:
    Bigint b2 = b; // b2 = abs(b)
    s = b2.s = r.s = 1;
    for (int i=r.len()-1; i>=0; i--) {
      int d=0, u=BIGMOD-1;
      while(d<u) {</pre>
        int m = (d+u+1)>>1;
        r.v[i] = m;
        if((r*b2) > (*this)) u = m-1;
        else d = m;
      r.v[i] = d;
    s = oriS:
    r.s = s * b.s;
    r.n();
    return r;
  Bigint operator % (const Bigint &b) {
    return (*this)-(*this)/b*b;
};
3.7 Linear Recurrence
```

```
// Usage: linearRec({0, 1}, {1, 1}, k) //k'th fib
typedef vector<ll> Poly;
//S:前i項的值,tr:遞迴系數,k:求第k項
ll linearRec(Poly& S, Poly& tr, ll k) {
  int n = tr.size()
  auto combine = [&](Poly& a, Poly& b) {
  Poly res(n * 2 + 1);
    rep(i,0,n+1) rep(j,0,n+1)
       res[i+j]=(res[i+j] + a[i]*b[j])%mod;
    for(int i = 2*n; i > n; --i) rep(j,0,n)
  res[i-1-j]=(res[i-1-j] + res[i]*tr[j])%mod;
     res.resize(n + 1);
    return res;
  Poly pol(n_+_1), e(pol);
  pol[0] = e[1] = 1;
  for (++k; k; k /= 2) {
  if (k % 2) pol = combine(pol, e);
     e = combine(e, e);
  11 \text{ res} = 0:
  rep(i,0,n) res=(res + pol[i+1]*S[i])%mod;
  return res;
```

#### 3.8 Miller Rabin

```
3 : 2, 7, 61
4 : 2, 13, 23, 1662803
6 : pirmes <= 13
// n < 4,759,123,141
// n < 1,122,004,669,633
// n < 3,474,749,660,383
                                             pirmes <= 13
// n < 2^64
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
// Make sure testing integer is in range [2, n-2] if
// you want to use magic.
LL magic[]={}
bool witness(LL a, LL n, LL u, int t){
  if(!a) return 0;
  LL x=mypow(a,u,n);
  for(int i=0;i<t;i++) {</pre>
    LL nx=(x*x)%n
     if(nx==1&&x!=1&&x!=n-1) return 1;
    x=nx;
  }
  return x!=1;
bool miller_rabin(LL n) {
```

```
National Taiwan Ocean University HongLongLong
  int s=(magic number size)
  // iterate s times of witness on n
  // return 1 if prime, 0 otherwise
  if(n<2) return 0;</pre>
  if(!(n\&1)) return n == 2;
  ll u=n-1; int t=0;
// n-1 = u*2^t
  while(!(u&1)) u>>=1, t++;
  while(s--){
    LL a=magic[s]%n;
    if(witness(a,n,u,t)) return 0;
  return 1;
       Faulhaber (\sum_{i=1}^{n} i^p)
/* faulhaber's formula -
 * cal power sum formula of all p=1\sim k in O(k^2) */
#define MAXK 2500
const int mod = 1000000007;
int b[MAXK]; // bernoulli number
int inv[MAXK+1]; // inverse
int cm[MAXK+1][MAXK+1]; // combinactories
int co[MAXK][MAXK+2]; // coeeficient of x^j when p=i
inline int getinv(int x) {
  int a=x, b=mod, a0=1, a1=0, b0=0, b1=1;
  while(b) {
    int q,t;
    q=a/b; t=b; b=a-b*q; a=t;
t=b0; b0=a0-b0*q; a0=t;
    t=b1; b1=a1-b1*q; a1=t;
  return a0<0?a0+mod:a0;</pre>
inline void pre() {
  /* combinational */
  for(int i=0;i<=MAXK;i++) {</pre>
    cm[i][0]=cm[i][i]=1;
    for(int j=1;j<i;j++)
  cm[i][j]=add(cm[i-1][j-1],cm[i-1][j]);</pre>
  /* inverse */
  for(int i=1;i<=MAXK;i++) inv[i]=getinv(i);</pre>
  /* bernoulli */
  b[0]=1; b[1]=getinv(2); // with b[1] = 1/2 for(int i=2;i<MAXK;i++) {
     if(i&1) { b[i]=0; continue; }
    b[i]=1;
    for(int j=0;j<i;j++)</pre>
       b[i]=sub(b[i],
                 mul(cm[i][j],mul(b[j], inv[i-j+1])));
  /* faulhaber */
  // sigma_x=1~n \{x^p\} =
// 1/(p+1) * sigma_j=0~p \{C(p+1,j)*Bj*n^(p-j+1)\}
  for(int i=1;i<MAXK;i++) {</pre>
    co[i][0]=0;
    for(int j=0;j<=i;j++)
  co[i][i-j+1]=mul(inv[i+1], mul(cm[i+1][j], b[j]))</pre>
/* sample usage: return f(n,p) = sigma_x=1\sim (x^p) */
inline int solve(int n,int p) {
  int sol=0,m=n;
  for(int i=1;i<=p+1;i++) {</pre>
    sol=add(sol,mul(co[p][i],m));
    m = mul(m, n);
  return sol;
3.10 Chinese Remainder
LL x[N],m[N];
LL CRT(LL x1, LL m1, LL x2, LL m2) {
            _gcd(m1, m2);
  if((x2 - x1) % g) return -1;// no sol
```

```
m1 /= g; m2 /= g;
pair<LL,LL> p = gcd(m1, m2);
```

```
LL lcm = m1 * m2 * g;
LL res = p.first * (x2 - x1) * m1 + x1;
   return (res % lcm + lcm) % lcm;
LL solve(int n){ // n>=2,be careful with no solution
   LL res=CRT(x[0],m[0],x[1],m[1]),p=m[0]/__gcd(m[0],m
       [1])*m[1];
   for(int i=2;i<n;i++){</pre>
     res=CRT(res,p,x[i],m[i]);
     p=p/__gcd(p,m[i])*m[i];
   return res;
}
```

### 3.11 Pollard Rho 找因數

```
// does not work when n is prime
LL f(LL x, LL mod){ return add(mul(x,x,mod),1,mod); }
LL pollard_rho(LL n) {
  if(!(n&1)) return 2;
  while(true){
    LL y=2, x=rand()%(n-1)+1, res=1;
    for(int sz=2; res==1; sz*=2) {
       for(int i=0; i<sz && res<=1; i++) {</pre>
        x = f(x, n);
        res = \_gcd(abs(x-y), n);
      }
      y = x;
     if (res!=0 && res!=n) return res;
}
```

### 3.12 Josephus Problem

```
int josephus(int n, int m){ //n人每m次
    int ans = 0;
    for (int i=1; i<=n; ++i)</pre>
        ans = (ans + m) \% i;
    return ans;
}
```

#### 3.13 Gaussian Elimination

```
const int GAUSS_MOD = 100000007LL;
 struct GAUSS{
      int n;
      vector<vector<int>> v;
      int ppow(int a , int k){
   if(k == 0) return 1;
           if(k % 2 == 0) return ppow(a * a % GAUSS_MOD ,
                k >> 1);
           if(k % 2 == 1) return ppow(a * a % GAUSS_MOD ,
    k >> 1) * a % GAUSS_MOD;
      vector<int> solve(){
           vector<int> ans(n);
           REP(now , 0 , n){
REP(i , now ,
                     i , now , n) if(v[now][now] == 0 && v[i ][now] != 0)
                swap(v[i] , v[now]); // det = -det;
if(v[now][now] == 0) return ans;
                int inv = ppow(v[now][now] , GAUSS_MOD - 2)
                REP(i , 0 , n) if(i != now){
   int tmp = v[i][now] * inv % GAUSS_MOD;
                     REP(j , now , n + 1) (v[i][j] += GAUSS_MOD - tmp * v[now][j] %
                          GAUSS_MOD) %= GAUSS_MOD;
                   , 0 , n) ans[i] = v[i][n + 1] * ppow(v[i])
                [i] , GAUSS_MOD - 2) % GAUSS_MOD;
           return ans;
      // gs.v.clear() , gs.v.resize(n , vector<int>(n + 1
             , 0));
} gs;
```

#### 3.14 ax+by=gcd

|PII gcd(int a, int b){

if(b == 0) return {1, 0};

PII q = gcd(b, a % b);

```
return {q.second, q.first - q.second * (a / b)};
3.15 Discrete sqrt
void calcH(LL &t, LL &h, const LL p) {
  LL tmp=p-1; for(t=0;(tmp&1)==0;tmp/=2) t++; h=tmp;
// solve equation x^2 \mod p = a
bool solve(LL a, LL p, LL &x, LL &y) {
  if(p == 2) { x = y = 1; return true; }
int p2 = p / 2, tmp = mypow(a, p2, p);
if (tmp == p - 1) return false;
  if ((p + 1) \% 4 == 0) {
     x=mypow(a,(p+1)/4,p); y=p-x; return true;
  } else {
     LL t, h, b, pb; calcH(t, h, p);
     if (t >= 2) {
       do \{b = rand() \% (p - 2) + 2;
       } while (mypow(b, p / 2, p)] = p - 1);
     pb = mypow(b, h, p);
} int s = mypow(a, h / 2, p);
for (int step = 2; step <= t; step++) {
  int ss = (((LL)(s * s) % p) * a) % p;</pre>
        for(int i=0;i<t-step;i++) ss=mul(ss,ss,p);</pre>
       if (ss + 1 == p) s = (s * pb) % p;
pb = ((LL)pb * pb) % p;
     x = ((LL)s * a) % p; y = p - x;
  } return true;
```

# 3.16 Romberg 定積分

```
// Estimates the definite integral of
// \cdot int_a^b f(x) dx
template<class T>
double romberg( T& f, double a, double b, double eps=1e
  vector<double>t; double h=b-a,last,curr; int k=1,i=1;
  t.push_back(h*(f(a)+f(b))/2);
  do{ last=t.back(); curr=0; double x=a+h/2;
for(int j=0;j<k;j++) curr+=f(x), x+=h;</pre>
     curr=(t[0] + h*curr)/2; double k1=4.0/3.0,k2
          =1.0/3.0;
     for(int j=0;j<i;j++){ double temp=k1*curr-k2*t[j];</pre>
     t[j]=curr; curr=temp; k2/=4*k1-k2; k1=k2+1; 
} t.push_back(curr); k*=2; h/=2; i++;
  }while( fabs(last-curr) > eps);
  return t.back();
```

### 3.17 Prefix Inverse

```
void solve( int m ){
  inv[1] = 1;
  for( int i = 2 ; i < m ; i ++ )
  inv[ i ] = ((LL)(m - m / i) * inv[m % i]) % m;</pre>
```

# 3.18 Roots of Polynomial 找多項式的根

```
const double eps = 1e-12;
const double inf = 1e+12;
double a[ 10 ], x[ 10 ];
int sign( double x ){return (x < -eps)?(-1):(x>eps);}
double f(double a[], int n, double x){
  double tmp=1,sum=0;
  for(int i=0;i<=n;i++)</pre>
  { sum=sum+a[i]*tmp; tmp=tmp*x; }
  return sum;
double binary(double l,double r,double a[],int n){
  int sl=sign(f(a,n,l)),sr=sign(f(a,n,r));
  if(sl==0) return l; if(sr==0) return r;
  if(sl*sr>0) return inf;
  while(r-l>eps){
    double mid=(l+r)/2;
    int ss=sign(f(a,n,mid));
    if(ss==0) return mid;
```

```
if(ss*sl>0) l=mid; else r=mid;
  return 1;
}
void solve(int n,double a[],double x[],int &nx){
  if(n==1){ x[1]=-a[0]/a[1]; nx=1; return; }
  double da[10], dx[10]; int ndx;
  for(int i=n;i>=1;i--) da[i-1]=a[i]*i;
  solve(n-1,da,dx,ndx);
  nx=0:
  if(ndx==0){
    double tmp=binary(-inf,inf,a,n);
    if (tmp<inf) x[++nx]=tmp;</pre>
    return;
  double tmp;
  tmp=binary(-inf,dx[1],a,n);
if(tmp<inf) x[++nx]=tmp;</pre>
  for(int i=1;i<=ndx-1;i++){</pre>
    tmp=binary(dx[i],dx[i+1],a,n);
    if(tmp<inf) x[++nx]=tmp;</pre>
  tmp=binary(dx[ndx],inf,a,n);
  if(tmp<inf) x[++nx]=tmp;</pre>
int main() {
  scanf("%d",&n);
  for(int i=n;i>=0;i--) scanf("%lf",&a[i]);
  int nx;
  solve(n,a,x,nx);
  for(int i=1;i<=nx;i++) printf("%.6f\n",x[i]);</pre>
```

#### 3.19 Primes

```
/* 12721, 13331, 14341, 75577, 123457, 222557, 556679 * 999983, 1097774749, 1076767633, 100102021, 999997771
* 1001010013, 1000512343, 987654361, 999991231
* 999888733, 98789101, 987777733, 999991921, 1010101333
* 1010102101, 1000000000039, 100000000000037
* 2305843009213693951, 4611686018427387847
* 9223372036854775783, 18446744073709551557 */
int mu[N], p_tbl[N];
vector<int> primes;
void sieve() {
  mu[1] = p_tbl[1] = 1;
   for( int i = 2 ; i < N ; i ++ ){
  if( !p_tbl[ i ] ){</pre>
        p_tbl[ i ] = i;
        primes.push_back( i );
        mu[i] = -1;
     for( int p : primes ){
  int x = i * p;
  if( x >= M ) break;
        p_{tbl}[x] = p;
        mu[x] = -mu[i]
        if( i % p == 0 ){
           mu[x] = 0;
           break;
vector<int> factor( int x ){
   vector<int> fac{ 1 };
   while(x > 1)
     int fn = SZ(fac), p = p_tbl[ x ], pos = 0;
     while( x \% p == 0){
        x /= p;
for( int i = 0 ; i < fn ; i ++ )
  fac.PB( fac[ pos ++ ] * p );</pre>
  } }
   return fac;
```

### 3.20 Result

- For  $n,m\in\mathbb{Z}^*$  and prime P,  $C(m,n)\mod P=\Pi(C(m_i,n_i))$  where  $m_i$  is the i-th digit of m in base P.
- Stirling approximation :  $n! \approx \sqrt{2\pi n} \left(\frac{n}{e}\right)^n e^{\frac{1}{12n}}$
- Stirling Numbers(permutation |P|=n with k cycles):  $S(n,k) = \text{coefficient of } x^k \text{ in } \Pi_{i=0}^{n-1}(x+i)$

### 4 Geometry

#### 4.1 Intersection of 2 lines

```
Pt interPnt( Pt p1, Pt p2, Pt q1, Pt q2){
  double f1 = ( p2 - p1 ) ^ ( q1 - p1 );
  double f2 = ( p2 - p1 ) ^ ( p1 - q2 );
  double f = ( f1 + f2 );
  if( fabs( f ) < eps ) return Pt( nan(""), nan("") );
  return q1 * ( f2 / f ) + q2 * ( f1 / f );
}</pre>
```

#### 4.2 halfPlaneIntersection

```
bool isin( Line 10, Line 11, Line 12 ){
  // Check inter(l1, l2) in l0
  bool res; Pt p = interPnt(l1, l2,
                                      res):
  return ((10.SE - 10.FI) \wedge (p - 10.FI)) > eps;
/* If no solution, check: 1. ret.size() < 3
 * Or more precisely, 2. interPnt(ret[0], ret[1])</pre>
 * in all the lines. (use (l.S - l.F) \land (p - l.F) \gt 0
/* --^- Line.FI --^- Line.SE --^- */
vector<Line> halfPlaneInter( vector<Line> lines ){
  int sz = lines.size();
  vector<double> ata(sz), ord(sz);
  for( int i=0; i<sz; i++) {</pre>
    ord[i] = i;
    Pt d = lines[i].SE - lines[i].FI;
    ata[i] = atan2(d.Y, d.X);
  return ata[i] < ata[j];</pre>
  });
  vector<Line> fin;
  for (int i=0; i<sz; i++)</pre>
    if (!i or fabs(ata[ord[i]] - ata[ord[i-1]]) > eps)
      fin.PB(lines[ord[i]]);
  deque<Line> dq;
  for (int i=0; i<(int)(fin.size()); i++) {
  while((int)(dq.size()) >= 2 and
        not isin(fin[i], dq[(int)(dq.size())-2]
                           dq[(int)(dq.size())-1]))
      dq.pop_back();
    while((int)(dq.size()) >= 2 and
        not isin(fin[i], dq[0], dq[1]))
      dq.pop_front();
    dq.push_back(fin[i]);
  while( (int)(dq.size()) >= 3 and
      not isin(dq[0], dq[(int)(dq.size())-2]
                       dq[(int)(dq.size())-1]))
    dq.pop_back();
  while( (int)(dq.size()) >= 3 and
```

```
not isin(dq[(int)(dq.size())-1], dq[0], dq[1]))
  dq.pop_front();
vector<Line> res(dq.begin(),dq.end());
return res;
}
```

#### 4.3 Convex Hull

```
double cross(Pt o, Pt a, Pt b){
  return (a-o) ^ (b-o);
vector<Pt> convex_hull(vector<Pt> pt){
  sort(pt.begin(),pt.end());
  int top=0;
  vector<Pt> stk(2*pt.size());
  for (int i=0; i<(int)pt.size(); i++){</pre>
    while (top'>= 2 && cross(stk[top-2],stk[top-1],pt[i
        ]) <= 0)
      top--;
    stk[top++] = pt[i];
  for (int i=pt.size()-2, t=top+1; i>=0; i--){
    while (top >= t && cross(stk[top-2],stk[top-1],pt[i
        ]) <= 0)
      top--;
    stk[top++] = pt[i];
  stk.resize(top-1);
  return stk;
```

#### 4.4 Convex Hull 3D

```
struct Pt{
   Pt cross(const Pt &p) const
   { return Pt(y * p.z - z * p.y, z * p.x - x * p.z, x * p.y - y * p.x); }
} info[N];
int mark[N][N],n, cnt;;
double mix(const Pt &a, const Pt &b, const Pt &c)
{ return a * (b ^ c); }
double area(int a, int b, int c)
{ return norm((info[b] - info[a]) ^ (info[c] - info[a])
double volume(int a, int b, int c, int d)
{ return mix(info[b] - info[a], info[c] - info[a], info
      [d] - info[a]); }
struct Face{
   int a, b, c; Face(){}
Face(int a, int b, int c): a(a), b(b), c(c) {}
   int &operator [](int k)
   { if (k == 0) return a; if (k == 1) return b; return
vector<Face> face;
void insert(int a, int b, int c)
{ face.push_back(Face(a, b, c)); }
void add(int v) {
   vector <Face> tmp; int a, b, c; cnt++;
for (int i = 0; i < SIZE(face); i++) {</pre>
      a = face[i][0]; b = face[i][1]; c = face[i][2];
      if(Sign(volume(v, a, b, c)) < 0)
mark[a][b] = mark[b][a] = mark[b][c] = mark[c][b] =</pre>
             mark[c][a] = mark[a][c] = cnt;
      else tmp.push_back(face[i]);
   } face = tmp;
   for (int i = 0; i < SIZE(tmp); i++) {</pre>
      a = face[i][0]; b = face[i][1]; c = face[i][2];
if (mark[a][b] == cnt) insert(b, a, v);
if (mark[b][c] == cnt) insert(c, b, v);
      if (mark[c][a] == cnt) insert(a, c, v);
}}
int Find(){
   for (int i = 2; i < n; i++) {
  Pt ndir = (info[0] - info[i]) ^ (info[1] - info[i])</pre>
      if (ndir == Pt()) continue; swap(info[i], info[2]);
for (int j = i + 1; j < n; j++) if (Sign(volume(0, 1, 2, j)) != 0) {</pre>
         swap(info[j], info[3]); insert(0, 1, 2); insert
(0, 2, 1); return 1;
} } return 0; }
```

```
int main() {
 for (; scanf("%d", &n) == 1; ) {
    for (int i = 0; i < n; i++) info[i].Input();
    sort(info, info + n); n = unique(info, info + n) -
    face.clear(); random_shuffle(info, info + n);
    if (Find()) { memset(mark, 0, sizeof(mark)); cnt =
      for (int i = 3; i < n; i++) add(i); vector<Pt>
          Ndir;
      for (int i = 0; i < SIZE(face); ++i) {
        p = p / norm( p ); Ndir.push_back(p);
} sort(Ndir.begin(), Ndir.end());
      int ans = unique(Ndir.begin(), Ndir.end()) - Ndir
    .begin();
printf("%d\n", ans)
} else printf("1\n");
                      ans);
} }
double calcDist(const Pt &p, int a, int b, int c)
{ return fabs(mix(info[a] - p, info[b] - p, info[c] - p
    ) / area(a, b, c)); }
//compute the minimal distance of center of any faces
double findDist() { //compute center of mass
 double totalWeight = 0; Pt center(.0, .0, .0);
Pt first = info[face[0][0]];
  for (int i = 0; i < SIZE(face); ++i) {</pre>
    double weight = mix(info[face[i][0]] - first, info[
         face[i][1]]
    - first, info[face[i][2]] - first);
totalWeight += weight; center = center + p * weight
 } center = center / totalWeight;
double res = 1e100; //compute distance
  for (int i = 0; i < SIZE(face); ++i)</pre>
    res = min(res, calcDist(center, face[i][0], face[i
        ][1], face[i][2]));
    return res; }
```

### 4.5 Intersection of 2 segments

# 4.6 Intersection of circle and segment

```
| bool Inter( const Pt& p1 , const Pt& p2 , Circle& cc ){
| Pt dp = p2 - p1;
| double a = dp * dp;
| double b = 2 * ( dp * ( p1 - cc.0 ) );
| double c = cc.0 * cc.0 + p1 * p1 - 2 * ( cc.0 * p1 )
| - cc.R * cc.R;
| double bb4ac = b * b - 4 * a * c;
| return !( fabs( a ) < eps or bb4ac < 0 );
| }
```

### 4.7 Intersection of 2 circles

### 4.8 Circle cover

```
| #define N 1021
| #define D double
| struct CircleCover{
| int C; Circ c[ N ]; //填入C(圓數量),c(圓陣列)
```

```
bool g[ N ][ N ], overlap[ N ][ N ];
// Area[i] : area covered by at least i circles
   D Area[ N ];
   void init( int _C ){ C = _C; }
   bool CCinter( Circ& a , Circ& b , Pt& p1 , Pt& p2 ){
      Pt o1 = a.0, o2 = b.0;
      D r1 = a.R , r2 = b.R;
if( norm( o1 - o2 ) > r1 + r2 ) return {};
if( norm( o1 - o2 ) < max(r1, r2) - min(r1, r2) )
      return {};
D d2 = ( o1 - o2 ) * ( o1 - o2 );
      D d = sqrt(d2);
      if( d > r1 + r2 ) return false;
      Pt u=(01+02)*0.5 + (01-02)*((r2*r2-r1*r1)/(2*d2));

D A=sqrt((r1+r2+d)*(r1-r2+d)*(r1+r2-d)*(-r1+r2+d));

Pt v=Pt( 01.Y-02.Y , -01.X + 02.X ) * A / (2*d2);
      p1 = u + v; p2 = u - v;
      return true;
   struct Teve {
      Pt p; D ang; int add;
      Teve() {}
      Teve(Pt _a, D _b, int _c):p(_a), ang(_b), add(_c){}
bool operator<(const Teve &a)const</pre>
   {return ang < a.ang;}
}eve[ N * 2 ];
    // strict: x = 0, otherwise x = -1
   bool disjuct( Circ& a, Circ &b, int x )
   {return sign( norm( a.0 - b.0 ) - a.R - b.R ) > x;}
bool contain( Circ& a, Circ &b, int x )
{return sign( a.R - b.R - norm( a.0 - b.0 ) ) > x;}
   bool contain(int i, int j){
      /* c[j] is non-strictly in c[i]. */
return (sign(c[i].R - c[j].R) > 0 ||
                  (sign(c[i].R - c[j].R) == 0 \& i < j) ) \& \&
                        contain(c[i], c[j], -1);
   void solve(){
      for( int i = 0 ; i <= C + 1 ; i ++ )
Area[ i ] = 0;
      for( int i = 0; i < C; i ++ )
for( int j = 0; j < C; j ++ )
overlap[i][j] = contain(i, j);
      for( int i = 0 ; i < C ; i ++ )
for( int j = 0 ; j < C ; j ++
            for( int i = 0 ; i < C ; i ++ ){
         int E = 0, cnt = 1;
for( int j = 0 ; j < C ; j ++ )
if( j != i && overlap[j][i] )
               cnt ++;
         for( int j = 0 ; j < C ; j
  if( i != j && g[i][j] ){
   Pt aa, bb;</pre>
               CCinter(c[i], c[j], aa, bb);

D A=atan2(aa.Y - c[i].0.Y, aa.X - c[i].0.X);

D B=atan2(bb.Y - c[i].0.Y, bb.X - c[i].0.X);
               eve[E ++] = Teve(bb, B, 1);
               eve[E ++] = Teve(aa, A, -1);
               if(B > A) cnt ++;
         if( E == 0 ) Area[ cnt ] += pi * c[i].R * c[i].R;
         else{
            sort( eve , eve + E );
            eve[E] = eve[0];
            for( int j = 0; j < E; j ++){
               cnt += eve[j].add;
Area[cnt] += (eve[j].p ^ eve[j + 1].p) * 0.5;
               D theta = eve[j + 1].ang - eve[j].ang; if (theta < 0) theta += 2.0 * pi;
               Area[cnt] +=
                  (theta - sin(theta)) * c[i].R*c[i].R * 0.5;
}}}};
```

#### 4.9 Convex Hull trick

```
/* Given a convexhull, answer querys in O(\lg N)
CH should not contain identical points, the area should
be > 0, min pair(x, y) should be listed first */
double det( const Pt& p1 , const Pt& p2 )
{ return p1.X * p2.Y - p1.Y * p2.X; }
```

```
struct Conv{
                                                                      return true:
  int n;
  vector<Pt> a;
  vector<Pt> upper, lower;
  Conv(vector < Pt > _a) : a(_a){}
                                                                    int get_tang(Pt vec){
    n = a.size();
    int ptr = 0;
for(int i=1; i<n; ++i) if (a[ptr] < a[i]) ptr = i;</pre>
    for(int i=0; i<=ptr; ++i) lower.push_back(a[i]);
for(int i=ptr; i<n; ++i) upper.push_back(a[i]);</pre>
                                                                      return ret.second;
    upper.push_back(a[0]);
  int sign( LL x ){ // fixed when changed to double
  return x < 0 ? -1 : x > 0; }
  pair<LL,int> get_tang(vector<Pt> &conv, Pt vec){
    int l = 0, r = (int)conv.size() - 2;
                                                                        if (p0 > p1) swap(p0, p1);
    for( ; l + 1 < r; ){
  int mid = (l + r) / 2;</pre>
      if(sign(det(conv[mid+1]-conv[mid],vec))>0)r=mid;
                                                                       return 1;
      else l = mid;
                                                                     }
    return max(make_pair(det(vec, conv[r]), r);
                                                                     return 0;
                make_pair(det(vec, conv[0]), 0));
                                                                 } };
  void upd_tang(const Pt &p, int id, int &i0, int &i1){
    if(det(a[i0] - p, a[id] - p) > 0) i0 = id;
    if(det(a[i1] - p, a[id] - p) < 0) i1 = id;
                                                                      sign1 ){
  void bi_search(int l, int r, Pt p, int &i0, int &i1){
    if(l == r) return;
upd_tang(p, l % n, i0, i1);
                                                                    vector<Line> ret;
    int sl=sign(det(a[l % n] - p, a[(l + 1) % n] - p));
                                                                    if( d_sq < eps ) return ret;</pre>
                                                                    double d = sqrt( d_sq );
Pt v = ( c2.0 - c1.0 ) / d;
    for( ; l + 1 < r; ) {
  int mid = (l + r) / 2;</pre>
      int smid=sign(det(a[mid%n]-p, a[(mid+1)%n]-p));
      if (smid == sl) l = mid;
      else r = mid;
    upd_tang(p, r % n, i0, i1);
  int bi_search(Pt u, Pt v, int l, int r) {
    int sl = sign(det(v - u, a[l % n] - u));
    for(; l + 1 < r; ) {
      int mid = (l + r) / 2;
                                                                         p2 = p1 + perp(c2.0 - c1.0);
      int smid = sign(det(v - u, a[mid % n] - u));
                                                                      ret.push_back( { p1 , p2 } );
      if (smid == sl) l = mid;
      else r = mid;
                                                                    return ret;
    return 1 % n;
  // 1. whether a given point is inside the CH
                                                                  4.11 KD Tree
 bool contain(Pt p) {
    if (p.X < lower[0].X || p.X > lower.back().X)
                                                                  const int MXN=100005;
                                                                  const int MXK=10;
                                                                  struct KDTree{
    int id = lower_bound(lower.begin(), lower.end(), Pt
         (p.X, -INF)) - lower.begin();
                                                                    struct Nd{
    if (lower[id].X == p.X) {
                                                                      LL x[MXK], mn[MXK], mx[MXK];
    if (lower[id].Y > p.Y) return 0;
}else if(det(lower[id-1]-p,lower[id]-p)<0)return 0;</pre>
                                                                      int id,f;
                                                                      Nd *1,*r;
    id = lower_bound(upper.begin(), upper.end(), Pt(p.X
                                                                    }tree[MXN],*root;
          INF), greater<Pt>()) - upper.begin();
    if (upper[id].X == p.X) {
  if (upper[id].Y < p.Y) return 0;</pre>
                                                                    LL dis(LL a[MXK],LL b[MXK]){
    }else if(det(upper[id-1]-p,upper[id]-p)<0)return 0;</pre>
                                                                      LL ret=0;
                                                                      return ret;
  // 2. Find 2 tang pts on CH of a given outside point
  // return true with i0, i1 as index of tangent points
  // return false if inside CH
                                                                      n=_n, k=_k;
 bool get_tang(Pt p, int &i0, int &i1) {
                                                                       for(int i=0;i<n;i++){</pre>
    if (contain(p)) return false;
                                                                         tree[i].id=i;
    i0 = i1 = 0;
    int id = lower_bound(lower.begin(), lower.end(), p)
    - lower.begin();
bi_search(0, id, p, i0, i1);
bi_search(id, (int)lower.size(), p, i0, i1);
                                                                      root=build(0,n-1,0);
                                                                    Nd* build(int l,int r,int d){
    id = lower_bound(upper.begin(), upper.end(), p,
                                                                      if(l>r) return NULL;
         greater<Pt>()) - upper.begin();
                                                                       if(d==k) d=0;
    bi_search((int)lower.size() - 1, (int)lower.size()
                                                                      int m=(l+r)>>1;
         - 1 + id, p, i0, i1)
    bi_search((int)lower.size() - 1 + id, (int)lower.
         size() - 1 + (int)upper.size(), p, i0, i1);
```

```
// 3. Find tangent points of a given vector
  // ret the idx of vertex has max cross value with vec
    pair<LL, int> ret = get_tang(upper, vec);
     ret.second = (ret.second+(int)lower.size()-1)%n;
     ret = max(ret, get_tang(lower, vec));
  // 4. Find intersection point of a given line
  // return 1 and intersection is on edge (i, next(i))
  // return 0 if no strictly intersection
  bool get_intersection(Pt u, Pt v, int &i0, int &i1){
   int p0 = get_tang(u - v), p1 = get_tang(v - u);
if(sign(det(v-u,a[p0]-u))*sign(det(v-u,a[p1]-u))<0){</pre>
      i0 = bi_search(u, v, p0, p1);
      i1 = bi\_search(u, v, p1, p0 + n);
4.10 Tangent line of two circles
vector<Line> go( const Cir& c1 , const Cir& c2 , int
   // sign1 = 1 for outer tang, -1 for inter tang
  double d_{sq} = norm2(c1.0 - c2.0);
  double c = (c1.R - sign1 * c2.R) / d;
  if( c * c > 1 ) return ret;

double h = sqrt( max( 0.0 , 1.0 - c * c ) );

for( int sign2 = 1 ; sign2 >= -1 ; sign2 -= 2 ){
    Pt n = { v.X * c - sign2 * h * v.Y
    v.Y * c + sign2 * h * v.X };
Pt p1 = c1.0 + n * c1.R;
    Pt p2 = c2.0 + n * (c2.R * sign1);
    if( fabs( p1.X - p2.X ) < eps and fabs( p1.Y - p2.Y ) < eps )
```

```
LL dis(LL a, LL b){return (a-b)*(a-b);}
  for(int i=0;i<k;i++) ret+=dis(a[i],b[i]);</pre>
void init(vector<vector<LL>>> &ip,int _n,int _k){
    copy(ip[i].begin(),ip[i].end(),tree[i].x);
  nth_element(tree+l,tree+m,tree+r+1,[&](const Nd &a,
      const Nd &b){return a.x[d]<b.x[d];});</pre>
  tree[m].f=d;
```

```
copy(tree[m].x,tree[m].x+k,tree[m].mn);
    copy(tree[m].x,tree[m].x+k,tree[m].mx);
    tree[m].l=build(l,m-1,d+1);
    if(tree[m].l){
      for(int i=0;i<k;i++){</pre>
        tree[m].mn[i]=min(tree[m].mn[i],tree[m].l->mn[i
        tree[m].mx[i]=max(tree[m].mx[i],tree[m].l->mx[i
             ]);
      }
    tree[m].r=build(m+1,r,d+1);
    if(tree[m].r){
      for(int i=0;i<k;i++){</pre>
        tree[m].mn[i]=min(tree[m].mn[i],tree[m].r->mn[i
        tree[m].mx[i]=max(tree[m].mx[i],tree[m].r->mx[i
      }
    }
    return tree+m;
  LL pt[MXK],md;
  int mID;
  bool touch(Nd *r){
    LL d=0;
    for(int i=0;i<k;i++){</pre>
      if(pt[i]<=r->mn[i]) d+=dis(pt[i],r->mn[i]);
        else if(pt[i]>=r->mx[i]) d+=dis(pt[i],r->mx[i])
    return d<md;</pre>
  void nearest(Nd *r){
    if(!rll!touch(r)) return;
    LL td=dis(r->x,pt);
    if(td<md) md=td,mID=r->id;
    nearest(pt[r->f]< r->x[r->f]?r->l:r->r);
    nearest(pt[r->f]< r->x[r->f]?r->r:r->l);
  pair<LL,int> query(vector<LL> &_pt,LL _md=1LL<<57){</pre>
    mID=-1, md=\_md;
    copy(_pt.begin(),_pt.end(),pt);
    nearest(root);
    return {md,mID};
}tree;
```

#### 4.12 Lower Concave Hull

```
const ll is_query = -(1LL<<62);</pre>
struct Line {
 11 m, b;
 mutable function<const Line*()> succ;
 bool operator<(const Line& rhs) const {</pre>
    if (rhs.b != is_query) return m < rhs.m;</pre>
    const Line* s = succ();
    return s ? b - s->b < (s->m - m) * rhs.m : 0;
}; // maintain upper hull for maximum
struct HullDynamic : public multiset<Line> {
 bool bad(iterator y) {
    auto z = next(y)
    if (y == begin()) {
      if (z == end()) return 0;
      return y->m == z->m \&\& y->b <= z->b;
    auto x = prev(y);
    if(z==end())return y->m==x->m&y->b<=x->b;
    return (x->b-y->b)*(z->m-y->m)>=
            (y->b-z->b)*(y->m-x->m);
 void insert_line(ll m, ll b) {
   auto y = insert(\{m, b\});
   y->succ = [=]{return next(y)==end()?0:&*next(y);};
    if(bad(y)) {erase(y); return; }
    while(next(y)!=end()&&bad(next(y)))erase(next(y));
    while(y!=begin()&&bad(prev(y)))erase(prev(y));
 ll eval(ll x) {
   auto l = *lower_bound((Line) {x, is_query});
    return 1.m * x + 1.b;
```

# 4.13 Min Enclosing Circle

```
struct Mec{
   // return pair of center and r
   static const int N = 101010;
   int n:
   Pt p[N], cen;
   double r2
   void init( int _n , Pt _p[] ){
     n = _n;
     memcpy( p , _p , sizeof(Pt) * n );
   double sqr(double a){ return a*a; }
   Pt center(Pt p0, Pt p1, Pt p2) {
     Pt a = p1-p0;
     Pt b = p2-p0;
     double c1=norm2( a ) * 0.5;
double c2=norm2( b ) * 0.5;
     double d = a \wedge b;
     double x = p0.X + (c1 * b.Y - c2 * a.Y) / d;
     double y = p0.Y + (a.X * c2 - b.X * c1) / d;
     return Pt(x,y);
   pair<Pt,double> solve(){
     random_shuffle(p,p+n);
     r2=0;
     for (int i=0; i<n; i++){
       if (norm2(cen-p[i]) <= r2) continue;</pre>
        cen = p[i];
        r2 = 0;
        for (int j=0; j<i; j++){
          if (norm2(cen-p[j]) <= r2) continue;</pre>
          cen=Pt((p[i].X+p[j].X)/2,(p[i].Y+p[j].Y)/2);
          r2 = norm2(cen-p[j]);
for (int k=0; k<j; k++){</pre>
            if (norm2(cen-p[k]) <= r2) continue;
cen = center(p[i],p[j],p[k]);</pre>
            r2 = norm2(cen-p[k]);
       }
     return {cen,sqrt(r2)};
} mec;
```

### 4.14 Min Enclosing Ball

```
// Pt : { x , y , z } #define N 202020
int n, nouter; Pt pt[ N ], outer[4], res;
double radius,tmp;
void ball() {
  Pt q[3]; double m[3][3], sol[3], L[3], det; int i,j; res.x = res.y = res.z = radius = 0;
  switch ( nouter ) {
     case 1: res=outer[0]; break;
     case 2: res=(outer[0]+outer[1])/2; radius=norm2(res
            outer[0]); break;
     case 3:
        for (i=0; i<2; ++i) q[i]=outer[i+1]-outer[0];</pre>
        for (i=0; i<2; ++i) for(j=0; j<2; ++j) m[i][j]=(q
     [i] * q[j])*2;
for (i=0; i<2; ++i) sol[i]=(q[i] * q[i]);</pre>
        if (fabs(det=m[0][0]*m[1][1]-m[0][1]*m[1][0])<eps
               return
        L[0]=(sol[0]*m[1][1]-sol[1]*m[0][1])/det;
L[1]=(sol[1]*m[0][0]-sol[0]*m[1][0])/det;
        res=outer[0]+q[0]*L[0]+q[1]*L[1];
        radius=norm2(res, outer[0]);
       break;
     case 4:
        for (i=0; i<3; ++i) q[i]=outer[i+1]-outer[0], sol
   [i]=(q[i] * q[i]);</pre>
        for (i=0;i<3;++i) for(j=0;j<3;++j) m[i][j]=(q[i]
        * q[j])*2;
det= m[0][0]*m[1][1]*m[2][2]
          + m[0][1]*m[1][2]*m[2][0]
          + m[0][2]*m[2][1]*m[1][0]
          - m[0][2]*m[1][1]*m[2][0]
```

```
- m[0][1]*m[1][0]*m[2][2]
- m[0][0]*m[1][2]*m[2][1];
       if ( fabs(det)<eps ) return;</pre>
       for (j=0; j<3; ++j) {
  for (i=0; i<3; ++i) m[i][j]=sol[i];
  L[j]=( m[0][0]*m[1][1]*m[2][2]</pre>
                  + m[0][1]*m[1][2]*m[2][0]
+ m[0][2]*m[2][1]*m[1][0]
                  - m[0][2]*m[1][1]*m[2][0]
- m[0][1]*m[1][0]*m[2][2]
                    m[0][0]*m[1][2]*m[2][1]
                ) / det;
          for (i=0; i<3; ++i) m[i][j]=(q[i] * q[j])*2;</pre>
       res=outer[0];
for (i=0; i<3; ++i ) res = res + q[i] * L[i];</pre>
       radius=norm2(res, outer[0]);
void minball(int n){ ball();
  if( nouter < 4 ) for( int i = 0 ; i < n ; i ++ )
     if( norm2(res, pt[i]) - radius > eps ){
       outer[ nouter ++ ] = pt[ i ]; minball(i); --
            nouter:
       if(i>0){ Pt Tt = pt[i];
          memmove(&pt[1], &pt[0], sizeof(Pt)*i); pt[0]=Tt
double solve(){
 // n points in pt
  random_shuffle(pt, pt+n); radius=-1;
  for(int i=0;i<n;i++) if(norm2(res,pt[i])-radius>eps)
    nouter=1, outer[0]=pt[i], minball(i);
  return sqrt(radius);
```

### 4.15 Min dist on Cuboid

### 4.16 Heart of Triangle

```
Pt inCenter( Pt &A, Pt &B, Pt &C) { // 内心 double a = norm(B-C), b = norm(C-A), c = norm(A-B); return (A * a + B * b + C * c) / (a + b + c); }

Pt circumCenter( Pt &a, Pt &b, Pt &c) { // 外心 Pt bb = b - a, cc = c - a; double db=norm2(bb), dc=norm2(cc), d=2*(bb ^ cc); return a-Pt(bb.Y*dc-cc.Y*db, cc.X*db-bb.X*dc) / d; }

Pt othroCenter( Pt &a, Pt &b, Pt &c) { // 垂心 Pt ba = b - a, ca = c - a, bc = b - c; double Y = ba.Y * ca.Y * bc.Y, A = ca.X * ba.Y - ba.X * ca.Y, x0= (Y+ca.X*ba.Y*b.X-ba.X*ca.Y*c.X) / A, y0= -ba.X * (x0 - c.X) / ba.Y + ca.Y; return Pt(x0, y0); }
```

## 5 Graph

#### 5.1 DominatorTree

```
const int MAXN = 100010;
 struct DominatorTree{
 #define REP(i,s,e) for(int i=(s);i<=(e);i++)</pre>
 #define REPD(i,s,e) for(int i=(s);i>=(e);i--)
   int n , m , s;
vector< int > g[ MAXN ] , pred[ MAXN ];
vector< int > cov[ MAXN ];
    int dfn[ MAXN ] , nfd[ MAXN ] , ts;
   int par[ MAXN ]; //idom[u] s到u的最後一個必經點int sdom[ MAXN ] , idom[ MAXN ]; int mom[ MAXN ] , mn[ MAXN ]; inline bool cmp( int u , int v )
    { return dfn[ u ] < dfn[ v ]; } int eval( int u ){
       if( mom[ u ] == u ) return u;
      int res = eval( mom[ u ] );
if(cmp( sdom[ mn[ mom[ u ] ] ] , sdom[ mn[ u ] ] ))
         mn[u] = mn[mom[u]];
       return mom[ u ] = res;
   void init( int _n , int _m , int _s ){
  ts = 0; n = _n; m = _m; s = _s;
  REP( i, 1, n ) g[ i ].clear(), pred[ i ].clear();
   void addEdge( int u , int v ){
  g[ u ].push_back( v );
  pred[ v ].push_back( u );
    void dfs( int u ){
      ts++;
       dfn['u ] = ts;
      nfd[ ts ] = u;
for( int v : g[ u ] ) if( dfn[ v ] == 0 ){
         par[ v ] = u;
          dfs(v);
      }
   }
    void build(){
      REP( i , 1 , n ){
   dfn[ i ] = nfd[ i ] = 0;
         cov[ i ].clear();
mom[ i ] = mn[ i ] = sdom[ i ] = i;
      dfs( s );
      REPD( i , n , 2 ){
  int u = nfd[ i ];
          if( u == 0 ) continue;
          for( int v : pred[ u ] ) if( dfn[ v ] ){
            eval( v );
if( cmp( sdom[ mn[ v ] ] , sdom[ u ] ) )
    sdom[ u ] = sdom[ mn[ v ] ];
         cov[ sdom[ u ] ].push_back( u );
         mom[ u ] = par[ u ];
for( int w : cov[ par[ u ] ){
            eval( w );
             if( cmp( sdom[ mn[ w ] ] , par[ u ] ) )
            idom[w] = mn[w];
else idom[w] = par[u];
         cov[ par[ u ] ].clear();
       REP( i , 2 , n ){
         int u = nfd[ i ];
if( u == 0 ) continue ;
if( idom[ u ] != sdom[ u ] )
             idom[\bar{u}] = idom[idom[u]];
      }
} domT;
```

### 5.2 MaxClique 最大團

```
#define N 111
struct MaxClique{ // 0-base
  typedef bitset< N > Int;
  Int linkto[ N ] , v[ N ];
  int n;
```

```
void init( int _n ){
    n = _n;
for( int i = 0 ; i < n ; i ++ ){</pre>
       linkto[ i ].réset();
       v[ i ].reset();
  void addEdge( int a , int b ){
  v[ a ][ b ] = v[ b ][ a ] = 1;
  int popcount(const Int& val)
  { return val.count(); } int lowbit(const Int& val)
  { return val._Find_first(); }
  int ans , stk[ N ];
int id[ N ] , di[ N ] , deg[ N ];
  Int cans;
  void maxclique(int elem_num, Int candi){
     if(elem_num > ans){
       ans = elem_num;
       cans.reset();
       for( int i = 0
                          i < elem_num ; i ++ )
         cans[ id[ stk[ i ] ] ] = 1;
         //potential,smaller_candi
     int potential = elem_num + popcount(candi);
     if(potential <= ans) return;</pre>
     int pivot = lowbit(candi);
     Int smaller_candi = candi & (~linkto[pivot]);
    while(smaller_candi.count() && potential > ans){
       int next = lowbit(smaller_candi);
       candi[next] = !candi[next];
       smaller_candi[ next ] = !smaller_candi[ next ];
       potential --
       if(next == pivot || (smaller_candi & linkto[next
            ]).count() ){
         stk[elem_num] = next;
         maxclique(elem_num + 1, candi & linkto[next]);
  int solve(){
    for( int i = 0 ; i < n ; i ++ ){
  id[ i ] = i;</pre>
       deg[i] = v[i].count();
     sort( id , id + n , [&](int id1, int id2){
            return deg[id1] > deg[id2]; } );
     for( int i = 0 ; i < n ; i ++ )
    di[id[i]] = i;
for(int i = 0; i < n; i ++ )
  for(int j = 0; j < n; j ++ )
    if(v[i][j])
    linkto[di[i]][di[j]] = 1;</pre>
     Int cand; cand.reset();
     for( int i = 0 ; i < n ; i ++ )</pre>
       cand[ i ] = 1;
     ans = 1:
    cans.reset(); cans[0] = 1;
    maxclique(0, cand);
    return ans;
} solver;
```

#### 5.3 Strongly Connected Component

```
int n, nScc, vst[MXN], bln[MXN];
vector<int> E[MXN], rE[MXN], vec;
void init(int _n){
  n = _n;
  for (int i=0; i<MXN; i++)</pre>
    E[i].clear(), rE[i].clear();
void addEdge(int u, int v){
 E[u].PB(v); rE[v].PB(u);
}
void DFS(int u){
  vst[u]=1;
  for (auto v : E[u]) if (!vst[v]) DFS(v);
  vec.PB(u);
void rDFS(int u){
  vst[u] = 1; bln[u] = nScc;
  for (auto v : rE[u]) if (!vst[v]) rDFS(v);
```

```
void solve(){
    nScc = 0;
    vec.clear();
    FZ(vst);
    for (int i=0; i<n; i++)
        if (!vst[i]) DFS(i);
    reverse(vec.begin(),vec.end());
    FZ(vst);
    for (auto v : vec)
        if (!vst[v]){
        rDFS(v); nScc++;
        }
    }
};</pre>
```

```
5.4 Dynamic MST
/* Dynamic MST 0( Q lg^2 Q )
 (qx[i], qy[i])->chg weight of edge No.qx[i] to qy[i]
 delete an edge: (i, \infty)
 add an edge: change from \infty to specific value
const int SZ=M+3*MXQ;
int a[N],*tz;
int find(int xx){
  int root=xx; while(a[root]) root=a[root];
  int next; while((next=a[xx])){a[xx]=root; xx=next; }
  return root;
bool cmp(int aa,int bb){ return tz[aa]<tz[bb]; }</pre>
int kx[N],ky[N],kt, vd[N],id[M], app[M];
bool extra[M];
void solve(int *qx,int *qy,int Q,int n,int *x,int *y,
    int *z,int m1,long long ans){
  if(Q==1){
    for(int i=1;i<=n;i++) a[i]=0;</pre>
    z[ qx[0] ]=qy[0]; tz = z;
for(int i=0;i<m1;i++) id[i]=i;</pre>
    sort(id,id+m1,cmp); int ri,rj;
     for(int i=0;i<m1;i++){</pre>
       ri=find(x[id[i]]); rj=find(y[id[i]]);
       if(ri!=rj){ ans+=z[id[i]]; a[ri]=rj; }
    printf("%lld\n",ans);
    return;
  int ri,rj;
  //contract
  kt=0:
  for(int i=1;i<=n;i++) a[i]=0;</pre>
  for(int i=0;i<Q;i++){</pre>
    ri=find(x[qx[i]]); rj=find(y[qx[i]]); if(ri!=rj) a[
         ri]=rj;
  int tm=0;
  for(int i=0;i<m1;i++) extra[i]=true;</pre>
  for(int i=0;i<Q;i++) extra[ qx[i] ]=false;</pre>
  for(int i=0;i<m1;i++) if(extra[i]) id[tm++]=i;</pre>
  tz=z; sort(id,id+tm,cmp);
  for(int i=0;i<tm;i++){</pre>
    ri=find(x[id[i]]); rj=find(y[id[i]]);
    if(ri!=rj){
       a[ri]=rj; ans += z[id[i]];
       kx[kt]=x[id[i]]; ky[kt]=y[id[i]]; kt++;
    }
  for(int i=1;i<=n;i++) a[i]=0;</pre>
  for(int i=0;i<kt;i++) a[ find(kx[i]) ]=find(ky[i]);</pre>
  int n2=0;
  for(int i=1;i<=n;i++) if(a[i]==0)</pre>
  vd[i]=++n2;
  for(int i=1;i<=n;i++) if(a[i])</pre>
  vd[i]=vd[find(i)];
int m2=0, *Nx=x+m1, *Ny=y+m1, *Nz=z+m1;
  for(int i=0;i<m1;i++) app[i]=-1;</pre>
  for(int i=0;i<Q;i++) if(app[qx[i]]==-1){</pre>
    Nx[m2]=vd[ x[ qx[i] ] ]; Ny[m2]=vd[ y[ qx[i] ] ];
    Nz[m2]=z[ qx[i] ];
app[qx[i]]=m2; m2++;
  for(int i=0;i<0;i++){ z[qx[i]]=qy[i]; qx[i]=app[qx[
       i]]; }
```

```
for(int i=1;i<=n2;i++) a[i]=0;</pre>
  for(int i=0;i<tm;i++)</pre>
    ri=find(vd[ x[id[i]] ]); rj=find(vd[ y[id[i]] ]);
    if(ri!=rj){
      a[ri]=rj; Nx[m2]=vd[ x[id[i]] ]
      Ny[m2]=vd[y[id[i]]]; Nz[m2]=z[id[i]]; m2++;
    }
  }
  int mid=Q/2;
  solve(qx,qy,mid,n2,Nx,Ny,Nz,m2,ans);
  solve(qx+mid,qy+mid,Q-mid,n2,Nx,Ny,Nz,m2,ans);
int x[SZ],y[SZ],z[SZ],qx[MXQ],qy[MXQ],n,m,Q;
void init(){
  scanf("%d%d",&n,&m);
  for(int i=0;i<m;i++) scanf("%d%d%d",x+i,y+i,z+i);</pre>
  scanf("%d",&Q):
  for(int i=0;i<Q;i++){ scanf("%d%d",qx+i,qy+i); qx[i</pre>
void work(){ if(Q) solve(qx,qy,Q,n,x,y,z,m,0); }
int main(){init(); work(); }
```

### 5.5 Maximum General graph Matching

```
const int N = 514, E = (2e5) * 2;
struct Graph{
  int to[E],bro[E],head[N],e;
  int lnk[N],vis[N],stp,n;
void init( int _n ){
    stp = 0; e = 1; n = _n;
    for( int i = 1 ; i <= n ; i ++ )
        lnk[i] = vis[i] = 0;
}</pre>
  void add_edge(int u,int v){
     to[e]=v,bro[e]=head[u],head[u]=e++;
     to[e]=u,bro[e]=head[v],head[v]=e++;
  bool dfs(int x){
     vis[x]=stp;
     for(int i=head[x];i;i=bro[i]){
       int v=to[i];
       if(!lnk[v]){
          lnk[x]=v, lnk[v]=x;
          return true;
       }else if(vis[lnk[v]]<stp){</pre>
          int w=lnk[v]:
          lnk[x]=v, lnk[v]=x, lnk[w]=0;
          if(dfs(w)){
            return true:
          lnk[w]=v, lnk[v]=w, lnk[x]=0;
       }
     return false;
  int solve(){
     int ans = 0;
     for(int i=1;i<=n;i++)</pre>
       if(!lnk[i]){
          stp++; ans += dfs(i);
     return ans;
} graph;
```

#### 5.6 Minimum General Weighted Matching

```
struct Graph {
    // Minimum General Weighted Matching (Perfect Match)
    static const int MXN = 105;
    int n, edge[MXN][MXN];
    int match[MXN],dis[MXN],onstk[MXN];
    vector<int> stk;
    void init(int _n) {
        n = _n;
        for( int i = 0 ; i < n ; i ++ )
            for( int j = 0 ; j < n ; j ++ )
            edge[ i ][ j ] = 0;
    }
    void add_edge(int u, int v, int w)
    { edge[u][v] = edge[v][u] = w; }</pre>
```

```
bool SPFA(int u){
     if (onstk[u]) return true;
     stk.PB(u);
     onstk[u] = 1;
     for (int v=0; v<n; v++){</pre>
       if (u != v && match[u] != v && !onstk[v]){
         int m = match[v];
         if (dis[m] > dis[u] - edge[v][m] + edge[u][v]){
           dis[m] = dis[u] - edge[v][m] + edge[u][v];
           onstk[v] = 1;
           stk.PB(v):
           if (SPFA(m)) return true;
           stk.pop_back();
           onstk[v] = 0;
     } } }
     onstk[u] = 0;
     stk.pop_back();
     return false;
  int solve() {
     // find a match
     for (int i=0; i<n; i+=2){
  match[i] = i+1;</pre>
       match[i+1] = i;
    while (true){
  int found = 0;
       for( int i = 0 ; i < n ; i ++ )</pre>
         onstk[i] = dis[i] = 0;
       for (int i=0; i<n; i++){</pre>
         stk.clear()
         if (!onstk[i] && SPFA(i)){
           found = 1
           while (SZ(stk)>=2){
              int u = stk.back(); stk.pop_back();
              int v = stk.back(); stk.pop_back();
             match[u] = v;
             match[v] = u;
       } } }
       if (!found) break;
     int ret = 0;
     for (int i=0; i< n; i++)
       ret += edge[i][match[i]];
     ret /= 2;
     return ret;
}graph;
```

#### 5.7 Maximum General Weighted Matching

```
struct WeightGraph
  static const int INF = INT_MAX;
  static const int N = 514;
  struct edge{
    int u,v,w; edge(){}
    edge(int ui,int vi,int wi)
      :u(ui),v(vi),w(wi){}
  };
  int n,n_x
  edge g[N*2][N*2];
  int lab[N*2];
  int match[N*2],slack[N*2],st[N*2],pa[N*2];
  int flo_from[N*2][N+1],S[N*2],vis[N*2];
  vector<int> flo[N*2];
  queue<int> q;
  int e_delta(const edge &e){
    return lab[e.u]+lab[e.v]-g[e.u][e.v].w*2;
  void update_slack(int u,int x){
    if(!slack[x]||e_delta(g[u][x])<e_delta(g[slack[x]][</pre>
        x]))slack[x]=u;
  void set_slack(int x){
    slack[x]=0;
    for(int u=1;u<=n;++u)</pre>
      if(g[u][x].w>0&&st[u]!=x&&S[st[u]]==0)
        update_slack(u,x);
  void q_push(int x){
    if(x<=n)q.push(x);</pre>
    else for(size_t i=0;i<flo[x].size();i++)</pre>
```

```
q_push(flo[x][i]);
void set_st(int x,int b){
  st[x]=b;
  if(x>n)for(size_t i=0;i<flo[x].size();++i)</pre>
    set_st(flo[x][i],b);
int get_pr(int b,int xr){
  int pr=find(flo[b].begin(),flo[b].end(),xr)-flo[b].
      begin()
  if(pr%2==1){
    reverse(flo[b].begin()+1,flo[b].end());
    return (int)flo[b].size()-pr;
  }else return pr;
}
void set_match(int u,int v){
  match[u]=g[u][v].v;
if(u<=n) return;</pre>
  edge e=g[u][v];
  int xr=flo_from[u][e.u],pr=get_pr(u,xr);
  for(int i=0;i<pr;++i)set_match(flo[u][i],flo[u][i</pre>
      ^1]);
  set_match(xr,v);
  rotate(flo[u].begin(),flo[u].begin()+pr,flo[u].end
void augment(int u,int v){
  for(;;){
    int xnv=st[match[u]];
    set_match(u,v);
    if(!xnv)return
    set_match(xnv,st[pa[xnv]]);
    u=st[pa[xnv]],v=xnv;
int get_lca(int u,int v){
  static int t=0;
  for(++t;ullv;swap(u,v)){
    if(u==0)continue;
    if(vis[u]==t)return u;
    vis[u]=t;
    u=st[match[u]];
    if(u)u=st[pa[u]];
  return 0;
void add_blossom(int u,int lca,int v){
  int b=n+1:
  while(b \le n_x \& st[b])++b;
  if(b>n_x)++n_x;
  lab[b]=0, S[b]=0;
  match[b]=match[lca];
  flo[b].clear();
  flo[b].push_back(lca);
  for(int x=u,y;x!=lca;x=st[pa[y]])
    flo[b].puśh_back(x),flo[b].push_back(y=st[match[x
         ]]),q_push(y)
  reverse(flo[b].begin()+1,flo[b].end());
  for(int x=v,y;x!=lca;x=st[pa[y]])
    flo[b].push_back(x),flo[b].push_back(y=st[match[x
         ]]),q_push(y);
  set_st(b,b);
  for(int x=1;x<=n_x;++x)g[b][x].w=g[x][b].w=0;
  for(int x=1;x<=n;++x)flo_from[b][x]=0;</pre>
  for(size_t i=0;i<flo[b].size();++i){</pre>
    int xs=flo[b][i];
    for(int x=1;x<=n_x;++x)</pre>
      if(g[b][x].w==0||e_delta(g[xs][x])<e_delta(g[b
           ][x]))
        g[b][x]=g[xs][x],g[x][b]=g[x][xs];
    for(int x=1;x<=n;++x)</pre>
      if(flo_from[xs][x])flo_from[b][x]=xs;
  set_slack(b);
void expand_blossom(int b){
  for(size_t i=0;i<flo[b].size();++i)</pre>
    set_st(flo[b][i],flo[b][i])
  int xr=flo_from[b][g[b][pa[b]].u],pr=get_pr(b,xr);
  for(int i=0;i<pr;i+=2){</pre>
    int xs=flo[b][i],xns=flo[b][i+1];
    pa[xs]=g[xns][xs].u;
    S[xs]=1,S[xns]=0;
```

```
slack[xs]=0,set_slack(xns);
    q_push(xns);
  S[xr]=1,pa[xr]=pa[b];
  for(size_t i=pr+1;i<flo[b].size();++i){</pre>
    int xs=flo[b][i];
    S[xs]=-1, set\_slack(xs);
  st[b]=0;
bool on_found_edge(const edge &e){
  int u=st[e.u],v=st[e.v];
  if(S[v]==-1){
    pa[v]=e.u,S[v]=1
    int nu=st[match[v]]
    slack[v]=slack[nu]=0;
  S[nu]=0,q_push(nu);
}else if(S[v]==0){
    int lca=get_lca(u,v);
    if(!lca)return augment(u,v),augment(v,u),true;
    else add_blossom(u,lca,v);
  return false;
bool matching(){
  memset(S+1,-1,sizeof(int)*n_x);
  memset(slack+1,0,sizeof(int)*n_x);
  q=queue<int>();
  for(int x=1;x<=n_x;++x)</pre>
    if(st[x]==x\&\&!match[x])pa[x]=0,S[x]=0,q_push(x);
  if(q.empty())return false;
  for(;;){
    while(q.size()){
      int u=q.front();q.pop();
      if(S[st[u]]==1)continue;
      for(int v=1; v<=n; ++v)
  if(g[u][v].w>0&&st[u]!=st[v]){
           if(e_delta(g[u][v])==0){
             if(on_found_edge(g[u][v]))return true;
          }else update_slack(u,st[v]);
    }
    int d=INF;
    for(int b=n+1;b<=n_x;++b)</pre>
      if(st[b]==b&&S[b]==1)d=min(d,lab[b]/2);
    for(int x=1;x<=n_x;++x)</pre>
      if(st[x]==x\&slack[x]){
        if(S[x]=-1)d=min(d,e_delta(g[slack[x]][x]));
        else if(S[x]==0)d=min(d,e_delta(g[slack[x]][x
             1)/2);
    for(int u=1;u<=n;++u){</pre>
      if(S[st[u]]==0){
        if(lab[u]<=d)return 0;</pre>
        lab[u]-=d;
      }else if(S[st[u]]==1)lab[u]+=d;
    for(int b=n+1;b<=n_x;++b)</pre>
      if(st[b]==b){
        if(S[st[b]]==0)lab[b]+=d*2;
        else if(S[st[b]]==1)lab[b]-=d*2;
    q=queue<int>();
    for(int x=1;x<=n_x;++x)</pre>
      if(st[x]==x&&slack[x]&&st[slack[x]]!=x&&e_delta
           (g[slack[x]][x])==0)
         if(on_found_edge(g[slack[x]][x]))return true;
    for(int b=n+1;b<=n_x;++b)</pre>
      if(st[b]==b&&S[b]==1&&lab[b]==0)expand_blossom(
          b);
  return false;
pair<long long,int> solve(){
  memset(match+1,0,sizeof(int)*n);
  int n_matches=0;
  long long tot_weight=0;
  for(int u=0;u<=n;++u)st[u]=u,flo[u].clear();</pre>
  int w_max=0;
  for(int u=1;u<=n;++u)</pre>
    for(int v=1;v<=n;++v){</pre>
      flo_from[u][v]=(u==v?u:0);
```

```
w_max=max(w_max,g[u][v].w);
}
for(int u=1;u<=n;++u)lab[u]=w_max;
while(matching())++n_matches;
for(int u=1;u<=n;++u)
    if(match[u]&&match[u]<u)
        tot_weight+=g[u][match[u]].w;
    return make_pair(tot_weight,n_matches);
}
void add_edge( int ui , int vi , int wi ){
    g[ui][vi].w = g[vi][ui].w = wi;
}
void init( int _n ){
    n = _n;
    for(int u=1;u<=n;++u)
        for(int v=1;v<=n;++v)
        g[u][v]=edge(u,v,0);
}
} graph;</pre>
```

## 5.8 Minimum Steiner Tree

```
// Minimum Steiner Tree 重要點的mst
// 0(V 3^T + V^2 2^T)
struct SteinerTree{
#define V 33
#define T 8
#define INF 1023456789
  int n , dst[V][V] , dp[1 << T][V] , tdst[V]; void init( int _n ){
    n = _n;
for( int i = 0 ; i < n ; i ++ ){</pre>
       for( int j = 0; j < n; j ++ )

dst[ i ][ j ] = INF;

dst[ i ][ i ] = 0;
    }
  void add_edge( int ui , int vi , int wi ){
  dst[ ui ][ vi ] = min( dst[ ui ][ vi ] , wi );
  dst[ vi ][ ui ] = min( dst[ vi ][ ui ] , wi );
  void shortest_path(){
     for( int k = 0 ; k < n ; k ++ )
        for( int i = 0 ; i < n ; i ++ )
          int solve( const vector<int>& ter ){
  int t = (int)ter.size();
     for( int i = 0 ; i < (1 << t) ; i ++ )
     for( int j = 0; j < n; j ++ )

dp[ i ][ j ] = INF;

for( int i = 0; i < n; i ++ )
       dp[0][i] = 0;
     for( int msk = 1 ; msk < ( 1 << t ) ; msk ++ ){
  if( msk == ( msk & (-msk) ) ){</pre>
          int who = __lg( msk );
          for( int i = 0 ; i < n ; i ++ )
  dp[ msk ][ i ] = dst[ ter[ who ] ][ i ];</pre>
          continue;
        for( int i = 0 ; i < n ; i ++ )</pre>
          for( int submsk = ( msk - 1 ) & msk ; submsk ;
                submsk = ( submsk - 1 ) & msk )
dp[ msk ][ i ] = min( dp[ msk ][ i ],
                                      dp[submsk][i] +
                                     dp[ msk ^ submsk ][ i ] );
        for( int i = 0; i < n; i ++){
          tdst[ i ] = INF;
          for( int j = 0 ; j < n ; j ++ )
  tdst[ i ] = min( tdst[ i ],</pre>
                            dp[ msk ][ j ] + dst[ j ][ i ] );
       for( int i = 0 ; i < n ; i ++ )
  dp[ msk ][ i ] = tdst[ i ];</pre>
     int ans = INF
     for( int i = 0 ; i < n ; i ++ )
       ans = min(ans, dp[(1 << t) - 1][i]);
     return ans;
```

```
} solver;
```

```
5.9 BCC based on vertex
struct BccVertex {
  int n,nScc,step,dfn[MXN],low[MXN];
  vector<int> E[MXN],sccv[MXN];
  int top,stk[MXN];
  void init(int _n) {
    n = _n; nScc = step = 0;
for (int i=0; i<n; i++) E[i].clear();
  void addEdge(int u, int v)
{ E[u].PB(v); E[v].PB(u); }
  void DFS(int u, int f) {
    dfn[u] = low[u] = step++;
    stk[top++] = u;
    for (auto v:E[ú]) {
      if (v == f) continue;
      if (dfn[v] == -1) {
        DFS(v,u);
         low[u] = min(low[u], low[v]);
         if (low[v] >= dfn[u]) {
           int z; //進到if裡面u為關節點
           sccv[nScc].clear();
           do {
             z = stk[--top]
             sccv[nScc].PB(z);
           } while (z != v);
           sccv[nScc++].PB(u);
      }else
        low[u] = min(low[u],dfn[v]);
  } }
  vector<vector<int>> solve() {
    vector<vector<int>> res;
    for (int i=0; i<n; i++)</pre>
      dfn[i] = low[i] = -1;
    for (int i=0; i<n; i++)
      if (dfn[i] == -1) {
        top = 0:
        DFS(i,i);
    REP(i,nScc) res.PB(sccv[i]);
    return res;
}graph;
```

#### 5.10 Min Mean Cycle

```
/* minimum mean cycle O(VE) */
struct MMC{
#define E 101010
#define V 1021
#define inf 1e9
#define eps 1e-6
  struct Edge { int v,u; double c; };
int n, m, prv[V][V], prve[V][V], vst[V];
   Edge e[E];
   vector<int> edgeID, cycle, rho;
   double d[V][V];
   void init( int _n )
   \{ n = _n; m = 0; \}
   // WARNING: TYPE matters
   void addEdge( int vi , int ui , double ci )
{ e[ m ++ ] = { vi , ui , ci }; }
   void bellman_ford() {
     for(int i=0; i<n; i++) d[0][i]=0;

for(int i=0; i<n; i++) {

fill(d[i+1], d[i+1]+n, inf);

for(int j=0; j<m; j++) {

  int v = e[j].v, u = e[j].u;

  if(d[i][v]<inf && d[i+1][u]>d[i][v]+e[j].c) {
              d[i+1][u] = d[i][v]+e[j].c;
prv[i+1][u] = v;
              prve[i+1][u] = j;
   double solve(){
     // returns inf if no cycle, mmc otherwise
      double mmc=inf;
      int st = -1;
      bellman_ford();
```

```
for(int i=0; i<n; i++) {</pre>
       double avg=-inf;
       for(int k=0; k<n; k++) {</pre>
         if(d[n][i]<inf-eps) avg=max(avg,(d[n][i]-d[k][i</pre>
              ])/(n-k));
         else avg=max(avg,inf);
      if (avg < mmc) tie(mmc, st) = tie(avg, i);</pre>
    fill(vst,0); edgeID.clear(); cycle.clear(); rho.
         clear();
    for (int i=n; !vst[st]; st=prv[i--][st]) {
      vst[st]++;
      edgeID.PB(prve[i][st]);
      rho.PB(st);
    while (vst[st] != 2) {
  if(rho.empty()) return inf;
      int v = rho.back(); rho.pop_back();
      cycle.PB(v);
      vst[v]++;
    reverse(ALL(edgeID));
    edgeID.resize(SZ(cycle));
    return mmc;
} mmc;
```

### Directed Graph Min Cost Cycle

```
// works in O(N M)
#define INF 1000000000000000LL
#define N 5010
#define M 200010
struct edge{
  int to; LL w;
  edge(int a=0, LL b=0): to(a), w(b){}
struct node{
  LL d; int u, next; node(LL a=0, int b=0, int c=0): d(a), u(b), next(c){}
struct DirectedGraphMinCycle{
  vector<edge> g[N], grev[N];
  LL dp[N][\bar{N}], p[N], d[N], mu;
  bool inq[N];
  int n, bn, bsz, hd[N];
  void b_insert(LL d, int u){
     int i = d/mu;
     if(i >= bn) return;
     b[++bsz] = node(d, u, hd[i]);
     hd[i] = bsz;
  void init( int _n ){
    n = _n;
for( int i = 1 ; i <= n ; i ++ )</pre>
       g[i].clear();
  void addEdge( int ai , int bi , LL ci )
  { g[ai].push_back(edge(bi,ci)); }
  LL solve(){
     fill(dp[0], dp[0]+n+1, 0);
for(int i=1; i<=n; i++){
       fill(dp[i]+1, dp[i]+n+1, INF);
for(int j=1; j<=n; j++) if(dp[i-1][j] < INF){
  for(int k=0; k<(int)g[j].size(); k++)
    dp[i][g[j][k].to] =min(dp[i][g[j][k].to],</pre>
                                          dp[i-1][j]+g[j][k].w);
       }
     mu=INF; LL bunbo=1;
     for(int i=1; i<=n; i++) if(dp[n][i] < INF){</pre>
       LL a=-INF, b=1;
       for(int j=0; j<=n-1; j++) if(dp[j][i] < INF){
  if(a*(n-j) < b*(dp[n][i]-dp[j][i])){</pre>
            a = dp[n][i]-dp[j][i];
             b = n-j;
          }
       if(mu*b > bunbo*a)
          mu = a, bunbo = b;
```

```
if(mu < 0) return -1; // negative cycle
if(mu == INF) return INF; // no cycle</pre>
      if(mu == 0) return 0;
       for(int i=1; i<=n; i++)</pre>
         for(int j=0; j<(int)g[i].size(); j++)
g[i][j].w *= bunbo;</pre>
      memset(p, 0, sizeof(p));
      queue<int> q;
       for(int i=1; i<=n; i++){</pre>
         q.push(i);
         inq[i] = true;
      while(!q.empty()){
         int i=q.front(); q.pop(); inq[i]=false;
for(int j=0; j<(int)g[i].size(); j++){
  if(p[g[i][j].to] > p[i]+g[i][j].w-mu){
               p[g[i][j].to] = p[i]+g[i][j].w-mu;
if(!inq[g[i][j].to]){
   q.push(g[i][j].to);
                  inq[g[i][j].to] = true;
      for(int i=1; i<=n; i++) grev[i].clear();</pre>
      for(int i=1; i<=n; i++)
  for(int j=0; j<(int)g[i].size(); j++){
    g[i][j].w += p[i]-p[g[i][j].to];
</pre>
            grev[g[i][j].to].push_back(edge(i, g[i][j].w));
      LL_mldc = n*mu;
      for(int i=1; i<=n; i++){</pre>
         bn=mldc/mu, bsz=0;
memset(hd, 0, sizeof(hd));
         fill(d+i+1, d+n+1, INF);
         b_insert(d[i]=0, i);
         for(int j=\bar{0}; j<=bn-1; j++) for(int k=hd[j]; k; k=
               b[k].next){
            int u = b[k].u;
            LL du = b[k].d;
            if(du > d[u]) continue;
for(int l=0; l<(int)g[u].size(); l++) if(g[u][l
    ].to > i){
               if(d[g[u][i].to] > du + g[u][i].w){
  d[g[u][i].to] = du + g[u][i].w;
                  b_insert(d[g[u][l].to], g[u][l].to);
         for(int j=0; j<(int)grev[i].size(); j++) if(grev[
    i][j].to > i)
            mldc=min(mldc,d[grev[i][j].to] + grev[i][j].w);
      return mldc / bunbo;
} graph;
 5.12 K-th Shortest Path
```

```
// time: O(|E| \setminus lg \mid E| + \mid V| \setminus lg \mid V| + K)
// memory: 0(|E| \langle lg |E| + |V|)
struct KSP{ // 1-base
  struct nd{
     nd(int ui = 0, int vi = 0, int di = INF)
     \{ u = ui; v = vi; d = di; \}
  struct heap{
     nd* edge; int dep; heap* chd[4];
  static int cmp(heap* a,heap* b)
  { return a->edge->d > b->edge->d; }
  struct node{
     int v; LL d; heap* H; nd* E;
     node(){}
     node(LL _d, int _v, nd* _E) { d =_d; v = _v; E = _E; }
     node(heap* _H, LL _d)
     \{ H = _H; d = _d; \}
     friend bool operator<(node a, node b)</pre>
     { return a.d > b.d; }
  int n, k, s, t, dst[ N ];
nd *nxt[ N ];
  vector<nd*> g[ N ], rg[ N ];
heap *nullNd, *head[ N ];
  void init( int _n , int _k , int _s , int _t ){
```

```
n = _n; k = _k; s = _s; t = _t;
for( int i = 1 ; i <= n ; i ++ ){
    g[ i ].clear(); rg[ i ].clear();
    nxt[ i ] = head[ i ] = NULL;
    dst[ i ] = -1;</pre>
                                                                            priority_queue<node> Q;
if( dst[ s ] == -1 ) return;
ans.push_back( dst[ s ] );
                                                                             if( head[s] != nullNd )
                                                                             Q.push(node(head[s], dst[s]+head[s]->edge->d));
for( int _ = 1 ; _ < k and not Q.empty() ; _ ++ ){
  node p = Q.top(), q; Q.pop();</pre>
  void addEdge( int ui , int vi , int di ){
  nd* e = new nd(ui, vi, di);
  g[_ui ].push_back( e );
                                                                               ans.push_back( p.d );
                                                                               if(head[ p.H->edge->v ] != nullNd){
                                                                                  q.H = head[ p.H->edge->v ];
     rg[ vi ].push_back( e );
                                                                                  q.d = p.d + q.H->edge->d;
                                                                                  Q.push(q):
  queue<int> dfsQ;
  void dijkstra(){
                                                                               for( int i = 0 ; i < 4 ; i ++ )
  if( p.H->chd[ i ] != nullNd ){
    while(dfsQ.size()) dfsQ.pop();
     priority_queue<node> Q;
                                                                                    q.H = p.H--chd[i];
     Q.push(node(0, t, NULL));
    while (!Q.empty()){
  node p = Q.top(); Q.pop();
  if(dst[p.v] != -1) continue;
                                                                                    q.d = p.d - p.H->edge->d + p.H->chd[i]->
                                                                                        edge->d;
                                                                                    Q.push( q );
       dst[ p.v ] = p.d;
nxt[ p.v ] = p.E;
                                                                          } }
                                                                                 }
                                                                          void solve(){
       dfsQ.push( p.v_);
                                                                             dijkstra();
                                                                             build();
       for(auto e: rg[ p.v ])
         Q.push(node(p.d + e->d, e->u, e));
                                                                             first_K();
  heap* merge(heap* curNd, heap* newNd){
                                                                       } solver;
     if(curNd == nullNd) return newNd;
     heap* root = new heap;
                                                                        5.13 SPFA
     memcpy(root, curNd, sizeof(heap));
                                                                       bool spfa(){
     if(newNd->edge->d < curNd->edge->d){
                                                                          deque<int> dq;
       root->edge = newNd->edge
                                                                          dis[0]=0;
       root->chd[2] = newNd->chd[2];
                                                                          dq.push_back(0);
       root->chd[3] = newNd->chd[3];
                                                                          inq[0]=1;
       newNd->edge = curNd->edge;
       newNd - chd[2] = curNd - chd[2];
                                                                          while(!dq.empty()){
                                                                             int u=dq.front();
       newNd - > chd[3] = curNd - > chd[3];
                                                                                  dq.pop_front();
                                                                             inq[u]=0;
for(auto i:edge[u]){
     if(root->chd[0]->dep < root->chd[1]->dep)
       root->chd[0] = merge(root->chd[0], newNd);
                                                                               if(dis[i.first]>i.second+dis[u]){
                                                                                  dis[i.first]=i.second+dis[u];
       root->chd[1] = merge(root->chd[1], newNd);
                                                                                  len[i.first]=len[u]+1;
     root->dep = max(root->chd[0]->dep, root->chd[1]->
                                                                                  if(len[i.first]>n) return 1;
          dep) + 1;
                                                                                  if(inq[i.first]) continue;
     return root;
                                                                                  if(!dq.empty()&&dis[dq.front()]>dis[i.first])
                                                                                    dq.push_front(i.first);
  vector<heap*> V;
  void build(){
                                                                                    dq.push_back(i.first);
    nullNd = new heap;
                                                                                  inq[i.first]=1;
    nullNd->dep = 0;
                                                                          } } }
    nullNd->edge = new nd;
                                                                          return 0;
     fill(nullNd->chd, nullNd->chd+4, nullNd);
     while(not dfsQ.empty()){
       int u = dfsQ.front(); dfsQ.pop();
if(!nxt[ u ]) head[ u ] = nullNd;
                                                                             String
       else head[ u ] = head[nxt[ u ]->v];
                                                                        6.1 PalTree
       V.clear();
       for( auto&& e : g[ u ] ){
                                                                       |// len[s]是對應的回文長度
         int v = e->v;
if( dst[ v ] == -1 ) continue;
e->d += dst[ v ] - dst[ u ];
if( nxt[ u ] != e ){
                                                                       // num[s]是有幾個回文後綴
                                                                       // cnt[s]是這個回文子字串在整個字串中的出現次數
                                                                       // fail[s]是他長度次長的回文後綴,aba的fail是a
            heap* p = new heap;
fill(p->chd, p->chd+4, nullNd);
                                                                        const int MXN = 1000010;
                                                                        struct PalT{
            p->dep = 1;
                                                                          int nxt[MXN][26],fail[MXN],len[MXN];
                                                                          int tot,lst,n,state[MXN],cnt[MXN],num[MXN];
int diff[MXN],sfail[MXN],fac[MXN],dp[MXN];
            p->edge = e;
            V.push_back(p);
                                                                          char s[MXN] = \{-1\};
                                                                          int newNode(int 1,int f){
  len[tot]=1,fail[tot]=f,cnt[tot]=num[tot]=0;
       if(V.empty()) continue;
make_heap(V.begin(), V.end(), cmp);
#define L(X) ((X<<1)+1)
#define R(X) ((X<<1)+2)
                                                                             memset(nxt[tot],0,sizeof(nxt[tot]));
diff[tot]=(l>0?l-len[f]:0);
       for( size_t i = 0 ; i < V.size() ; i ++ ){</pre>
                                                                             sfail[tot]=(l>0&&diff[tot]==diff[f]?sfail[f]:f);
          if(L(i) < V.size()) V[i] -> chd[2] = V[L(i)];
                                                                             return tot++;
          else V[i]->chd[2]=nullNd;
         if(R(i) < V.size()) V[i]->chd[3] = V[R(i)];
                                                                          int getfail(int x){
         else V[i]->chd[3]=nullNd;
                                                                             while(s[n-len[x]-1]!=s[n]) x=fail[x];
                                                                             return x;
       head[u] = merge(head[u], V.front());
  } }
                                                                          int getmin(int v){
  vector<LL> ans;
                                                                             dp[v]=fac[n-len[sfail[v]]-diff[v]];
  void first_K(){
                                                                             if(diff[v]==diff[fail[v]])
    ans.clear();
                                                                                  dp[v]=min(dp[v],dp[fail[v]]);
```

```
return dp[v]+1;
  int push(){
    int c=s[n]-'a',np=getfail(lst);
    if(!(lst=nxt[np][c])){
      lst=newNode(len[np]+2,nxt[getfail(fail[np])][c]);
      nxt[np][c]=lst; num[lst]=num[fail[lst]]+1;
    for(int v=lst;len[v]>0;v=sfail[v])
         fac[n]=min(fac[n],getmin(v));
    return ++cnt[lst],lst;
  void init(const char *_s){
    tot=lst=n=0:
    newNode(0,1), newNode(-1,1);
    for(;_s[n];) s[n+1]=_s[n],++n,state[n-1]=push();
for(int i=tot-1;i>1;i--) cnt[fail[i]]+=cnt[i];
}palt;
```

#### 6.2 KMP

```
int failure[MAXN];
void KMP(string& t, string& p)
    if (p.size() > t.size()) return;
    for (int i=1, j=failure[0]=-1; i<p.size(); ++i)</pre>
        while (j \ge 0 \& p[j+1] != p[i])
             j = failure[j];
        if (p[j+1] == p[i]) j++;
        failure[i] = j;
    for (int i=0, j=-1; i<t.size(); ++i)</pre>
        while (j \ge 0 \& p[j+1] != t[i])
             j = failure[j];
        if (p[j+1] == t[i]) j++;
        if (j == p.size()-1)
             cout << i - p.size() + 1<<" ";</pre>
             j = failure[j];
}
    }
        }
```

### 6.3 SAIS

```
const int N = 300010;
struct SA{
#define REP(i,n) for ( int i=0; i<int(n); i++ )</pre>
#define REP1(i,a,b) for ( int i=(a); i<=int(b); i++ )
  bool _t[N*2];
  int _s[N*2], _sa[N*2], _c[N*2], x[N], _p[N], _q[N*2],
  hei[N], r[N];
int operator [] (int i){ return _sa[i]; }
void build(int *s, int n, int m){
    memcpy(_s, s, sizeof(int) * n);
}
    sais(_s, _sa, _p, _q, _t, _c, n, m);
    mkhei(n);
  void mkhei(int n){
    REP(i,n) r[\_sa[i]] = i;
    hei[0] = 0;
    REP(i,n) if(r[i]) {
      int ans = i>0? max(hei[r[i-1]] - 1, 0) : 0;
      hei[r[i]] = ans;
    }
  void sais(int *s, int *sa, int *p, int *q, bool *t,
       int *c, int n, int z){
    bool uniq = t[n-1] = true, neq;
    int nn = 0, nmxz = -1, *nsa = sa + n, *ns = s + n,
         lst = -1;
#define MSO(x,n) memset((x),0,n*sizeof(*(x)))
#define MAGIC(XD) MS0(sa, n); \
    memcpy(x, c, sizeof(int) * z); \
    memcpy(x + 1, c, sizeof(int) * (z - 1)); \
    REP(i,n) if(sa[i] && !t[sa[i]-1]) sa[x[s[sa[i
    ]-1]]++] = sa[i]-1; \
memcpy(x, c, sizeof(int) * z); \
```

```
for(int i = n - 1; i >= 0; i--) if(sa[i] && t[sa[i
]-1]) sa[--x[s[sa[i]-1]]] = sa[i]-1;
    MSO(c, z);
    REP(i,n) uniq \&= ++c[s[i]] < 2;
    REP(i,z-1) c[i+1] += c[i];
    MAGIC(REP1(i,1,n-1) if(t[i] && !t[i-1]) sa[--x[s[i
    ]]]=p[q[i]=nn++]=i);
REP(i, n) if (sa[i] && t[sa[i]] && !t[sa[i]-1]) {
      neq=lst<0|lmemcmp(s+sa[i],s+lst,(p[q[sa[i]]+1]-sa[i])|
           [i])*sizeof(int));
      ns[q[lst=sa[i]]]=nmxz+=neq;
    sais(ns, nsa, p + nn, q + n, t + n, c + z, nn, nmxz
    + 1);
MAGIC(for(int i = nn - 1; i >= 0; i--) sa[--x[s[p[
        nsa[i]]]] = p[nsa[i]];
  }
}sa;
int H[ N ], SA[ N ];
void suffix_array(int* ip, int len) {
  // should padding a zero in the back
  // ip is int array, len is array length
  // ip[0..n-1] != 0, and ip[len] = 0
  ip[len++] = 0;
  sa.build(ip, ĺen, 128);
  for (int i=0; i<len; i++) {</pre>
    H[i] = sa.hei[i + 1];
    SA[i] = sa.\_sa[i + 1];
  // resulting height, sa array \in [0,len)
}
```

#### 6.4 SuffixAutomata

```
// any path start from root forms a substring of S
// occurrence of P : iff SAM can run on input word P
// number of different substring : ds[1]-1
// total length of all different substring : dsl[1]
// max/min length of state i : mx[i]/mx[mom[i]]+1
// assume a run on input word P end at state i:
// number of occurrences of P : cnt[i]
// first occurrence position of P : fp[i]-IPI+1
// all position of P : fp of "dfs from i through rmom"
const int MXM = 1000010:
struct SAM{
  int tot, root, lst, mom[MXM], mx[MXM]; //ind[MXM]
int nxt[MXM][33]; //cnt[MXM],ds[MXM],dsl[MXM],fp[MXM]
  // bool v[MXM]
  int newNode(){
    int res = ++tot;
    fill(nxt[res], nxt[res]+33, 0);
mom[res] = mx[res] = 0; //cnt=ds=dsl=fp=v=0
    return res:
  }
  void init(){
    tot = 0;
    root = newNode();
    lst = root;
  void push(int c){
    int p = lst;
    int np = newNode(); //cnt[np]=1
mx[np] = mx[p]+1; //fp[np]=mx[np]-1
    for(; p && nxt[p][c] == 0; p = mom[p])
      nxt[p][c] = np
    if(p == 0) mom[np] = root;
      int q = nxt[p][c];
       if(mx[p]+1 == mx[q]) mom[np] = q;
       else{
         int nq = newNode(); //fp[nq]=fp[q]
         mx[nq] = mx[p]+1;
         for(int i = 0; i < 33; i++)
           nxt[nq][i] = nxt[q][i];
         mom[nq] = mom[q];
         mom[q] = nq;
         mom[np] = nq;
         for(; p && nxt[p][c] == q; p = mom[p])
           nxt[p][c] = nq;
```

left=right=0; z[0]=len;

```
for(i=1;i<len;i++) {</pre>
                                                                       j=max(min(z[i-left],right-i),0);
    lst = np;
                                                                       for(;i+j<len&&s[i+j]==s[j];j++);</pre>
  void calc(){
                                                                       z[i]=j:
                                                                       if(i+z[i]>right) {
    calc(root);
    iota(ind,ind+tot,1);
                                                                         right=i+z[i];
    sort(ind,ind+tot,[&](int i,int j){return mx[i]<mx[j</pre>
                                                                         left=i;
          ];});
     for(int i=tot-1;i>=0;i--)
    cnt[mom[ind[i]]]+=cnt[ind[i]];
                                                                  6.7
                                                                          BWT
  void calc(int x){
                                                                  struct BurrowsWheeler{
    v[x]=ds[x]=1;dsl[x]=0; //rmom[mom[x]].push_back(x);
                                                                  #define SIGMA 26
    for(int i=1; i<=26; i++){
                                                                  #define BASE 'a'
                                                                    vector<int> v[ SIGMA ];
       if(nxt[x][i]){
         if(!v[nxt[x][i]]) calc(nxt[x][i]);
                                                                    void BWT(char* ori, char* res){
                                                                       // make ori -> ori + ori
         ds[x] += ds[nxt[x][i]];
         dsl[x]+=ds[nxt[x][i]]+dsl[nxt[x][i]];
                                                                       // then build suffix array
  void push(char *str){
  for(int i = 0; str[i]; i++)
                                                                    void iBWT(char* ori, char* res){
                                                                       for( int i = 0 ; i < SIGMA ; i ++ )
       push(str[i]-'a'+\bar{1});
                                                                         v[ i ].clear();
                                                                       int len = strlen( ori );
for( int i = 0 ; i < len ; i ++</pre>
} sam;
                                                                         v[ ori[i] - BÁSE ].push_back( i );
6.5 Aho-Corasick
                                                                       vector<int> a;
                                                                       for( int i = 0 , ptr = 0 ; i < SIGMA ; i ++ )
for( auto j : v[ i ] ){</pre>
struct ACautomata{
  struct Node{
                                                                           a.push_back( j );
                                                                           ori[ ptr ++ ] = BASE + i;
     int cnt,i;
    Node *go[26], *fail, *dic;
                                                                      for( int i = 0 , ptr = 0 ; i < len ; i ++ ){
  res[ i ] = ori[ a[ ptr ] ];
  ptr = a[ ptr ];</pre>
    Node (){
       cnt = 0; fail = 0; dic=0;
       memset(go,0,sizeof(go));
  }pool[1048576],*root;
                                                                       res[ len ] = 0;
  int nMem,n_pattern;
  Node* new_Node(){
                                                                  } bwt;
    pool[nMem] = Node()
     return &pool[nMem++];
                                                                  6.8 ZValue Palindrome
                                                                  void z_value_pal(char *s,int len,int *z){
  void init() {nMem=0;root=new_Node();n_pattern=0;}
  void add(const string &str) { insert(root,str,0); }
void insert(Node *cur, const string &str, int pos){
                                                                    len=(len<<1)+1;
                                                                    for(int i=len-1;i>=0;i--)
    for(int i=pos;i<str.size();i++){</pre>
                                                                       s[i]=i&1?s[i>>1]:'@';
       if(!cur->go[str[i]-'a'])
  cur->go[str[i]-'a'] = new_Node();
                                                                    z[0]=1;
                                                                    for(int i=1,l=0,r=0;i<len;i++){</pre>
       cur=cur->go[str[i]-'a'];
                                                                       z[i]=i < r?min(z[l+l-i],r-i):1;
                                                                       while(i-z[i]>=0\&i+z[i]<len\&s[i-z[i]]==s[i+z[i]])
    cur->cnt++; cur->i=n_pattern++;
                                                                           ++z[i];
                                                                       if(i+z[i]>r) l=i,r=i+z[i];
                                                                  } }
  void make_fail(){
    queue<Node*> que;
                                                                         Smallest Rotation
    que.push(root);
    while (!que.empty()){
  Node* fr=que.front(); que.pop();
                                                                  //rotate(begin(s),begin(s)+minRotation(s),end(s))
       for (int i=0; i<26; i++){
                                                                  int minRotation(string s) {
         if (fr->go[i]){
                                                                    int a = 0, N = s.size(); s += s;
           Node *ptr = fr->fail;
                                                                    rep(b,0,N) rep(k,0,N) {
           while (ptr && !ptr->go[i]) ptr = ptr->fail;
                                                                       if(a+k == b \mid \mid s[a+k] < s[b+k])
           fr->go[i]->fail=ptr=(ptr?ptr->go[i]:root);
fr->go[i]->dic=(ptr->cnt?ptr:ptr->dic);
                                                                         \{b += \max(0, k-1); break;\}
                                                                       if(s[a+k] > s[b+k]) {a = b; break;}
           que.push(fr->go[i]);
                                                                    } return a:
                                                                  }
  void query(string s){
       Node *cur=root;
                                                                  6.10 Cyclic LCS
       for(int i=0;i<(int)s.size();i++){</pre>
           while(cur&&!cur->go[s[i]-'a']) cur=cur->fail;
cur=(cur?cur->go[s[i]-'a']:root);
                                                                  #define L 0
                                                                  #define LU 1
           if(cur->i>=0) ans[cur->i]++;
                                                                  #define U 2
           for(Node *tmp=cur->dic;tmp;tmp=tmp->dic)
                                                                  const int mov[3][2]=\{0,-1,-1,-1,-1,0\};
                ans[tmp->i]++;
                                                                  int al,bl;
  } }// ans[i] : number of occurrence of pattern i
                                                                  char a[MAXL*2],b[MAXL*2]; // 0-indexed
                                                                  int dp[MAXL*2][MAXL]
}AC;
                                                                  char pred[MAXL*2][MAXL];
6.6 Z Value
                                                                  inline int lcs_length(int r) {
                                                                    int i=r+al, j=bl, l=0;
char s[MAXN]:
                                                                    while(i>r) {
int len,z[MAXN];
                                                                       char dir=pred[i][j];
void Z_value() { //z[i] = lcp(s[1...],s[i...])
  int i,j,left,right;
                                                                       if(dir==LU) l++;
                                                                       i+=mov[dir][0];
```

j+=mov[dir][1];

```
return 1:
inline void reroot(int r) { // r = new base row
  int i=r,j=1
  while(j<=bl&&pred[i][j]!=LU) j++;</pre>
  if(j>bl) return;
  pred[i][j]=L;
while(i<2*al&&j<=bl) {</pre>
     if(pred[i+1][j]==U) {
       pred[i][j]=L;
    } else if(j<bl&&pred[i+1][j+1]==LU) {</pre>
       i++;
       j++
       pred[i][j]=L;
     } else {
} } }
int cyclic_lcs() {
  // a, b, al, bl should be properly filled
  // note: a WILL be altered in process
                  concatenated after itself
  char tmp[MAXL];
  if(al>bl)
     swap(al,bl)
     strcpy(tmp,a);
     strcpy(a,b);
     strcpy(b,tmp);
  strcpy(tmp,a);
  strcat(a,tmp);
  // basic lcs
  for(int i=0;i<=2*al;i++) {</pre>
     d\hat{p}[i][0]=\hat{0};
    pred[i][0]=U;
  for(int j=0;j<=bl;j++) {
  dp[0][j]=0;
  pred[0][j]=L;</pre>
  for(int i=1;i<=2*al;i++) {
    for(int j=1;j<=bl;j++) {
  if(a[i-1]==b[j-1]) dp[i][j]=dp[i-1][j-1]+1;</pre>
       else dp[i][j]=max(dp[i-1][j],dp[i][j-1]);
if(dp[i][j-1]==dp[i][j]) pred[i][j]=L;
else if(a[i-1]==b[j-1]) pred[i][j]=LU;
       else pred[i][j]=U;
  } }
// do cyclic lcs
  int clcs=0;
  for(int i=0;i<al;i++) {</pre>
    clcs=max(clcs,lcs_length(i));
     reroot(i+1);
  // recover a
  a[al]='\0':
  return clcs;
```

### 7 Data Structure

### 7.1 Segment tree

```
struct seg_tree{
  static const int MXN=1e5+5,NO_TAG=0; // to be set
  11 a[MXN],val[MXN*4],tag[MXN*4],v;
  int n,ql,qr
  void push(int i,int l,int r){
    if(tag[i]!=NO_TAG){
      val[i]+=tag[i]; // update by tag
      if(l!=r){
        tag[cl(i)]+=tag[i]; // push
        tag[cr(i)]+=tag[i]; // push
      tag[i]=NO_TAG;
 } }
 void pull(int i,int l,int r){
   int mid=(l+r)>>1;
    push(cl(i),l,mid);push(cr(i),mid+1,r);
    val[i]=max(val[cl(i)],val[cr(i)]); // pull
```

```
void build(int i,int l,int r){
     if(l==r){
       val[i]=a[l]; // set value
       return;
     int mid=(l+r)>>1;
    build(cl(i),l,mid);build(cr(i),mid+1,r);
    pull(i,l,r);
  void update(int i,int l,int r){
    push(i,l,r);
     if(al<=l&&r<=ar){
       tag[i]+=v; // update tag
       return;
     int mid=(l+r)>>1;
     if(ql<=mid) update(cl(i),l,mid);</pre>
    if(qr>mid) update(cr(i),mid+1,r);
    pull(i,l,r);
  void query(int i,int l,int r){
    push(i,l,r);
     if(ql<=l&&r<=qr){
       v=max(v,val[i]); // update answer
       return:
     int mid=(l+r)>>1;
     if(ql<=mid) query(cl(i),l,mid);</pre>
     if(qr>mid) query(cr(i),mid+1,r);
} }tree;
7.2 Treap
struct Treap{
  int sz , val , pri , tag;
Treap *l , *r;
  Treap( int _val ){
    val = _val; sz = 1;
pri = rand(); l = r = NULL; tag = 0;
};
void push( Treap * a ){
  if( a->tag ){
    Treap *swp = a \rightarrow l; a \rightarrow l = a \rightarrow r; a \rightarrow r = swp;
     int swp2;
    if( a->l ) a->l->tag ^= 1;
if( a->r ) a->r->tag ^= 1;
    a \rightarrow tag = 0;
inline int Size( Treap * a ){ return a ? a->sz : 0; }
void pull( Treap * a ){
  a\rightarrow sz = Size(a\rightarrow l) + Size(a\rightarrow r) + 1;
Treap* merge( Treap *a , Treap *b ){
  if( !a || !b ) return a ? a : b;
  if( a->pri > b->pri ){
    push( a );
     a \rightarrow r = merge(a \rightarrow r, b);
    pull( a );
     return a;
  }else{
     push( b );
     b->l = merge( a , b->l );
    pull( b );
    return b;
void split_kth( Treap *t , int k, Treap*&a, Treap*&b ){
  if(!t){ a = b = NULL; return; }
  push( t )
  if( Size( t->l ) + 1 <= k ){
     split_kth(t->r, k-Size(t->l)-1, a->r, b)
    pull( a );
  }else{
     split_kth(t->l,k,a,b->l);
    pull( b );
} }
void split_key(Treap *t, int k, Treap*&a, Treap*&b){
  if(!t){ a = b = NULL; return; }
```

push(t);

void chroot(Splay \*x){

access(x);

```
splay(x);
  if(k \le t - val)
                                                                 x \rightarrow rev \stackrel{\wedge}{=} 1;
    split_key(t->l,k,a,b->l);
                                                                 x->push(); x->pull();
    pull(b);
                                                               }
                                                               void link(Splay *x, Splay *y){
  else{
                                                                 access(x);
    a = t;
                                                                 splay(x)
    split_key(t->r,k,a->r,b);
                                                                 chroot(y);
    pull(a);
                                                                 x->setCh(y, 1);
                                                               void cut_p(Splay *y) {
7.3
      Link-Cut Tree
                                                                 access(y);
                                                                 splay(y)
const int MXN = 100005
                                                                 y->push();
const int MEM = 100005;
                                                                 y - ch[0] = y - ch[0] - f = nil;
struct Splay {
  static Splay nil, mem[MEM], *pmem;
Splay *ch[2], *f;
                                                               void cut(Splay *x, Splay *y){
                                                                 chroot(x);
  int val, rev, size;
                                                                 cut_p(y);
  Splay (int _val=-1) : val(_val), rev(0), size(1)
{ f = ch[0] = ch[1] = &nil; }
                                                               Splay* get_root(Splay *x) {
  bool isr()
                                                                 access(x);
  { return f->ch[0] != this && f->ch[1] != this; }
                                                                 splay(x);
  int dir()
                                                                 for(; x \rightarrow ch[0] != nil; x = x \rightarrow ch[0])
  { return f->ch[0] == this ? 0 : 1; }
                                                                    x->push();
  void setCh(Splay *c, int d){
                                                                 splay(x);
    ch[d] = c;
                                                                 return x;
    if (c != &nil) c->f = this;
                                                               bool conn(Splay *x, Splay *y) {
    pull();
                                                                 x = get_root(x);
                                                                 y = get_root(y)
  void push(){
    if( !rev ) return;
                                                                 return x == y;
    swap(ch[0], ch[1]);
    if (ch[0] != &nil) ch[0]->rev ^= 1;
                                                               Splay* lca(Splay *x, Splay *y) {
    if (ch[1] != &nil) ch[1]->rev ^= 1;
                                                                 access(x);
                                                                 access(y);
    rev=0;
                                                                 splay(x);
                                                                 if (x->f == nil) return x;
  void pull(){
    size = ch[0] -> size + ch[1] -> size + 1;
                                                                 else return x->f;
    if (ch[0] != &nil) ch[0]->f = this;
    if (ch[1] != &nil) ch[1]->f = this;
                                                               7.4 Disjoint Set
} Splay::nil, Splay::mem[MEM], *Splay::pmem = Splay::
                                                               struct DisjointSet{
                                                                 // save() is like recursive
Splay *nil = &Splay::nil;
                                                                 // undo() is like return
void rotate(Splay *x){
                                                                 int n, fa[ N ], sz[ N ];
  Splay *p = x \rightarrow f;
                                                                 vector< pair<int*,int> > h;
  int d = x->dir();
                                                                 vector<int> sp;
  if (!p->isr()) p->f->setCh(x, p->dir());
                                                                 void init( int tn ){
  else x->f = p->f
                                                                    n=tn;
  p->setCh(x->ch[!d], d);
                                                                    for( int i = 0 ; i < n ; i ++ ){
  x->setCh(p, !d);
                                                                      fa[ i ]=i;
sz[ i ]=1;
 p->pull(); x->pull();
vector<Splay*> splayVec;
                                                                    sp.clear(); h.clear();
void splay(Splay *x){
  splayVec.clear();
                                                                 void assign( int *k, int v ){
  for (Splay *q=x;; q=q->f){
                                                                    h.PB( {k, *k} );
    splayVec.push_back(q);
                                                                    *k = v;
    if (q->isr()) break;
                                                                 void save(){ sp.PB(SZ(h)); }
  reverse(begin(splayVec), end(splayVec));
for (auto it : splayVec) it->push();
                                                                 void undo(){
                                                                    assert(!sp.empty());
  while (!x->isr()) {
                                                                    int last=sp.back(); sp.pop_back();
    if (x->f->isr()) rotate(x)
                                                                    while( SZ(h)!=last ){
    else if (x->dir()==x->f->dir())
                                                                      auto x=h.back(); h.pop_back();
      rotate(x->f),rotate(x);
                                                                      *x.first = x.second;
    else rotate(x),rotate(x);
                                                                 } }
                                                                 int f( int x ){
                                                                   while( fa[x] != x) x = fa[x];
int id(Splay *x) { return x - Splay::mem + 1; }
                                                                    return x;
Splay* access(Splay *x){
  Splay *q = nil;
for (;x!=nil;x=x->f){
                                                                 void uni( int x , int y ){
                                                                    x = f(x); y = f(y);
    splay(x);
                                                                   if( x == y ) return;
if( sz[ x ] < sz[ y ] ) swap( x, y );
    x->setCh(q, 1);
    q = x;
                                                                    assign( \&sz[x], sz[x] + sz[y]);
                                                                    assign( &fa[ y ] , x);
  return q;
                                                                 } }djs;
```

#### 7.5 Black Magic

```
#include <bits/extc++.h>
using namespace __gnu_pbds;
typedef tree<int,null_type,less<int>,rb_tree_tag,
    tree_order_statistics_node_update> set_t;
#include <ext/pb_ds/assoc_container.hpp>
typedef cc_hash_table<int,int> umap_t;
typedef priority_queue<int> heap;
#include<ext/rope>
using namespace __gnu_cxx;
int main(){
  // Insert some entries into s.
  set_t s; s.insert(12); s.insert(505);
  // The order of the keys should be: 12, 505.
  assert(*s.find_by_order(0) == 12);
  assert(*s.find_by_order(3) == 505);
  // The order of the keys should be: 12, 505.
 assert(s.order_of_key(12) == 0);
assert(s.order_of_key(505) == 1);
  // Erase an entry.
  s.erase(12);
  // The order of the keys should be: 505.
  assert(*s.find_by_order(0) == 505);
  // The order of the keys should be: 505.
  assert(s.order_of_key(505) == 0);
 heap h1 , h2; h1.join( h2 );
  rope<char> r[ 2 ];
 r[1] = r[0]; // persistenet
string t = "abc";
r[1].insert(0, t.c_str());
r[1].erase(1, 1);
  cout << r[ 1 ].substr( 0 , 2 );</pre>
```

### 8 Others

## 8.1 Find max tangent(x,y is increasing)

```
typedef long long LL;
const int MAXN = 100010;
struct Coord{
 LL x, y;
Coord operator - (Coord ag) const{
    Coord res;
    res.x = x - ag.x;
res.y = y - ag.y;
    return res;
}sum[MAXN], pnt[MAXN], ans, calc;
inline bool cross(Coord a, Coord b, Coord c){
 return (c.y-a.y)*(c.x-b.x) > (c.x-a.x)*(c.y-b.y);
int main(){
 int n, 1, np, st, ed, now;
scanf("%d %d\n", &n, &1);
  sum[0].x = sum[0].y = np = st = ed = 0;
  for (int i = 1, v; i \le n; i++){
    scanf("%d", &v);
    sum[i].y = sum[i - 1].y + v;
sum[i].x = i;
 ans.x = now = 1;
 ans.y = -1;
  for (int i = 0; i <= n - 1; i++){
    while (np > 1 \&\&
            cross(pnt[np - 2], pnt[np - 1], sum[i]))
    if (np < now \&\& np != 0) now = np;
    pnt[np++] = sum[i];
    while (now < np &&
            !cross(pnt[now - 1], pnt[now], sum[i + 1]))
    calc = sum[i + l] - pnt[now - 1];
    if (ans.y * calc.x < ans.x * calc.y){
      ans = calc;
      st = pnt[now - 1].x;
      ed = i + 1;
 }
```

#### 8.2 Exact Cover Set

```
// given n*m 0-1 matrix
// find a set of rows s.t.
// for each column, there's exactly one 1
#define N 1024 //row
#define M 1024 //column
#define NM ((N+2)*(M+2))
char A[N][M]; //n*m 0-1 matrix
int used[N]; //answer: the row used
int id[N][M]
int L[NM],R[NM],D[NM],U[NM],C[NM],S[NM],ROW[NM];
void remove(int c){
  L[R[c]]=L[c]; R[L[c]]=R[c];

for( int i=D[c]; i!=c; i=D[i] )

for( int j=R[i]; j!=i; j=R[j] ){
       U[D[j]]=U[j]; D[U[j]]=D[j]; S[C[j]]--;
void resume(int c){
  for( int i=D[c]; i!=c; i=D[i] )
  for( int j=L[i]; j!=i; j=L[j] ){
       U[D[j]]=D[Ū[j]]=j; S[C[j]]++;
  L[R[c]]=R[L[c]]=c;
int dfs(){
  if(R[0] = 0) return 1;
   int md=100000000,c;
   for( int i=R[0]; i!=0; i=R[i] )
  if(S[i]<md){ md=S[i]; c=i; }</pre>
   if(md==0) return 0;
   remove(c);
for( int i=D[c]; i!=c; i=D[i] ){
     used[ROW[i]]=1;
     for( int j=R[i]; j!=i; j=R[j] ) remove(C[j]);
     if(dfs()) return 1;
     for( int j=L[i]; j!=i; j=L[j] ) resume(C[j]);
used[ROW[i]]=0;
   resume(c):
   return 0;
int exact_cover(int n,int m){
  for( int i=0; i<=m; i++ ){
   R[i]=i+1; L[i]=i-1; U[i]=D[i]=i;
   S[i]=0; C[i]=i;</pre>
   R[m]=0; L[0]=m;
   int t=m+1;
   for( int i=0; i<n; i++ ){
     int k=-1;
                 j=0; j<m; j++ ){
     for( int
        if(!A[i][j]) continue;
        if(k==-1) L[t]=R[t]=t
        else{ L[t]=k; R[t]=R[k]; }
k=t; D[t]=j+1; U[t]=U[j+1];
        L[R[t]]=R[L[t]]=U[D[t]]=D[U[t]]=t;
        C[t]=j+1; S[C[t]]++; ROW[t]=i; id[i][j]=t++;
   for( int i=0; i<n; i++ ) used[i]=0;
   return dfs();
}
```

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