

# Contents

<b>1 Basic</b>	
1.1 Increase Stack Size	1
1.2 Misc	1
1.3 check	1
1.4 python-related	1
<b>2 flow</b>	
2.1 MinCostFlow	1
2.2 Dinic	1
2.3 Kuhn Munkres 最大完美二分匹配	1
2.4 Max flow with lower/upper bound	2
2.5 Flow Method	2
<b>3 Math</b>	
3.1 FFT	3
3.2 NTT	3
3.3 Fast Walsh Transform	3
3.4 Poly operator	3
3.5 $O(1)$ mul	4
3.6 Linear Recurrence	4
3.7 Miller Rabin	4
3.8 Faulhaber ( $\sum_{i=1}^n i^p$ )	5
3.9 Chinese Remainder	5
3.10 Pollard Rho	5
3.11 Josephus Problem	5
3.12 Gaussian Elimination	5
3.13 $ax+by=gcd$	5
3.14 Discrete sqrt	5
3.15 Prefix Inverse	6
3.16 Roots of Polynomial 找多項式的根	6
3.17 Primes	6
3.18 Result	6
<b>4 Geometry</b>	
4.1 definition	7
4.2 Intersection of 2 lines	7
4.3 halfPlaneIntersection	7
4.4 Convex Hull	7
4.5 Intersection of 2 segments	7
4.6 Intersection of circle and segment	7
4.7 Intersection of 2 circles	7
4.8 Circle cover	8
4.9 Convex Hull trick	8
4.10 Tangent line of two circles	8
4.11 KD Tree	9
4.12 Lower Concave Hull	9
4.13 Min Enclosing Circle	10
4.14 Min Enclosing Ball	10
4.15 Min dist on Cuboid	10
4.16 Heart of Triangle	10
<b>5 Graph</b>	
5.1 DominatorTree	11
5.2 MaximumClique 最大團	11
5.3 MaximalClique 極大團	11
5.4 Strongly Connected Component	11
5.5 Dynamic MST	12
5.6 Maximum General graph Matching	12
5.7 Minimum General Weighted Matching	12
5.8 Minimum Steiner Tree	13
5.9 BCC based on vertex	13
5.10 Min Mean Cycle	14
5.11 Directed Graph Min Cost Cycle	14
5.12 K-th Shortest Path	15
5.13 SPFA	16
5.14 差分約束	16
<b>6 String</b>	
6.1 PalTree	16
6.2 KMP	16
6.3 SAIS	16
6.4 SuffixAutomata	17
6.5 Aho-Corasic	17
6.6 Z Value	18
6.7 ZValue Palindrome	18
6.8 Smallest Rotation	18
6.9 Cyclic LCS	18
<b>7 Data Structure</b>	
7.1 Segment tree	18
7.2 Treap	19
7.3 Disjoint Set	19
7.4 Black Magic	19
<b>8 Others</b>	
8.1 Find max tangent(x,y is increasing)	19
8.2 Exact Cover Set	20

## 1 Basic

### 1.1 Increase Stack Size

```
//stack resize (linux)
#include <sys/resource.h>
void increase_stack_size() {
    const rlim_t ks = 64*1024*1024;
    struct rlimit rl;
    int res=getrlimit(RLIMIT_STACK, &rl);
    if(res==0){
        if(rl.rlim_cur<ks){
            rl.rlim_cur=ks;
            res=setrlimit(RLIMIT_STACK, &rl);
        } } }
```

### 1.2 Misc

```
編譯參數: -std=c++14 -Wall -Wshadow (-fsanitize=
undefined)
//check special cases for example (n==1)
//check size arrays

#include <random>
mt19937 gen(0x5EED);
int randint(int lb, int ub)
{ return uniform_int_distribution<int>(lb, ub)(gen); }

#define SECS ((double)clock() / CLOCKS_PER_SEC)

struct KeyHasher {
    size_t operator()(const Key& k) const {
        return k.first + k.second * 100000;
    } };
typedef unordered_map<Key,int,KeyHasher> map_t;

__builtin_popcountll //換成二進位有幾個1
__builtin_clzll //返回左起第一個1之前0的個數
__builtin_parityll //返回1的個數的奇偶性
```

### 1.3 check

```
for ((i=0;;i++))
do
    echo "$i"
    python3 gen.py > input
    ./ac < input > ac.out
    ./wa < input > wa.out
    diff ac.out wa.out || break
done
```

### 1.4 python-related

```
parser:
int(eval(num.replace("/", "/")))

from fractions import Fraction
from decimal import Decimal, getcontext
getcontext().prec = 250 # set precision

itwo = Decimal(0.5)
two = Decimal(2)

N = 200
def angle(cosT):
    """given cos(theta) in decimal return theta"""
    for i in range(N):
        cosT = ((cosT + 1) / two) ** itwo
        sinT = (1 - cosT * cosT) ** itwo
        return sinT * (2 ** N)
pi = angle(Decimal(-1))
```

## 2 flow

### 2.1 MinCostFlow

```
struct MinCostMaxFlow{
    typedef int Tcost;
    static const int MAXV = 20010;
    static const int INFF = 1000000;
    static const Tcost INFC = 1e9;
    struct Edge{
```

```

int v, cap;
Tcost w;
int rev;
Edge(){}
Edge(int t2, int t3, Tcost t4, int t5)
: v(t2), cap(t3), w(t4), rev(t5) {}
};
int V, s, t;
vector<Edge> g[MAXV];
void init(int n, int _s, int _t){
    V = n; s = _s; t = _t;
    for(int i = 0; i <= V; i++) g[i].clear();
}
void addEdge(int a, int b, int cap, Tcost w){
    g[a].push_back(Edge(b, cap, w, (int)g[b].size()));
    g[b].push_back(Edge(a, 0, -w, (int)g[a].size()-1));
}
Tcost d[MAXV];
int id[MAXV], mom[MAXV];
bool inqu[MAXV];
queue<int> q;
pair<int, Tcost> solve(){
    int mxf = 0; Tcost mnc = 0;
    while(1){
        fill(d, d+1+V, INF);
        fill(inqu, inqu+1+V, 0);
        fill(mom, mom+1+V, -1);
        mom[s] = s;
        d[s] = 0;
        q.push(s); inqu[s] = 1;
        while(q.size()){
            int u = q.front(); q.pop();
            inqu[u] = 0;
            for(int i = 0; i < (int) g[u].size(); i++){
                Edge &e = g[u][i];
                int v = e.v;
                if(e.cap > 0 && d[v] > d[u]+e.w){
                    d[v] = d[u]+e.w;
                    mom[v] = u;
                    id[v] = i;
                    if(!inqu[v]) q.push(v), inqu[v] = 1;
                }
            }
            if(mom[t] == -1) break;
            int df = INF;
            for(int u = t; u != s; u = mom[u])
                df = min(df, g[mom[u]][id[u]].cap);
            for(int u = t; u != s; u = mom[u]){
                Edge &e = g[mom[u]][id[u]];
                e.cap -= df;
                g[e.v][e.rev].cap += df;
            }
            mxf += df;
            mnc += df*d[t];
        }
        return {mxf, mnc};
    }
} }flow;

```

## 2.2 Dinic

```

const int MXN = 10000;
struct Dinic{
    struct Edge{ int v, f, re; };
    int n, s, t, level[MXN];
    vector<Edge> E[MXN];
    void init(int _n, int _s, int _t){
        n = _n; s = _s; t = _t;
        for (int i=0; i<n; i++) E[i].clear();
    }
    void add_edge(int u, int v, int f){
        E[u].PB({v, f, SZ(E[v])});
        E[v].PB({u, 0, SZ(E[u])-1});
    }
    bool BFS(){
        for (int i=0; i<n; i++) level[i] = -1;
        queue<int> que;
        que.push(s);
        level[s] = 0;
        while (!que.empty()){
            int u = que.front(); que.pop();
            for (auto it : E[u]){
                if (it.f > 0 && level[it.v] == -1){
                    level[it.v] = level[u]+1;

```

```

                que.push(it.v);
            }
        }
        return level[t] != -1;
    }
    int DFS(int u, int nf){
        if (u == t) return nf;
        int res = 0;
        for (auto &it : E[u]){
            if (it.f > 0 && level[it.v] == level[u]+1){
                int tf = DFS(it.v, min(nf, it.f));
                res += tf; nf -= tf; it.f -= tf;
                E[it.v][it.re].f += tf;
                if (nf == 0) return res;
            }
        }
        if (!res) level[u] = -1;
        return res;
    }
    int flow(int res=0){
        while (BFS())
            res += DFS(s, 2147483647);
        return res;
    }
} }flow;

```

## 2.3 Kuhn Munkres 最大完美二分匹配

```

struct KM{ // max weight, for min negate the weights
    static const int MXN = 2001; // 1-based
    static const ll INF = 0x3f3f3f3f;
    int n, mx[MXN], my[MXN], pa[MXN];
    ll g[MXN][MXN], lx[MXN], ly[MXN], sy[MXN];
    bool vx[MXN], vy[MXN];
    void init(int _n) {
        n = _n;
        for(int i=1; i<=n; i++) fill(g[i], g[i]+n+1, 0);
    }
    void addEdge(int x, int y, ll w) {g[x][y] = w;}
    void augment(int y) {
        for(int x, z; y; y = z)
            x=pa[y], z=mx[x], my[y]=x, mx[x]=y;
    }
    void bfs(int st) {
        for(int i=1; i<=n; ++i) sy[i]=INF, vx[i]=vy[i]=0;
        queue<int> q; q.push(st);
        for(;;) {
            while(q.size()) {
                int x=q.front(); q.pop(); vx[x]=1;
                for(int y=1; y<=n; ++y) if(!vy[y]){
                    ll t = lx[x]+ly[y]-g[x][y];
                    if(t==0){
                        pa[y]=x;
                        if(!my[y]){augment(y);return;}
                        vy[y]=1, q.push(my[y]);
                    }else if(sy[y]>t) pa[y]=x, sy[y]=t;
                }
            }
            ll cut = INF;
            for(int y=1; y<=n; ++y)
                if(!vy[y]&&cut>sy[y]) cut=sy[y];
            for(int j=1; j<=n; ++j){
                if(vx[j]) lx[j] -= cut;
                if(vy[j]) ly[j] += cut;
                else sy[j] -= cut;
            }
            for(int y=1; y<=n; ++y) if(!vy[y]&&sy[y]==0){
                if(!my[y]){augment(y);return;}
                vy[y]=1, q.push(my[y]);
            }
        }
    }
    ll solve(){
        fill(mx, mx+n+1, 0); fill(my, my+n+1, 0);
        fill(ly, ly+n+1, 0); fill(lx, lx+n+1, -INF);
        for(int x=1; x<=n; ++x) for(int y=1; y<=n; ++y)
            lx[x] = max(lx[x], g[x][y]);
        for(int x=1; x<=n; ++x) bfs(x);
        ll ans = 0;
        for(int y=1; y<=n; ++y) ans += g[my[y]][y];
        return ans;
    }
} }graph;

```

## 2.4 Max flow with lower/upper bound

```

// flow use ISAP
// Max flow with lower/upper bound on edges
// source = 1 , sink = n

```

```

int in[ N ] , out[ N ];
int l[ M ] , r[ M ] , a[ M ] , b[ M ]; //0-base, a下界, b
上界
int solve(){
    flow.init( n );    //n為點的數量, m為邊的數量, 點是1-
    base
    for( int i = 0 ; i < m ; i ++ ){
        in[ r[ i ] ] += a[ i ];
        out[ l[ i ] ] += a[ i ];
        flow.addEdge( l[ i ] , r[ i ] , b[ i ] - a[ i ] );
        // flow from l[i] to r[i] must in [a[i], b[i]]
    }
    int nd = 0;
    for( int i = 1 ; i <= n ; i ++ ){
        if( in[ i ] < out[ i ] ){
            flow.addEdge( i , flow.t , out[ i ] - in[ i ] );
            nd += out[ i ] - in[ i ];
        }
        if( out[ i ] < in[ i ] )
            flow.addEdge( flow.s , i , in[ i ] - out[ i ] );
    }
    // original sink to source
    flow.addEdge( n , 1 , INF );
    if( flow.maxflow() != nd )
        // no solution
        return -1;
    int ans = flow.G[ 1 ].back().c; // source to sink
    flow.G[ 1 ].back().c = flow.G[ n ].back().c = 0;
    // take out super source and super sink
    for( size_t i = 0 ; i < flow.G[ flow.s ].size() ; i
        ++ ){
        flow.G[ flow.s ][ i ].c = 0;
        Edge &e = flow.G[ flow.s ][ i ];
        flow.G[ e.v ][ e.r ].c = 0;
    }
    for( size_t i = 0 ; i < flow.G[ flow.t ].size() ; i
        ++ ){
        flow.G[ flow.t ][ i ].c = 0;
        Edge &e = flow.G[ flow.t ][ i ];
        flow.G[ e.v ][ e.r ].c = 0;
    }
    flow.addEdge( flow.s , 1 , INF );
    flow.addEdge( n , flow.t , INF );
    flow.reset();
    return ans + flow.maxflow();
}

```

## 2.5 Flow Method

Maximize  $c^T x$  subject to  $Ax \leq b$ ,  $x \geq 0$ ;  
 with the corresponding symmetric dual problem,  
 Minimize  $b^T y$  subject to  $A^T y \geq c$ ,  $y \geq 0$ .

Maximize  $c^T x$  subject to  $Ax \leq b$ ;  
 with the corresponding asymmetric dual problem,  
 Minimize  $b^T y$  subject to  $A^T y = c$ ,  $y \geq 0$ .

Minimum vertex cover on bipartite graph =  
 Maximum matching on bipartite graph

找出最小點覆蓋，做完dinic之後，從源點dfs只走還有流量的邊，紀錄每個點有沒有被走到，左邊沒被走到的點跟右邊被走到的點就是答案

Maximum density subgraph (  $\sum W_e + \sum W_v$  ) /  $|V|$

Binary search on answer:

For a fixed D, construct a Max flow model as follow:  
 Let S be Sum of all weight( or inf)

1. from source to each node with cap = S
2. For each (u,v,w) in E, (u→v, cap=w), (v→u, cap=w)
3. For each node v, from v to sink with cap = S + 2 \* D - deg[v] - 2 \* (W of v)

where deg[v] =  $\sum$  weight of edge associated with v  
 If maxflow < S \* |V|, D is an answer.

Requiring subgraph: all vertex can be reached from  
 source with  
 edge whose cap > 0.

## 3 Math

### 3.1 FFT

```

// const int MAXN = 262144;
// (must be 2^k)
// before any usage, run pre_fft() first
typedef long double ld;
typedef complex<ld> cplx; //real() , imag()
const ld PI = acos(-1);
const cplx I(0, 1);
cplx omega[MAXN+1];
void pre_fft(){
    for(int i=0; i<=MAXN; i++)
        omega[i] = exp(i * 2 * PI / MAXN * I);
}
// n must be 2^k
void fft(int n, cplx a[], bool inv=false){
    int basic = MAXN / n;
    int theta = basic;
    for (int m = n; m >= 2; m >>= 1) {
        int mh = m >> 1;
        for (int i = 0; i < mh; i++) {
            cplx w = omega[inv ? MAXN-(i*theta%MAXN) : i*theta%MAXN];
            for (int j = i; j < n; j += m) {
                int k = j + mh;
                cplx x = a[j] - a[k];
                a[j] += a[k];
                a[k] = w * x;
            }
            theta = (theta * 2) % MAXN;
        }
    }
    int i = 0;
    for (int j = 1; j < n - 1; j++) {
        for (int k = n >> 1; k > (i ^ k); k >>= 1);
        if (j < i) swap(a[i], a[j]);
    }
    if(inv) for (i = 0; i < n; i++) a[i] /= n;
}
cplx arr[MAXN+1];
inline void mul(int _n, ll a[], int _m, ll b[], ll ans[])
{
    int n=1, sum=_n+_m-1;
    while(n<sum)
        n<<=1;
    for(int i=0; i<n; i++)
    {
        double x=(i<_n?a[i]:0), y=(i<_m?b[i]:0);
        arr[i]=complex<double>(x+y, x-y);
    }
    fft(n, arr);
    for(int i=0; i<n; i++)
        arr[i]=arr[i]*arr[i];
    fft(n, arr, true);
    for(int i=0; i<sum; i++)
        ans[i]=(long long int)(arr[i].real()/4+0.5);
}

```

### 3.2 NTT

```

// Remember coefficient are mod P
/* p=a*2^n+1
n    2^n    p    a    root
16   65536   65537   1    3
20   1048576 7340033  7    3 */
// (must be 2^k)
template<LL P, LL root, int MAXN>
struct NTT{
    static LL bigmod(LL a, LL b) {
        LL res = 1;
        for (LL bs = a; b; b >>= 1, bs = (bs * bs) % P)
            if(b&1) res=(res*bs)%P;
        return res;
    }
    static LL inv(LL a, LL b) {
        if(a==1)return 1;
        return (((LL)(a-inv(b%a,a))*b+1)/a)%b;
    }
    LL omega[MAXN+1];
    NTT() {
        omega[0] = 1;
    }
}

```

```

    LL r = bigmod(root, (P-1)/MAXN);
    for (int i=1; i<=MAXN; i++)
        omega[i] = (omega[i-1]*r)%P;
}
// n must be 2^k
void tran(int n, LL a[], bool inv_ntt=false){
    int basic = MAXN / n, theta = basic;
    for (int m = n; m >= 2; m >>= 1) {
        int mh = m >> 1;
        for (int i = 0; i < mh; i++) {
            LL w = omega[i*theta*MAXN];
            for (int j = i; j < n; j += m) {
                int k = j + mh;
                LL x = a[j] - a[k];
                if (x < 0) x += P;
                a[j] += a[k];
                if (a[j] > P) a[j] -= P;
                a[k] = (w * x) % P;
            }
        }
        theta = (theta * 2) % MAXN;
    }
    int i = 0;
    for (int j = 1; j < n - 1; j++) {
        for (int k = n >> 1; k > (i ^ k); k >>= 1);
        if (j < i) swap(a[i], a[j]);
    }
    if (inv_ntt) {
        LL ni = inv(n,P);
        reverse(a+1, a+n);
        for (i = 0; i < n; i++)
            a[i] = (a[i] * ni) % P;
    }
}
};
const LL P=2013265921, root=31;
const int MAXN=4194304;
NTT<P, root, MAXN> ntt;

```

### 3.3 Fast Walsh Transform

```

/* xor convolution:
* x = (x0,x1) , y = (y0,y1)
* z = ( x0y0 + x1y1 , x0y1 + x1y0 )
* =>
* x' = ( x0+x1 , x0-x1 ) , y' = ( y0+y1 , y0-y1 )
* z' = ( ( x0+x1 )( y0+y1 ) , ( x0-x1 )( y0-y1 ) )
* z = (1/2) * z'
* or convolution:
* x = (x0, x0+x1), inv = (x0, x1-x0) w/o final div
* and convolution:
* x = (x0+x1, x1), inv = (x0-x1, x1) w/o final div */
typedef long long LL;
const int MAXN = (1<<20)+10;
const LL MOD = 1e9+7;
inline LL inv( LL x ) {
    return mypow( x , MOD-2 );
}
inline void fwt( LL x[ MAXN ] , int N , bool inv=0 ) {
    for (int d = 1 ; d < N ; d <= 1 ) {
        int d2 = d<<1;
        for (int s = 0 ; s < N ; s += d2 )
            for (int i = s , j = s+d ; i < s+d ; i++, j++) {
                LL ta = x[ i ] , tb = x[ j ];
                x[ i ] = ta+tb;
                x[ j ] = ta-tb;
                if ( x[ i ] >= MOD ) x[ i ] -= MOD;
                if ( x[ j ] < 0 ) x[ j ] += MOD;
            }
    }
    if (inv)
        for (int i = 0 ; i < N ; i++) {
            x[ i ] *= inv( N );
            x[ i ] %= MOD;
        }
}

```

### 3.4 Poly operator

```

struct PolyOp {
#define FOR(i, c) for (int i = 0; i < (c); ++i)
    NTT<P, root, MAXN> ntt;
    static int nxt2k(int x) {
        int i = 1; for (; i < x; i <= 1); return i;
    }
};

```

```

}
// c[i]=sum{j=0~i}a[j]*b[i-j] -> c[i+j]=a[i]*b[j](加
    分卷積)
// if c[i-j]=a[i]*b[j] (減法卷積)
// (轉換成加法卷積) -> reverse(a); c=mul(a,b);
    reverse( c );
void Mul(int n, LL a[], int m, LL b[], LL c[]) {
    static LL aa[MAXN], bb[MAXN];
    int N = nxt2k(n+m);
    copy(a, a+n, aa); fill(aa+n, aa+N, 0);
    copy(b, b+m, bb); fill(bb+m, bb+N, 0);
    ntt.tran(N, aa); ntt.tran(N, bb);
    FOR(i, N) c[i] = aa[i] * bb[i] % P;
    ntt.tran(N, c, 1);
}
void Inv(int n, LL a[], LL b[]) {
    // ab = aa^-1 = 1 mod x^(n/2)
    // (b - a^-1)^2 = 0 mod x^n
    // bb - a^-2 + 2 ba^-1 = 0
    // bba - a^-1 + 2b = 0
    // bba + 2b = a^-1
    static LL tmp[MAXN];
    if (n == 1) {b[0] = ntt.inv(a[0], P); return;}
    Inv((n+1)/2, a, b);
    int N = nxt2k(n*2);
    copy(a, a+n, tmp);
    fill(tmp+n, tmp+N, 0);
    fill(b+n, b+N, 0);
    ntt.tran(N, tmp); ntt.tran(N, b);
    FOR(i, N) {
        LL t1 = (2 - b[i] * tmp[i]) % P;
        if (t1 < 0) t1 += P;
        b[i] = b[i] * t1 % P;
    }
    ntt.tran(N, b, 1);
    fill(b+n, b+N, 0);
}
void Div(int n, LL a[], int m, LL b[], LL d[], LL r
    []) {
    // Ra = Rb * Rd mod x^(n-m+1)
    // Rd = Ra * Rb^-1 mod
    static LL aa[MAXN], bb[MAXN], ta[MAXN], tb[MAXN];
    if (n < m) {copy(a, a+n, r); fill(r+n, r+m, 0);
        return;}
    // d: n-1 - (m-1) = n-m (n-m+1 terms)
    copy(a, a+n, aa); copy(b, b+m, bb);
    reverse(aa, aa+n); reverse(bb, bb+m);
    Inv(n-m+1, bb, tb);
    Mul(n-m+1, ta, n-m+1, tb, d);
    fill(d+n-m+1, d+n, 0); reverse(d, d+n-m+1);
    // r: m-1 - 1 = m-2 (m-1 terms)
    Mul(m, b, n-m+1, d, ta);
    FOR(i, n) { r[i] = a[i] - ta[i]; if (r[i] < 0) r[i]
        += P; }
}
void dx(int n, LL a[], LL b[]) { REP(i, 1, n-1) b[i]
    -1] = i * a[i] % P; }
void Sx(int n, LL a[], LL b[]) {
    b[0] = 0;
    FOR(i, n) b[i+1] = a[i] * ntt.inv(i+1, P) % P;
}
void Ln(int n, LL a[], LL b[]) {
    // Integral a' a^-1 dx
    static LL a1[MAXN], a2[MAXN], b1[MAXN];
    int N = nxt2k(n*2);
    dx(n, a, a1); Inv(n, a, a2);
    Mul(n-1, a1, n, a2, b1);
    Sx(n+n-1-1, b1, b);
    fill(b+n, b+N, 0);
}
void Exp(int n, LL a[], LL b[]) {
    // Newton method to solve g(a(x)) = ln b(x) - a(x)
    = 0
    // b' = b - g(b(x)) / g'(b(x))
    // b' = b (1 - lnb + a)
    static LL lnb[MAXN], c[MAXN], tmp[MAXN];
    assert(a[0] == 0); // dont know exp(a[0]) mod P
    if (n == 1) {b[0] = 1; return;}
    Exp((n+1)/2, a, b);
    fill(b+(n+1)/2, b+n, 0);
    Ln(n, b, lnb);
}

```

```

fill(c, c+n, 0); c[0] = 1;
FOR(i, n) {
    c[i] += a[i] - lnb[i];
    if (c[i] < 0) c[i] += P;
    if (c[i] >= P) c[i] -= P;
}
Mul(n, b, n, c, tmp);
copy(tmp, tmp+n, b);
}
} polyop;

```

### 3.5 O(1)mul

```

LL mul(LL x, LL y, LL mod) {
    LL ret = x*y - (LL)((long double)x/mod*y)*mod;
    // LL ret = x*y - (LL)((long double)x*y/mod + 0.5)*mod;
    return ret < 0 ? ret + mod : ret;
}

```

### 3.6 Linear Recurrence

```

// Usage: linearRec({0, 1}, {1, 1}, k) // k'th fib
typedef vector<ll> Poly;
// S: 前i項的值, tr: 遞迴係數, k: 求第k項
ll linearRec(Poly& S, Poly& tr, ll k) {
    int n = tr.size();
    auto combine = [&](Poly& a, Poly& b) {
        Poly res(n * 2 + 1);
        rep(i, 0, n+1) rep(j, 0, n+1)
            res[i+j] = (res[i+j] + a[i]*b[j])%mod;
        for(int i = 2*n; i > n; --i) rep(j, 0, n)
            res[i-1-j] = (res[i-1-j] + res[i]*tr[j])%mod;
        res.resize(n + 1);
        return res;
    };
    Poly pol(n + 1, e(pol));
    pol[0] = e[1] = 1;
    for (++k; k; k /= 2) {
        if (k % 2) pol = combine(pol, e);
        e = combine(e, e);
    }
    ll res = 0;
    rep(i, 0, n) res = (res + pol[i+1]*S[i])%mod;
    return res;
}

```

### 3.7 Miller Rabin

```

// n < 4,759,123,141      3 : 2, 7, 61
// n < 1,122,004,669,633  4 : 2, 13, 23, 1662803
// n < 3,474,749,660,383  6 : pimes <= 13
// n < 2^64               7 :
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
// Make sure testing integer is in range [2, n-2] if
// you want to use magic.
LL magic[] = {}
bool witness(LL a, LL n, LL u, int t) {
    if(!a) return 0;
    LL x = mypow(a, u, n);
    for(int i = 0; i < t; i++) {
        LL nx = mul(x, x, n);
        if(nx == 1 && x != 1 && x != n-1) return 1;
        x = nx;
    }
    return x != 1;
}
bool miller_rabin(LL n) {
    int s = (magic number size)
    // iterate s times of witness on n
    if(n < 2) return 0;
    if(!(n & 1)) return n == 2;
    ll u = n-1; int t = 0;
    // n-1 = u*2^t
    while(!(u & 1)) u >>= 1, t++;
    while(s--){
        LL a = magic[s]%n;
        if(witness(a, n, u, t)) return 0;
    }
    return 1;
}

```

### 3.8 Faulhaber ( $\sum_{i=1}^n i^p$ )

```

/* faulhaber' s formula -
 * cal power sum formula of all p=1~k in O(k^2) */
#define MAXK 2500
const int mod = 1000000007;
int b[MAXK]; // bernoulli number
int inv[MAXK+1]; // inverse
int cm[MAXK+1][MAXK+1]; // combinactories
int co[MAXK][MAXK+2]; // coefficient of x^j when p=i
inline int getinv(int x) {
    int a=x, b=mod, a0=1, a1=0, b0=0, b1=1;
    while(b) {
        int q, t;
        q=a/b; t=b; b=a-b*q; a=t;
        t=b0; b0=a0-b0*q; a0=t;
        t=b1; b1=a1-b1*q; a1=t;
    }
    return a0 < 0 ? a0 + mod : a0;
}
inline void pre() {
    /* combinational */
    for(int i=0; i<=MAXK; i++) {
        cm[i][0] = cm[i][i] = 1;
        for(int j=1; j<i; j++)
            cm[i][j] = add(cm[i-1][j-1], cm[i-1][j]);
    }
    /* inverse */
    for(int i=1; i<=MAXK; i++) inv[i] = getinv(i);
    /* bernoulli */
    b[0] = 1; b[1] = getinv(2); // with b[1] = 1/2
    for(int i=2; i<=MAXK; i++) {
        if(i & 1) { b[i] = 0; continue; }
        b[i] = 1;
        for(int j=0; j<i; j++)
            b[i] = sub(b[i], mul(cm[i][j], mul(b[j], inv[i-j+1])));
    }
    /* faulhaber */
    // sigma_x=1~n {x^p} =
    // 1/(p+1) * sigma_j=0~p {C(p+1, j)*B_j*n^(p-j+1)}
    for(int i=1; i<=MAXK; i++) {
        co[i][0] = 0;
        for(int j=0; j<=i; j++)
            co[i][i-j+1] = mul(inv[i+1], mul(cm[i+1][j], b[j]));
    }
}
/* sample usage: return f(n,p) = sigma_x=1~n (x^p) */
inline int solve(int n, int p) {
    int sol = 0, m = n;
    for(int i=1; i<=p+1; i++) {
        sol = add(sol, mul(co[p][i], m));
        m = mul(m, n);
    }
    return sol;
}

```

### 3.9 Chinese Remainder

```

LL x[N], m[N];
LL CRT(LL x1, LL m1, LL x2, LL m2) {
    LL g = __gcd(m1, m2);
    if((x2 - x1) % g) return -1; // no sol
    m1 /= g; m2 /= g;
    pair<LL, LL> p = gcd(m1, m2);
    LL lcm = m1 * m2 * g;
    LL res = p.first * (x2 - x1) * m1 + x1;
    return (res % lcm + lcm) % lcm;
}
LL solve(int n) { // n>=2, be careful with no solution
    LL res = CRT(x[0], m[0], x[1], m[1]), p = m[0] / __gcd(m[0], m[1]) * m[1];
    for(int i=2; i<=n; i++){
        res = CRT(res, p, x[i], m[i]);
        p = p / __gcd(p, m[i]) * m[i];
    }
    return res;
}

```



### 3.10 Pollard Rho

```
// does not work when n is prime
LL f(LL x, LL mod){ return add(mul(x,x,mod),1,mod); }
LL pollard_rho(LL n) {
    if(!(n&1)) return 2;
    while(true){
        LL y=2, x=rand()%(n-1)+1, res=1;
        for(int sz=2; res==1; sz*=2) {
            for(int i=0; i<sz && res<=1; i++) {
                x = f(x, n);
                res = __gcd(abs(x-y), n);
            }
            y = x;
        }
        if (res!=0 && res!=n) return res;
    }
}
```

### 3.11 Josephus Problem

```
int josephus(int n, int m){ //n人每m次
    int ans = 0;
    for (int i=1; i<=n; ++i)
        ans = (ans + m) % i;
    return ans;
}
```

### 3.12 Gaussian Elimination

```
const int GAUSS_MOD = 100000007LL;
struct GAUSS{
    int n;
    vector<vector<int>>> v;
    int ppow(int a, int k){
        if(k == 0) return 1;
        if(k % 2 == 0) return ppow(a * a % GAUSS_MOD, k >> 1);
        if(k % 2 == 1) return ppow(a * a % GAUSS_MOD, k >> 1) * a % GAUSS_MOD;
    }
    vector<int> solve(){
        vector<int> ans(n);
        REP(now, 0, n){
            REP(i, now, n) if(v[now][now] == 0 && v[i][now] != 0)
                swap(v[i], v[now]); // det = -det;
            if(v[now][now] == 0) return ans;
            int inv = ppow(v[now][now], GAUSS_MOD - 2);
            REP(i, 0, n) if(i != now){
                int tmp = v[i][now] * inv % GAUSS_MOD;
                REP(j, now, n + 1) (v[i][j] += GAUSS_MOD - tmp * v[now][j] % GAUSS_MOD) %= GAUSS_MOD;
            }
            REP(i, 0, n) ans[i] = v[i][n + 1] * ppow(v[i][i], GAUSS_MOD - 2) % GAUSS_MOD;
        }
        return ans;
    }
    // gs.v.clear(), gs.v.resize(n, vector<int>(n + 1, 0));
} gs;
```

### 3.13 ax+by=gcd

```
PII gcd(int a, int b){
    if(b == 0) return {1, 0};
    PII q = gcd(b, a % b);
    return {q.second, q.first - q.second * (a / b)};
}
```

### 3.14 Discrete sqrt

```
void calcH(LL &t, LL &h, const LL p) {
    LL tmp=p-1; for(t=0;(tmp&1)==0;tmp/=2) t++; h=tmp;
}
// solve equation x^2 mod p = a
bool solve(LL a, LL p, LL &x, LL &y) {
    if(p == 2) { x = y = 1; return true; }
    int p2 = p / 2, tmp = mypow(a, p2, p);
    if (tmp == p - 1) return false;
```

```
if ((p + 1) % 4 == 0) {
    x=mypow(a,(p+1)/4,p); y=p-x; return true;
} else {
    LL t, h, b, pb; calcH(t, h, p);
    if (t >= 2) {
        do {b = rand() % (p - 2) + 2;
            } while (mypow(b, p / 2, p) != p - 1);
        pb = mypow(b, h, p);
        int s = mypow(a, h / 2, p);
        for (int step = 2; step <= t; step++) {
            int ss = (((LL)(s * s) % p) * a) % p;
            for(int i=0;i<t-step;i++) ss=mul(ss,ss,p);
            if (ss + 1 == p) s = (s * pb) % p;
            pb = ((LL)pb * pb) % p;
        } x = ((LL)s * a) % p; y = p - x;
    } return true;
}
```

### 3.15 Prefix Inverse

```
void solve( int m ){
    inv[ 1 ] = 1;
    for( int i = 2 ; i < m ; i ++ )
        inv[ i ] = ((LL)(m - m / i) * inv[m % i]) % m;
}
```

### 3.16 Roots of Polynomial 找多項式的根

```
const double eps = 1e-12;
const double inf = 1e+12;
double a[ 10 ], x[ 10 ]; // a[0..n](coef) must be filled
int n; // degree of polynomial must be filled
int sign( double x ){return (x < -eps)?(-1):(x>eps);}
double f(double a[], int n, double x){
    double tmp=1,sum=0;
    for(int i=0;i<=n;i++){
        sum=sum+a[i]*tmp; tmp=tmp*x;
    }
    return sum;
}
double binary(double l,double r,double a[],int n){
    int sl=sign(f(a,n,l)),sr=sign(f(a,n,r));
    if(sl==0) return l; if(sr==0) return r;
    if(sl*sr>0) return inf;
    while(r-l>eps){
        double mid=(l+r)/2;
        int ss=sign(f(a,n,mid));
        if(ss==0) return mid;
        if(ss*sl>0) l=mid; else r=mid;
    }
    return l;
}
void solve(int n,double a[],double x[],int &nx){
    if(n==1){ x[1]=-a[0]/a[1]; nx=1; return; }
    double da[10], dx[10]; int ndx;
    for(int i=n;i>=1;i--) da[i-1]=a[i]*i;
    solve(n-1,da,dx,ndx);
    nx=0;
    if(ndx==0){
        double tmp=binary(-inf,inf,a,n);
        if (tmp<inf) x[++nx]=tmp;
        return;
    }
    double tmp;
    tmp=binary(-inf,dx[1],a,n);
    if(tmp<inf) x[++nx]=tmp;
    for(int i=1;i<=ndx-1;i++){
        tmp=binary(dx[i],dx[i+1],a,n);
        if(tmp<inf) x[++nx]=tmp;
    }
    tmp=binary(dx[ndx],inf,a,n);
    if(tmp<inf) x[++nx]=tmp;
} // roots are stored in x[1..nx]
```

### 3.17 Primes

```
/* 12721, 13331, 14341, 75577, 123457, 222557, 556679
* 999983, 1097774749, 1076767633, 100102021, 999997771
* 1001010013, 1000512343, 987654361, 999991231
* 999888733, 98789101, 98777733, 999991921, 1010101333
* 1010102101, 1000000000039, 100000000000037
* 2305843009213693951, 4611686018427387847
```

```

* 9223372036854775783, 18446744073709551557 */
int mu[ N ], p_tbl[ N ];
vector<int> primes;
void sieve() {
    mu[ 1 ] = p_tbl[ 1 ] = 1;
    for( int i = 2 ; i < N ; i ++ ){
        if( !p_tbl[ i ] ){
            p_tbl[ i ] = i;
            primes.push_back( i );
            mu[ i ] = -1;
        }
        for( int p : primes ){
            int x = i * p;
            if( x >= M ) break;
            p_tbl[ x ] = p;
            mu[ x ] = -mu[ i ];
            if( i % p == 0 ){
                mu[ x ] = 0;
                break;
            }
        }
    }
}
vector<int> factor( int x ){
    vector<int> fac{ 1 };
    while( x > 1 ){
        int fn = SZ(fac), p = p_tbl[ x ], pos = 0;
        while( x % p == 0 ){
            x /= p;
            for( int i = 0 ; i < fn ; i ++ )
                fac.PB( fac[ pos ++ ] * p );
        }
    }
    return fac;
}

```

### 3.18 Result

- Lucas' Theorem :  
For  $n, m \in \mathbb{Z}^*$  and prime  $P$ ,  $C(m, n) \bmod P = \prod(C(m_i, n_i))$  where  $m_i$  is the  $i$ -th digit of  $m$  in base  $P$ .
- Stirling approximation :  
$$n! \approx \sqrt{2\pi n} \left(\frac{n}{e}\right)^n e^{\frac{1}{12n}}$$
- Stirling Numbers(permutation  $|P| = n$  with  $k$  cycles):  
 $S(n, k) = \text{coefficient of } x^k \text{ in } \prod_{i=0}^{n-1} (x + i)$
- Stirling Numbers(Partition  $n$  elements into  $k$  non-empty set):  
$$S(n, k) = \frac{1}{k!} \sum_{j=0}^k (-1)^{k-j} \binom{k}{j} j^n$$
- Pick' s Theorem :  $A = i + b/2 - 1$   
其面積  $A$  和內部格點數目  $i$ 、邊上格點數目  $b$  的關係
- Catalan number :  $C_n = \binom{2n}{n} / (n + 1)$   
$$C_n^{n+m} - C_{n+1}^{n+m} = (m + n)! \frac{n-m+1}{n+1} \quad \text{for } n \geq m$$
  
$$C_n = \frac{1}{n+1} \binom{2n}{n} = \frac{(2n)!}{(n+1)!n!}$$
  
$$C_0 = 1 \quad \text{and} \quad C_{n+1} = 2 \binom{2n+1}{n+2} C_n$$
  
$$C_0 = 1 \quad \text{and} \quad C_{n+1} = \sum_{i=0}^n C_i C_{n-i} \quad \text{for } n \geq 0$$
- Euler Characteristic:  
planar graph:  $V - E + F - C = 1$   
convex polyhedron:  $V - E + F = 2$   
 $V, E, F, C$ : number of vertices, edges, faces(regions), and components
- Kirchhoff's theorem :  
 $A_{ii} = \deg(i), A_{ij} = (i, j) \in E ? -1 : 0$ , Deleting any one row, one column, and cal the  $\det(A)$
- Polya' theorem ( $c$  為方法數,  $m$  為總數):  
$$\left( \sum_{i=1}^m c^{gcd(i, m)} \right) / m$$
- 錯排公式: ( $n$  個人中, 每個人皆不再原來位置的組合數):  
$$dp[0] = 1; dp[1] = 0;$$
  
$$dp[i] = (i - 1) * (dp[i - 1] + dp[i - 2]);$$
- Bell 數 (有  $n$  個人, 把他們拆組的方法總數) :  
$$B_0 = 1$$
  
$$B_n = \sum_{k=0}^n s(n, k) \quad (\text{second - stirling})$$
  
$$B_{n+1} = \sum_{k=0}^n \binom{n}{k} B_k$$
- Wilson's theorem :  
$$(p - 1)! \equiv -1 \pmod{p}$$
- Fermat's little theorem :  
$$a^p \equiv a \pmod{p}$$

## 4 Geometry

### 4.1 definition

```

typedef long double ld;
const ld eps = 1e-8;
int dcmp(ld x) {
    if(abs(x) < eps) return 0;
    else return x < 0 ? -1 : 1;
}
struct Pt {
    ld x, y;
    Pt(ld _x=0, ld _y=0):x(_x), y(_y) {}

    Pt operator+(const Pt &a) const {
        return Pt(x+a.x, y+a.y);
    }
    Pt operator-(const Pt &a) const {
        return Pt(x-a.x, y-a.y);
    }
    Pt operator*(const ld &a) const {
        return Pt(x*a, y*a);
    }
    Pt operator/(const ld &a) const {
        return Pt(x/a, y/a);
    }
    ld operator*(const Pt &a) const {
        return x*a.x + y*a.y;
    }
    ld operator^(const Pt &a) const {
        return x*a.y - y*a.x;
    }
    bool operator<(const Pt &a) const {
        return x < a.x || (x == a.x && y < a.y);
        //return dcmp(x-a.x) < 0 || (dcmp(x-a.x) == 0 &&
        //    dcmp(y-a.y) < 0);
    }
    bool operator==(const Pt &a) const {
        return dcmp(x-a.x) == 0 && dcmp(y-a.y) == 0;
    }
};
ld norm2(const Pt &a) {
    return a*a;
}
ld norm(const Pt &a) {
    return sqrt(norm2(a));
}
Pt perp(const Pt &a) {
    return Pt(-a.y, a.x);
}
Pt rotate(const Pt &a, ld ang) {
    return Pt(a.x*cos(ang)-a.y*sin(ang), a.x*sin(ang)+a.y*cos(ang));
}
struct Line {
    Pt s, e, v; // start, end, end-start
    ld ang;
    Line(Pt _s=Pt(0, 0), Pt _e=Pt(0, 0)):s(_s), e(_e) { v = e-s; ang = atan2(v.y, v.x); }

    bool operator<(const Line &L) const {
        return ang < L.ang;
    }
};
struct Circle {
    Pt o; ld r;
    Circle(Pt _o=Pt(0, 0), ld _r=0):o(_o), r(_r) {}
};

```

### 4.2 Intersection of 2 lines

```

Pt LLIntersect(Line a, Line b) {
    Pt p1 = a.s, p2 = a.e, q1 = b.s, q2 = b.e;
    ld f1 = (p2-p1)^(q1-p1), f2 = (p2-p1)^(p1-q2), f;
    if(dcmp(f=f1+f2) == 0)
        return dcmp(f1)?Pt(NAN, NAN):Pt(INFINITY, INFINITY);
    return q1*(f2/f) + q2*(f1/f);
}

```

### 4.3 halfPlaneIntersection

// for point or line solution, change > to >=

```

bool onleft(Line L, Pt p) {
    return dcmp(L.v^(p-L.s)) > 0;
}
// assume that Lines intersect
vector<Pt> HPI(vector<Line>& L) {
    sort(L.begin(), L.end()); // sort by angle
    int n = L.size(), fir, las;
    Pt *p = new Pt[n];
    Line *q = new Line[n];
    q[fir=las=0] = L[0];
    for(int i = 1; i < n; i++) {
        while(fir < las && !onleft(L[i], p[las-1])) las--;
        while(fir < las && !onleft(L[i], p[fir])) fir++;
        q[++las] = L[i];
        if(dcmp(q[las].v^q[las-1].v) == 0) {
            las--;
            if(onleft(q[las], L[i].s)) q[las] = L[i];
        }
        if(fir < las) p[las-1] = LLIntersect(q[las-1], q[las]);
    }
    while(fir < las && !onleft(q[fir], p[las-1])) las--;
    if(las-fir <= 1) return {};
    p[las] = LLIntersect(q[las], q[fir]);
    int m = 0;
    vector<Pt> ans(las-fir+1);
    for(int i = fir; i <= las; i++) ans[m++] = p[i];
    return ans;
}

```

#### 4.4 Convex Hull

```

double cross(Pt o, Pt a, Pt b){
    return (a-o) ^ (b-o);
}
vector<Pt> convex_hull(vector<Pt> pt){
    sort(pt.begin(), pt.end());
    int top=0;
    vector<Pt> stk(2*pt.size());
    for (int i=0; i<(int)pt.size(); i++){
        while (top >= 2 && cross(stk[top-2], stk[top-1], pt[i]) <= 0)
            top--;
        stk[top++] = pt[i];
    }
    for (int i=pt.size()-2, t=top+1; i>=0; i--){
        while (top >= t && cross(stk[top-2], stk[top-1], pt[i]) <= 0)
            top--;
        stk[top++] = pt[i];
    }
    stk.resize(top-1);
    return stk;
}

```

#### 4.5 Intersection of 2 segments

```

int ori( const Pt& o , const Pt& a , const Pt& b ){
    LL ret = ( a - o ) ^ ( b - o );
    return (ret > 0) - (ret < 0);
}
// p1 == p2 || q1 == q2 need to be handled
bool banana( const Pt& p1 , const Pt& p2 ,
              const Pt& q1 , const Pt& q2 ){
    if( ( ( p2 - p1 ) ^ ( q2 - q1 ) ) == 0 ){ // parallel
        if( ori( p1 , p2 , q1 ) ) return false;
        return ( ( p1 - q1 ) * ( p2 - q1 ) ) <= 0 ||
               ( ( p1 - q2 ) * ( p2 - q2 ) ) <= 0 ||
               ( ( q1 - p1 ) * ( q2 - p1 ) ) <= 0 ||
               ( ( q1 - p2 ) * ( q2 - p2 ) ) <= 0;
    }
    return (ori( p1, p2, q1 ) * ori( p1, p2, q2 ) <= 0) &&
           (ori( q1, q2, p1 ) * ori( q1, q2, p2 ) <= 0);
}

```

#### 4.6 Intersection of circle and segment

```

bool Inter( const Pt& p1 , const Pt& p2 , Circle& cc ){
    Pt dp = p2 - p1;
    double a = dp * dp;
    double b = 2 * ( dp * ( p1 - cc.o ) );

```

```

    double c = cc.o * cc.o + p1 * p1 - 2 * ( cc.o * p1 )
              - cc.R * cc.R;
    double bb4ac = b * b - 4 * a * c;
    return !( fabs( a ) < eps or bb4ac < 0 );
}

```

#### 4.7 Intersection of 2 circles

#### 4.8 Circle cover

```

#define N 1021
#define D double
struct CircleCover{
    int C; Circ c[ N ]; //填入C(圓數量),c(圓陣列)
    bool g[ N ][ N ], overlap[ N ][ N ];
    // Area[i] : area covered by at least i circles
    D Area[ N ];
    void init( int _C ){ C = _C; }
    bool CCinter( Circ& a , Circ& b , Pt& p1 , Pt& p2 ){
        Pt o1 = a.o , o2 = b.o;
        D r1 = a.R , r2 = b.R;
        if( norm( o1 - o2 ) > r1 + r2 ) return {};
        if( norm( o1 - o2 ) < max(r1, r2) - min(r1, r2) )
            return {};
        D d2 = ( o1 - o2 ) * ( o1 - o2 );
        D d = sqrt(d2);
        if( d > r1 + r2 ) return false;
        Pt u=(o1+o2)*0.5 + (o1-o2)*((r2*r2-r1*r1)/(2*d2));
        D A=sqrt((r1+r2+d)*(r1-r2+d)*(r1+r2-d)*(-r1+r2+d));
        Pt v=Pt( o1.Y-o2.Y , -o1.X + o2.X ) * A / (2*d2);
        p1 = u + v; p2 = u - v;
        return true;
    }
}
struct Teve {
    Pt p; D ang; int add;
    Teve() {}
    Teve(Pt _a, D _b, int _c):p(_a), ang(_b), add(_c){}
    bool operator<(const Teve &a)const {
        return ang < a.ang;
    }
}eve[ N * 2 ];
// strict: x = 0, otherwise x = -1
bool disjuct( Circ& a, Circ &b, int x )
{return sign( norm( a.o - b.o ) - a.R - b.R ) > x;}
bool contain( Circ& a, Circ &b, int x )
{return sign( a.R - b.R - norm( a.o - b.o ) ) > x;}
bool contain(int i, int j){
    /* c[j] is non-strictly in c[i]. */
    return (sign(c[i].R - c[j].R) > 0 ||
            (sign(c[i].R - c[j].R) == 0 && i < j) ) &&
           contain(c[i], c[j], -1);
}
void solve(){
    for( int i = 0; i <= C + 1; i ++ )
        Area[ i ] = 0;
    for( int i = 0; i < C; i ++ )
        for( int j = 0; j < C; j ++ )
            overlap[i][j] = contain(i, j);
    for( int i = 0; i < C; i ++ )
        for( int j = 0; j < C; j ++ )
            g[i][j] = !(overlap[i][j] || overlap[j][i] ||
                       disjuct(c[i], c[j], -1));
    for( int i = 0; i < C; i ++ ){
        int E = 0, cnt = 1;
        for( int j = 0; j < C; j ++ )
            if( j != i && overlap[j][i] )
                cnt ++;
        for( int j = 0; j < C; j ++ )
            if( i != j && g[i][j] ){
                Pt aa, bb;
                CCinter(c[i], c[j], aa, bb);
                D A=atan2(aa.Y - c[i].o.Y, aa.X - c[i].o.X);
                D B=atan2(bb.Y - c[i].o.Y, bb.X - c[i].o.X);
                eve[E ++] = Teve(bb, B, 1);
                eve[E ++] = Teve(aa, A, -1);
                if(B > A) cnt ++;
            }
        if( E == 0 ) Area[ cnt ] += pi * c[i].R * c[i].R;
        else{
            sort( eve , eve + E );
            eve[E] = eve[0];
            for( int j = 0; j < E; j ++ ){
                cnt += eve[j].add;

```



```

    Area[cnt] += (eve[j].p ^ eve[j + 1].p) * 0.5;
    D theta = eve[j + 1].ang - eve[j].ang;
    if (theta < 0) theta += 2.0 * pi;
    Area[cnt] +=
        (theta - sin(theta)) * c[i].R*c[i].R * 0.5;
}
}
}
}
}

```

#### 4.9 Convex Hull trick

```

/* Given a convexhull, answer queries in O(lg N)
CH should not contain identical points, the area should
be > 0, min pair(x, y) should be listed first */
double det( const Pt& p1 , const Pt& p2 )
{ return p1.X * p2.Y - p1.Y * p2.X; }
struct Conv{
    int n;
    vector<Pt> a;
    vector<Pt> upper, lower;
    Conv(vector<Pt> _a) : a(_a){
        n = a.size();
        int ptr = 0;
        for(int i=1; i<n; ++i) if (a[ptr] < a[i]) ptr = i;
        for(int i=0; i<=ptr; ++i) lower.push_back(a[i]);
        for(int i=ptr; i<n; ++i) upper.push_back(a[i]);
        upper.push_back(a[0]);
    }
    int sign( LL x ){ // fixed when changed to double
        return x < 0 ? -1 : x > 0; }
    pair<LL,int> get_tang(vector<Pt> &conv, Pt vec){
        int l = 0, r = (int)conv.size() - 2;
        for( ; l + 1 < r; ){
            int mid = (l + r) / 2;
            if(sign(det(conv[mid+1]-conv[mid],vec))>0)r=mid;
            else l = mid;
        }
        return max(make_pair(det(vec, conv[r]), r),
            make_pair(det(vec, conv[0]), 0));
    }
    void upd_tang(const Pt &p, int id, int &i0, int &i1){
        if(det(a[i0] - p, a[id] - p) > 0) i0 = id;
        if(det(a[i1] - p, a[id] - p) < 0) i1 = id;
    }
    void bi_search(int l, int r, Pt p, int &i0, int &i1){
        if(l == r) return;
        upd_tang(p, l % n, i0, i1);
        int sl=sign(det(a[l % n] - p, a[(l + 1) % n] - p));
        for( ; l + 1 < r; ){
            int mid = (l + r) / 2;
            int smid=sign(det(a[mid%n]-p, a[(mid+1)%n]-p));
            if (smid == sl) l = mid;
            else r = mid;
        }
        upd_tang(p, r % n, i0, i1);
    }
    int bi_search(Pt u, Pt v, int l, int r) {
        int sl = sign(det(v - u, a[l % n] - u));
        for( ; l + 1 < r; ){
            int mid = (l + r) / 2;
            int smid = sign(det(v - u, a[mid % n] - u));
            if (smid == sl) l = mid;
            else r = mid;
        }
        return l % n;
    }
    // 1. whether a given point is inside the CH
    bool contain(Pt p) {
        if (p.X < lower[0].X || p.X > lower.back().X)
            return 0;
        int id = lower_bound(lower.begin(), lower.end(), Pt
            (p.X, -INF)) - lower.begin();
        if (lower[id].X == p.X) {
            if (lower[id].Y > p.Y) return 0;
        }else if(det(lower[id-1]-p,lower[id]-p)<0)return 0;
        id = lower_bound(upper.begin(), upper.end(), Pt(p.X
            , INF), greater<Pt>()) - upper.begin();
        if (upper[id].X == p.X) {
            if (upper[id].Y < p.Y) return 0;
        }else if(det(upper[id-1]-p,upper[id]-p)<0)return 0;
        return 1;
    }
    // 2. Find 2 tang pts on CH of a given outside point
    // return true with i0, i1 as index of tangent points

```

```

    // return false if inside CH
    bool get_tang(Pt p, int &i0, int &i1) {
        if (contain(p)) return false;
        i0 = i1 = 0;
        int id = lower_bound(lower.begin(), lower.end(), p)
            - lower.begin();
        bi_search(0, id, p, i0, i1);
        bi_search(id, (int)lower.size(), p, i0, i1);
        id = lower_bound(upper.begin(), upper.end(), p,
            greater<Pt>()) - upper.begin();
        bi_search((int)lower.size() - 1, (int)lower.size()
            - 1 + id, p, i0, i1);
        bi_search((int)lower.size() - 1 + id, (int)lower.
            size() - 1 + (int)upper.size(), p, i0, i1);
        return true;
    }
    // 3. Find tangent points of a given vector
    // ret the idx of vertex has max cross value with vec
    int get_tang(Pt vec){
        pair<LL, int> ret = get_tang(upper, vec);
        ret.second = (ret.second+(int)lower.size()-1)%n;
        ret = max(ret, get_tang(lower, vec));
        return ret.second;
    }
    // 4. Find intersection point of a given line
    // return 1 and intersection is on edge (i, next(i))
    // return 0 if no strictly intersection
    bool get_intersection(Pt u, Pt v, int &i0, int &i1){
        int p0 = get_tang(u - v), p1 = get_tang(v - u);
        if(sign(det(v-u,a[p0]-u))*sign(det(v-u,a[p1]-u)<0){
            if (p0 > p1) swap(p0, p1);
            i0 = bi_search(u, v, p0, p1);
            i1 = bi_search(u, v, p1, p0 + n);
            return 1;
        }
        return 0;
    }
}

```

#### 4.10 Tangent line of two circles

```

vector<Line> go( const Cir& c1 , const Cir& c2 , int
    sign1 ){
    // sign1 = 1 for outer tang, -1 for inter tang
    vector<Line> ret;
    double d_sq = norm2( c1.0 - c2.0 );
    if( d_sq < eps ) return ret;
    double d = sqrt( d_sq );
    Pt v = ( c2.0 - c1.0 ) / d;
    double c = ( c1.R - sign1 * c2.R ) / d;
    if( c * c > 1 ) return ret;
    double h = sqrt( max( 0.0 , 1.0 - c * c ) );
    for( int sign2 = 1 ; sign2 >= -1 ; sign2 -= 2 ){
        Pt n = { v.X * c - sign2 * h * v.Y ,
            v.Y * c + sign2 * h * v.X };
        Pt p1 = c1.0 + n * c1.R;
        Pt p2 = c2.0 + n * ( c2.R * sign1 );
        if( fabs( p1.X - p2.X ) < eps and
            fabs( p1.Y - p2.Y ) < eps )
            p2 = p1 + perp( c2.0 - c1.0 );
        ret.push_back( { p1 , p2 } );
    }
    return ret;
}

```

#### 4.11 KD Tree

```

const int MXN=100005;
const int MXK=10;
struct KDTree{
    struct Nd{
        LL x[MXK],mn[MXK],mx[MXK];
        int id,f;
        Nd *l,*r;
    }tree[MXN],*root;
    int n,k;
    LL dis(LL a,LL b){return (a-b)*(a-b);}
    LL dis(LL a[MXK],LL b[MXK]){
        LL ret=0;
        for(int i=0;i<k;i++) ret+=dis(a[i],b[i]);
        return ret;
    }
    void init(vector<vector<LL>> &ip,int _n,int _k){

```

```

n=_n,k=_k;
for(int i=0;i<n;i++){
    tree[i].id=i;
    copy(ip[i].begin(),ip[i].end(),tree[i].x);
}
root=build(0,n-1,0);
}
Nd* build(int l,int r,int d){
    if(l>r) return NULL;
    if(d==k) d=0;
    int m=(l+r)>>1;
    nth_element(tree+l,tree+m,tree+r+1,&)(const Nd &a,
        const Nd &b){return a.x[d]<b.x[d];});
    tree[m].f=d;
    copy(tree[m].x,tree[m].x+k,tree[m].mn);
    copy(tree[m].x,tree[m].x+k,tree[m].mx);
    tree[m].l=build(l,m-1,d+1);
    if(tree[m].l){
        for(int i=0;i<k;i++){
            tree[m].mn[i]=min(tree[m].mn[i],tree[m].l->mn[i]);
            tree[m].mx[i]=max(tree[m].mx[i],tree[m].l->mx[i]);
        }
    }
    tree[m].r=build(m+1,r,d+1);
    if(tree[m].r){
        for(int i=0;i<k;i++){
            tree[m].mn[i]=min(tree[m].mn[i],tree[m].r->mn[i]);
            tree[m].mx[i]=max(tree[m].mx[i],tree[m].r->mx[i]);
        }
    }
    return tree+m;
}
LL pt[MXK],md;
int mID;
bool touch(Nd *r){
    LL d=0;
    for(int i=0;i<k;i++){
        if(pt[i]<=r->mn[i]) d+=dis(pt[i],r->mn[i]);
        else if(pt[i]>=r->mx[i]) d+=dis(pt[i],r->mx[i]);
    }
    return d<md;
}
void nearest(Nd *r){
    if(!r||!touch(r)) return;
    LL td=dis(r->x,pt);
    if(td<md) md=td,mID=r->id;
    nearest(pt[r->f]<r->x[r->f]?r->l:r->r);
    nearest(pt[r->f]<r->x[r->f]?r->r:r->l);
}
pair<LL,int> query(vector<LL> &_pt,LL _md=1LL<<57){
    mID=-1,md=_md;
    copy(_pt.begin(),_pt.end(),pt);
    nearest(root);
    return {md,mID};
}
}tree;

```

#### 4.12 Lower Concave Hull

```

const ll is_query = -(1LL<<62);
struct Line {
    ll m, b;
    mutable function<const Line*> succ;
    bool operator<(const Line& rhs) const {
        if (rhs.b != is_query) return m < rhs.m;
        const Line* s = succ();
        return s ? b - s->b < (s->m - m) * rhs.m : 0;
    }
}; // maintain upper hull for maximum
struct HullDynamic : public multiset<Line> {
    bool bad(iterator y) {
        auto z = next(y);
        if (y == begin()) {
            if (z == end()) return 0;
            return y->m == z->m && y->b <= z->b;
        }
        auto x = prev(y);

```

```

        if(z==end())return y->m==x->m&&y->b<=x->b;
        return (x->b-y->b)*(z->m-y->m)>=
            (y->b-z->b)*(y->m-x->m);
    }
    void insert_line(ll m, ll b) {
        auto y = insert({m, b});
        y->succ = [=]{return next(y)==end()?0:*next(y);};
        if(bad(y)) {erase(y); return; }
        while(next(y)!=end()&&bad(next(y)))erase(next(y));
        while(y!=begin()&&bad(prev(y)))erase(prev(y));
    }
    ll eval(ll x) {
        auto l = *lower_bound((Line) {x, is_query});
        return l.m * x + l.b;
    }
};

```

#### 4.13 Min Enclosing Circle

```

struct Mec{
    // return pair of center and r
    static const int N = 101010;
    int n;
    Pt p[ N ], cen;
    double r2;
    void init( int _n , Pt _p[] ){
        n = _n;
        memcpy( p , _p , sizeof(Pt) * n );
    }
    double sqr(double a){ return a*a; }
    Pt center(Pt p0, Pt p1, Pt p2) {
        Pt a = p1-p0;
        Pt b = p2-p0;
        double c1=norm2( a ) * 0.5;
        double c2=norm2( b ) * 0.5;
        double d = a ^ b;
        double x = p0.X + (c1 * b.Y - c2 * a.Y) / d;
        double y = p0.Y + (a.X * c2 - b.X * c1) / d;
        return Pt(x,y);
    }
    pair<Pt,double> solve(){
        random_shuffle(p,p+n);
        r2=0;
        for (int i=0; i<n; i++){
            if (norm2(cen-p[i]) <= r2) continue;
            cen = p[i];
            r2 = 0;
            for (int j=0; j<i; j++){
                if (norm2(cen-p[j]) <= r2) continue;
                cen=Pt((p[i].X+p[j].X)/2,(p[i].Y+p[j].Y)/2);
                r2 = norm2(cen-p[j]);
                for (int k=0; k<j; k++){
                    if (norm2(cen-p[k]) <= r2) continue;
                    cen = center(p[i],p[j],p[k]);
                    r2 = norm2(cen-p[k]);
                }
            }
        }
        return {cen,sqrt(r2)};
    }
} mec;

```

#### 4.14 Min Enclosing Ball

```

// Pt : { x , y , z }
#define N 202020
int n, nouter; Pt pt[ N ], outer[4], res;
double radius,tmp;
void ball() {
    Pt q[3]; double m[3][3], sol[3], L[3], det;
    int i,j; res.x = res.y = res.z = radius = 0;
    switch ( nouter ) {
        case 1: res=outer[0]; break;
        case 2: res=(outer[0]+outer[1])/2; radius=norm2(res, outer[0]); break;
        case 3:
            for (i=0; i<2; ++i) q[i]=outer[i+1]-outer[0];
            for (i=0; i<2; ++i) for(j=0; j<2; ++j) m[i][j]=(q[i] * q[j])*2;
            for (i=0; i<2; ++i) sol[i]=(q[i] * q[i]);
            if (fabs(det=m[0][0]*m[1][1]-m[0][1]*m[1][0])<eps)
                return;

```

```

L[0]=(sol[0]*m[1][1]-sol[1]*m[0][1])/det;
L[1]=(sol[1]*m[0][0]-sol[0]*m[1][0])/det;
res=outer[0]+q[0]*L[0]+q[1]*L[1];
radius=norm2(res, outer[0]);
break;
case 4:
for (i=0; i<3; ++i) q[i]=outer[i+1]-outer[0], sol
[i]=(q[i] * q[i]);
for (i=0; i<3; ++i) for (j=0; j<3; ++j) m[i][j]=(q[i]
* q[j])*2;
det= m[0][0]*m[1][1]*m[2][2]
+ m[0][1]*m[1][2]*m[2][0]
+ m[0][2]*m[1][0]*m[2][1]
- m[0][2]*m[1][1]*m[2][0]
- m[0][1]*m[1][0]*m[2][2]
- m[0][0]*m[1][2]*m[2][1];
if ( fabs(det)<eps ) return;
for (j=0; j<3; ++j) {
for (i=0; i<3; ++i) m[i][j]=sol[i];
L[j]=( m[0][0]*m[1][1]*m[2][2]
+ m[0][1]*m[1][2]*m[2][0]
+ m[0][2]*m[1][0]*m[2][1]
- m[0][2]*m[1][1]*m[2][0]
- m[0][1]*m[1][0]*m[2][2]
- m[0][0]*m[1][2]*m[2][1]
) / det;
for (i=0; i<3; ++i) m[i][j]=(q[i] * q[j])*2;
} res=outer[0];
for (i=0; i<3; ++i) res = res + q[i] * L[i];
radius=norm2(res, outer[0]);
}}
void minball(int n){ ball();
if( nouter < 4 ) for( int i = 0 ; i < n ; i ++ )
if( norm2(res, pt[i]) - radius > eps ){
outer[ nouter ++ ] = pt[ i ]; minball(i); --
nouter;
if(i>0){ Pt Tt = pt[i];
memmove(&pt[1], &pt[0], sizeof(Pt)*i); pt[0]=Tt
;
}}}
double solve(){
// n points in pt
random_shuffle(pt, pt+n); radius=-1;
for(int i=0; i<n; i++) if(norm2(res, pt[i])>radius)
nouter=1, outer[0]=pt[i], minball(i);
return sqrt(radius);
}

```

#### 4.15 Min dist on Cuboid

```

typedef LL T;
T r;
void turn(T i, T j, T x, T y, T z,
T x0, T y0, T L, T W, T H) {
if (z==0) { T R = x*x+y*y; if (R<r) r=R; return; }
if(i>=0 && i<2) turn(i+1, j, x0+L+z, y, x0+L-x,
x0+L, y0, H, W, L);
if(j>=0 && j<2) turn(i, j+1, x, y0+W+z, y0+W-y,
x0, y0+W, L, H, W);
if(i<=0 && i>-2) turn(i-1, j, x0-z, y, x-x0,
x0-H, y0, H, W, L);
if(j<=0 && j>-2) turn(i, j-1, x, y0-z, y-y0,
x0, y0-H, L, H, W);
}
T solve(T L, T W, T H,
T x1, T y1, T z1, T x2, T y2, T z2){
if( z1!=0 && z1!=H ){
if( y1==0 || y1==W )
swap(y1,z1), swap(y2,z2), swap(W,H);
else swap(x1,z1), swap(x2,z2), swap(L,H);
}
if (z1==H) z1=0, z2=H-z2;
r=INF; turn(0,0,x2-x1,y2-y1,z2,-x1,-y1,L,W,H);
return r;
}

```

#### 4.16 Heart of Triangle

```

Pt inCenter( Pt &A, Pt &B, Pt &C) { // 內心
double a = norm(B-C), b = norm(C-A), c = norm(A-B);
return (A * a + B * b + C * c) / (a + b + c);
}

```

```

Pt circumCenter( Pt &a, Pt &b, Pt &c) { // 外心
Pt bb = b - a, cc = c - a;
double db=norm2(bb), dc=norm2(cc), d=2*(bb ^ cc);
return a-Pt(bb.Y*dc-cc.Y*db, cc.X*db-bb.X*dc) / d;
}
Pt orthoCenter( Pt &a, Pt &b, Pt &c) { // 垂心
Pt ba = b - a, ca = c - a, bc = b - c;
double Y = ba.Y * ca.Y * bc.Y,
A = ca.X * ba.Y - ba.X * ca.Y,
x0= (Y+ca.X*ba.Y*b.X-ba.X*ca.Y*c.X) / A,
y0= -ba.X * (x0 - c.X) / ba.Y + ca.Y;
return Pt(x0, y0);
}

```

## 5 Graph

### 5.1 DominatorTree

```

const int MAXN = 100010;
struct DominatorTree{
#define REP(i,s,e) for(int i=(s);i<=(e);i++)
#define REPD(i,s,e) for(int i=(s);i>=(e);i--)
int n, m, s;
vector< int > g[ MAXN ], pred[ MAXN ];
vector< int > cov[ MAXN ];
int dfn[ MAXN ], nfd[ MAXN ], ts;
int par[ MAXN ]; //idom[u] s到u的最後一個必經點
int sdom[ MAXN ], idom[ MAXN ];
int mom[ MAXN ], mn[ MAXN ];
inline bool cmp( int u, int v )
{ return dfn[ u ] < dfn[ v ]; }
int eval( int u ){
if( mom[ u ] == u ) return u;
int res = eval( mom[ u ] );
if(cmp( sdom[ mn[ mom[ u ] ] ], sdom[ mn[ u ] ] ))
mn[ u ] = mn[ mom[ u ] ];
return mom[ u ] = res;
}
void init( int _n, int _m, int _s ){
ts = 0; n = _n; m = _m; s = _s;
REP( i, 1, n ) g[ i ].clear(), pred[ i ].clear();
}
void addEdge( int u, int v ){
g[ u ].push_back( v );
pred[ v ].push_back( u );
}
void dfs( int u ){
ts++;
dfn[ u ] = ts;
nfd[ ts ] = u;
for( int v : g[ u ] ) if( dfn[ v ] == 0 ){
par[ v ] = u;
dfs( v );
}
}
void build(){
REP( i, 1, n ){
dfn[ i ] = nfd[ i ] = 0;
cov[ i ].clear();
mom[ i ] = mn[ i ] = sdom[ i ] = i;
}
dfs( s );
REPD( i, n, 2 ){
int u = nfd[ i ];
if( u == 0 ) continue;
for( int v : pred[ u ] ) if( dfn[ v ] ){
eval( v );
if( cmp( sdom[ mn[ v ] ], sdom[ u ] ) )
sdom[ u ] = sdom[ mn[ v ] ];
}
cov[ sdom[ u ] ].push_back( u );
mom[ u ] = par[ u ];
for( int w : cov[ par[ u ] ] ){
eval( w );
if( cmp( sdom[ mn[ w ] ], par[ u ] ) )
idom[ w ] = mn[ w ];
else idom[ w ] = par[ u ];
}
}
cov[ par[ u ] ].clear();
}
REP( i, 2, n ){
int u = nfd[ i ];

```

```

    if( u == 0 ) continue ;
    if( idom[ u ] != sdom[ u ] )
        idom[ u ] = idom[ idom[ u ] ];
}
} domT;

```

## 5.2 MaximumClique 最大團

```

#define N 111
struct MaxClique{ // 0-base
    typedef bitset<N> Int;
    Int linkto[N] , v[N];
    int n;
    void init(int _n){
        n = _n;
        for(int i = 0 ; i < n ; i++){
            linkto[i].reset(); v[i].reset();
        }
    }
    void addEdge(int a , int b)
    { v[a][b] = v[b][a] = 1; }
    int popcount(const Int& val)
    { return val.count(); }
    int lowbit(const Int& val)
    { return val._Find_first(); }
    int ans , stk[N];
    int id[N] , di[N] , deg[N];
    Int cans;
    void maxclique(int elem_num, Int candi){
        if(elem_num > ans){
            ans = elem_num; cans.reset();
            for(int i = 0 ; i < elem_num ; i++){
                cans[id[stk[i]]] = 1;
            }
            int potential = elem_num + popcount(candi);
            if(potential <= ans) return;
            int pivot = lowbit(candi);
            Int smaller_candi = candi & (~linkto[pivot]);
            while(smaller_candi.count() && potential > ans){
                int next = lowbit(smaller_candi);
                candi[next] = !candi[next];
                smaller_candi[next] = !smaller_candi[next];
                potential --;
                if(next == pivot || (smaller_candi & linkto[next]).count()){
                    stk[elem_num] = next;
                    maxclique(elem_num + 1, candi & linkto[next]);
                }
            }
        }
    }
    int solve(){
        for(int i = 0 ; i < n ; i++){
            id[i] = i; deg[i] = v[i].count();
        }
        sort(id , id + n , [&](int id1, int id2){
            return deg[id1] > deg[id2]; });
        for(int i = 0 ; i < n ; i++) di[id[i]] = i;
        for(int i = 0 ; i < n ; i++){
            for(int j = 0 ; j < n ; j++){
                if(v[i][j]) linkto[di[i]][di[j]] = 1;
            }
        }
        Int cand; cand.reset();
        for(int i = 0 ; i < n ; i++) cand[i] = 1;
        ans = 1;
        cans.reset(); cans[0] = 1;
        maxclique(0, cand);
        return ans;
    }
} solver;

```

## 5.3 MaximalClique 極大團

```

#define N 80
struct MaxClique{ // 0-base
    typedef bitset<N> Int;
    Int lnk[N] , v[N];
    int n;
    void init(int _n){
        n = _n;
        for(int i = 0 ; i < n ; i++){
            lnk[i].reset(); v[i].reset();
        }
    }

```

```

}
void addEdge(int a , int b)
{ v[a][b] = v[b][a] = 1; }
int ans , stk[N] , id[N] , di[N] , deg[N];
Int cans;
void dfs(int elem_num, Int candi, Int ex){
    if(candi.none() && ex.none()){
        cans.reset();
        for(int i = 0 ; i < elem_num ; i++){
            cans[id[stk[i]]] = 1;
        }
        ans = elem_num; // cans is a maximal clique
        return;
    }
    int pivot = (candilex)._Find_first();
    Int smaller_candi = candi & (~lnk[pivot]);
    while(smaller_candi.count()){
        int nxt = smaller_candi._Find_first();
        candi[nxt] = smaller_candi[nxt] = 0;
        ex[nxt] = 1;
        stk[elem_num] = nxt;
        dfs(elem_num+1, candi&lnk[nxt], ex&lnk[nxt]);
    }
}
int solve(){
    for(int i = 0 ; i < n ; i++){
        id[i] = i; deg[i] = v[i].count();
    }
    sort(id , id + n , [&](int id1, int id2){
        return deg[id1] > deg[id2]; });
    for(int i = 0 ; i < n ; i++) di[id[i]] = i;
    for(int i = 0 ; i < n ; i++){
        for(int j = 0 ; j < n ; j++){
            if(v[i][j]) lnk[di[i]][di[j]] = 1;
        }
    }
    ans = 1; cans.reset(); cans[0] = 1;
    dfs(0, Int(string(n, '1')), 0);
    return ans;
}
} solver;

```

## 5.4 Strongly Connected Component

```

struct Scc{
    int n, nScc, vst[MXN], bln[MXN];
    vector<int> E[MXN], rE[MXN], vec;
    void init(int _n){
        n = _n;
        for (int i=0; i<MXN; i++){
            E[i].clear(), rE[i].clear();
        }
    }
    void addEdge(int u, int v){
        E[u].PB(v); rE[v].PB(u);
    }
    void DFS(int u){
        vst[u]=1;
        for (auto v : E[u]) if (!vst[v]) DFS(v);
        vec.PB(u);
    }
    void rDFS(int u){
        vst[u] = 1; bln[u] = nScc;
        for (auto v : rE[u]) if (!vst[v]) rDFS(v);
    }
    void solve(){
        nScc = 0;
        vec.clear();
        FZ(vst);
        for (int i=0; i<n; i++){
            if (!vst[i]) DFS(i);
            reverse(vec.begin(), vec.end());
            FZ(vst);
            for (auto v : vec)
                if (!vst[v]){
                    rDFS(v); nScc++;
                }
        }
    }
};

```

## 5.5 Dynamic MST

```

/* Dynamic MST O( Q lg^2 Q )
(qx[i], qy[i])->chg weight of edge No.qx[i] to qy[i]
delete an edge: (i, \infty)
add an edge: change from \infty to specific value */

```

```

const int SZ=M+3*MXQ;
int a[N],*tz;
int find(int xx){
    int root=xx; while(a[root]) root=a[root];
    int next; while((next=a[xx])){a[xx]=root; xx=next; }
    return root;
}
bool cmp(int aa,int bb){ return tz[aa]<tz[bb]; }
int kx[N],ky[N],kt, vd[N],id[M], app[M];
bool extra[M];
void solve(int *qx,int *qy,int Q,int n,int *x,int *y,
    int *z,int m1,long long ans){
    if(Q==1){
        for(int i=1;i<=n;i++) a[i]=0;
        z[ qx[0] ]=qy[0]; tz = z;
        for(int i=0;i<m1;i++) id[i]=i;
        sort(id,id+m1,cmp); int ri,rj;
        for(int i=0;i<m1;i++){
            ri=find(x[id[i]]); rj=find(y[id[i]]);
            if(ri!=rj){ ans+=z[id[i]]; a[ri]=rj; }
        }
        printf("%lld\n",ans);
        return;
    }
    int ri,rj;
    //contract
    kt=0;
    for(int i=1;i<=n;i++) a[i]=0;
    for(int i=0;i<Q;i++){
        ri=find(x[qx[i]]); rj=find(y[qx[i]]); if(ri!=rj) a[ri]=rj;
    }
    int tm=0;
    for(int i=0;i<m1;i++) extra[i]=true;
    for(int i=0;i<Q;i++) extra[ qx[i] ]=false;
    for(int i=0;i<m1;i++) if(extra[i]) id[tm++]=i;
    tz=z; sort(id,id+tm,cmp);
    for(int i=0;i<tm;i++){
        ri=find(x[id[i]]); rj=find(y[id[i]]);
        if(ri!=rj){
            a[ri]=rj; ans += z[id[i]];
            kx[kt]=x[id[i]]; ky[kt]=y[id[i]]; kt++;
        }
    }
    for(int i=1;i<=n;i++) a[i]=0;
    for(int i=0;i<kt;i++) a[ find(kx[i]) ]=find(ky[i]);
    int n2=0;
    for(int i=1;i<=n;i++) if(a[i]==0)
        vd[i]++;n2;
    for(int i=1;i<=n;i++) if(a[i])
        vd[i]=vd[find(i)];
    int m2=0, *Nx=x+m1, *Ny=y+m1, *Nz=z+m1;
    for(int i=0;i<m1;i++) app[i]=-1;
    for(int i=0;i<Q;i++) if(app[qx[i]]==-1){
        Nx[m2]=vd[ x[ qx[i] ] ]; Ny[m2]=vd[ y[ qx[i] ] ];
        Nz[m2]=z[ qx[i] ];
        app[qx[i]]=m2; m2++;
    }
    for(int i=0;i<Q;i++){ z[ qx[i] ]=qy[i]; qx[i]=app[qx[i]]; }
    for(int i=1;i<=n2;i++) a[i]=0;
    for(int i=0;i<tm;i++){
        ri=find(vd[ x[id[i]] ]); rj=find(vd[ y[id[i]] ]);
        if(ri!=rj){
            a[ri]=rj; Nx[m2]=vd[ x[id[i]] ];
            Ny[m2]=vd[ y[id[i]] ]; Nz[m2]=z[id[i]]; m2++;
        }
    }
    int mid=Q/2;
    solve(qx,qy,mid,n2,Nx,Ny,Nz,m2,ans);
    solve(qx+mid,qy+mid,Q-mid,n2,Nx,Ny,Nz,m2,ans);
}
int x[SZ],y[SZ],z[SZ],qx[MXQ],qy[MXQ],n,m,Q;
void init(){
    scanf("%d%d",&n,&m);
    for(int i=0;i<m;i++) scanf("%d%d%d",x+i,y+i,z+i);
    scanf("%d",&Q);
    for(int i=0;i<Q;i++){ scanf("%d%d",qx+i,qy+i); qx[i]--; }
}
void work(){ if(Q) solve(qx,qy,Q,n,x,y,z,m,0); }

```

## 5.6 Maximum General graph Matching

```

const int N = 514, E = (2e5) * 2;
struct Graph{
    int to[E],bro[E],head[N],e;
    int lnk[N],vis[N],stp,n;
    void init( int _n ){
        stp = 0; e = 1; n = _n;
        for( int i = 1 ; i <= n ; i ++ )
            lnk[i] = vis[i] = 0;
    }
    void add_edge(int u,int v){
        to[e]=v,bro[e]=head[u],head[u]=e++;
        to[e]=u,bro[e]=head[v],head[v]=e++;
    }
    bool dfs(int x){
        vis[x]=stp;
        for(int i=head[x];i;i=bro[i]){
            int v=to[i];
            if(!lnk[v]){
                lnk[x]=v,lnk[v]=x;
                return true;
            }else if(vis[lnk[v]]<stp){
                int w=lnk[v];
                lnk[x]=v,lnk[v]=x,lnk[w]=0;
                if(dfs(w)){
                    return true;
                }
                lnk[w]=v,lnk[v]=w,lnk[x]=0;
            }
        }
        return false;
    }
    int solve(){
        int ans = 0;
        for(int i=1;i<=n;i++)
            if(!lnk[i]){
                stp++; ans += dfs(i);
            }
        return ans;
    }
} graph;

```

## 5.7 Minimum General Weighted Matching

```

struct Graph {
    // Minimum General Weighted Matching (Perfect Match)
    static const int MXN = 105;
    int n, edge[MXN][MXN];
    int match[MXN],dis[MXN],onstk[MXN];
    vector<int> stk;
    void init(int _n) {
        n = _n;
        for( int i = 0 ; i < n ; i ++ )
            for( int j = 0 ; j < n ; j ++ )
                edge[ i ][ j ] = 0;
    }
    void add_edge(int u, int v, int w)
    { edge[u][v] = edge[v][u] = w; }
    bool SPFA(int u){
        if (onstk[u]) return true;
        stk.PB(u);
        onstk[u] = 1;
        for (int v=0; v<n; v++){
            if (u != v && match[u] != v && !onstk[v]){
                int m = match[v];
                if (dis[m] > dis[u] - edge[v][m] + edge[u][v]){
                    dis[m] = dis[u] - edge[v][m] + edge[u][v];
                    onstk[v] = 1;
                    stk.PB(v);
                    if (SPFA(m)) return true;
                    stk.pop_back();
                    onstk[v] = 0;
                }
            }
        }
        onstk[u] = 0;
        stk.pop_back();
        return false;
    }
    int solve() {
        // find a match
        for (int i=0; i<n; i+=2){
            match[i] = i+1;
            match[i+1] = i;
        }
    }
}

```



```

while (true){
    int found = 0;
    for( int i = 0 ; i < n ; i ++ )
        onstk[ i ] = dis[ i ] = 0;
    for (int i=0; i<n; i++){
        stk.clear();
        if (!onstk[i] && SPFA(i)){
            found = 1;
            while (SZ(stk)>=2){
                int u = stk.back(); stk.pop_back();
                int v = stk.back(); stk.pop_back();
                match[u] = v;
                match[v] = u;
            } }
        if (!found) break;
    }
    int ret = 0;
    for (int i=0; i<n; i++)
        ret += edge[i][match[i]];
    ret /= 2;
    return ret;
}
}graph;

```

## 5.8 Minimum Steiner Tree

```

// Minimum Steiner Tree 重要點的mst
// O(V 3^T + V^2 2^T)
struct SteinerTree{
#define V 33
#define T 8
#define INF 1023456789
    int n , dst[V][V] , dp[1 << T][V] , tdst[V];
    void init( int _n ){
        n = _n;
        for( int i = 0 ; i < n ; i ++ ){
            for( int j = 0 ; j < n ; j ++ )
                dst[ i ][ j ] = INF;
            dst[ i ][ i ] = 0;
        }
        void add_edge( int ui , int vi , int wi ){
            dst[ ui ][ vi ] = min( dst[ ui ][ vi ] , wi );
            dst[ vi ][ ui ] = min( dst[ vi ][ ui ] , wi );
        }
        void shortest_path(){
            for( int k = 0 ; k < n ; k ++ )
                for( int i = 0 ; i < n ; i ++ )
                    for( int j = 0 ; j < n ; j ++ )
                        dst[ i ][ j ] = min( dst[ i ][ j ] ,
                            dst[ i ][ k ] + dst[ k ][ j ] );
        }
        int solve( const vector<int>& ter ){
            int t = (int)ter.size();
            for( int i = 0 ; i < ( 1 << t ) ; i ++ )
                for( int j = 0 ; j < n ; j ++ )
                    dp[ i ][ j ] = INF;
            for( int i = 0 ; i < n ; i ++ )
                dp[ 0 ][ i ] = 0;
            for( int msk = 1 ; msk < ( 1 << t ) ; msk ++ ){
                if( msk == ( msk & (-msk) ) ){
                    int who = __lg( msk );
                    for( int i = 0 ; i < n ; i ++ )
                        dp[ msk ][ i ] = dst[ ter[ who ] ][ i ];
                    continue;
                }
                for( int i = 0 ; i < n ; i ++ )
                    for( int submsk = ( msk - 1 ) & msk ; submsk ;
                        submsk = ( submsk - 1 ) & msk )
                        dp[ msk ][ i ] = min( dp[ msk ][ i ] ,
                            dp[ submsk ][ i ] +
                            dp[ msk ^ submsk ][ i ] );
                for( int i = 0 ; i < n ; i ++ ){
                    tdst[ i ] = INF;
                    for( int j = 0 ; j < n ; j ++ )
                        tdst[ i ] = min( tdst[ i ] ,
                            dp[ msk ][ j ] + dst[ j ][ i ] );
                }
                for( int i = 0 ; i < n ; i ++ )
                    dp[ msk ][ i ] = tdst[ i ];
            }
            int ans = INF;
            for( int i = 0 ; i < n ; i ++ )

```

```

        ans = min( ans , dp[ ( 1 << t ) - 1 ][ i ] );
        return ans;
    } }solver;

```

## 5.9 BCC based on vertex

```

struct BccVertex {
    int n,nScc,step,dfn[MXN],low[MXN];
    vector<int> E[MXN],sccv[MXN];
    int top,stk[MXN];
    void init(int _n) {
        n = _n; nScc = step = 0;
        for (int i=0; i<n; i++) E[i].clear();
    }
    void addEdge(int u, int v)
    { E[u].PB(v); E[v].PB(u); }
    void DFS(int u, int f) {
        dfn[u] = low[u] = step++;
        stk[top++] = u;
        for (auto v:E[u]) {
            if (v == f) continue;
            if (dfn[v] == -1) {
                DFS(v,u);
                low[u] = min(low[u], low[v]);
                if (low[v] >= dfn[u]) {
                    int z; //進到if裡面u為關節點
                    sccv[nScc].clear();
                    do {
                        z = stk[--top];
                        sccv[nScc].PB(z);
                    } while (z != v);
                    sccv[nScc++].PB(u);
                }
            } else
                low[u] = min(low[u],dfn[v]);
        }
    }
    vector<vector<int>> solve() {
        vector<vector<int>> res;
        for (int i=0; i<n; i++)
            dfn[i] = low[i] = -1;
        for (int i=0; i<n; i++)
            if (dfn[i] == -1) {
                top = 0;
                DFS(i,i);
            }
        REP(i,nScc) res.PB(sccv[i]);
        return res;
    }
}graph;

```

## 5.10 Min Mean Cycle

```

/* minimum mean cycle O(VE) */
struct MMC{
#define E 101010
#define V 1021
#define inf 1e9
#define eps 1e-6
    struct Edge { int v,u; double c; };
    int n, m, prv[V][V], prve[V][V], vst[V];
    Edge e[E];
    vector<int> edgeID, cycle, rho;
    double d[V][V];
    void init( int _n )
    { n = _n; m = 0; }
    // WARNING: TYPE matters
    void addEdge( int vi , int ui , double ci )
    { e[ m ++ ] = { vi , ui , ci }; }
    void bellman_ford() {
        for(int i=0; i<n; i++) d[0][i]=0;
        for(int i=0; i<n; i++) {
            fill(d[i+1], d[i+1]+n, inf);
            for(int j=0; j<m; j++) {
                int v = e[j].v, u = e[j].u;
                if(d[i][v]<inf && d[i+1][u]>d[i][v]+e[j].c) {
                    d[i+1][u] = d[i][v]+e[j].c;
                    prv[i+1][u] = v;
                    prve[i+1][u] = j;
                }
            }
        }
    }
    double solve(){
        // returns inf if no cycle, mmc otherwise
        double mmc=inf;

```

```

int st = -1;
bellman_ford();
for(int i=0; i<n; i++) {
    double avg=-inf;
    for(int k=0; k<n; k++) {
        if(d[n][i]<inf-eps) avg=max(avg,(d[n][i]-d[k][i])/(n-k));
        else avg=max(avg,inf);
    }
    if (avg < mmc) tie(mmc, st) = tie(avg, i);
}
fill(vst,0); edgeID.clear(); cycle.clear(); rho.clear();
for (int i=n; !vst[st]; st=prv[i--][st]) {
    vst[st]++;
    edgeID.PB(prve[i][st]);
    rho.PB(st);
}
while (vst[st] != 2) {
    if(rho.empty()) return inf;
    int v = rho.back(); rho.pop_back();
    cycle.PB(v);
    vst[v]++;
}
reverse(ALL(edgeID));
edgeID.resize(SZ(cycle));
return mmc;
} }mmc;

```

### 5.11 Directed Graph Min Cost Cycle

```

// works in O(N M)
#define INF 1000000000000000LL
#define N 5010
#define M 200010
struct edge{
    int to; LL w;
    edge(int a=0, LL b=0): to(a), w(b){}
};
struct node{
    LL d; int u, next;
    node(LL a=0, int b=0, int c=0): d(a), u(b), next(c){}
}b[M];
struct DirectedGraphMinCycle{
    vector<edge> g[N], grev[N];
    LL dp[N][N], p[N], d[N], mu;
    bool inq[N];
    int n, bn, bsz, hd[N];
    void b_insert(LL d, int u){
        int i = d/mu;
        if(i >= bn) return;
        b[++bsz] = node(d, u, hd[i]);
        hd[i] = bsz;
    }
    void init( int _n ){
        n = _n;
        for( int i = 1 ; i <= n ; i ++ )
            g[ i ].clear();
    }
    void addEdge( int ai , int bi , LL ci )
    { g[ai].push_back(edge(bi,ci)); }
    LL solve(){
        fill(dp[0], dp[0]+n+1, 0);
        for(int i=1; i<=n; i++){
            fill(dp[i]+1, dp[i]+n+1, INF);
            for(int j=1; j<=n; j++) if(dp[i-1][j] < INF){
                for(int k=0; k<(int)g[j].size(); k++)
                    dp[i][g[j][k].to] =min(dp[i][g[j][k].to],
                    dp[i-1][j]+g[j][k].w);
            }
        }
        mu=INF; LL bunbo=1;
        for(int i=1; i<=n; i++) if(dp[n][i] < INF){
            LL a=-INF, b=1;
            for(int j=0; j<=n-1; j++) if(dp[j][i] < INF){
                if(a*(n-j) < b*(dp[n][i]-dp[j][i])){
                    a = dp[n][i]-dp[j][i];
                    b = n-j;
                }
            }
            if(mu*b > bunbo*a)
                mu = a, bunbo = b;
        }
        if(mu < 0) return -1; // negative cycle

```

```

if(mu == INF) return INF; // no cycle
if(mu == 0) return 0;
for(int i=1; i<=n; i++){
    for(int j=0; j<(int)g[i].size(); j++){
        g[i][j].w *= bunbo;
    }
    memset(p, 0, sizeof(p));
    queue<int> q;
    for(int i=1; i<=n; i++){
        q.push(i);
        inq[i] = true;
    }
    while(!q.empty()){
        int i=q.front(); q.pop(); inq[i]=false;
        for(int j=0; j<(int)g[i].size(); j++){
            if(p[g[i][j].to] > p[i]+g[i][j].w-mu){
                p[g[i][j].to] = p[i]+g[i][j].w-mu;
                if(!inq[g[i][j].to]){
                    q.push(g[i][j].to);
                    inq[g[i][j].to] = true;
                }
            }
        }
    }
    for(int i=1; i<=n; i++) grev[i].clear();
    for(int i=1; i<=n; i++){
        for(int j=0; j<(int)g[i].size(); j++){
            g[i][j].w += p[i]-p[g[i][j].to];
            grev[g[i][j].to].push_back(edge(i, g[i][j].w));
        }
    }
    LL mlldc = n*mu;
    for(int i=1; i<=n; i++){
        bn=mlldc/mu, bsz=0;
        memset(hd, 0, sizeof(hd));
        fill(d+i+1, d+n+1, INF);
        b_insert(d[i]=0, i);
        for(int j=0; j<=bn-1; j++) for(int k=hd[j]; k; k=
            b[k].next){
            int u = b[k].u;
            LL du = b[k].d;
            if(du > d[u]) continue;
            for(int l=0; l<(int)g[u].size(); l++) if(g[u][l].to > i){
                if(d[g[u][l].to] > du + g[u][l].w){
                    d[g[u][l].to] = du + g[u][l].w;
                    b_insert(d[g[u][l].to], g[u][l].to);
                }
            }
        }
        for(int j=0; j<(int)grev[i].size(); j++) if(grev[i][j].to > i)
            mlldc=min(mlldc,d[grev[i][j].to] + grev[i][j].w);
    }
    return mlldc / bunbo;
} }graph;

```

### 5.12 K-th Shortest Path

```

// time: O(|E| \lg |E| + |V| \lg |V| + K)
// memory: O(|E| \lg |E| + |V|)
struct KSP{ // 1-base
    struct nd{
        int u, v; ll d;
        nd(int ui = 0, int vi = 0, ll di = INF)
        { u = ui; v = vi; d = di; }
    };
    struct heap{
        nd* edge; int dep; heap* chd[4];
    };
    static int cmp(heap* a,heap* b)
    { return a->edge->d > b->edge->d; }
    struct node{
        int v; ll d; heap* H; nd* E;
        node(){
            node(ll _d, int _v, nd* _E)
            { d = _d; v = _v; E = _E; }
            node(heap* _H, ll _d)
            { H = _H; d = _d; }
        }
        friend bool operator<(node a, node b)
        { return a.d > b.d; }
    };
    int n, k, s, t, dst[ N ];
    nd *nxt[ N ];
    vector<nd*> g[ N ], rg[ N ];
    heap *nullNd, *head[ N ];
    void init( int _n , int _k , int _s , int _t ){
        n = _n; k = _k; s = _s; t = _t;
        for( int i = 1 ; i <= n ; i ++ ){

```

```

    g[ i ].clear(); rg[ i ].clear();
    nxt[ i ] = NULL; head[ i ] = NULL;
    dst[ i ] = -1;
} }
void addEdge( int ui , int vi , ll di ){
    nd* e = new nd(ui, vi, di);
    g[ ui ].push_back( e );
    rg[ vi ].push_back( e );
}
queue<int> dfsQ;
void dijkstra(){
    while(dfsQ.size()) dfsQ.pop();
    priority_queue<node> Q;
    Q.push(node(0, t, NULL));
    while (!Q.empty()){
        node p = Q.top(); Q.pop();
        if(dst[p.v] != -1) continue;
        dst[ p.v ] = p.d;
        nxt[ p.v ] = p.E;
        dfsQ.push( p.v );
        for(auto e: rg[ p.v ])
            Q.push(node(p.d + e->u, e->u, e));
    }
}
heap* merge(heap* curNd, heap* newNd){
    if(curNd == nullNd) return newNd;
    heap* root = new heap;
    memcpy(root, curNd, sizeof(heap));
    if(newNd->edge->d < curNd->edge->d){
        root->edge = newNd->edge;
        root->chd[2] = newNd->chd[2];
        root->chd[3] = newNd->chd[3];
        newNd->edge = curNd->edge;
        newNd->chd[2] = curNd->chd[2];
        newNd->chd[3] = curNd->chd[3];
    }
    if(root->chd[0]->dep < root->chd[1]->dep)
        root->chd[0] = merge(root->chd[0], newNd);
    else
        root->chd[1] = merge(root->chd[1], newNd);
    root->dep = max(root->chd[0]->dep, root->chd[1]->
        dep) + 1;
    return root;
}
vector<heap*> V;
void build(){
    nullNd = new heap;
    nullNd->dep = 0;
    nullNd->edge = new nd;
    fill(nullNd->chd, nullNd->chd+4, nullNd);
    while(not dfsQ.empty()){
        int u = dfsQ.front(); dfsQ.pop();
        if(!nxt[ u ]) head[ u ] = nullNd;
        else head[ u ] = head[nxt[ u ]->v];
        V.clear();
        for( auto&& e : g[ u ] ){
            int v = e->v;
            if( dst[ v ] == -1 ) continue;
            e->d += dst[ v ] - dst[ u ];
            if( nxt[ u ] != e ){
                heap* p = new heap;
                fill(p->chd, p->chd+4, nullNd);
                p->dep = 1;
                p->edge = e;
                V.push_back(p);
            }
        }
        if(V.empty()) continue;
        make_heap(V.begin(), V.end(), cmp);
#define L(X) ((X<<1)+1)
#define R(X) ((X<<1)+2)
        for( size_t i = 0 ; i < V.size() ; i ++ ){
            if(L(i) < V.size()) V[i]->chd[2] = V[L(i)];
            else V[i]->chd[2]=nullNd;
            if(R(i) < V.size()) V[i]->chd[3] = V[R(i)];
            else V[i]->chd[3]=nullNd;
        }
        head[u] = merge(head[u], V.front());
    }
}
vector<ll> ans;
void first_K(){
    ans.clear();
    priority_queue<node> Q;
    if( dst[ s ] == -1 ) return;

```

```

    ans.push_back( dst[ s ] );
    if( head[s] != nullNd )
        Q.push(node(head[s], dst[s]+head[s]->edge->d));
    for( int _ = 1 ; _ < k and not Q.empty() ; _ ++ ){
        node p = Q.top(); q; Q.pop();
        ans.push_back( p.d );
        if(head[ p.H->edge->v ] != nullNd){
            q.H = head[ p.H->edge->v ];
            q.d = p.d + q.H->edge->d;
            Q.push(q);
        }
        for( int i = 0 ; i < 4 ; i ++ )
            if( p.H->chd[ i ] != nullNd ){
                q.H = p.H->chd[ i ];
                q.d = p.d - p.H->edge->d + p.H->chd[ i ]->
                    edge->d;
                Q.push( q );
            }
    }
    void solve(){ // ans[i] stores the i-th shortest path
        dijkstra();
        build();
        first_K(); // ans.size() might less than k
    }
} solver;

```

### 5.13 SPFA

```

bool spfa(){
    deque<int> dq;
    dis[0]=0;
    dq.push_back(0);
    inq[0]=1;
    while(!dq.empty()){
        int u=dq.front();
        dq.pop_front();
        inq[u]=0;
        for(auto i:edge[u]){
            if(dis[i.first]>i.second+dis[u]){
                dis[i.first]=i.second+dis[u];
                len[i.first]=len[u]+1;
                if(len[i.first]>n) return 1;
                if(inq[i.first]) continue;
                if(!dq.empty()&&dis[dq.front()]>dis[i.first])
                    dq.push_front(i.first);
                else
                    dq.push_back(i.first);
                inq[i.first]=1;
            }
        }
    }
    return 0;
}

```

### 5.14 差分約束

約束條件  $V_j - V_i \leq W$  建邊  $V_i \rightarrow V_j$  權重為  $W$  → bellman-ford or spfa

## 6 String

### 6.1 PalTree

```

// len[s]是對應的回文長度
// num[s]是有幾個回文後綴
// cnt[s]是這個回文字串在整個字串中的出現次數
// fail[s]是他長度次長的回文後綴, aba的fail是a
const int MXN = 1000010;
struct PalT{
    int nxt[MXN][26], fail[MXN], len[MXN];
    int tot, lst, n, state[MXN], cnt[MXN], num[MXN];
    int diff[MXN], sfail[MXN], fac[MXN], dp[MXN];
    char s[MXN]={-1};
    int newNode(int l, int f){
        len[tot]=l, fail[tot]=f, cnt[tot]=num[tot]=0;
        memset(nxt[tot], 0, sizeof(nxt[tot]));
        diff[tot]=(l>0?l-len[f]:0);
        sfail[tot]=(l>0&&diff[tot]==diff[f]?sfail[f]:f);
        return tot++;
    }
    int getfail(int x){
        while(s[n-len[x]-1]!=s[n]) x=fail[x];
        return x;
    }
    int getmin(int v){
        dp[v]=fac[n-len[sfail[v]]-diff[v]];
        if(diff[v]==diff[fail[v]])
            dp[v]=min(dp[v], dp[fail[v]]);
    }
}

```

```

    return dp[v]+1;
}
int push(){
    int c=s[n]-'a',np=getfail(lst);
    if(! (lst=nxt[np][c])){
        lst=newNode(len[np]+2,nxt[getfail(fail[np])][c]);
        nxt[np][c]=lst; num[lst]=num[fail[lst]]+1;
    }
    fac[n]=n;
    for(int v=lst;len[v]>0;v=sfail[v])
        fac[n]=min(fac[n],getmin(v));
    return ++cnt[lst],lst;
}
void init(const char *_s){
    tot=lst=n=0;
    newNode(0,1),newNode(-1,1);
    for(;_s[n];) s[n+1]=_s[n],++n,state[n-1]=push();
    for(int i=tot-1;i>1;i--) cnt[fail[i]]+=cnt[i];
}
}palt;

```

## 6.2 KMP

/\*  
len-failure[k]:  
在k結尾的情況下，這個子字串可以由開頭  
長度為(len-failure[k])的部分重複出現來表達

failure[k]:  
failure[k]為次長相同前綴後綴  
如果我們不只想求最多，而且以0-base做為考量  
，那可能的長度由大到小會是  
failuer[k]、failure[failuer[k]-1]  
、failure[failure[failuer[k]-1]-1]..  
直到有值為0為止

```

*/
int failure[MXN];
void KMP(string& t, string& p)
{
    if (p.size() > t.size()) return;
    for (int i=1, j=failure[0]=-1; i<p.size(); ++i)
    {
        while (j >= 0 && p[j+1] != p[i])
            j = failure[j];
        if (p[j+1] == p[i]) j++;
        failure[i] = j;
    }
    for (int i=0, j=-1; i<t.size(); ++i)
    {
        while (j >= 0 && p[j+1] != t[i])
            j = failure[j];
        if (p[j+1] == t[i]) j++;
        if (j == p.size()-1)
        {
            cout << i - p.size() + 1 << " ";
            j = failure[j];
        }
    }
}

```

## 6.3 SAIS

```

const int N = 300010;
struct SA{
#define REP(i,n) for ( int i=0; i<int(n); i++ )
#define REP1(i,a,b) for ( int i=(a); i<=int(b); i++ )
    bool _t[N*2];
    int _s[N*2], _sa[N*2], _c[N*2], x[N], _p[N], _q[N*2],
        hei[N], r[N];
    int operator [] (int i){ return _sa[i]; }
    void build(int *s, int n, int m){
        memcpy(_s, s, sizeof(int) * n);
        sais(_s, _sa, _p, _q, _t, _c, n, m);
        mkhei(n);
    }
    void mkhei(int n){
        REP(i,n) r[_sa[i]] = i;
        hei[0] = 0;
        REP(i,n) if(r[i]) {
            int ans = i>0 ? max(hei[r[i-1]] - 1, 0) : 0;
            while(_s[i+ans] == _s[_sa[r[i]-1]+ans]) ans++;
            hei[r[i]] = ans;
        }
    }
}

```

```

}
void sais(int *s, int *sa, int *p, int *q, bool *t,
    int *c, int n, int z){
    bool uniq = t[n-1] = true, neq;
    int nn = 0, nmzx = -1, *nsa = sa + n, *ns = s + n,
        lst = -1;
#define MS0(x,n) memset((x),0,n*sizeof(*(x)))
#define MAGIC(XD) MS0(sa, n); \
    memcpy(x, c, sizeof(int) * z); \
    XD; \
    memcpy(x + 1, c, sizeof(int) * (z - 1)); \
    REP(i,n) if(sa[i] && !t[sa[i]-1]) sa[x[s[sa[i]
        ]-1]]++ = sa[i]-1; \
    memcpy(x, c, sizeof(int) * z); \
    for(int i = n - 1; i >= 0; i--) if(sa[i] && t[sa[i]
        ]-1]) sa[--x[s[sa[i]-1]]] = sa[i]-1;
    MS0(c, z);
    REP(i,n) uniq &= ++c[s[i]] < 2;
    REP(i,z-1) c[i+1] += c[i];
    if (uniq) { REP(i,n) sa[--c[s[i]]] = i; return; }
    for(int i = n - 2; i >= 0; i--) t[i] = (s[i]==s[i
        +1] ? t[i+1] : s[i]<s[i+1]);
    MAGIC(REP1(i,1,n-1) if(t[i] && !t[i-1]) sa[--x[s[i]
        ]]] = p[q[i]=nn++] = i);
    REP(i, n) if (sa[i] && t[sa[i]] && !t[sa[i]-1]) {
        neq=lst<0 || memcmp(s+sa[i],s+lst,(p[q[sa[i]]+1]-sa
            [i])*sizeof(int));
        ns[q[lst=sa[i]]]=nmzx+=neq;
    }
    sais(ns, nsa, p + nn, q + n, t + n, c + z, nn, nmzx
        + 1);
    MAGIC(for(int i = nn - 1; i >= 0; i--) sa[--x[p[
        nsa[i]]]] = p[nsa[i]]);
}
}sa;
int H[ N ], SA[ N ];
void suffix_array(int* ip, int len) {
    // should padding a zero in the back
    // ip is int array, len is array length
    // ip[0..n-1] != 0, and ip[len] = 0
    ip[len++] = 0;
    sa.build(ip, len, 128);
    for (int i=0; i<len; i++) {
        H[i] = sa.hei[i + 1];
        SA[i] = sa._sa[i + 1];
    }
    // resulting height, sa array \in [0,len)
}

```

## 6.4 SuffixAutomata

```

// any path start from root forms a substring of S
// occurrence of P : iff SAM can run on input word P
// number of different substring : ds[1]-1
// total length of all different substring : dsl[1]
// max/min length of state i : mx[i]/mx[mom[i]]+1
// assume a run on input word P end at state i:
// number of occurrences of P : cnt[i]
// first occurrence position of P : fp[i]-|P|+1
// all position of P : fp of "dfs from i through rmom"
const int MXM = 1000010;
struct SAM{
    int tot, root, lst, mom[MXM], mx[MXM]; //ind[MXM]
    int nxt[MXM][33]; //cnt[MXM],ds[MXM],dsl[MXM],fp[MXM]
    // bool v[MXM]
    int newNode(){
        int res = ++tot;
        fill(nxt[res], nxt[res]+33, 0);
        mom[res] = mx[res] = 0; //cnt=ds=dsl=fp=v=0
        return res;
    }
    void init(){
        tot = 0;
        root = newNode();
        lst = root;
    }
    void push(int c){
        int p = lst;
        int np = newNode(); //cnt[np]=1
        mx[np] = mx[p]+1; //fp[np]=mx[np]-1
        for(; p && nxt[p][c] == 0; p = mom[p])
            nxt[p][c] = np;
    }
}

```

```

if(p == 0) mom[np] = root;
else{
    int q = nxt[p][c];
    if(mx[p]+1 == mx[q]) mom[np] = q;
    else{
        int nq = newNode(); //fp[nq]=fp[q]
        mx[nq] = mx[p]+1;
        for(int i = 0; i < 33; i++){
            nxt[nq][i] = nxt[q][i];
            mom[nq] = mom[q];
            mom[q] = nq;
            mom[np] = nq;
            for(; p && nxt[p][c] == q; p = mom[p])
                nxt[p][c] = nq;
        }
        lst = np;
    }
}
void calc(){
    calc(root);
    iota(ind,ind+tot,1);
    sort(ind,ind+tot,[&](int i,int j){return mx[i]<mx[j];});
    for(int i=tot-1;i>=0;i--)
        cnt[mom[ind[i]]]+=cnt[ind[i]];
}
void calc(int x){
    v[x]=ds[x]=1;dsl[x]=0; //rmom[mom[x]].push_back(x);
    for(int i=1;i<=26;i++){
        if(nxt[x][i]){
            if(!v[nxt[x][i]]) calc(nxt[x][i]);
            ds[x]+=ds[nxt[x][i]];
            dsl[x]+=ds[nxt[x][i]]+dsl[nxt[x][i]];
        }
    }
}
void push(char *str){
    for(int i = 0; str[i]; i++)
        push(str[i]-'a'+1);
}
}
} sam;

```

## 6.5 Aho-Corasick

```

struct ACautomata{
    struct Node{
        int cnt,i;
        Node *go[26], *fail, *dic;
        Node (){
            cnt = 0; fail = 0; dic=0;
            memset(go,0,sizeof(go));
        }
    }pool[1048576],*root;
    int nMem,n_pattern;
    Node* new_Node(){
        pool[nMem] = Node();
        return &pool[nMem++];
    }
    void init() {nMem=0;root=new_Node();n_pattern=0;}
    void add(const string &str) {insert(root,str,0); }
    void insert(Node *cur, const string &str, int pos){
        for(int i=pos;i<str.size();i++){
            if(!cur->go[str[i]-'a'])
                cur->go[str[i]-'a'] = new_Node();
            cur=cur->go[str[i]-'a'];
        }
        cur->cnt++; cur->i=n_pattern++;
    }
    void make_fail(){
        queue<Node*> que;
        que.push(root);
        while (!que.empty()){
            Node* fr=que.front(); que.pop();
            for (int i=0; i<26; i++){
                if (fr->go[i]){
                    Node *ptr = fr->fail;
                    while (ptr && !ptr->go[i]) ptr = ptr->fail;
                    fr->go[i]->fail=ptr=(ptr?ptr->go[i]:root);
                    fr->go[i]->dic=(ptr->cnt?ptr:ptr->dic);
                    que.push(fr->go[i]);
                }
            }
        }
    }
    void query(string s){
        Node *cur=root;
        for(int i=0;i<(int)s.size();i++){
            while(cur&&!cur->go[s[i]-'a']) cur=cur->fail;

```

```

        cur=(cur?cur->go[s[i]-'a']:root);
        if(cur->i>=0) ans[cur->i]++;
        for(Node *tmp=cur->dic;tmp;tmp=tmp->dic)
            ans[tmp->i]++;
    } } // ans[i] : number of occurrence of pattern i
} AC;

```

## 6.6 Z Value

```

char s[MAXN];
int len,z[MAXN];
void Z_value() { //z[i] = lcp(s[1...],s[i...])
    int i,j,left,right;
    left=right=0; z[0]=len;
    for(i=1;i<len;i++) {
        j=max(min(z[i-left],right-i),0);
        for(;i+j<len&&s[i+j]==s[j];j++);
        z[i]=j;
        if(i+z[i]>right) {
            right=i+z[i];
            left=i;
        }
    }
}

```

## 6.7 ZValue Palindrome

```

void z_value_pal(char *s,int len,int *z){
    len=(len<1)+1;
    for(int i=len-1;i>=0;i--)
        s[i]=i&1?s[i>1]:'@';
    z[0]=1;
    for(int i=1,l=0,r=0;i<len;i++){
        z[i]=i<r?min(z[l+l-i],r-i):1;
        while(i-z[i]>=0&&i+z[i]<len&&s[i-z[i]]==s[i+z[i]])
            ++z[i];
        if(i+z[i]>r) l=i,r=i+z[i];
    }
}

```

## 6.8 Smallest Rotation

```

//rotate(begin(s),begin(s)+minRotation(s),end(s))
int minRotation(string s) {
    int a = 0, N = s.size(); s += s;
    rep(b,0,N) rep(k,0,N) {
        if(a+k == b || s[a+k] < s[b+k])
            {b += max(0, k-1); break;}
        if(s[a+k] > s[b+k]) {a = b; break;}
    } return a;
}

```

## 6.9 Cyclic LCS

```

#define L 0
#define LU 1
#define U 2
const int mov[3][2]={0,-1, -1,-1, -1,0};
int al,bl;
char a[MAXL*2],b[MAXL*2]; // 0-indexed
int dp[MAXL*2][MAXL];
char pred[MAXL*2][MAXL];
inline int lcs_length(int r) {
    int i=r+al,j=bl,l=0;
    while(i>r) {
        char dir=pred[i][j];
        if(dir==LU) l++;
        i+=mov[dir][0];
        j+=mov[dir][1];
    }
    return l;
}
inline void reroot(int r) { // r = new base row
    int i=r,j=1;
    while(j<=bl&&pred[i][j]!=LU) j++;
    if(j>bl) return;
    pred[i][j]=L;
    while(i<2*al&&j<=bl) {
        if(pred[i+1][j]==U) {
            i++;
            pred[i][j]=L;
        } else if(j<bl&&pred[i+1][j+1]==LU) {
            i++;
            j++;
            pred[i][j]=L;
        }
    }
}

```



```

    } else {
        j++;
    } }
int cyclic_lcs() {
    // a, b, al, bl should be properly filled
    // note: a WILL be altered in process
    // -- concatenated after itself
    char tmp[MAXL];
    if(al>bl) {
        swap(al,bl);
        strcpy(tmp,a);
        strcpy(a,b);
        strcpy(b,tmp);
    }
    strcpy(tmp,a);
    strcat(a,tmp);
    // basic lcs
    for(int i=0;i<=2*al;i++) {
        dp[i][0]=0;
        pred[i][0]=U;
    }
    for(int j=0;j<=bl;j++) {
        dp[0][j]=0;
        pred[0][j]=L;
    }
    for(int i=1;i<=2*al;i++) {
        for(int j=1;j<=bl;j++) {
            if(a[i-1]==b[j-1]) dp[i][j]=dp[i-1][j-1]+1;
            else dp[i][j]=max(dp[i-1][j],dp[i][j-1]);
            if(dp[i][j-1]==dp[i][j]) pred[i][j]=L;
            else if(a[i-1]==b[j-1]) pred[i][j]=LU;
            else pred[i][j]=U;
        }
    }
    // do cyclic lcs
    int clcs=0;
    for(int i=0;i<al;i++) {
        clcs=max(clcs,lcs_length(i));
        reroot(i+1);
    }
    // recover a
    a[al]='\0';
    return clcs;
}

```

## 7 Data Structure

### 7.1 Segment tree

```

struct seg_tree{
    static const int MXN=1e5+5,NO_TAG=0; // to be set
    ll a[MXN],val[MXN*4],tag[MXN*4],v;
    int n,ql,qr;
    void push(int i,int l,int r){
        if(tag[i]!=NO_TAG){
            val[i]+=tag[i]; // update by tag
            if(l!=r){
                tag[cl(i)]+=tag[i]; // push
                tag[cr(i)]+=tag[i]; // push
            }
            tag[i]=NO_TAG;
        }
    }
    void pull(int i,int l,int r){
        int mid=(l+r)>>1;
        push(cl(i),l,mid);push(cr(i),mid+1,r);
        val[i]=max(val[cl(i)],val[cr(i)]); // pull
    }
    void build(int i,int l,int r){
        if(l==r){
            val[i]=a[l]; // set value
            return;
        }
        int mid=(l+r)>>1;
        build(cl(i),l,mid);build(cr(i),mid+1,r);
        pull(i,l,r);
    }
    void update(int i,int l,int r){
        push(i,l,r);
        if(ql<=l&&r<=qr){
            tag[i]+=v; // update tag
            return;
        }
        int mid=(l+r)>>1;

```

```

        if(ql<=mid) update(cl(i),l,mid);
        if(qr>mid) update(cr(i),mid+1,r);
        pull(i,l,r);
    }
    void query(int i,int l,int r){
        push(i,l,r);
        if(ql<=l&&r<=qr){
            v=max(v,val[i]); // update answer
            return;
        }
        int mid=(l+r)>>1;
        if(ql<=mid) query(cl(i),l,mid);
        if(qr>mid) query(cr(i),mid+1,r);
    } }tree;

```

### 7.2 Treap

```

struct Treap{
    int sz , val , pri , tag;
    Treap *l , *r;
    Treap( int _val ){
        val = _val; sz = 1;
        pri = rand(); l = r = NULL; tag = 0;
    }
};
void push( Treap * a ){
    if( a->tag ){
        Treap *swp = a->l; a->l = a->r; a->r = swp;
        int swp2;
        if( a->l ) a->l->tag ^= 1;
        if( a->r ) a->r->tag ^= 1;
        a->tag = 0;
    }
}
inline int Size( Treap * a ){ return a ? a->sz : 0; }
void pull( Treap * a ){
    a->sz = Size( a->l ) + Size( a->r ) + 1;
}
Treap* merge( Treap *a , Treap *b ){
    if( !a || !b ) return a ? a : b;
    if( a->pri > b->pri ){
        push( a );
        a->r = merge( a->r , b );
        pull( a );
        return a;
    }else{
        push( b );
        b->l = merge( a , b->l );
        pull( b );
        return b;
    }
}
void split_kth( Treap *t , int k , Treap*&a , Treap*&b ){
    if( !t ){ a = b = NULL; return; }
    push( t );
    if( Size( t->l ) + 1 <= k ){
        a = t;
        split_kth( t->r , k - Size( t->l ) - 1 , a->r , b );
        pull( a );
    }else{
        b = t;
        split_kth( t->l , k , a , b->l );
        pull( b );
    }
}
void split_key( Treap *t , int k , Treap*&a , Treap*&b ){
    if( !t ){ a = b = NULL; return; }
    push( t );
    if( k<=t->val ){
        b = t;
        split_key( t->l , k , a , b->l );
        pull( b );
    }
    else{
        a = t;
        split_key( t->r , k , a->r , b );
        pull( a );
    }
}

```

### 7.3 Disjoint Set

```

struct DisjointSet{
    // save() is like recursive
    // undo() is like return

```

```

int n, fa[ N ], sz[ N ];
vector< pair<int*,int> > h;
vector<int> sp;
void init( int tn ){
    n=tn;
    for( int i = 0 ; i < n ; i ++ ){
        fa[ i ]=i;
        sz[ i ]=1;
    }
    sp.clear(); h.clear();
}
void assign( int *k, int v ){
    h.PB( {k, *k} );
    *k = v;
}
void save(){ sp.PB(SZ(h)); }
void undo(){
    assert(!sp.empty());
    int last=sp.back(); sp.pop_back();
    while( SZ(h)!=last ){
        auto x=h.back(); h.pop_back();
        *x.first = x.second;
    }
}
void uni( int x , int y ){
    x = f( x ); y = f( y );
    if( x == y ) return;
    if( sz[ x ] < sz[ y ] ) swap( x, y );
    assign( &sz[ x ], sz[ x ] + sz[ y ] );
    assign( &fa[ y ], x );
} }djs;

```

## 7.4 Black Magic

```

#include <bits/extc++.h>
using namespace __gnu_pbds;
typedef tree<int,null_type,less<int>,rb_tree_tag,
    tree_order_statistics_node_update> set_t;
#include <ext/pb_ds/assoc_container.hpp>
typedef cc_hash_table<int,int> umap_t;
typedef priority_queue<int> heap;
#include<ext/rope>
using namespace __gnu_cxx;
int main(){
    // Insert some entries into s.
    set_t s; s.insert(12); s.insert(505);
    // The order of the keys should be: 12, 505.
    assert(*s.find_by_order(0) == 12);
    assert(*s.find_by_order(3) == 505);
    // The order of the keys should be: 12, 505.
    assert(s.order_of_key(12) == 0);
    assert(s.order_of_key(505) == 1);
    // Erase an entry.
    s.erase(12);
    // The order of the keys should be: 505.
    assert(*s.find_by_order(0) == 505);
    // The order of the keys should be: 505.
    assert(s.order_of_key(505) == 0);

    heap h1 , h2; h1.join( h2 );

    rope<char> r[ 2 ];
    r[ 1 ] = r[ 0 ]; // persistenet
    string t = "abc";
    r[ 1 ].insert( 0 , t.c_str() );
    r[ 1 ].erase( 1 , 1 );
    cout << r[ 1 ].substr( 0 , 2 );
}

```

## 8 Others

### 8.1 Find max tangent(x,y is increasing)

```

const int MAXN = 100010;
Pt sum[MAXN], pnt[MAXN], ans, calc;
inline bool cross(Pt a, Pt b, Pt c){
    return (c.y-a.y)*(c.x-b.x) > (c.x-a.x)*(c.y-b.y);
} //pt[0]=(0,0);pt[i]=(i,pt[i-1].y+dy[i-1]),i=1~n;dx>=1
double find_max_tan(int n,int l,LL dy[]){
    int np, st, ed, now;
    sum[0].x = sum[0].y = np = st = ed = 0;
    for (int i = 1, v; i <= n; i++){
        sum[i].x=i,sum[i].y=sum[i-1].y+dy[i-1];

```

```

ans.x = now = 1,ans.y = -1;
for (int i = 0; i <= n - 1; i++){
    while(np>1&&cross(pnt[np-2],pnt[np-1],sum[i]))
        np--;
    if (np < now && np != 0) now = np;
    pnt[np++] = sum[i];
    while(now<np&&!cross(pnt[now-1],pnt[now],sum[i+1]))
        now++;
    calc = sum[i + 1] - pnt[now - 1];
    if (ans.y * calc.x < ans.x * calc.y)
        ans = calc,st = pnt[now - 1].x,ed = i + 1;
}
return (double)(sum[ed].y-sum[st].y)/(sum[ed].x-sum[st].x);
}

```

### 8.2 Exact Cover Set

```

// given n*m 0-1 matrix
// find a set of rows s.t.
// for each column, there's exactly one 1
#define N 1024 //row
#define M 1024 //column
#define NM ((N+2)*(M+2))
char A[N][M]; //n*m 0-1 matrix
int used[N]; //answer: the row used
int id[N][M];
int L[NM],R[NM],D[NM],U[NM],C[NM],S[NM],ROW[NM];
void remove(int c){
    L[R[c]]=L[c]; R[L[c]]=R[c];
    for( int i=D[c]; i!=c; i=D[i] )
        for( int j=R[i]; j!=i; j=R[j] ){
            U[D[j]]=U[j]; D[U[j]]=D[j]; S[C[j]]--;
        }
}
void resume(int c){
    for( int i=D[c]; i!=c; i=D[i] )
        for( int j=L[i]; j!=i; j=L[j] ){
            U[D[j]]=D[U[j]]=j; S[C[j]]++;
        }
    L[R[c]]=R[L[c]]=c;
}
int dfs(){
    if(R[0]==0) return 1;
    int md=100000000,c;
    for( int i=R[0]; i!=0; i=R[i] )
        if(S[i]<md){ md=S[i]; c=i; }
    if(md==0) return 0;
    remove(c);
    for( int i=D[c]; i!=c; i=D[i] ){
        used[ROW[i]]=1;
        for( int j=R[i]; j!=i; j=R[j] ) remove(C[j]);
        if(dfs()) return 1;
        for( int j=L[i]; j!=i; j=L[j] ) resume(C[j]);
        used[ROW[i]]=0;
    }
    resume(c);
    return 0;
}
int exact_cover(int n,int m){
    for( int i=0; i<=m; i++ ){
        R[i]=i+1; L[i]=i-1; U[i]=D[i]=i;
        S[i]=0; C[i]=i;
    }
    R[m]=0; L[0]=m;
    int t=m+1;
    for( int i=0; i<n; i++ ){
        int k=-1;
        for( int j=0; j<m; j++ ){
            if(!A[i][j]) continue;
            if(k== -1) L[t]=R[t]=t;
            else{ L[t]=k; R[t]=R[k]; }
            k=t; D[t]=j+1; U[t]=U[j+1];
            L[R[t]]=R[L[t]]=U[D[t]]=D[U[t]]=t;
            C[t]=j+1; S[C[t]]++; ROW[t]=i; id[i][j]=t++;
        }
    }
    for( int i=0; i<n; i++ ) used[i]=0;
    return dfs();
}

```