

**The Effects of Visual and Design Features on the Perception of Correlation in
Scatterplots**

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Contents

Contents	2
List of figures	8
List of tables	14
Abstract	15
Lay abstract	16
Declaration of originality	17
Copyright statement	18
Acknowledgements	19
1 Introduction	20
1.1 Research Motivation	21
1.2 Overview of Contributions	21
1.3 Included Publications	22
1.4 Overview of Thesis	23
2 Related Work	25
2.1 Data Visualisation: A Brief History	25
2.2 Relatedness & Correlation	31
2.3 Visualising Correlation	32
2.3.1 History	33
2.3.2 Present Landscape	35
2.3.3 Scatterplots	37
2.4 Correlation Perception	38
2.5 Correlation Cognition	40

2.6	The Underestimation of Correlation in Positively Correlated Scatterplots	41
2.6.1	Underestimation: Potential Consequences	41
2.7	Data Visualisation Literacy	44
2.8	Objectives and Contributions	45
3	General Methodology	47
3.1	Introduction	47
3.2	Experimental Methods	47
3.2.1	Experimental Design	47
3.2.2	Tools for Testing	48
3.2.3	Recruitment & Participants	50
3.2.4	Creating Stimuli	52
3.3	Analytical Methods	53
3.3.1	Linear Mixed-Effects Models	53
3.3.2	Ordinal Modelling	55
3.3.3	Model Construction	55
3.3.4	Effects Sizes	56
3.3.5	Reporting Analyses	56
3.4	Computational Methods	56
3.4.1	Executable Reporting	57
3.4.2	Containerised Environments	57
3.5	Reproducibility In This Thesis	60
3.5.1	Sharing Data and Code	60
3.5.2	Executable Papers and Docker Containers	61
3.5.3	Pre-Registration of Hypotheses and Analysis Plans	61
3.5.4	Experimental Resources	61
3.6	Conclusion	62
4	Adjusting the Opacities of Scatterplot Points Can Affect Correlation Estimates	63
4.1	Abstract	63
4.2	Preface: Learning From an Early Pilot Study	63
4.2.1	Pilot Study: Results	65
4.2.2	Pilot Study: Discussion	65

4.3	Introduction	66
4.3.1	Overview	66
4.4	Related Work	67
4.4.1	Transparency, Contrast, Opacity, and Formal Definitions	67
4.4.2	Effects of Point Opacity on Correlation Estimation	69
4.5	General Methods	70
4.5.1	Open Research	71
4.6	Experiment 1: Uniform Opacity Adjustments	72
4.6.1	Introduction	72
4.6.2	Method	73
4.6.3	Results	73
4.6.4	Discussion	75
4.7	Experiment 2: Spatially-Dependent Opacity Adjustments	77
4.7.1	Introduction	77
4.7.2	Method	77
4.7.3	Results	79
4.7.4	Discussion	81
4.8	General Discussion	82
4.8.1	Training	84
4.8.2	Limitations	84
4.8.3	Future Work	85
4.9	Conclusion	86
5	Adjusting the Sizes of Scatterplot Points Can Correct for a Historic Correlation Underestimation Bias	87
5.1	Abstract	87
5.2	Introduction	87
5.3	Related Work	88
5.3.1	Point Size and the Perception of Correlation in Scatterplots	88
5.4	Hypotheses	89
5.5	Method	90
5.5.1	Open Research	90

5.5.2 Stimuli	90
5.5.3 Dot Pitch in Crowdsourced Experiments	90
5.5.4 Point Visibility Testing	92
5.5.5 Design	92
5.5.6 Procedure	92
5.5.7 Participants	93
5.6 Results	93
5.7 Discussion	95
5.7.1 Increased Correlation Estimation Accuracy	95
5.7.2 Constant Correlation Estimation Precision	96
5.7.3 Training	97
5.7.4 Limitations	97
5.7.5 Future Work	98
5.8 Conclusion	98
6 Interactions of Opacity and Size Adjustments	99
6.1 Abstract	99
6.2 Introduction	99
6.3 Related Work	100
6.3.1 Opacity and Contrast	100
6.3.2 Point Size	101
6.4 Hypotheses	101
6.5 Method	102
6.5.1 Open Research	102
6.5.2 Stimuli	103
6.5.3 Point Visibility Testing	104
6.5.4 Dot Pitch	104
6.5.5 Design	105
6.5.6 Procedure	105
6.5.7 Participants	105
6.6 Results	106
6.7 Discussion	108

6.7.1	Combining Manipulations	109
6.7.2	Estimation Precision	110
6.7.3	Relative Contributions of Opacity and Size Decay	111
6.7.4	Mechanisms	113
6.7.5	Limitations	113
6.7.6	Future Work	114
6.8	Conclusion	116
7	Visual Features Affecting Perceptual Estimates Also Affect Beliefs About Correlations	117
7.1	Abstract	117
7.2	Introduction	118
7.3	Related Work	119
7.3.1	Scatterplots: Developments in This Thesis	119
7.3.2	Perception & Cognition in Data Visualisation	120
7.4	Open Research	121
7.5	Pre-Study: Investigating Beliefs About Relatedness Statements	121
7.5.1	Hypotheses	122
7.5.2	Method	122
7.5.3	Results	123
7.5.4	Selecting Statements for the Main Experiment	123
7.5.5	Discussion	124
7.6	Main Experiment: Alternative Scatterplot Designs and Beliefs	124
7.6.1	Hypotheses	125
7.6.2	Method	126
7.6.3	Results	128
7.6.4	Discussion	131
7.7	General Discussion	133
7.8	Future Work	134
7.9	Limitations	135
7.10	Conclusion	135
8	Conclusion	136
8.1	Main Findings	136

8.2 Relationship to Prior Work	139
8.3 Reproducibility	140
8.4 Contributions	140
8.5 Implications	141
8.5.1 For Design	141
8.5.2 For Society	141
8.6 Limitations	142
8.7 Future Directions	142
8.8 Closing Remarks	143
References	144
Appendices	167
A First appendix	168

Word count: 1000

List of figures

2.1	Reproduced in Tufte and Graves-Morris, 1983 [166] from Funkhouser, 1936 [50].	26
2.2	Scheiner's (1626) plot detailing how configurations of sunspots change over time. This technique would later be referred to as the “principle of small multiples” by Tufte.	27
2.3	John Snow's (1854) map of cholera cases in Soho, London. Using this data visualisation, Snow was able to demonstrate a link between cholera cases and a contaminated water supply.	28
2.4	Charles Joseph Minard's (1869) flow diagram of Napoleon's botched invasion of Russia in 1812-1813. This diagram shows Napoleon's advance and retreat on Moscow. The width of the orange and black columns encodes the size of the Grande Armée. The temperature scale on the lower portion of the graph illustrates the weather conditions during the retreat, with a freezing Russian winter causing high rates of attrition.	29
2.5	Florence Nightingale's (1858) polar area chart illustrates the causes for mortality among British soldiers during the Crimean War. Data visualisations of this type were used to illustrate that in reality, more British soldiers died from preventable disease than were killed by the enemy, and were used as part of a campaign to improve sanitation among soldiers.	29
2.6	Major milestones in the development of data visualisation illustrated using a rug plot and density estimate. This figure is taken from Friendly (2008) [48].	30
2.7	Anscombe's quartet [6]. Each scatterplot depicts datasets with identical means ($x = 9$, $y = 7.5$), regression coefficients (y on $x = 0.5$), standard errors (0.118), and Pearson's r values (≈ 0.816).	32
2.8	Francis Galton's original plot comparing the semi-interquartile ranges of stature (height) and cubit (forearm length). The plot has been recreated using ggplot2 on the right.	33
2.9	Hertzsprung-Russell diagram of colour index against stellar luminosity. The code for this plot was taken from John Russell's (no relation) blog [138].	35

2.10	From left to right: Parallel Coordinates Plot (PCP) built with the GGally package [142]; Stacked area plot; radial stacked bar plot (doughnut plot); radar plot. Each plot uses the same dataset with an r value of 0.6.	36
2.11	Hexbin (left) and Kernel Density Estimation (KDE, right) plots. These are binned, density-flavoured versions of standard scatterplots.	37
2.12	Scatterplot with Pearson's $r = 0.4$ (left) and Pearson's $r = 0.95$ (right). Note how as the objective correlation increases, the point cloud tends towards a straight line.	39
2.13	Using an equation supplied in Rensink (2017) [134], and recorded median correlation estimates in Bobko (1979 [16]), the nature of correlation underestimation may be visualised. The solid lines represents underestimation behaviour, while the dashed diagonal line represents theoretically perfect estimation.	42
2.14	Two visualisations taken from the BBC News article "Covid: Twelve charts on how Covid changed our lives", published on the 15th of December, 2020 [37].	43
2.15	Participants were instructed to use the following interface to complete the Subjective Graph Literacy scale [54] before completing trials in each experiment.	45
3.1	An example of the slider participants used to estimate correlation in Experiments 1-4. .	48
3.2	Examples of the stimuli used in Experiment 1, demonstrated with an r value of 0.8. Here, "opacity" refers to the alpha value used by ggplot.	48
3.3	Examples of the stimuli used in Experiment 2, demonstrated with an r value of 0.8. . . .	49
3.4	Examples of the stimuli used in Experiment 3, demonstrated with an r value of 0.6. . . .	49
3.5	Examples of the stimuli used in Experiment 4, demonstrated with an r value of 0.6. . . .	49
3.6	Examples of the stimuli for Experiment 5 using an r value of 0.6. Group A saw the alternative scatterplot presented on the right, while group B saw the typical design on the left.	50
3.7	The basic design of scatterplots in experiments 1 to 4.	52
3.8	Visualising random intercepts and slopes for a theoretical experiment with 4 participants. The grand mean of the dependent variable is shown as a solid line, while each separate random intercept is drawn with dashed lines. Each line has a different gradient, representing different random slopes for each participant. This graphic was inspired by those featured in Brown, 2021 [22].	54

3.9	Peng's (2011) Reproducibility Spectrum. This figure has been reproduced from Peng (2011) [123].	60
4.1	Examples of the experimental stimuli used in the pilot study (opacity decay factor). On the left, the opacity decay function is visible. Note the linear scaling used.	64
4.2	Adjusting point opacity to address overplotting. Contrast between the points and the background is full (alpha = 1, full opacity points, left) or low (alpha = .1, low opacity points, right). The dataset used has 20,000 points.	67
4.3	The relationship between alpha values and rendered point opacity. Higher alpha values result in greater contrast between the foreground (scatterplot point) and background. When alpha = 0, the foreground is ignored and the background is rendered.	68
4.4	Participants viewed these plots for at least eight seconds before being allowed to continue to the practice trials.	71
4.5	An example of a visual mask displayed for 2.5 seconds before each experimental trial. .	72
4.6	Examples of the stimuli used in Experiment 1, demonstrated with an r value of 0.8. Here, “opacity” refers to the alpha value used by ggplot2.	73
4.7	Estimated marginal means for the four conditions tested in Experiment 1. 95% confidence intervals are shown. The vertical dashed line represents no estimation error. The overestimation zone is included to facilitate comparison to later work.	74
4.8	Participants' mean errors in correlation estimates grouped by factor and by r value. The dashed horizontal line represents perfect estimation. Participants were most accurate when presented with the plots featuring higher global point opacity. Error bars show standard deviations of estimates.	75
4.9	Using an r value of 0.2 to demonstrate the relationship between the size of a point's residual and the alpha value (opacity) rendered.	78
4.10	Examples of the stimuli used in Experiment 2, demonstrated with an r value of 0.8. Here, “opacity” refers to the alpha value used by ggplot2.	79
4.11	Estimated marginal means for the four conditions tested in Experiment 2. 95% confidence intervals are shown. The vertical dashed line represents no estimation error. . . .	80

4.12	Participants' mean errors in correlation estimates grouped by factor and by r value. The dashed horizontal line represents perfect estimation. Participants were most accurate when presented with the plots featuring the non-linear opacity decay function. Error bars show standard deviations of estimates.	81
4.13	Plot showing a 95% prediction ellipse over a scatterplot with an r value of 0.8.	83
5.1	An example of a bubble chart. This plot compares car engine displacement (cubic inches) with fuel efficiency (miles per gallon). Additionally, the number of cylinders in the car's engine are encoded with point size. One can see from this plot that vehicles with higher engine displacement and lower fuel efficiency tend to have a greater number of engine cylinders.	88
5.2	Examples of the stimuli used in Experiment 3, demonstrated with an r value of 0.8. . . .	91
5.3	A mock-up of the screen scale [105] task used to infer the sizes of participants' monitors. .	91
5.4	Estimated marginal means for the four conditions tested in Experiment 3. 95% confidence intervals are shown. The vertical dashed line represents no estimation error. The overestimation zone is included to facilitate comparison to later work.	93
5.5	Participants' mean errors in correlation estimates grouped by condition and by r value. The dashed horizontal line represents perfect estimation. Participants were most accurate when presented with the plots featuring the non-linear size decay function. Error bars show standard deviations of estimates.	95
6.1	Examples of the stimuli used in Experiment 4, demonstrated with an r value of 0.8. . . .	103
6.2	Estimated marginal means for the four conditions tested in Experiment 4. 95% confidence intervals are shown. The vertical dashed line represents no estimation error. . . .	106
6.3	Comparing mean errors in correlation estimation by trial number. Points represent unsigned mean errors for each trial number. The plotted line is the locally estimated smoothed curve, with the ribbon representing standard errors.	107
6.4	Participants' mean errors in correlation estimates grouped by factor and by r value. The dashed horizontal line represents perfect estimation. Participants were most accurate when presented with the plots in the congruent, typical orientation condition. Error bars show standard deviations of estimates.	108

6.5	Plotting errors in r estimation against objective r values for opacity and size decay functions. On the left, opacity and size decay functions in combination in the typical orientation congruent condition from the current experiment. The plots in the centre show estimation error for opacity and size decay functions in isolation from previous chapters. The right-hand plot averages the comparative baseline (standard scatterplot) conditions from the previous two chapters.	111
6.6	Power is the difference between what is observed when a decay function/combination of decay functions is used and what is observed when no manipulation is used. The dashed line represents the power that would be required to correct for the observed underestimation of correlation in scatterplots. The integral of each power curve over r is provided, as well as the difference between this integral and the integral of each required-power curve over r	112
7.1	Top row: Examples of scatterplot manipulations from previous work using an r value of 0.6. Bottom row: the corresponding correlation estimation behaviour across values of r between 0.2 and 0.99. The dashed diagonal line represents hypothetically accurate estimation, while the solid line is what is observed when participants are asked to estimate correlation.	119
7.2	Examples of the experimental stimuli for Experiment 5 using an r value of 0.6. Group A saw the alternative scatterplot presented on the left, while group B saw the typical design on the right. The labels below the plots are included for the reader's convenience, and were not a part of the experimental stimuli.	127
7.3	Dot plots for pre- and post-plot viewing ratings of strength of relatedness for standard and alternative scatterplot conditions. Mean ratings are also shown as points.	130
7.4	Histograms illustrating the magnitudes of the difference between pre- and post-plot viewing ratings of strength of relatedness for standard and alternative scatterplots. Median values are plotted as points.	131

- 7.5 Illustrating how differences in beliefs about strength of relatedness change as a function of participants' scores on the graph literacy test (left), their scores on the defensive confidence test (centre), and their ratings of statement emotional valence (right). Locally smoothed curves with 95% CI ribbons are shown separately for standard and alternative scatterplot viewing conditions. Lower ratings of Difference in Beliefs (*y* axis) corresponds to lower levels of belief change between pre- and post-scatterplot viewing times. 132
- 8.1 Estimated Marginal Mean errors in correlation estimation for experiments 1 to 4. This figure is an amalgamation of Figures 4.7, 4.11, 5.4, and 6.2. 95% confidence intervals are shown as error bars. The vertical dashed line represents no estimation error. 138

List of tables

4.1	Estimated Marginal Means (EMMs) of correlation estimation error for plot size and the presence of the opacity decay function.	65
4.2	Contrasts between levels of the size factor and opacity decay factor.	65
4.3	Contrasts between different levels of the opacity factor in Experiment 1.	74
4.4	Cohen's <i>d</i> effect sizes and summary statistics for levels of the opacity factor in Experiment 1. Each effect size is compared to the reference level, full contrast (alpha = 1).	75
4.5	Contrasts between different levels of opacity decay function in Experiment 2.	79
4.6	Cohen's <i>d</i> effect sizes and summary statistics for the opacity decay function factor in Experiment 2. Each effect size is compared to the reference level, full contrast (alpha = 1).	80
5.1	Contrasts between different levels of the size decay factor in Experiment 3.	94
5.2	Cohen's <i>d</i> effect sizes (left) and summary statistics (right) for levels of the size decay factor in Experiment 3. Each effect size is compared to the reference level, termed, "Standard Size".	94
6.1	Contrasts between different levels of the opacity and size decay factors in Experiment 4.	106
6.2	Significances of fixed effects and the interaction between them. Semi-partial R ² for each fixed effect and the interaction term is also displayed in lieu of effect sizes.	107
7.1	Statements with neutral average emotional valence ratings.	124
7.2	Statistics for the significant main effect of rating time. Odds ratio and the equivalent Cohen's <i>d</i> value is also supplied.	129
7.3	Statistics for the significant main effect of rating time and the significant interaction between rating time and condition on the difference between pre- and post-scatterplot viewing ratings for standard and alternative plots. Odds ratios and equivalent Cohen's <i>d</i> effect sizes are also shown.	129

Abstract

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Lay abstract

This is lay abstract text.

Declaration of originality

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Acknowledgements

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Chapter 1

Introduction

Data visualisation is the practice of pictorially presenting patterns otherwise described by numbers or words, and has been employed in one form or another for thousands of years [10]. Lists or matrices of numbers may be able to communicate simple trends, but for more complex patterns, harnessing the human visual system [101] through visualisation is crucial for the communication of science and data. Effective data visualisation is able to reduce cognitive and perceptual loads on viewers, regardless of their background levels of statistical knowledge or experience. This outsourcing, while efficient, leaves the viewer vulnerable to the design choices that were made when creating the visualisation. For this reason, understanding *how* design choices affect interpretation is crucial to designing better data visualisations, while simultaneously facilitating the inoculation of viewers against poor or malevolent design practices.

One cannot underestimate the importance and ubiquity of data visualisations. They are used to communicate with experts, with those with no background in science, and with everyone in between. They are relied upon to communicate vital public health information [31, 80, 88], to present evidence in court cases [16, 95], and to facilitate collaboration and encourage engagement on climate change issues [110, 145]. The study of data visualisation therefore has the potential to change the ways in which all science is communicated with members of the public.

Interaction with a data visualisation involves steps from perception (viewing), to cognition (interpreting), to behaviour (deciding and acting). These stages must be examined both separately and together, as there are bottom-up and top-down interactions present. In this thesis, I present a series of experiments whose aim was to investigate and address a long-standing perceptual bias in scatter-plots, a common form of data visualisation [47]. Following successful attempts to address this bias, I further investigated the impacts these attempts may have had on cognition and behaviour, showing

that small, simple changes in the visual features of data visualisations can have profound effects on the ways we think.

1.1 Research Motivation

Data visualisations were once the preserve of the academic and professional classes. Now, however, visualisation is everywhere. This became especially apparent to me during the COVID-19 pandemic, when people such as my parents, professionals with no background in mathematics or statistics, were exposed to data visualisations on a daily basis. Despite not being able to fully articulate the complexities of what they saw, they were still able to derive meaning from the visualisations they engaged with. The ability of data visualisation to transcend language and mathematical prowess motivated my study of them; concepts such as exponential growth or the relatedness between variables can be displayed in simple ways that do not rely on an underlying understanding of exponents or correlation coefficients, for example.

Using data visualisation in such a way effectively provides a cognitive proxy for viewers. This is an efficient way to communicate, however still necessitates the presence of accurate and reliable perceptions. While investigating the assumptions data visualisation designers make about the people who view their visualisations, I uncovered a historical perceptual bias that was still being described in the literature, seemingly with no attempt at correction. This bias was the underestimation of correlation in positively correlated scatterplots. In the literature, this robust effect had been observed in a number of experimental paradigms, including direct estimation [16, 32, 35, 83, 84, 99, 154] and estimation via bisection tasks [134], and as of 2021, no attempts had been made to correct for it. The goal of this thesis was therefore to collect empirical evidence on the perception of correlation in positively correlated scatterplots, to use that information to create novel visualisations with a view to correcting for the underestimation, and to further investigate the potential for these visualisations to affect what people think and believe.

1.2 Overview of Contributions

Through this thesis, I present a series of empirical experiments that provide knowledge about how changing visual features in scatterplots can affect how people interpret the strength of the correlation

displayed. I demonstrate that systematically reducing the opacities and sizes of points on scatterplots as a function of their distances from a regression line can significantly increase estimates of correlation and partially correct for the underestimation bias. Following this, I show that consequences of employing these techniques are not limited to perception, but in fact can be extended into a cognitive space to change what people think and believe. I utilise large-sample size, controlled experiments with lay populations to provide generalisable conclusions and recommendations for visualisation designers and researchers. Through this thesis, I provide a framework for the large-scale testing of data visualisations with lay audiences. Additionally, I hope to provide an example of a project conducted entirely in an open and reproducible manner.

1.3 Included Publications

The research described in Chapter 4, Chapter 5, Chapter 6, and Chapter 7 in this thesis is adapted from earlier publications. To avoid repetition, information and discussion that would be repeated has been consolidated into the literature review and general methodology chapters. *Gabriel Strain* is the primary author of all included papers.

- *The Effects of Contrast on Correlation Perception in Scatterplots* [157] is reproduced in Chapter 4. Sections 4.5, 4.6.2, 4.7.2, 4.6.3, 4.7.3, 4.6.4, 4.7.4, and 4.8 contain minimally altered parts of the published article.
- *Adjusting Point Size to Facilitate More Accurate Correlation Perception in Scatterplots* [156] is reproduced in Chapter 5. Sections 5.5, 5.6, and 5.7 contain minimally altered parts of the published article.
- *Effects of Point Size and Opacity Adjustments in Scatterplots* [158] is reproduced in Chapter 6. Sections 6.5, 6.6, and 6.7 contain minimally altered parts of the published article.
- *Effects of Alternative Scatterplot Designs on Belief* [155] is reproduced in Chapter 7. Sections 7.5, 7.6.2, 7.6.3, and 7.6.4 contain minimally altered parts of the published article.

1.4 Overview of Thesis

In Chapter 2, I conduct a review of the literature and provide the necessary context for the following experimental chapters. I begin by exploring the history and significance of data visualisation, before discussing concepts of correlation and relatedness, and their visualisation using scatterplots. I discuss a long-standing perceptual bias in the interpretation of correlation in scatterplots, and thereby motivate novel experimental work that seeks to address this bias. I provide real-world motivation by exploring the potential for perceptual biases to have effects on people’s lives and livelihoods. Finally, I provide a discussion on issues of graph and statistical literacy as they pertain to the interpretation of data visualisation.

Chapter 3 explores the general methodological approaches taken by this thesis, with a particular focus on transparency and reproducibility. This thesis embodies these principles, and justification for them along practical and pedagogical lines is presented. This chapter also contains detailed instructions for the full reproduction of this thesis, and provides context for the approaches to experimental design, recruitment, and analysis featured in the following chapters.

Chapter 4 presents a pair of experiments that establish the effects of point opacity on correlation perception in positively correlated scatterplots. The first experiment demonstrates that lowering point opacity in a uniform manner can increase participants’ levels of error on a correlation estimation task. The second experiment describes how employing a function relating point opacity to residual error can correct for the correlation underestimation bias by shifting estimates upwards. Specifically, lowering point opacity as a function of residual error provides significant levels of correction for the underestimation bias, and demonstrates that, in contrast to previous work, changing visual features can alter perceptions of correlation. This finding was foundational for the remainder of the work described in this thesis. This chapter also includes a discussion of a very early pilot study; I describe why this experiment was not included in published work, and offer reflections on what I learnt from conducting it in Section 4.2.

In Chapter 5, I present an experiment that expands the technique described in the previous chapter to point size. I show that reducing the sizes of scatterplot points as a function of residual error is able to alter participants’ estimates of correlation to a greater degree than functions relating point opacity and residual error. Additionally, I begin to explore how changing different visual features of scatterplots can affect the shape of the curve that describes how people relate subjective and

objective r values.

Chapter 6 describes an experiment in which point opacity and size adjustments are combined. This experiment includes conditions in which point opacity and size are increased and decreased together with residual error (congruent conditions), and conditions in which one increases while another decreases (incongruent conditions). I demonstrate that opacity and size adjustments interact in a non-linear fashion, and explore the potential for further tuning of these manipulations to create perceptually optimised scatterplots. This experiment features the most dramatic manipulation of correlation judgements seen in this thesis.

In Chapter 7, I extend the perceptual effects described in previous chapters to a cognitive space. In a single experiment, I explore the potential for scatterplots using point opacity and size manipulations to change participants' beliefs about the levels of relatedness between variables. I show that using such manipulations is able to change beliefs to a significantly greater degree compared to standard, unaltered scatterplots. This chapter also describes a pre-study that explores thoughts and feelings about relatedness to ascertain a variable pair that is particularly vulnerable to belief-change. The main experiment demonstrates that visual features that affect perceptions of correlation also affect beliefs about those correlations.

Chapter 8 presents a synthesis of the empirical work described in this thesis. I present discussions of the theoretical and practical implications of my findings, along with an exploration of future directions that my work could inform.

Chapter 2

Related Work

2.1 Data Visualisation: A Brief History

Data visualisation, which can be thought of as the practice of representing information in a visual modality [65], is difficult to concretely define, classify, and categorise. For most of us, vision is the primary way in which we interact with the world around us; in this way, data visualisation may be thought of as an extension of art and the written word. Both art and writing are ancient phenomena, with evidence for the former being found in the prehistoric period some 66,000 years ago [152], and evidence for the latter emerging as Mesopotamian cuneiform around 3200 B.C.E [144]. Broadly, the literature indicates that art emerged prior to the written word; this speaks volumes of the human instinct to represent our thoughts, feelings, emotions, and the ways that we interact with the world pictorially. This instinct has not waned, and modern computing makes it easier than ever for those of us with no technical or artistic skills to create graphics and visualisations that *tell stories* about our data in ways which are both beautiful and informative.

When, then, should we consider to be the emergence of data visualisation as a human practice? Schmandt-Besserat [143, 144] considers clay counting tokens to be the direct precursor of the written word; while the evidence for this link is controversial [136], the existence of such tokens is not. With each shape of token representing a certain amount of a certain good (measures of grain, jars of oil, etc.), this system could be considered a very early, simple form of data visualisation (or physicalisation, see Jansen et al., 2015 [72]). Similarly, there is limited evidence of prehistoric cartographic drawings [106], which may also be considered a form of, or related to, data visualisation. While I am not asserting that data visualisation is older than writing, or that ancient map drawings are equivalent to modern graphics, the existence of these representations emphasises the convenience that

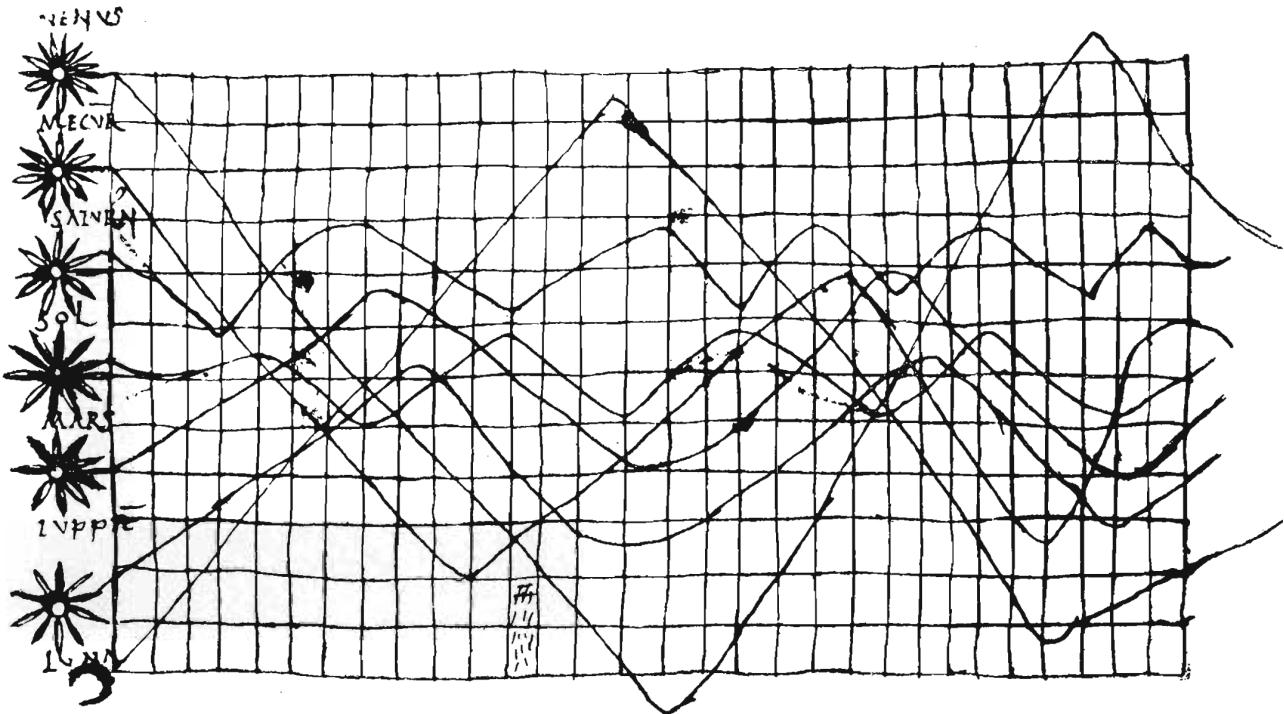


Figure 2.1. Reproduced in Tufte and Graves-Morris, 1983 [166] from Funkhouser, 1936 [50].

symbols and signs represent for humans; making sense of our world and the relationships therein is often easier through pictures as opposed to only words and/or numbers, a principle which I consider key for this thesis.

Moving on, then, to the kind of pictorial representation that modern-day students and scientists would firmly recognise as a “data visualisation”. Tufte and Graves-Morris, in 1983’s seminal *The Visual Display of Quantitative Information* [166], describe an unattributed time series illustration from the 10th or 11th century, itself described by Funkhouser in 1936 [50] as being discovered by Sigmund Günther in 1877. This illustration is included here in Figure 2.1.

This illustration purports to show the movements of planetary bodies as a function of time, although Funkhouser considered it little more than a “schematic diagram...for illustrative purposes” [50]. Regardless, the recognisable grid lines and sinusoidal variation in the curves are ideas that would not appear again for another 600-700 years, after which they would become mainstream visualisation techniques. In the mid-14th century, French philosopher Nicole Oresme demonstrated an understanding of graphing by plotting proto-bar charts, and by the 16th century, advances in cartography, photography, and mathematics laid the ground for an explosion in data visualisation.

The 17th century saw the birth of geometry and coordinate systems, error measurement, probability, and demographic statistics. With these scientific advancements came the advancements in data visualisation needed to communicate these concepts. For example, in 1626, Scheiner used what

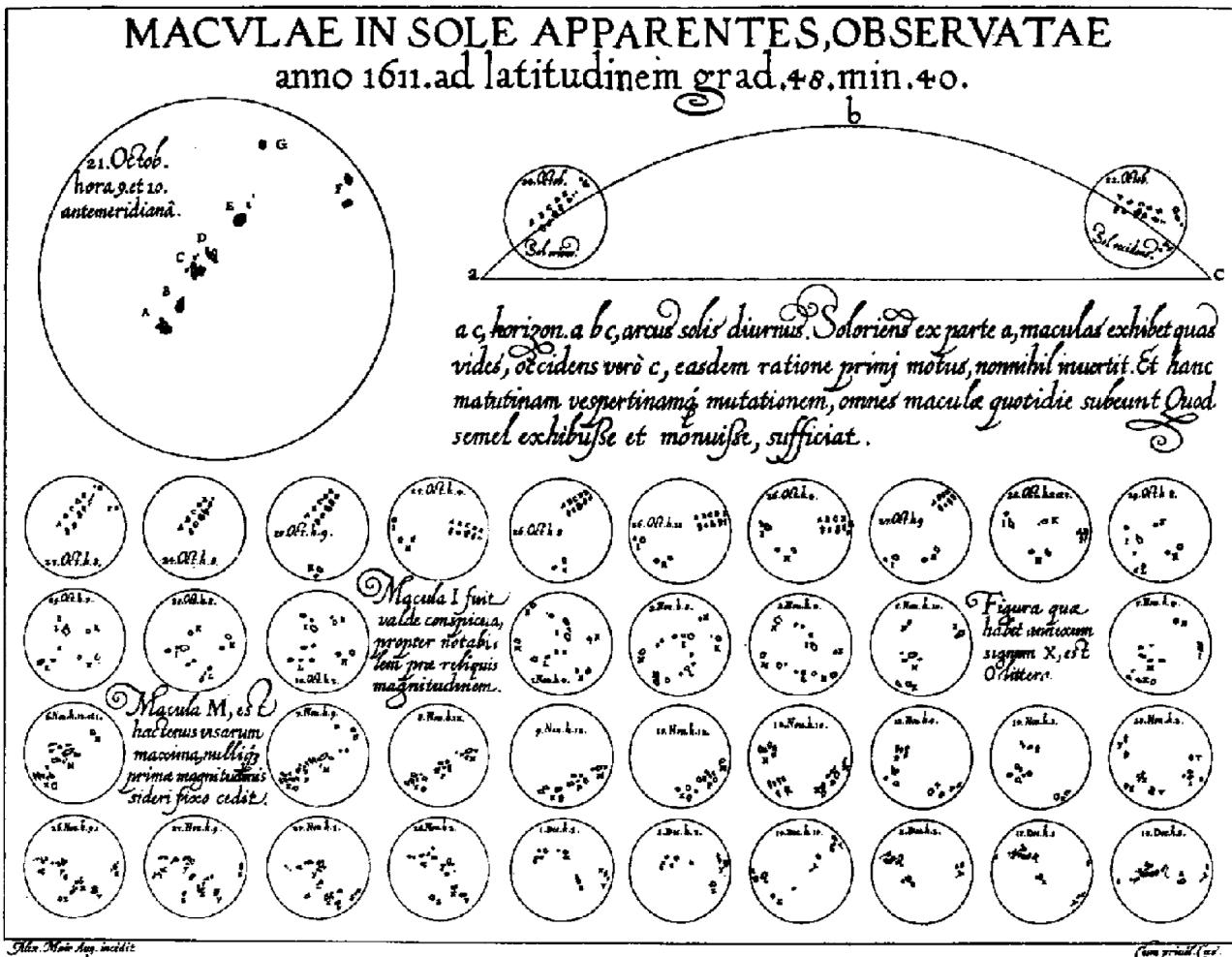


Figure 2.2. Scheiner's (1626) plot detailing how configurations of sunspots change over time. This technique would later be referred to as the "principle of small multiples" by Tufte.

Tufte would later term the "principle of small multiples" [166] to illustrate how configurations of sunspots change over time (see Figure 2.2).

The latter half of the 19th century, the so-called "Golden Age of Statistical Graphics" [48] saw the rise of forms of data visualisation that begin to look remarkably similar to the graphs and informatics seen in mass media and scientific publication today. The most notable examples of these are John Snow's cholera map, which was able to link the incidence of cholera to a contaminated water pump in London (Figure 2.3), Charles Joseph Minard's flow chart of the Napoleonic invasion of Russia (Figure 2.4), and Florence Nightingale's rose diagrams (polar area charts in the modern parlance, see Figure 2.5). In each of these graphs, visualisation is used with different intent. In John Snow's cholera map, visualisation was used to track cases of a deadly disease, and facilitated a novel linkage between cholera and contaminated drinking water. In Charles Joseph Minard's flow chart of Napoleon's failed 1812 invasion of Russia, a total of six variables are displayed to tell the data story, allowing the viewer to appreciate the movements of the Grande Armée, its diminishing size owing to attrition, and the freezing temperatures that largely caused that attrition. Florence Nightingale's polar area chart de-

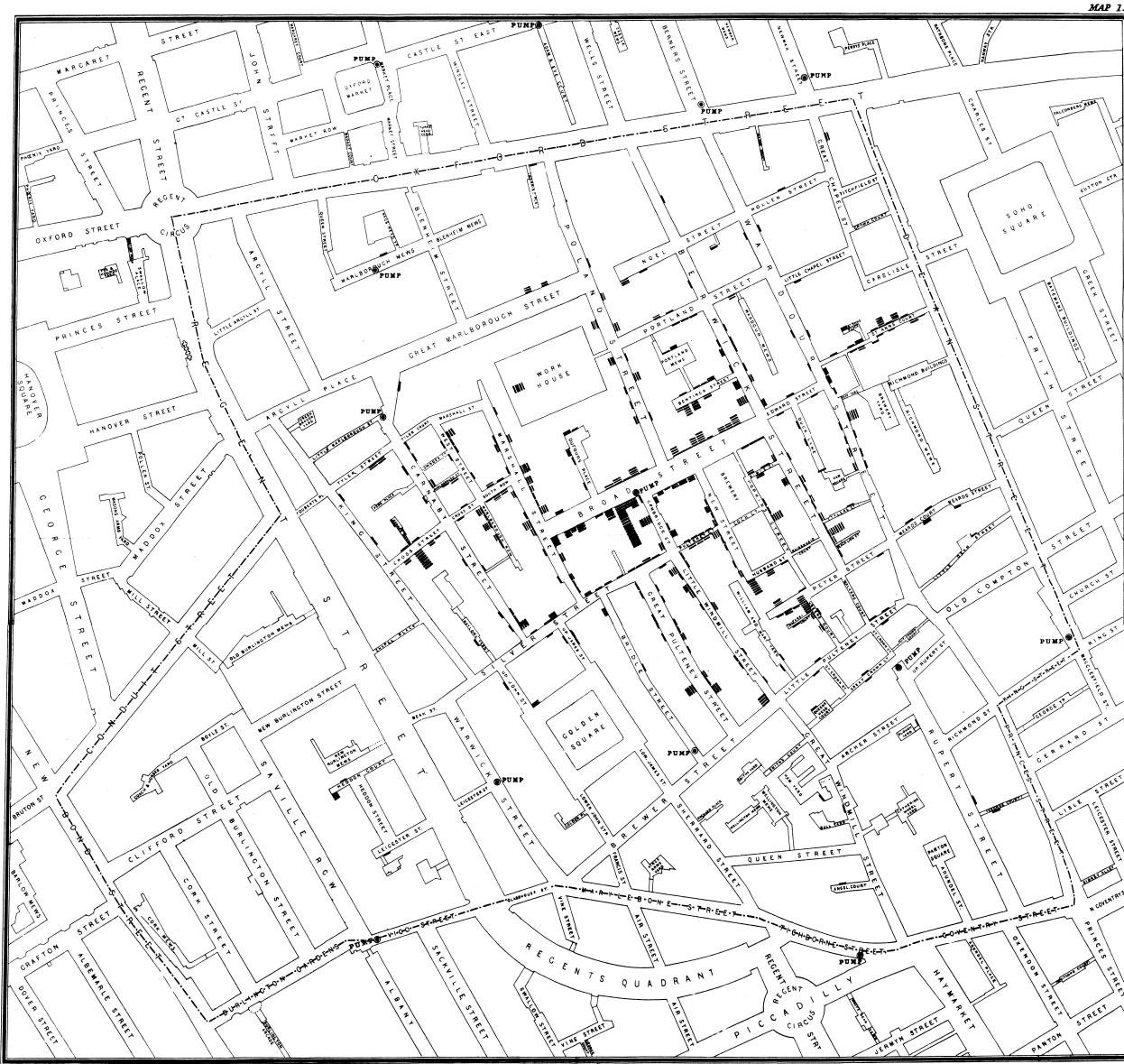


Figure 2.3. John Snow's (1854) map of cholera cases in Soho, London. Using this data visualisation, Snow was able to demonstrate a link between cholera cases and a contaminated water supply.

picts the causes of mortality amongst British troops in the Crimean War; charts such as this were used to successfully campaign for better sanitation in hospitals and the front lines.

In all of these visualisations, data was used to accentuate storytelling. In some cases, this lead to critical discoveries that saved lives, and in others, it simply facilitated a greater understanding and appreciation of the data. In either case, visualisation is used effectively to appeal to our affinity for visual storytelling. An appetite for precision defined the approach to statistical thinking, and by extension, data visualisation, in the first half of the 20th century. Statistical graphics finally became mainstream, and were implemented in curricula and used in government, commerce, and finance. This period also marks the beginning of graphical methods being used to generate new scientific insights, a trend which would only accelerate throughout the next century.

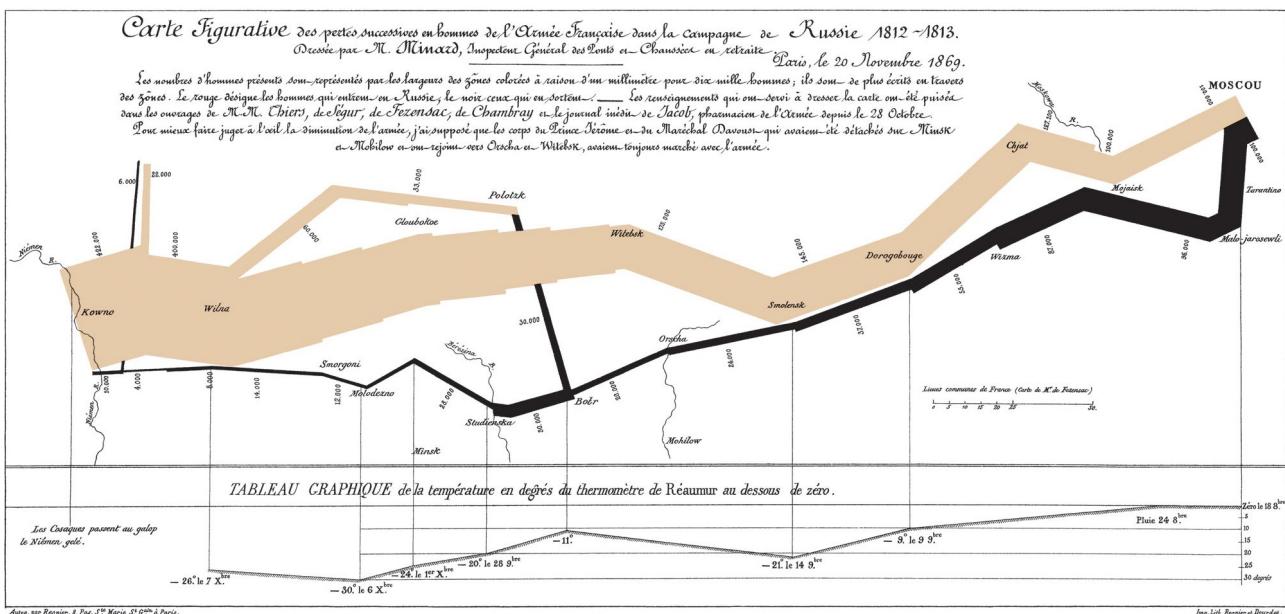


Figure 2.4. Charles Joseph Minard's (1869) flow diagram of Napoleon's botched invasion of Russia in 1812-1813. This diagram shows Napoleon's advance and retreat on Moscow. The width of the orange and black columns encodes the size of the Grande Armée. The temperature scale on the lower portion of the graph illustrates the weather conditions during the retreat, with a freezing Russian winter causing high rates of attrition.

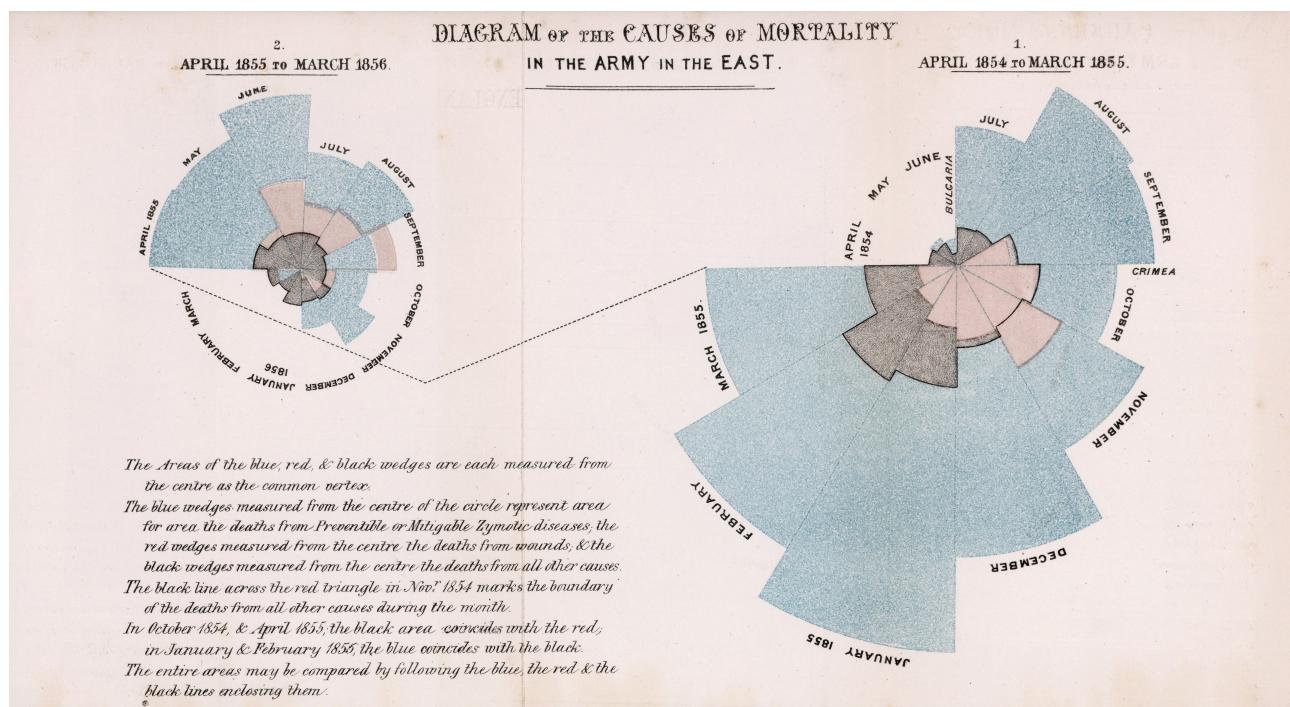


Figure 2.5. Florence Nightingale's (1858) polar area chart illustrates the causes for mortality among British soldiers during the Crimean War. Data visualisations of this type were used to illustrate that in reality, more British soldiers died from preventable disease than were killed by the enemy, and were used as part of a campaign to improve sanitation among soldiers.

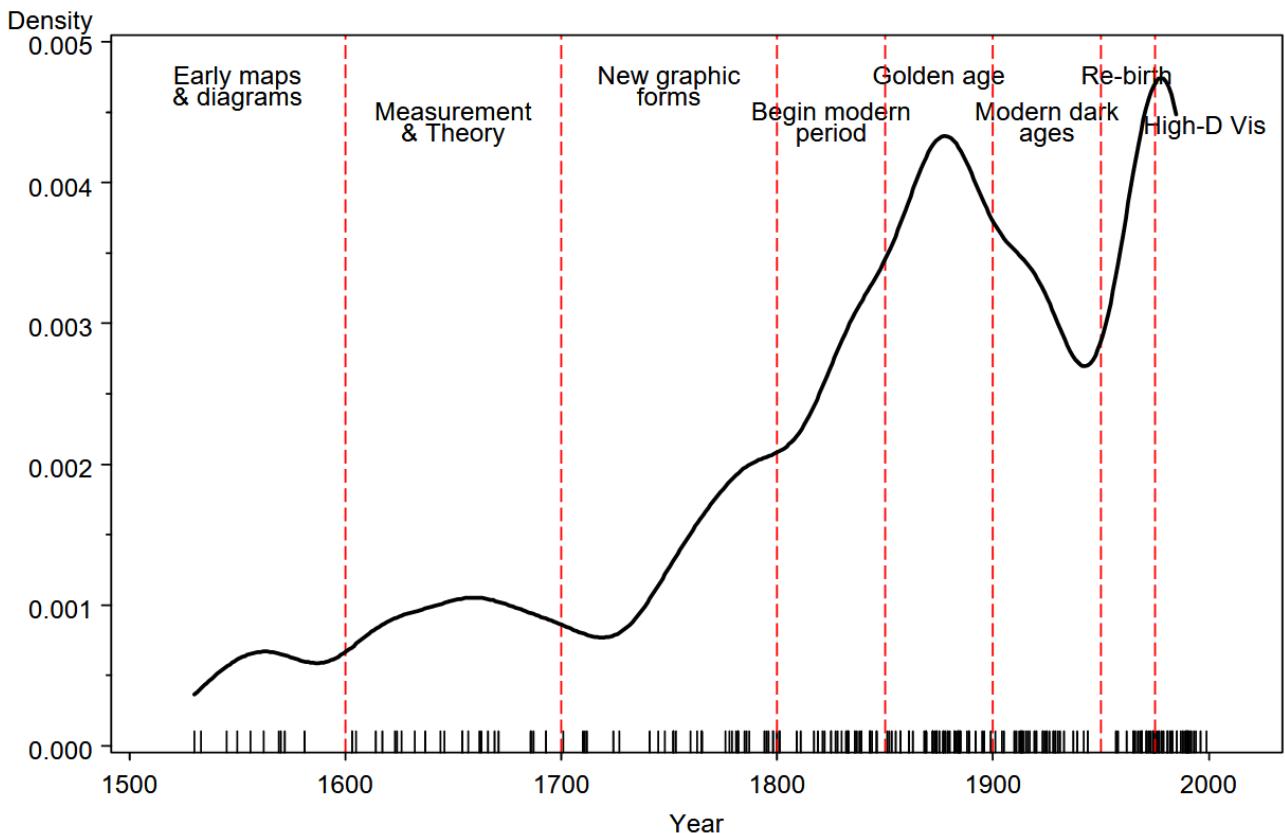


Figure 2.6. Major milestones in the development of data visualisation illustrated using a rug plot and density estimate. This figure is taken from Friendly (2008) [48].

Significant developments in the latter half of the 20th century laid the final brick in the foundations of what would become the modern data visualisation landscape. John W. Tukey's *The Future of Data Analysis* [167] proposed a separation between data analysis and mathematical statistics. This seminal work would become hugely influential, and Tukey would go on to invent a great number of analysis-driven data visualisations, including stem-leaf plots and box plots, both of which are now commonplace in software packages and statistics education. In 1967, Jacques Bertin [14] published *Sémiologie graphique (Semiology of Graphics)*, organising the perceptual elements of data visualisations according to their features and their relationships to the underlying data; this work would be influential for Leland Wilkinson's *Grammar of Graphics* [175], which in turn influenced the ggplot2 package [174] that is used extensively in this thesis.

Since then, both the practice and study of data visualisation have become thoroughly mainstream. Students are taught to visualise data early on, and the propagation of both software and powerful computing hardware have brought advanced techniques, such as high dimensional visualisation and massive datasets, into the home. To summarise the timeline of developments in data visualisation over the last 500 years, I include a rug and density plot from Friendly (2008) [48] in Figure 2.6. Recounting a full and detailed history of the practice and study of data visualisation would require much

more than a single thesis, and has been done to a very high standard elsewhere [48, 49]. Given that this thesis is focused on the perception of correlation in scatterplots, the remainder of the chapter is primarily limited to discussions of relatedness, correlation, and scatterplot visualisations.

2.2 Relatedness & Correlation

In Experiment 5, correlation was operationalised as *Strength of Relatedness*. To make my research accessible, I start from here; two (or more) things are related if a change in one is associated with a change in the other(s). In reality, however, the data visualisation under investigation, scatterplots, do not visualise relatedness, but correlation. Correlation refers to a specific statistical relationship, which I explore from first principles below.

Francis Galton was the first to formally introduce the concept of *co-relation* in 1888 [52]. He derived a definition of correlation as a by-product of his invention of statistical regression, first describing this new property by way of anthropometric data relating to the measurement of different parts of the body. Early in this brief, but significant, paper, Galton puts forward a basic definition of correlation:

“Two variable organs [of the body] are said to be co-related when the variation of the one is accompanied on the average by more or less variation of the other, and in the same direction.”

Measuring a variety of anatomical distances, including head lengths and heights of a range of “not wholly fully-grown” males, Galton first provides medians and semi-interquartile ranges as a measure of error. Plotting the semi-interquartile ranges (Q-units) of two variables against each other and calculating the slope of the line allowed Galton to devise a new, unitless measure of co-relatedness, which he termed r . This has since been lauded as a prime example of a mathematical discovery made based on observed data. This method is imprecise, as it requires intuition of a line-of-best-fit based on a hand drawn plot, but is credited as the first full conceptual definition of a measure of correlation; it should be noted that there was no concept of negative correlation at this point. Galton’s work did not take place in isolation, and the preceding 60 years featured famous names, such as Gauss and Darwin, skirting around the idea of correlation without recognising its importance; Lee Rodgers & Nicewander (1988) [85] provide an excellent overview of this period.

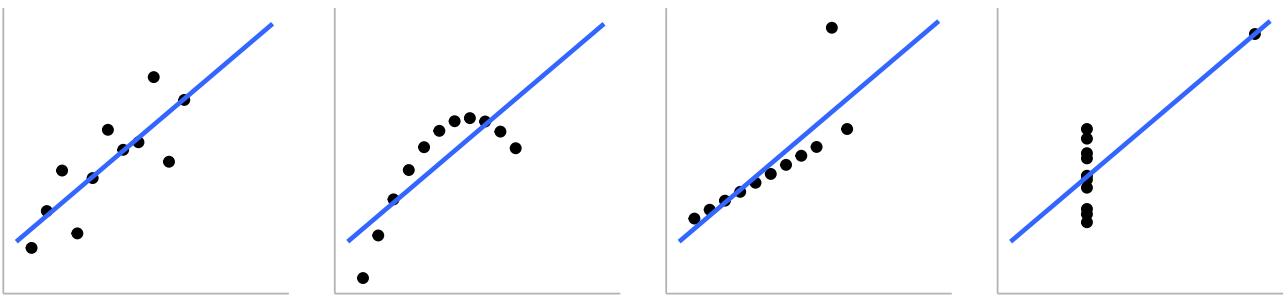


Figure 2.7. Anscombe's quartet [6]. Each scatterplot depicts datasets with identical means ($x = 9$, $y = 7.5$), regression coefficients (y on $x = 0.5$), standard errors (0.118), and Pearson's r values (≈ 0.816).

Building on Galton's groundbreaking work, his younger, more mathematically-minded student, Karl Pearson, developed the Pearson Product-Moment correlation in 1895 [85] based on initial formulae by Bravais (1844) [20]. This measure has been remarkably persistent, remaining unchanged for well over a century. In fact, many other measures of correlation, such as Spearman's ρ , the point-biserial correlation, and the ϕ coefficient are actually special cases of Pearson's r applied to different types of data [63]; such is the dominance of the measure. Equation 2.1 defines Pearson's r :

$$r = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2 \sum_{i=1}^n (y_i - \bar{y})^2}} \quad (2.1)$$

In this equation, \bar{x} and \bar{y} are the means of each variable. x_i and y_i are the individual values of each variable in question. In the numerator, the sum of the products of the difference between each value of x and y and their means are found; this value represents the degree of deviation of all points from the regression line. Then, in the denominator, this value is divided by the magnitude of the sum of the product of said deviations; squaring and finding the square root of these values provides the unsigned magnitude. In statistical language, Pearson's r finds the covariance of two variables, then divides this value by the product of both variables' standard deviations. Completing this calculation provides a measure of the overall distance between the observed values and a fitted least squares regression line, and provides a single value of r that describes how strongly related two variables are.

2.3 Visualising Correlation

Of course, while mathematically sound, a single value provides no information about the distribution of variables from which it was derived. To do this, the data must be examined visually. Here arises a parallel; in much the same way as Francis Galton used a proto-scatterplot to formulate his definition

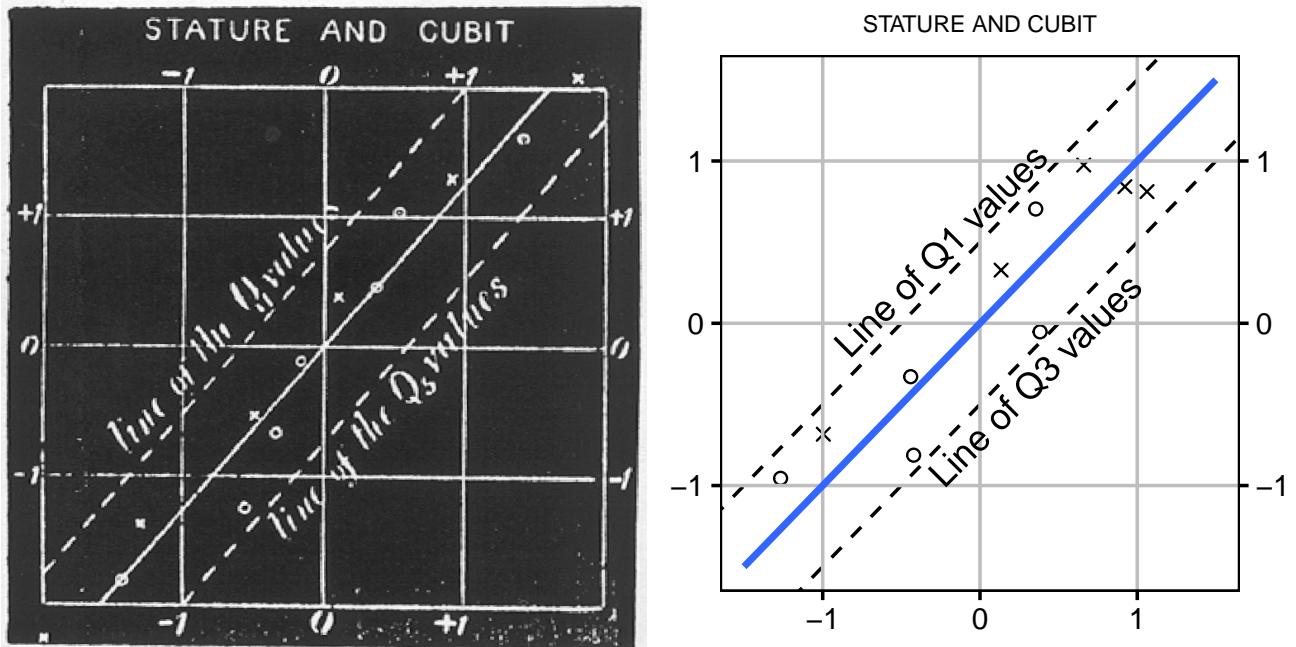


Figure 2.8. Francis Galton’s original plot comparing the semi-interquartile ranges of stature (height) and cubit (forearm length). The plot has been recreated using `ggplot2` on the right.

of correlation, so must data visualisation be used to tell the story behind a value of Pearson’s r . The need for visualisation is most viscerally illustrated by Anscombe’s quartet [6], which is recreated in Figure 2.7 using a dataset from the `datasets` core package [130] in R. Anscombe’s quartet [6] describes four simple datasets that are identical with regards to a range of statistical measures. They feature the same number of observations, the same means, regression coefficients, regression line equations, sums of squares, estimated standard errors, and correlation coefficients. A simple examination of these statistics would lead to the conclusion that the datasets are identical; in reality, there are significant differences between them that can only be seen via visualisation.

In this thesis, the primary concern is correlation and the ways in which people interpret it from scatterplots. The remainder of this section examines the history and current landscape of correlation visualisations. Starting with Galton’s scatterplot precursors, I then go on to discuss the development of the familiar modern scatterplot through the examination of a number of impressive use cases. Following that, I review the current landscape of correlation visualisation, including the more recent use of other, non-scatterplot graphs.

2.3.1 History

As mentioned in Section 2.2, Francis Galton based his initial formulation of the correlation coefficient on hand-drawn plots of the semi-interquartile ranges of two variables. Figure 2.8 contains a negative

scan of Galton's original plot, along with a modern recreation using ggplot2. Despite the importance of graphics like these for Galton's discovery of regression, statistical correlation, and the relationship between these and the bivariate density function [47], an earlier example of a scatterplot can be found in work on the orbits of twin stars by John F. W. Herschel [64]. Unfortunately, this scatterplot was never printed in Herschel's 1833 manuscript, however it can be inferred thanks to a detailed description of both the figure and the logic behind it. In short, Herschel wished to ascertain the orbits of binary star systems by using (often imprecise) astronomical measurements of certain angles and distances made over a long period of time. It is the imprecision in measurement which necessitated data visualisation, as precise measurements would allow common astronomical principles to provide precise solutions. First specifying the axes, angles of position (y) and date of observation (x), and grid lines, Herschel then describes plotting points and drawing, by hand, a line-of-best-fit. A particularly enlightening quote [64], with original emphasis, is reproduced below:

“Our next step, then, must be to draw, by the mere judgement of the eye, and with a free but careful hand, not *through*, but *among* them, a curve presenting as few and slight departures from them as possible, consistently with this character of large and graceful sinuosity, which must be preserved at all hazards...”

From this smoothed-by-eye line, Herschel was able to calculate the parameters that determined the rotation of the γ Virginis system. Herschel beat out Galton by more than 50 years to claim the first scatterplot, in a remarkable feat of using graphing to solve an astronomical problem. Just a few decades after Galton had discovered the concept of correlation, yet another astronomical example of a scatterplot can be found in the Hertzsprung-Russell diagram, created independently by both Ejnar Hertzsprung and Henry Norris Russell in 1911-1913 [103]. This type of scatterplot, which still sees use in modern astronomy, plots stellar luminosity against colour (temperature). I have plotted an HR diagram in Figure 2.9 using the HYG database¹.

The clear band that can be seen in Figure 2.9 from top-left to bottom right are “main sequence” stars. It was only by visualisation that astronomers were able to determine that there were laws that govern the formation and evolution of stars. Spence and Garrison [151] conducted a detailed analysis of the history and development of HR diagrams, and conclude that they represent a “shining example of the power of graphic display”. Again, it was the ability of the data visualisation to facilitate pattern recognition in its human viewers that was so crucial to its (continuing) success.

¹<https://www.astronexus.com/projects/hyg>

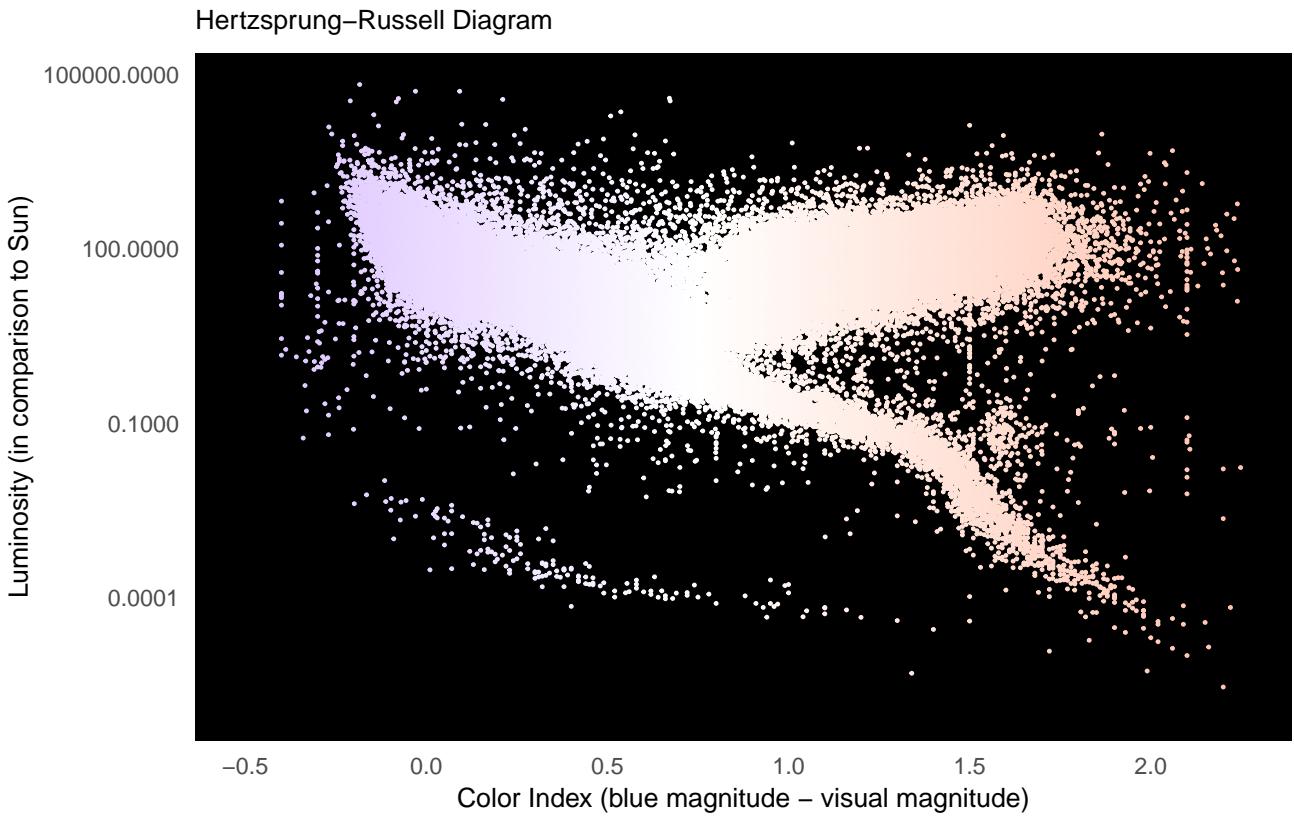


Figure 2.9. Hertzsprung–Russell diagram of colour index against stellar luminosity. The code for this plot was taken from John Russell’s (no relation) blog [138].

While this section has not been an exhaustive list of every scatterplot and scatterplot-related plot that has prominently featured in scientific publishing since Herschel’s initial description and Galton’s initial formulation of correlation, I hope that I have conveyed the importance of this visualisation type. As we will see in Section 2.3.2, the landscape of correlation visualisation is now much broader, however the humble scatterplot still remains a crucial part of the visualiser’s toolbox. From its origins as a way of inferring astronomical relationships, to its use in the discovery of correlation, the standard scatterplot remains largely unchanged to this day; this thesis charts the development of a new type of scatterplot that draws on key elements of human perception to increase its utility in correlation visualisation, however it is important to pay homage to the history of the visualisation as a valuable tool for those willing and able to use graphing to solve scientific problems.

2.3.2 Present Landscape

When I began this project in the autumn of 2021, I believed that scatterplots were the first and last word when it came to visualising correlation. I soon discovered that this was not the case; as scientists often do, a whole host of other visualisation types had been designed or adapted in an effort to better visualise correlation. Parallel Coordinates Plots (PCPs) [62], which are popular in the In-

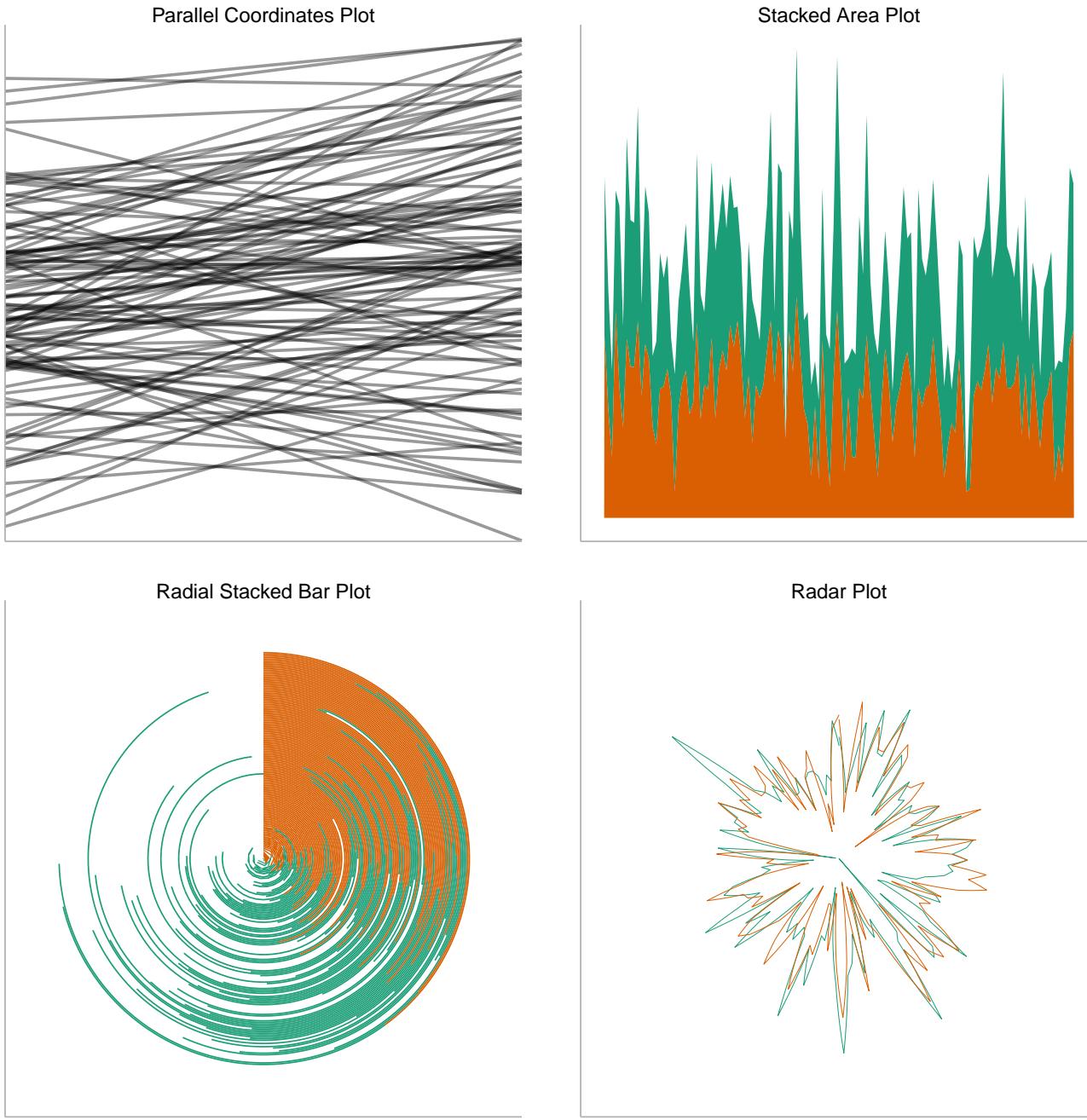


Figure 2.10. From left to right: Parallel Coordinates Plot (PCP) built with the `GGally` package [142]; Stacked area plot; radial stacked bar plot (doughnut plot); radar plot. Each plot uses the same dataset with an r value of 0.6.

foVis community, are most often used for multidimensional data. Applying them to bivariate data results in plots that perform (almost) as well as scatterplots with regards to visualising the correlation between two variables [76]. Figure 2.10 illustrates parallel coordinates plots, along with 3 other types of correlation visualisation, with an r value of 0.6. Despite the existence of plots such as these, scatterplots remain by far the most popular way of visualising correlation. With the exception of the charts featured in Figure 2.10, many other ways of visualising correlation are density-flavoured remixes of traditional scatterplots, such as Hexbin plots and Kernel Density Estimate plots (see Figure 2.11). While there may be many, often more visually appealing ways of visualising correlation, traditional scatterplots offer a number of distinct advantages.

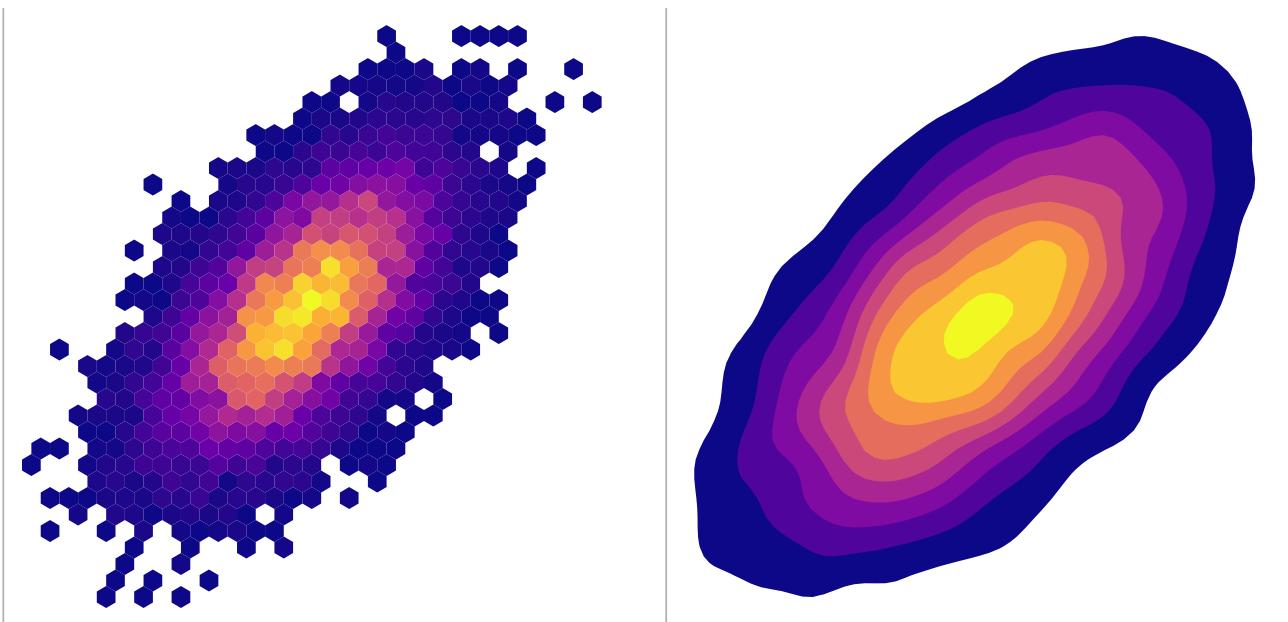


Figure 2.11. Hexbin (left) and Kernel Density Estimation (KDE, right) plots. These are binned, density-flavoured versions of standard scatterplots.

2.3.3 Scatterplots

In a large scale study, Harrison et al. [59] tested a range of correlation visualisation types, including scatterplots and those depicted in Figure 2.10, to see if precision in correlation estimation could be modelled by Weber’s law [135]. In the context of correlation estimation from scatterplots, Weber’s law states that the differential change in perception between scatterplots with two correlation values can be described by applying an experimentally-derived constant (k) to the actual difference between those correlations [59, 135]. This law may also be applied to the discrimination of other physical properties, such as weight [36]. In Harrison et al.’s study [59], itself a replication of a previous work investigating Weber’s law for the modelling of perceived correlation [135], participants were asked to make discriminative judgements between side-by-side correlation visualisations. An adaptive staircase procedure was employed to infer just-noticeable differences (JNDs) for correlation perception with nine correlation visualisation types. Both positive and negative correlations were tested. Fitting linear models to the JND data revealed that correlation discrimination in the nine different visualisations tested (scatterplots, PCPs, stacked area charts, stacked line charts, stacked bar charts, donut charts, radar charts, line graphs, and ordered line graphs) could be modelled using Weber’s law. This study also provides a ranking of the tested visualisation types with regards to participant’s correlation discrimination performance; overall, traditional scatterplots outperformed all other visualisation types with positively correlated data, and were tied with PCPs for negatively correlated data.

Rensink (2014) [133] measured correlation discriminability for scatterplots presented at 100, 400, or 1600 milliseconds. This study found that performance was almost identical for scatterplots presented for either 400 or 1600 milliseconds, and that there was only a small deterioration in performance for those presented for 100 milliseconds. The lack of performance improvement for longer presentation times both facilitates the rapid collection of large amounts of data and speaks to the intuitive nature of correlation perception in scatterplots.

Finally, scatterplots, in addition to outperforming other visualisation types and featuring rapid interpretation, are also ubiquitous. In 1983, scatterplots were estimated to account for between 70% and 80% of data visualisations in scientific publications [166]. With the advances in computers and graphing techniques seen since then, this proportion is certainly lower today. Regardless, scatterplots, their derivatives, and their remixes can be seen not only in scientific and technical publication, but also in the news and mass media. This combination of advantages makes them particularly suitable for scientific study, and was part of the reason I chose to pursue this project.

Despite this host of advantages, the estimation of correlation suffers from routine underestimation by viewers. To understand this bias with a view to correcting for it, I first explore the perception of correlation more generally, before briefly discussing the more cognitive aspects of the measure, which are especially relevant for the experiment described in Chapter 7. I then deal with the problem itself; the underestimation of correlation in positively correlated scatterplots, before finishing this chapter by discussing data visualisation and statistical literacy and stating the research objectives and contributions of this thesis.

2.4 Correlation Perception

Understanding the perception of correlation represents a complex problem. Despite decades of research into correlation, scatterplots, and the perception of information from visualisations, there is no definitive solution to this problem. Circumstantial evidence exists, however, that points to correlation perception being driven by the shape of the probability distribution represented by the scatterplot.

Firstly, increasing the x and y scales of a scatterplot such that the size of the point cloud increases is associated with an increase in viewers' judgements of bivariate association, despite the objective r

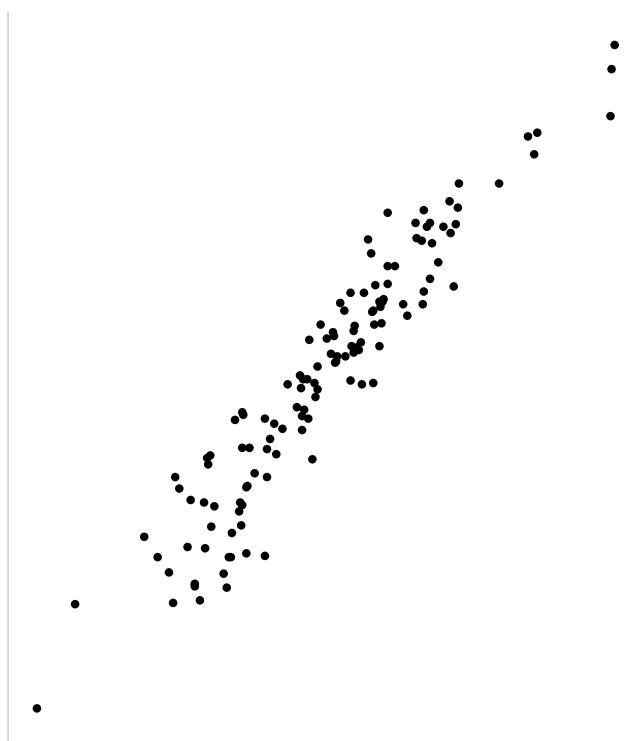
Low Correlation: Pearson's $r = 0.4$ High Correlation: Pearson's $r = 0.95$ 

Figure 2.12. Scatterplot with Pearson's $r = 0.4$ (left) and Pearson's $r = 0.95$ (right). Note how as the objective correlation increases, the point cloud tends towards a straight line.

value remaining the same [32]. This finding suggests that it is the area of the point cloud that viewers may use to judge correlation. This area is easier to visualise in hexbin or KDE plots (see Figure 2.11). With enough data, the point cloud of a scatterplot will tend towards a straight line as the r value increases, decreasing the area of the point cloud in the same way. Figure 2.12 illustrates this.

In 1997, Meyer et al. [99] asked participants to provide estimates of correlation from scatterplots. They found that people's estimates of correlation could be related to the objective correlation using a psychophysical function taking the following form:

$$r_{est} = 1 - aX^b \quad (2.2)$$

a and b are constant coefficients, while X represents the mean absolute vertical distance between the scatterplot points and a regression line. This modelling provides further evidence that estimates of correlation are based on perceptions of the physical arrangement of scatterplot points. This finding received later support from a study investigating the hypothesis that visual features may be used to judge correlations in scatterplots [180]. In that study, it was found that, among other equally predictive visual features, the standard deviation of all perpendicular distances between scatterplot points and the regression line is predictive of performance on a correlation estimation task.

Additionally, equations described by Rensink (2017) [134] linking subjective and objective r values include a u parameter. For both discrimination and magnitude judgements, this parameter is small when $r = 1$ and increases as r approaches 0. This quantity is indifferent to the type of visualisation used (e.g. line graphs and bar charts [59] or augmented stripplots [134]), and is functionally similar to that found in work mentioned above [32, 99, 180]. Regarding scatterplots, this quantity represents the average distance between data points and the regression line, and can be thought of as closely approximating the width of the underlying probability distribution. Findings from a convolutional neural network that learnt visual features related to correlation perception also support the idea that viewers are using an aspect of the shape of the point cloud to judge correlation, or some measure of what has been termed *dot entropy* [181], again considered a candidate visual proxy for correlation judgements [132, 134].

While none of this is strong evidence that people actually use the width of the probability distribution represented by the arrangement of scatterplot points as a basis for their judgements of correlation, the fact that such measures are *predictive* of performance suggests that they are at least a good proxy for what is really going on. This naturally led me to believe that ways in which these perceptions may be altered might also result in changes in how viewers interpret correlation.

2.5 Correlation Cognition

Thinking about the ways in which variables are related to one another is more complicated than simply perceiving the differences in relatedness. Particularly due to the speeds with which people are able to judge correlations [133], experiments that aim to investigate cognition require more complicated designs. Generally, interacting with data visualisation is a complex process involving bottom-up and top-down mechanisms [45, 134, 179]. Visualisation design methodologies often begin by clarification of the problem space [94, 107]; the problem I chose to address was the underestimation of correlation in positively correlated scatterplots. Due to this problem being perceptual in nature, I began by focusing on, and attempting to exploit, aspects of human perception. Only once I felt that this had been explored did I begin examining this problem from a top-down, cognitive point of view; these efforts are detailed in Chapter 7. It should be noted at this point that Chapter 7 is not a full account of how correlation is thought about by viewers, but rather a foray into how the findings of the previous chapters may be *extended* into a cognitive space.

2.6 The Underestimation of Correlation in Positively Correlated Scatterplots

While the previous section may have conveyed the idea of scatterplots as being “data visualisation done right”, that is not the full story. Since at least the 1960s, studies asking participants to simply provide a numerical estimate for the correlation displayed in positively correlated scatterplots [16, 32, 35, 41, 83, 84, 98, 154] and those asking participants to complete a bisection task comparing adjacent scatterplots [134] have found consistent levels of underestimation of the strength of the relationship between the two variables, particularly when $0.2 < r < 0.6$. Where sufficient data are available to recreate a full estimation curve, the nature of this underestimation has been visualised in Figure 2.13. Included are estimation curves derived from medians described by Bobko (1979 [16]) and from an equation relating subjective to objective correlation derived from JND studies by Rensink (2017 [134]).

The underestimation of correlation in positively correlated scatterplots was also observed in Experiments 1 to 4 in the current paper, and is detailed in each experimental chapter. Chapter 8 includes figures detailing the underestimation observed throughout this thesis in comparison to previous work. Despite this long-standing finding, no efforts have been made to correct for this perceptual underestimation. Quite why this perceptual bias correction has never been attempted, however, is unclear. The following section argues in favour of seeking design additions to correct for this.

2.6.1 Underestimation: Potential Consequences

As detailed at the beginning of this chapter, humans have a natural inclination towards the pictorial representation of data. There is evidence that statistical information is more persuasive than anecdotal and causal information [67, 68], and that, under certain conditions, data visualisations are more persuasive than tabular data [119]. Data visualisations have been submitted as evidence in court cases [16], and play key roles in organisational decision making, including in healthcare [127]. Data visualisation also plays a huge role in the communication of vital public health information, including during the COVID-19 pandemic [31]; from personal experience, this was when data visualisation entered the public conscience for many. Figure 2.14 includes examples of two COVID-19 visualisations created by the BBC.

All this is to say that the underestimation of correlation matters. It is not unreasonable to imagine

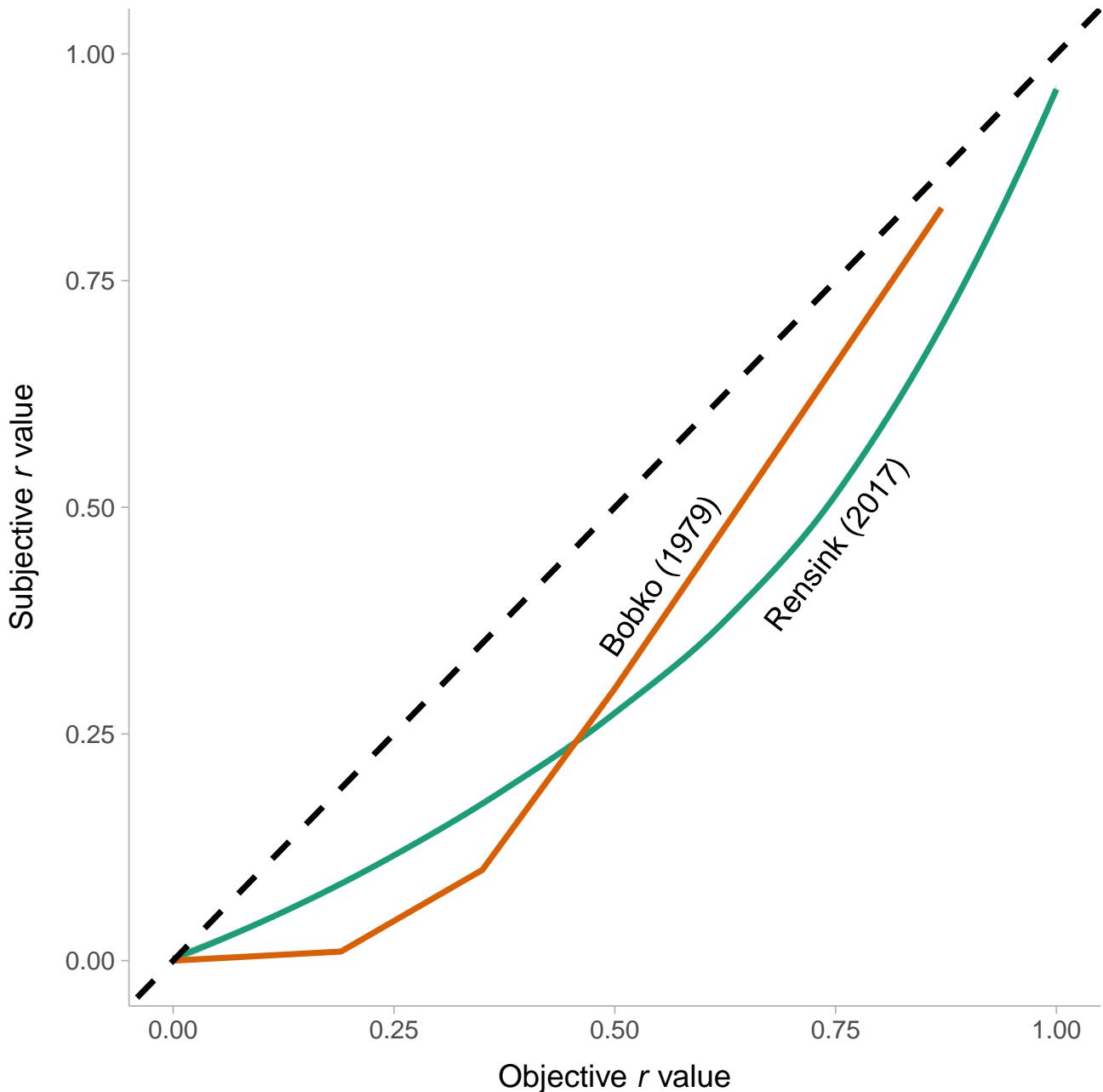
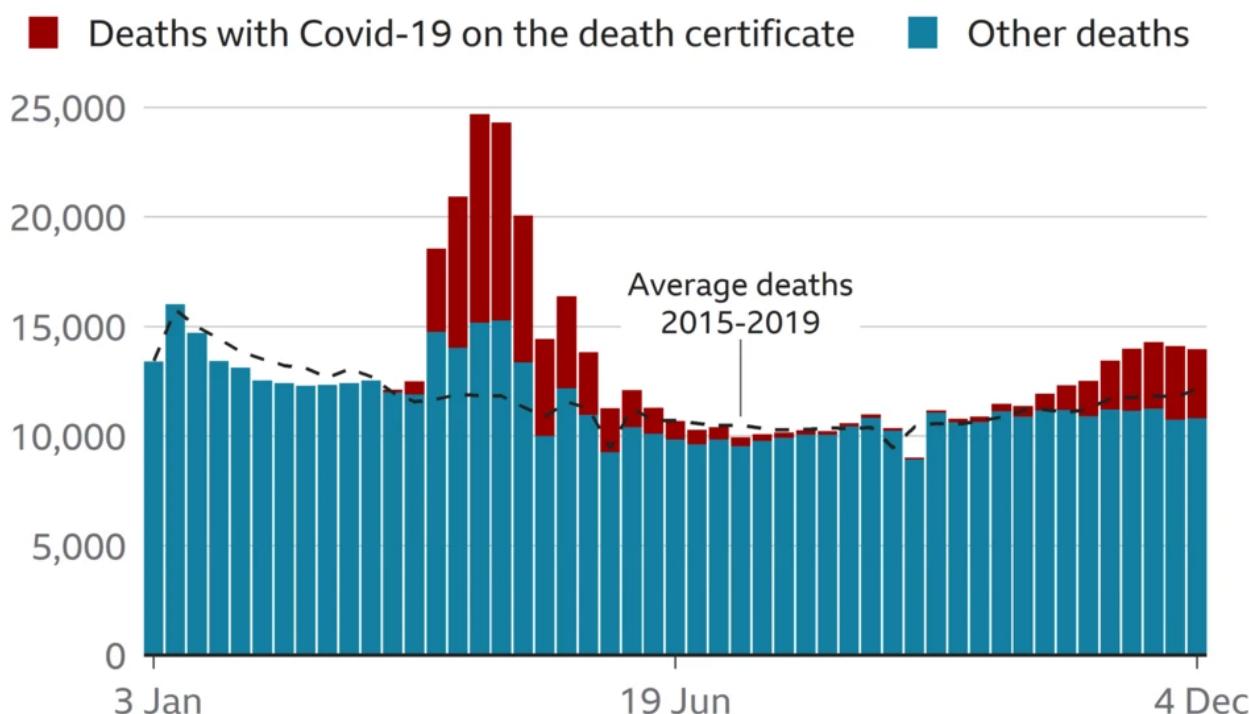


Figure 2.13. Using an equation supplied in Rensink (2017) [134], and recorded median correlation estimates in Bobko (1979 [16]), the nature of correlation underestimation may be visualised. The solid lines represents underestimation behaviour, while the dashed diagonal line represents theoretically perfect estimation.

that some policy decisions may have been made based at least in part on data visualisations; the long-standing finding that lay people and those educated in statistics underestimate the levels of relatedness between variables displayed in scatterplots is not trivial, and has real impact on people's lives. Those who design data visualisations have a responsibility to design with *all* viewers in mind, not merely those who have made statistics a part of their working lives. Designing in this way requires us to understand, on perceptual and cognitive levels, how visualisations are interpreted, and to apply this understanding to scaffold the hidden processes that allow pictorial representations to convey more than words and numbers alone ever could.

There have been more than 79,000 excess deaths

Weekly death registrations in the UK

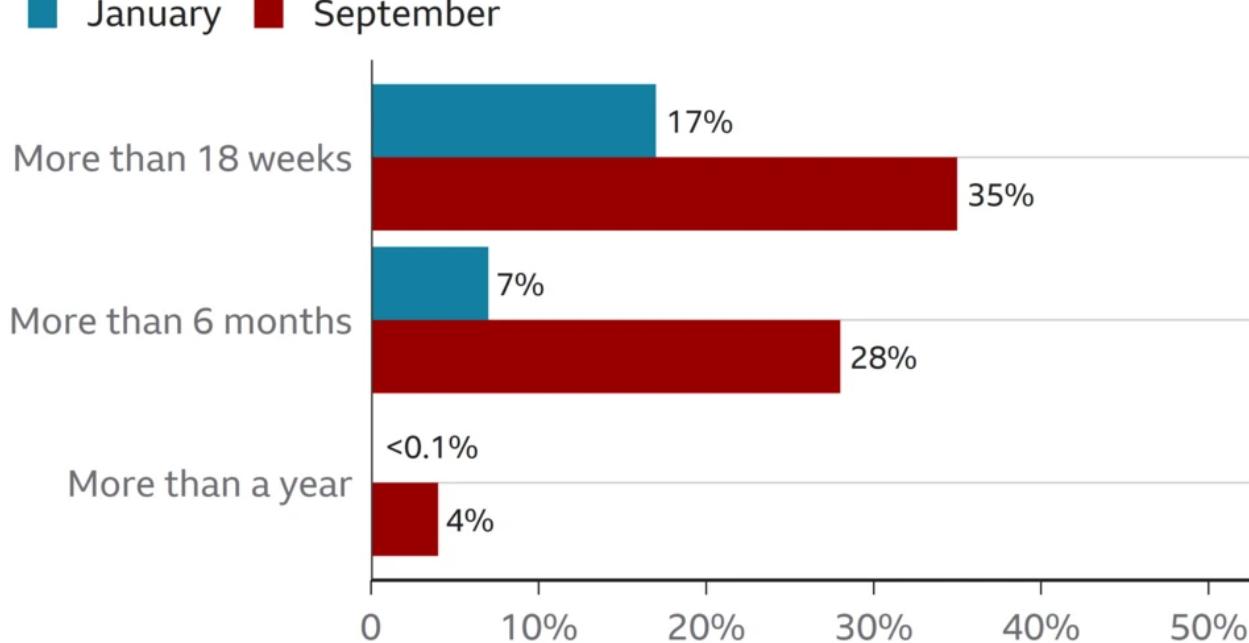


Source: ONS, NRS, NISRA

BBC

Almost 1.5m patients have been waiting more than six months for hospital appointments

Hospital appointment waits from referral to treatment, England



Source: NHS England

BBC

Figure 2.14. Two visualisations taken from the BBC News article "Covid: Twelve charts on how Covid changed our lives", published on the 15th of December, 2020 [37].

2.7 Data Visualisation Literacy

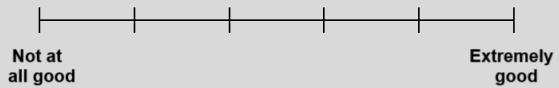
Proponents of Inclusive Design [30] argue for replacing the idea that people are disabled by physical and mental impediments with the idea that we are all, at different times, disabled by poor design and a lack of consideration of the full range of human capabilities. This concept led me to design and test novel scatterplot visualisations solely with lay populations; this is in stark contrast to most of the correlation perception studies cited in this thesis, which overwhelmingly take place with university staff and students in statistics and psychology departments. While I would argue that this leads to more generalisable results, especially as I did not pursue purely perceptual conclusions (rather used aspects of perception to inform design), it is also important to recognise and account for the differences in ability in lay populations. Research has shown that those with a greater level of skill in working with data visualisations tend to benefit more from their provision [55], and that differences in these skills may account for variations in experimental results regarding judgements of magnitude [19].

With this in mind, some measure of a participant's ability to comprehend the data visualisation displayed was required. Throughout this thesis, Garcia-Retamero's 5-item Subjective Graph Literacy (SGL) scale [54] was employed to accomplish this. Based on Galesic and Garcia-Retamero's 13-item graph literacy scale, which is itself based on work by Friel's work on graph comprehension [46], the SGL scale is quick to complete (< 1 minute), and correlates strongly with an objective measure of graph literacy [55] that has been shown to accurately predict to extent to which participants may benefit from visual aids [55]. Using the 13-item scale, it has also been shown that those with low graph literacy suffer from higher levels of misinterpretation and more often rely on misleading mappings [112].

The SGL was administered in each experiment described in this thesis, and takes the form of 5 questions to which participants must provide an answer. Generally, the scale ascertains each participant's self-reported competencies when working with a small range of visualisations. The SGL test portion of each experiment is shown in Figure 2.15.

Please answer the following questions about using graphs and charts. Click on the slider next to each question to indicate your response. Press the spacebar when you have finished:

How good are you at working with bar charts?



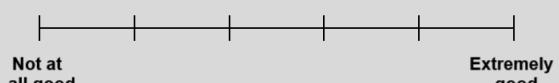
How good are you at working with line plots?



How good are you at working with pie charts?



How good are you at inferring the size of a bar in a bar chart?



How good are you at determining the difference between two bars in a bar chart?



Figure 2.15. Participants were instructed to use the following interface to complete the Subjective Graph Literacy scale [54] before completing trials in each experiment.

2.8 Objectives and Contributions

In this thesis, I aim to answer three high-level research questions:

1. Are there effects of changing the sizes and opacities of scatterplot points on performance on a correlation estimation task?
2. Can these changes be used to correct for a historic underestimation bias?
3. Are these effects only perceptual, or do participants integrate them cognitively such that they may influence beliefs about correlations?

The justifications and motivations for these research questions may be found in the experimental chapters where they are most relevant. Both in answering the above questions and in conducting this project more generally, I hope to make the following contributions:

1. There are clear and strong effects of changing the opacities and sizes of points on scatterplots, both in uniform ways, and using functions that draw inspiration from the nature of correlation perception.
2. These effects can be used to correct for the historic underestimation bias, without removing data, although significantly more work is needed to find the exact combinations of size and

- opacity adjustments that produce perceptually-optimised estimates.
3. These effects do extent into a cognitive space, and are able to influence people's beliefs about the levels of relatedness between variables.
 4. This project is exemplar of one done in an entirely open and reproducible manner; the latter half of Chapter 3 is concerned with this.

Chapter 3

General Methodology

3.1 Introduction

In this chapter I describe my research methodologies. The experiments described in Chapter 4, Chapter 5, and Chapter 6 share most aspects of experimental method, and are therefore described in full in this chapter. Chapter 7 features a different methodology, and is described therein. This chapter discusses experimental designs, the tools used to build and run the experiments, the approach to statistical analyses, and the computational methods and practices employed, particularly with regards to reproducibility and open science.

3.2 Experimental Methods

It is important to acknowledge that the way in which we conduct experiments influences what research questions we can ask and the conclusions that we may draw. The decisions that lead us to designing experiments in certain ways must be based not only on theory, but also on the external constraints imposed on (and by) the research team. Concerns such as time, practicality, and cost must be addressed, and a compromise between research that is *valuable* and research that is *doable* must be reached.

3.2.1 Experimental Design

All but the final experiment utilised within-participants designs. In such a design, each participant is exposed to each level of the intervention. This is in contrast with between-participants designs,

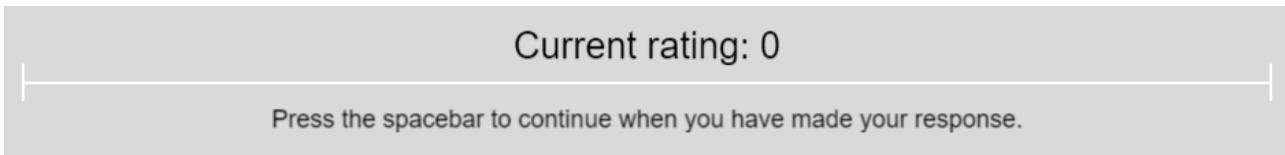


Figure 3.1. An example of the slider participants used to estimate correlation in Experiments 1-4.

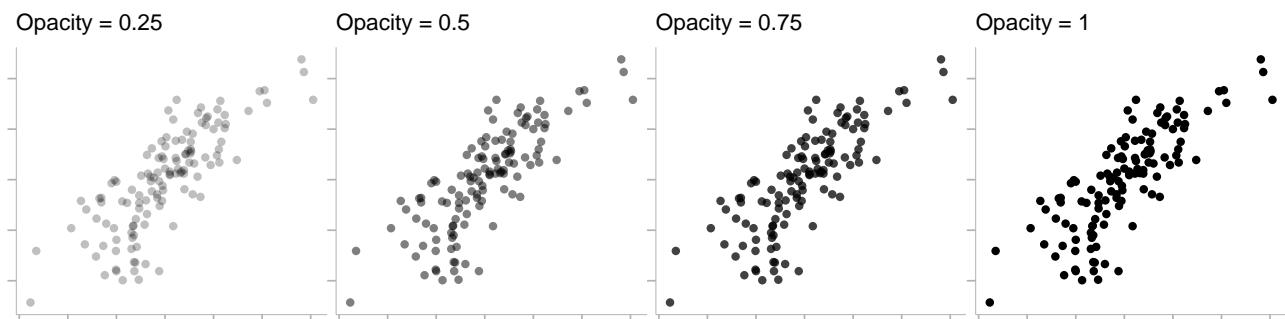


Figure 3.2. Examples of the stimuli used in Experiment 1, demonstrated with an r value of 0.8. Here, “opacity” refers to the alpha value used by ggplot.

where separate groups are exposed to only a single level of the intervention each. Where possible, within-participants designs are preferred. These designs do not rely on random allocation, and as each participant is able to provide as many data points as there are experimental items in levels [28], offer a significant boost in statistical power over between-participant designs where each participant may only provide data points for a portion of the total experimental items. In Experiments 1 to 3, each participant saw all experimental stimuli and provided a judgement of correlation using a sliding scale between 0 and 1 (see Figure 3.1). Experiment 1 featured a single factor of global scatterplot point opacity with 4 levels (see Figure 3.2). Experiment 2 featured a single factor of scatterplot point design regarding opacity with 4 levels (see Figure 3.3). Experiment 3 featured a single factor of scatterplot point design regarding size with 4 levels (see Figure 3.4). Experiment 4 featured a factorial 2×2 design; IV_1 was the scatterplot point opacity design used with 2 levels, and IV_2 was the scatterplot point size design used with 2 levels (see Figure 3.5). Experiment 5 is a departure from the shared experimental paradigm of the previous experiments, and features a 1 factor, 2 level between-participants design; group A saw scatterplots designed to elicit greater levels of belief change compared to typical scatterplots, which were shown to group B (see Figure 3.6).

3.2.2 Tools for Testing

However we design experiments, software plays a crucial role in allowing us to carry them out. Fortunately, there is a wealth of tools available to facilitate the testing of visualisations both in traditional lab-based tests and in online experiments. Following the principles of open and reproducible

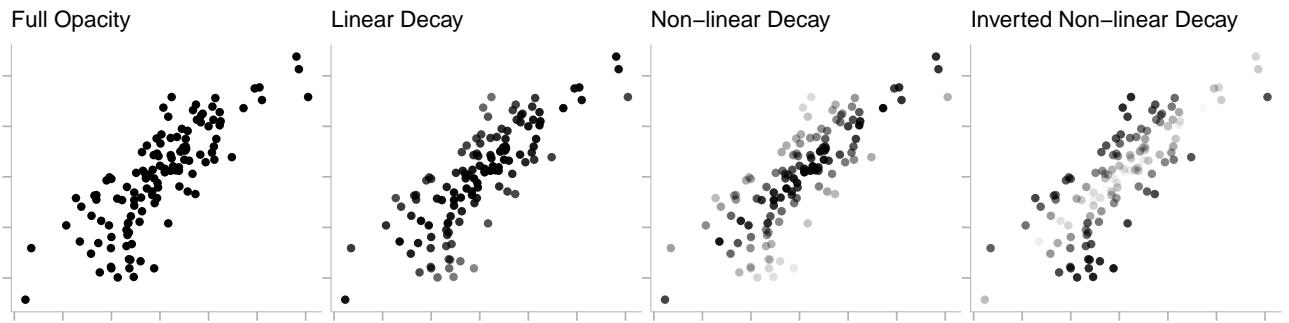


Figure 3.3. Examples of the stimuli used in Experiment 2, demonstrated with an r value of 0.8.

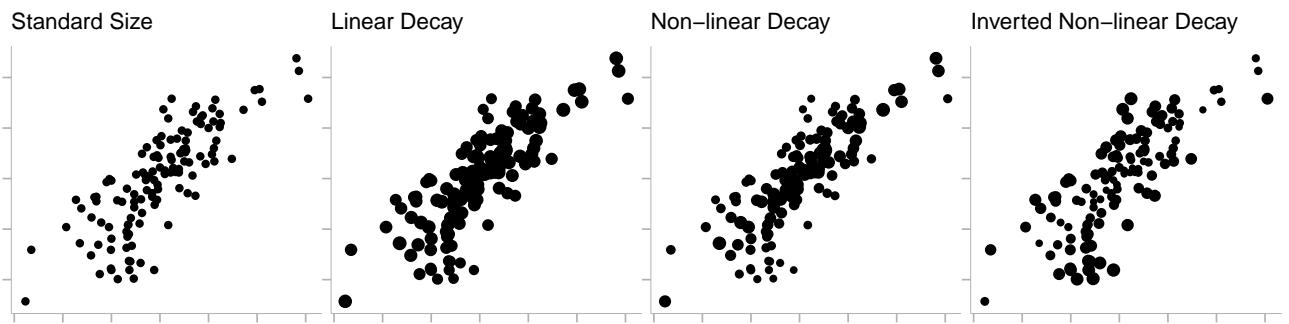


Figure 3.4. Examples of the stimuli used in Experiment 3, demonstrated with an r value of 0.6.

research [9], closed-source software, such as Gorilla [7] or E-prime [40] was discounted, as these rely on paid licences and do not allow for the sharing of code with future researchers. I settled on using PsychoPy [121] due to its open-source status, flexibility regarding graphical and code-based experimental design, and high level of timings accuracy [21]. Using such a open-source tool not only facilitated my own learning with regard to experiment building, but also enables the contribution of further examples of visualisation studies by hosting the resulting experiments online for use and modification by future researchers.

I elected to pursue online testing throughout this thesis. Doing so is much quicker than carrying out in-person lab-based testing, facilitating the collection of data from a much larger number of participants. This reduces the chances of detecting false positives during analysis and ensures adequate levels of power despite the potential for small effects sizes (see Section 3.2.3). Online testing also

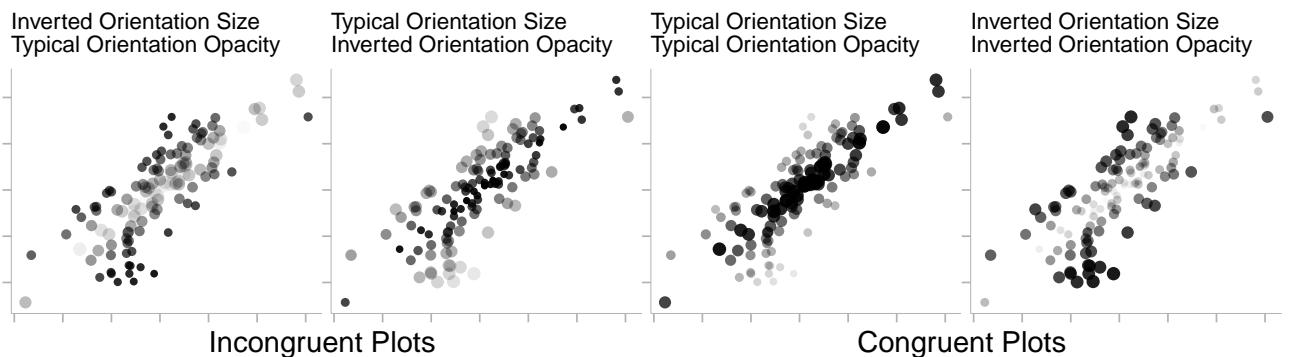
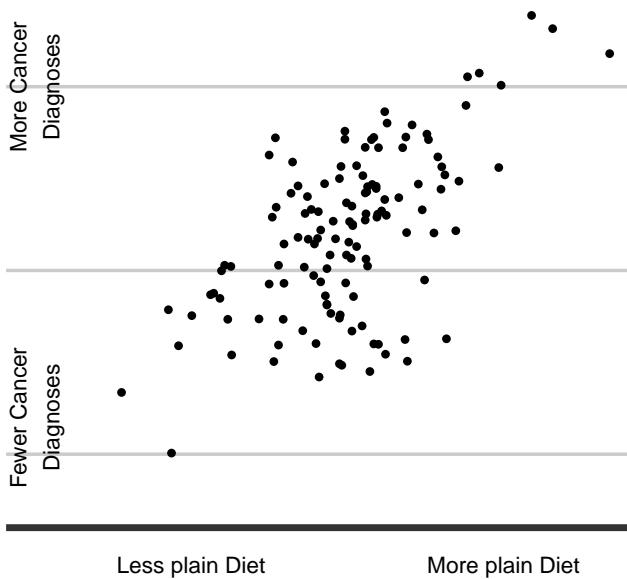


Figure 3.5. Examples of the stimuli used in Experiment 4, demonstrated with an r value of 0.6.

Spicy Foods

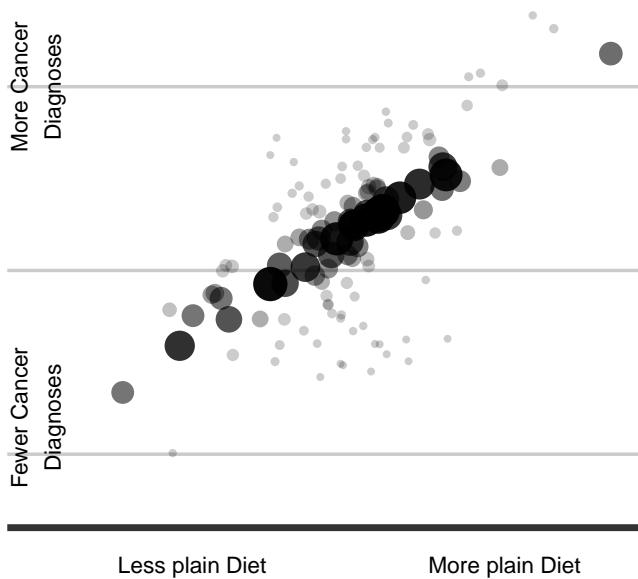
Higher consumption of plain (non-spicy) foods is associated with a higher risk of certain types of cancer.



Source: NHS England
Typical Scatterplot

Spicy Foods

Higher consumption of plain (non-spicy) foods is associated with a higher risk of certain types of cancer.



Source: NHS England
Atypical Scatterplot

Figure 3.6. Examples of the stimuli for Experiment 5 using an r value of 0.6. Group A saw the alternative scatterplot presented on the right, while group B saw the typical design on the left.

affords access to diverse groups of participants across our populations of interest, especially when compared to the relatively homogeneous student populations usually accessed in the lab by doctoral researchers. Research has identified online experimentation as producing reliable results that closely match those found in traditional lab-based experiments [8, 66, 128], especially with large sample sizes. Due to its integration with PsychoPy, Pavlovia was used to host all the experiments described in this thesis. Section 3.5.4 contains links to all experiments publicly hosted on Pavlovia's GitLab instance; these links are also provided as experiments are described in each chapter.

3.2.3 Recruitment & Participants

Recruitment of participants online is possible through a range of service providers, each with advantages and disadvantages. Research evaluating a number of these providers recently found that Prolific [129] and CloudResearch provide the highest quality data for the lowest cost [39]; I elected to use the former due to my familiarity with the system. Despite these findings, there has also been evidence of low data quality and skewed demographics affecting both general crowdsourcing platforms, such as Amazon's MTurk, and those tailored specifically towards academic research. On the 24th of July, 2021, the Prolific.co platform went viral on social media [27], leading to a participant

pool heavily skewed towards young people identifying as female. At the time, Prolific did not manually balance the participants recruited for a study. This was addressed in the pilot study (see Section 4.2) by preventing participants who joined after this date from participating, in addition to manually requesting a 1:1 ratio of male to female participants. The demographic issues settled quickly, however the screened 1:1 ratio was maintained for the remainder of the experiments.

The first experiment conducted was a pilot study (see Section 4.2 for full details) investigating a very early iteration of the point opacity manipulation in combination with exploratory work around plot size and correlation estimation. At the time, I was relatively naive to the intricacies of recruiting research participants online, and thus experienced issues regarding participant engagement. Each experiment included attention check questions in which participants were instructed to ignore the stimulus and provide a specific answer. The advert for each experiment stated that failure of more than 2 attention check items would result in a submission being rejected. This pilot study suffered from a rejection rate of 57.5%, indicating very low levels of participant engagement. For the following studies, published guidelines [120] were followed to address these issues; specifically, it was required that participants:

- Had previously completed at least 100 studies on Prolific.
- Had an acceptance rate of at least 99% for those studies.¹

Following implementation of these pre-screen criteria, the rejection rate for the next experiment fell to ~5%. Rejection rates were similar for the remainder of experiments. Exact numbers of accepted and rejected participants can be found in the **Participants** sections of each experiment.

Each full experiment recruited until 150 participants had completed successfully. Due to the novelty of this work, it was difficult to get a sense of the effect sizes that would be seen. I assumed a small effect size (Cohen's $d \sim 0.2$), and aimed to recruit enough participants to adequately power the experiments [23]. NB: I did not conduct an *a priori* power analysis. A post-hoc power analysis of the first experiment revealed a power of 0.54. Effect sizes were larger in the subsequent experiments, however to facilitate comparison, it was decided that $n = 150$ would remain the target recruitment rate.

¹this is a more strict rate than the 95% recommended by Peer et al. [120].



Figure 3.7. The basic design of scatterplots in experiments 1 to 4.

3.2.4 Creating Stimuli

All stimuli were created using `ggplot2` [174] in R. Specific versions numbers are provided with regard to the specific visualisations produced for each experiment. Identical principles were followed regarding data visualisation design for each experiment bar the last, which is discussed *in situ*.

Experiments were designed with the intention of isolating and addressing a perceptual effect; the underestimation of correlation in positively correlated scatterplots. To achieve this, confounding extraneous design factors were removed, including axis labels, tick labels, grid lines, and titles. The axis ticks themselves were preserved. Figure 3.7 demonstrates the basic design of the scatterplots used in experiments 1 to 4.

A single random seed was used to generate scatterplot datasets throughout this thesis. 45 r values were uniformly generated between 0.2 and 0.99, as there is evidence that little correlation is perceived below $r = 0.2$ [16, 32, 154]. The data that forms the scatterplots was randomly generated from bivariate normal distributions with standard deviations of 1 in each direction. Scatterplots always had a 1:1 aspect ratio, and were configured such that they occupied the same proportion of the experimental window regardless of the size or resolution of a participant's monitor. From Chapter 5 onwards, a measure of dot pitch is included, which facilitates the approximation of the physical size of scatterplot points on-screen; where available, this is included in discussions and analyses.

3.3 Analytical Methods

All analyses in this thesis were conducted using R (version 4.5.0 [130]). To investigate whether the experimental manipulations have actual effects on the interpretations participants provide, appropriate statistical testing must be employed. This involves taking into account the variability in responses that can be attributed to an experimental manipulation against the backdrop of other variability inherent in the dataset. Traditional analysis of the data collected throughout this thesis would involve the use of repeated measures analysis of variance (ANOVA). This technique assesses whether there are significant differences in means of dependent variables between conditions. While these techniques are commonplace, they do not allow for comparisons of differences across the full range of individual participant responses, nor do they allow for simultaneous consideration of by-item and by-participant variance. It is for these reasons that linear mixed-effects models were used throughout. Linear mixed-effects modelling is a reliable approach that is resistant to a variety of distributional assumption violations [141], and facilitates the appreciation of the data story in a broader and more detailed fashion.

3.3.1 Linear Mixed-Effects Models

In a mixed-effects modelling paradigm, a distinction is made between variability that is thought to arise as a result of an experimental manipulation (fixed effects), and that which arises due to differences between, for example, participants or particular experimental items (random effects). When a variable is manipulated by a researcher in an experiment, each level of that variable is present, meaning it is appropriate to be modelled as a fixed effect. When only a *subset* of levels of a variable

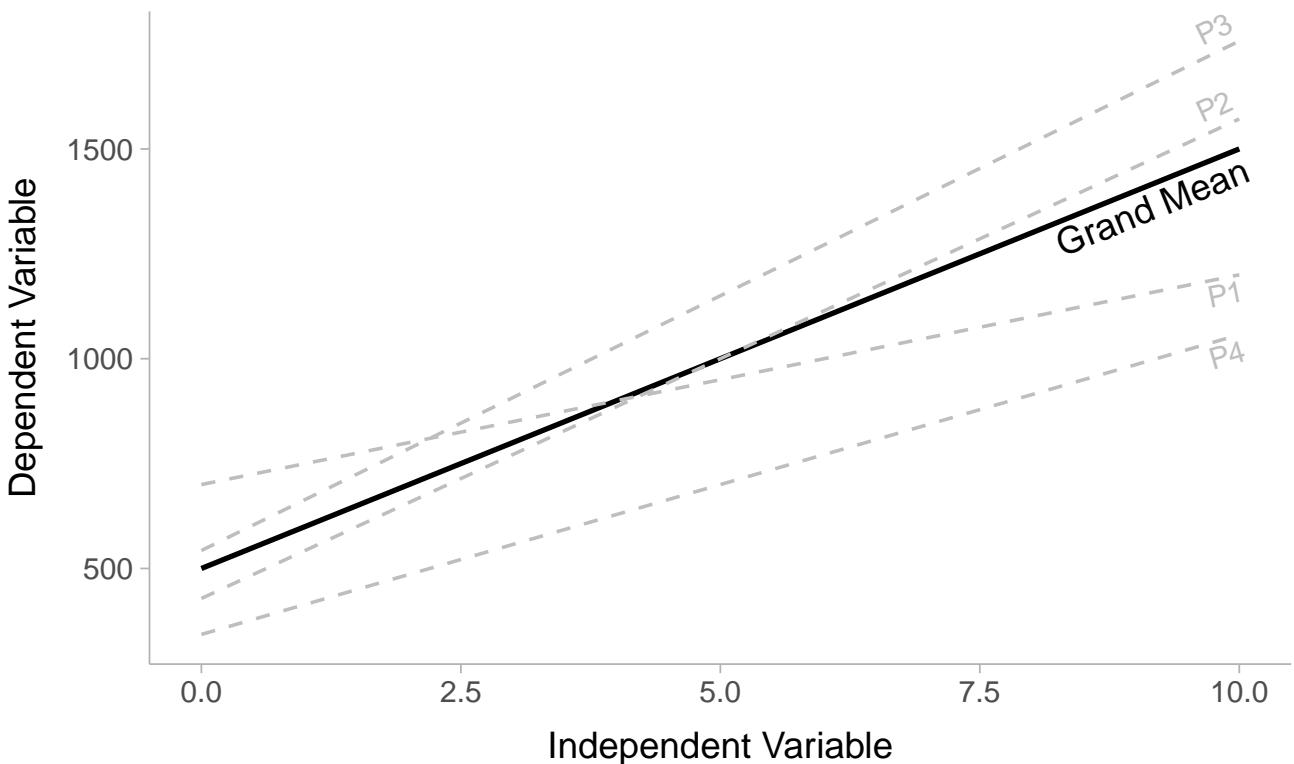


Figure 3.8. Visualising random intercepts and slopes for a theoretical experiment with 4 participants. The grand mean of the dependent variable is shown as a solid line, while each separate random intercept is drawn with dashed lines. Each line has a different gradient, representing different random slopes for each participant. This graphic was inspired by those featured in Brown, 2021 [22].

is present, such as a sample of all possible participants or experimental items, then this variable is appropriate for modelling as a random effect. Typically, mixed-effects models require the specification of *intercepts*; these are different baselines for each participant or item that reflect random deviations from the mean of the dependent variable. Mixed-effects models may also specify random *slopes*; these are differences in the magnitude of the difference between levels of the independent variable for each random effect [22]. Figure 3.8 visualises these concepts.

Throughout the course of this thesis, analyses attempt to model both random intercepts and slopes in order to capture the maximum amount of variability present in our datasets. In order to ascertain the goodness-of-fit of models, their ability to explain variance is compared to that of a nested null model [149]; such a model is identical bar the removal of the fixed effect of interest. The likelihood ratio test (LRT) is used here to assess goodness-of-fit. In cases where a model has in total more than two levels (here, all experiments bar experiment 5), the *emmeans* package [87] is used to calculate estimated marginal means between levels of fixed effects.

3.3.2 Ordinal Modelling

In experiment 5, participants used Likert scales to provide responses. These scales capture whether one rating is higher or lower than another, however they do not quantify the magnitude of the difference between levels of rating. Metric modelling, such as linear regression, treats the response options to a Likert scale as if they were numeric. Doing so assumes equal levels of difference between ratings, when in reality there is no theoretical reason to make this assumption. Metric modelling is therefore considered inappropriate for modelling responses to Likert scale data [89]. In light of these issues, the `ordinal` package [29] in R was used to build cumulative link mixed-effects models for the analysis of Likert scale data. This allows for the treatment of Likert responses as ordered factors as opposed to continuous response scales.

3.3.3 Model Construction

Choices are inherent in every type of statistical analysis, and can play a large role in the conclusions that are drawn from them. In linear mixed-effects modelling, deciding *what* is a fixed or random effect is straightforward; deciding *how to specify* random effects is a more complicated matter. Barr et al. [11] argue that for fully repeated measures designs, a maximal model should be preferred; one with random intercepts and slopes for each participant and experimental item. More recently, Bates et al. [12] have argued that attempting to specify maximal models for insufficiently rich datasets may lead to overfitting and unreliable conclusions. In light of this I sought a more systematic approach to selecting the random effects structure of a given model.

In an attempt to balance simplicity, explanatory power, and model convergence (whether or not a solution can be found), the `buildmer` package [169] in R was used to automate the selection of model specifications. Having been provided with a maximal model, `buildmer` uses stepwise regression to select the most complex model structure that successfully converges. Following this, random effects terms that fail to explain a significant amount of variance in the dataset are dropped; this stepwise elimination of terms is evaluated using successive likelihood ratio tests. This results in a model that captures the maximal amount of feasible variability while minimising redundancy. Note that `buildmer` was not relied upon as a modelling *panacea*; models are still based on theoretical underpinnings and are evaluated critically.

3.3.4 Effects Sizes

My approach to effects sizes evolved throughout the course of the research project due to reviewer feedback and a growing appreciation of the complexities of effect sizes when discussing linear mixed-effects models. Experiments 1, 2, and 3 featured a condition with no scatterplot manipulation present (henceforth referred to as a *baseline*); accordingly, the `EMAtools` package [77] was used to calculate equivalent Cohen's d effect sizes of manipulation-present conditions relative to the baseline. Experiment 4 did not feature a baseline condition, meaning Cohen's d was deemed inappropriate. The `r2glmm` package [71] was used instead to calculate semi-partial R^2 . In lieu of a traditional measure of effect size, this demonstrated the unique variance in the dependent variable explained by each level of the independent [108]. Experiment 5 features a much simpler modelling situation, and returns to providing equivalent Cohen's d values for the pre- vs. post- plot viewing conditions, this time calculated by converting odds ratios using the `effectsize` package [13]. More details on specific calculations, measures, and conclusions can be found *in situ*.

3.3.5 Reporting Analyses

Throughout this thesis, a broad approach to the reporting of statistical analyses was taken; while I consider our analytical methods and conclusions valid, I also present a range of statistics to allow the reader to draw their own conclusions, should they wish. Statistical results are visualised where appropriate, and where visualisation aids understanding and interpretation. In addition, details about model structures and the issues I tackled when modelling are included for transparency [97].

3.4 Computational Methods

The approach to computational methods in this thesis sought to marry practicality, simplicity, and reproducibility. Often, this meant that what would otherwise be a makeshift script followed by copy-pasting of results into Overleaf ended up being an involved exercise in literate programming [79] and code wrangling. This involved effort and time, particularly in the early stages of the project, however has yielded a number of benefits. Many of the techniques developed early in the project proved to be instrumental later on, resulting in time-savings overall. Additionally, these techniques, principles, and practices are shared to enable future researchers to learn, where I struggled. In this section,

I detail my approach to computational methods, including how the idea of **executable papers** was utilised, and how containerised environments were used to capture a freeze-frame of the analyses.

3.4.1 Executable Reporting

Each paper published throughout this project, and this thesis, has been written to be executable. Packaging research in such a way means a lay person can follow simple instructions to recreate the work, while also facilitating and encouraging literate programming, or the close alignment of documentation and underlying code [125].

The use of a literate programming paradigm to generate reports (usually using LaTeX) has a rich history. This section focuses on this history as it pertains to the language used throughout this project, R. Sweave [86], written in 2002, allowed R code to be integrated into LaTeX documents. This was followed by Yihui Xie's knitr [177], which expanded Sweave functionality and improved integration with tools such as pandoc [91]. knitr uses Rmarkdown [178] to mix markdown-flavoured text with code chunks into a document that can be rendered into an appropriately-formatted conference or journal pdf; this workflow was used for the papers associated with experiments 1, 2, and 3. Quarto [4], released in 2022, further expands on Rmarkdown functionality, and removes reliance on R or Rstudio. Quarto was used for the remainder of the papers associated with this project, and for the present thesis.

Writing executable or dynamic documents allows results to be re-generated whenever the document is rendered. This includes any associated data visualisation and statistical modelling. Structuring documents like this effectively “opens up” research by allowing others to view the code that performed the analysis and generated the data visualisations, in addition to guarding against accusations of questionable research practices (QRPs) through high levels of transparency [69]. This paradigm also allows for the caching of computationally expensive statistical models.

3.4.2 Containerised Environments

Providing the code associated with a project, even when that code is integrated into a literately programmed executable paper, is necessary, but not sufficient, for enabling adequate reproducibility. Previous work has found many instances where publicly-accessible code could not reproduce the

results included in the corresponding document or failed to run entirely [34, 139, 165]. Poor programming practices accounted for a significant portion of these problems, highlighting the issue of researchers without technical backgrounds being expected to produce high quality technical documentation. Elsewhere, differences in computational environment, package versions, and operating systems have been identified as responsible for the non-replication of results. Large research projects, such as this, can include hundreds of functions from scores of packages, meaning that small changes can critically break code.

These issues were addressed using containers, specifically, those created by Docker [17, 96]. 1979 saw the development of `chroot` (`change root`), which is able to isolate an application's file access to a 'chroot jail'. Since then, we have seen the rapid development and uptake of containerisation software, mostly within the software development and security communities. Docker, released in 2014, is a popular, lightweight containerisation tool that enables a precise recreation of computational environments. Recording software versions and dependencies avoids the potential for broken code in the future, and publicly hosting papers as GitHub repositories that build into Docker containers ensures that future researchers can interact with code and data in the same computational environment used when carrying out the research. While virtual machines make isolated sections of hardware available, containers abstract protected parts of the operating system [96]. This makes containers smaller and more lightweight than full virtual machines, while still conferring the advantages of virtualisation. For the Docker implementation here, portable R environments provided by the Rocker project [18] are used. These environments are agnostic regarding the host operating system, allowing the reader to reproduce the analyses featured here in a replica of the computational environment they were conducted in.

Building Docker containers is facilitated through a Dockerfile. This file instructs Docker to build a container with the appropriate version of R, the files required, and the correct package versions used during analysis. Below is the Dockerfile used to reproduce this thesis.

I first specify the Rocker image that will form the basis of the container. This includes the version of R required (version 4.4.2), the Rstudio Integrated Development Environment (IDE), Quarto, and the tidyverse package.

```
FROM rocker/version:4.4.2
```

Next, I add the files and folders required, including the Quarto document and related files, chapter

folder, bibliography, additional scripts, LaTeX class file and template, the folders containing the cached models and raw data, and the R project file:

```
ADD thesis.qmd /home/rstudio/  
  
ADD _quarto.yml /home/rstudio/  
  
ADD chapters_quarto/ /home/rstudio/chapters_quarto/  
  
ADD thesis.bib /home/studio/  
  
ADD reformat_tex.R /home/studio/  
  
ADD finalise_thesis.R /home/rstudio/  
  
ADD helper_functions.R /home/rstudio/  
  
ADD uom_thesis_casson.cls /home/rstudio/  
  
ADD main.tex /home/rstudio/  
  
ADD data/ /home/rstudio/data/  
  
ADD cache/ /home/rstudio/cache/  
  
ADD thesis.Rproj /home/rstudio/
```

Finally, I add the specific versions of the R packages used throughout the course of this thesis. For brevity, I only display the addition of the first three here:

```
RUN R -e "devtools::install_version('MASS', version = '7.3-60', dependencies = T)"  
  
RUN R -e "devtools::install_version('buildmer', version = '2.10.1', dependencies  
= T)"  
  
RUN R -e "devtools::install_version('emmeans', version = '1.8.8', dependencies =  
T)"  
  
...
```

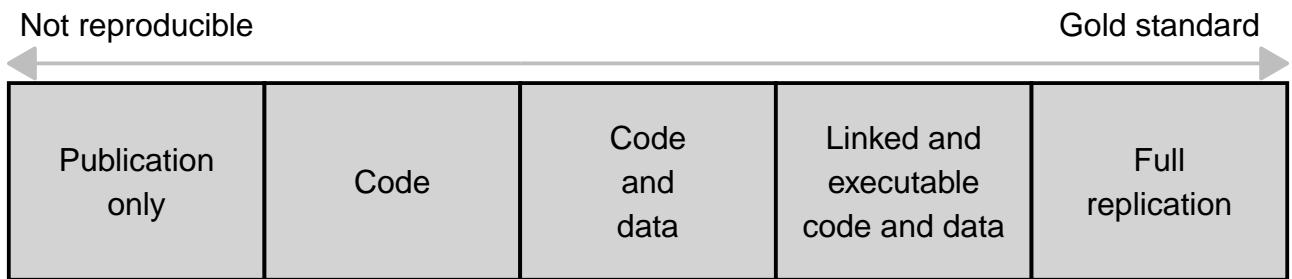


Figure 3.9. Peng’s (2011) Reproducibility Spectrum. This figure has been reproduced from Peng (2011) [123].

3.5 Reproducibility In This Thesis

Reproducibility is a broad spectrum [123] (see Figure 3.9). As discussed above, even when code and data are provided, results are often not replicable, and this is before issues around poor research practice, inappropriate analysis, and dishonest science even rear their heads. While for most, the reproducibility crisis [114] crystallised in the early 2010s [122], serious concerns had been voiced since at least the late 1960s [137]. Since coming into the wider academic consciousness, numerous studies have identified reasons for the crisis, ranging from poor practice (e.g. Potti et al., 2006 [1]) to outright deception and fabrication (e.g. the Woo-Suk Hwang scandal [140]). These issues led this project to strive for a gold standard [123] of reproducibility throughout. In this section, I detail how this was accomplished, and in doing so, expose my work to welcome critique.

3.5.1 Sharing Data and Code

The open and public sharing of data and code facilitates external assessment [5, 78] and secondary use of data [161], and guards against reproducibility issues [102]. Quite aside from external motivating factors, I found that developing and embedding the reproducibility practices described here have resulted in longer term savings in time and effort. Favouring a gold-standard reproducible approach is also a way of “paying it forward”; having come from a non-technical background, I found previous work that adhered to the same standard critical for my own learning and development. GitHub is used to host both this thesis and the papers associated with the project; links to these repositories can be found throughout. I favour permissive and lenient licencing, such as the MIT licence [163] for GitHub repositories and the CC-BY 4.0 license for pre-registrations. These enable future researchers to re-use data and code while providing clear guidance for appropriate use and facilitating long-term sustainability [73].

3.5.2 Executable Papers and Docker Containers

As detailed above, Quarto and Docker were used to produce executable journal/conference papers for each of the published works this thesis describes. For simplicity, all analyses from these papers have been repeated using up to date packages here. Accordingly, a single implementation of Docker to is provided to reproduce this thesis. All statistics have been checked against those provided in the original analyses, and repositories for the corresponding papers are provided complete with original Docker implementations.

3.5.3 Pre-Registration of Hypotheses and Analysis Plans

Often touted as a low-cost entry point into reproducible research practices [90], pre-registration is the practice of formally clarifying hypotheses and analysis plans prior to data collection. While this may not be able to prevent research fraud and QRPs entirely, it does lend credibility to the researcher [118]. All hypotheses and analysis plans were pre-registered with the Open Science Framework [117]. Pre-registrations are embargoed by the research team prior to data collection, and then made public in a frozen state following publication of the corresponding research. Where I felt it necessary to deviate from these plans, details are provided in the methods sections of the corresponding experiments.

3.5.4 Experimental Resources

Everything needed to run each experiment is included in the corresponding GitLab repository. Links to these repositories are also provided in the sections concerning each experiment.

Chapter 4

Experiment 1:

https://gitlab.pavlovia.org/Strain/exp_uniform_adjustments

Experiment 2:

https://gitlab.pavlovia.org/Strain/exp_spatially_dependent

Chapter 5

Experiment 3:

https://gitlab.pavlovia.org/Strain/exp_size_only

Chapter 6

Experiment 4:

https://gitlab.pavlovia.org/Strain/size_and_opacity_additive_exp

Chapter 7

Experiment 5 Pre-Study:

https://gitlab.pavlovia.org/Strain/beliefs_scatterplots_pretest

Experiment 5 Main Study (Group A):

https://gitlab.pavlovia.org/Strain/atypical_scatterplots_main_a

Experiment 5 Main Study (Group B):

https://gitlab.pavlovia.org/Strain/atypical_scatterplots_main_t

3.6 Conclusion

In this chapter, I have established the broad methodological approach taken by this thesis. This project sought to investigate novel ways of visualising data and their effects on perception and cognition. I have provided justifications for the designs used, the methodological challenges faced, and how the use of a broad array of tools and techniques was able to overcome these challenges. Throughout, I have detailed how I have learnt from my mistakes. Open research and reproducibility is at the core of the work described here, and I hope this thesis can serve as an example for future work facing similar challenges and with similar commitments to open science. To this end, I have produced a template to facilitate future reproducible theses. FAIR (Findable, Accessible, Interopable, and Reusable) data principles [176] are satisfied through public sharing of data and code, literate programming, and containerisation.

Chapter 4

Adjusting the Opacities of Scatterplot Points

Can Affect Correlation Estimates

4.1 Abstract

Scatterplots are common data visualisations utilised for communication with experts and lay people alike. Despite being widely studied, people tend to underestimate the level of correlation displayed in them. The weight of evidence points toward changes in the opacities of scatterplot points being unable to change perceptions of correlation, however this was not tested rigorously using systematic adjustments. Drawing on evidence that the shape of a scatterplot's point cloud may drive correlation perception, I conducted exploratory work addressing this underestimation bias. In two experiments (total $N = 300$), evidence is provided that changing the opacities of scatterplot points *can* have small effects on participants' performance on a correlation estimation task. The systematic adjustment of point opacity as a function of residual distance is able to alter estimates sufficiently to correct for the underestimation bias. In this chapter, I also present an early pilot study that was ultimately not included in any published works.

4.2 Preface: Learning From an Early Pilot Study

The research proposal that kickstarted this project in 2021 set out a plan to investigate the perception of correlation in scatterplots as a function of screen size. This proposal is included in the supplemental materials. This was prompted by recent research demonstrating consistent perceptual biases in scatterplots due to geometric scaling [173], the growing prevalence of data visualisations

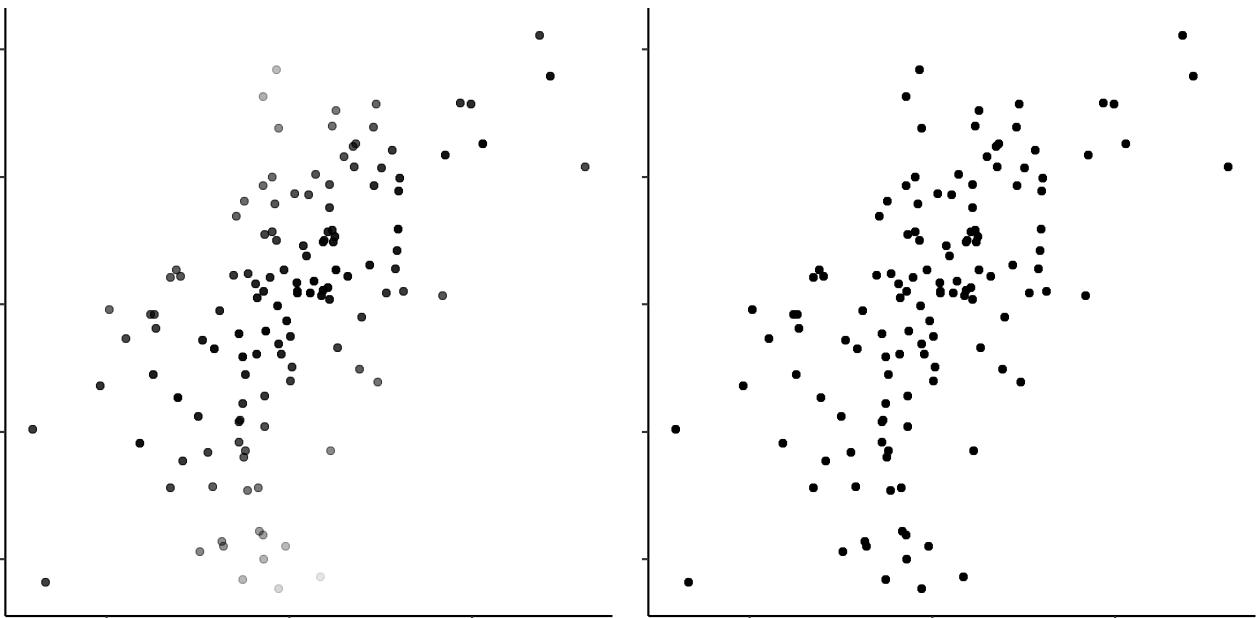


Figure 4.1. Examples of the experimental stimuli used in the pilot study (opacity decay factor). On the left, the opacity decay function is visible. Note the linear scaling used.

in lay people's daily lives due to the COVID-19 pandemic, and the increasing adoption of wearable devices [147]. The first experiment conducted therefore examined how perceptions of correlation changed according to the size of a scatterplot. Additionally, a very early version of the opacity decay factor from Experiment 2 was included, however the implementation of this factor was immature. In Experiment 2 onwards, if a scatterplot point resided in a particular place on a scatterplot, it would always have the same opacity or size. In the pilot study, the code that set the opacity of each point always scaled the opacity values such that the point with the highest residual had the lowest possible opacity, and vice versa, resulting in the plots seen in Figure 4.1.

This provided no consistency between different experimental stimuli, making it difficult to comment on the effects of changing levels of opacity in various parts of a plot on correlation estimation. This was later addressed by hardcoding residual size to a specific value of opacity or size. The pilot also suffered extensively from poor data quality. Of the 260 participants tested, data from only 118 was included in the final analyses due to failed attention checks. It is for this reason that the pre-screen requirements detailed in Section 3.2.3 were implemented.

Participants viewed 180 experimental plots in a 3x2 factorial design. The first independent variable, plot size, had three levels, 63%, 100%, and 252% scales. The second was the presence or absence of the opacity decay function (see Figure 4.1). I aimed to recruit 150 participants, but stopped after 118 due to ongoing data quality issues. Nevertheless, the results provided were crucial in informing the future direction of the research project. I present these results in brief below.

Table 4.1. Estimated Marginal Means (EMMs) of correlation estimation error for plot size and the presence of the opacity decay function.

(a) EMMs for plot size factor.			(b) EMMs for presence or absence of opacity decay function.		
Size	Mean	Standard Error	Decay	Mean	Standard Error
Large (252%)	0.12	0.014	Absent	0.13	0.015
Medium (100%)	0.12	0.014	Present	0.11	0.013
Small (62%)	0.13	0.014			

Table 4.2. Contrasts between levels of the size factor and opacity decay factor.

(a) Contrasts for size decay factor.		(b) Contrasts for opacity decay factor.					
Contrast		Statistics		Contrast		Statistics	
		Z ratio	p			Z ratio	p
Large (252%)	Medium (100%)	-0.94	0.618	Absent	Present	3.65	<0.001
Large (252%)	Small (52%)	-3.56	0.001				
Medium (100%)	Small (52%)	-2.63	0.023				

4.2.1 Pilot Study: Results

To investigate the effects of plot size and the presence or absence of an opacity decay manipulation on participants' estimates of correlation, a linear mixed effects model was built whereby participants' errors in correlation estimation were predicted by plot size and the presence or absence of the opacity decay function. This model features random intercepts for participants and items, as well as random slopes for both participants and items relevant to the presence or absence of the opacity decay function. A likelihood ratio test between the experimental model and a null model with the fixed effects removed revealed that the experimental model explained significantly more variance than the null ($\chi^2(3) = 26.38, p < .001$). There was no interaction between plot size and the presence or absence of the opacity decay function. The `emmeans` [87] package was used to explore estimated marginal means (see Table 4.1) and contrasts (see Table 4.2) separately for each factor.

4.2.2 Pilot Study: Discussion

For the factor of plot size, the effect observed was driven by significant differences in correlation estimation error between large and small plots and between medium and small plots. There were no significant differences in correlation estimation performance between large and medium plots. Participants estimated more accurately when the plot was large and when the decay function was present. Participants still underestimated correlation in all conditions. The finding that estimation error was lower for larger plots is in line with previous evidence that geometrically scaling a scat-

terplot up can increase perceptions of the strength of the correlation displayed [173]. Despite the statistical significance of this finding, we elected at this point to abandon the plot size factor due to the extremely small effect (see Table 4.1) and lack of novelty compared to the effects of the opacity decay function.

The impact of even an immature point opacity decay function on correlation estimation was a novel finding that I felt deserved further, and more rigorous, study. Its implementation was based on findings that changing the opacities of scatterplot points could bias estimates of means [70], and on limited evidence for the perception of correlation being based on the perceived width of a probability distribution represented by the arrangement of scatterplot points. I did not foresee the decay function, being novel, having a greater effect on correlation estimation than the established effect of plot size. Once evidence had been found that changing the opacity of points in scatterplots could have effects on correlation estimation, in opposition to previous research [131, 133], the door was opened for a rigorous investigation into how this worked and how it could be used systematically to correct for the historic underestimation bias.

4.3 Introduction

Findings from the pilot study suggest that changing the opacities of points in scatterplots is able to change participants' estimates of the correlation being displayed. The effect found in that study was too small to make a real difference with regards to correcting for the underestimation bias, and does not provide information on *how* changing opacity might change the percept (only that *it does*). Failing to understand the ways in which opacity is able to change the perception of correlation prevents future work from tuning what was a small effect in the pilot study into something with real potential for producing more perceptually optimised scatterplots.

4.3.1 Overview

In two experiments, the opacities of points in scatterplots were manipulated while participants were asked to make judgements of correlation. In the first, point opacity is changed in a uniform manner, while in the second, point opacity is systematically altered as a function of the size of a particular point's residual. By comparing participants' performance on a correlation estimation task for data-

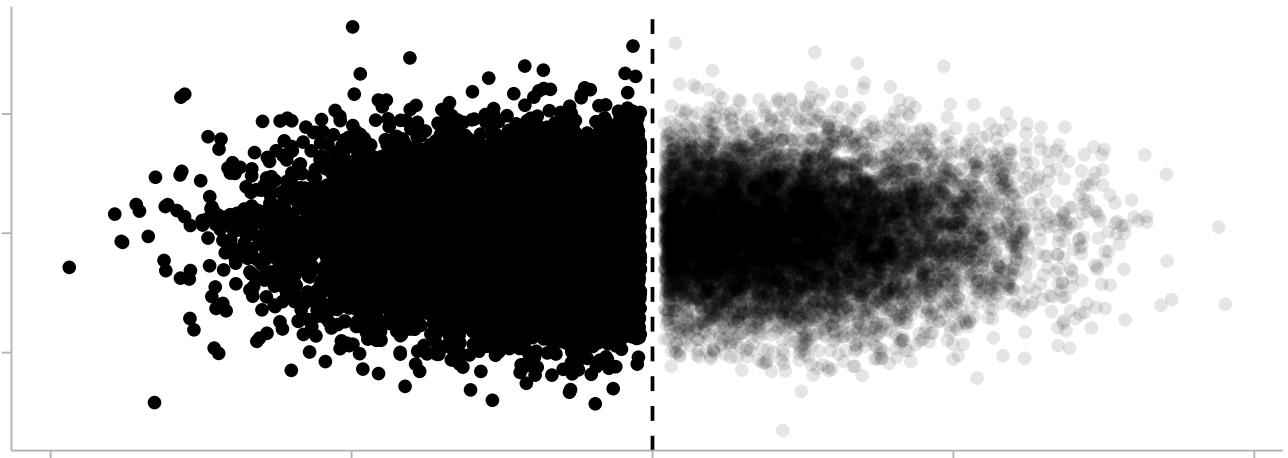


Figure 4.2. Adjusting point opacity to address overplotting. Contrast between the points and the background is full (alpha = 1, full opacity points, left) or low (alpha = .1, low opacity points, right). The dataset used has 20,000 points.

identical scatterplots that vary only in the opacities of their points, it is demonstrated that; lower global point opacity results in greater errors in the estimation of positive correlation (Experiment 1); and lowering point opacity as a function of the size of a point’s residual is able to bias estimates of positive correlation upwards to partially correct for a historic underestimation bias (Experiment 2).

4.4 Related Work

4.4.1 Transparency, Contrast, Opacity, and Formal Definitions

The original paper that this chapter is based on is titled “The Effects of Contrast on Correlation Perception in Scatterplots”. In response to reviewer comments to the paper that forms Chapter 6, the term “contrast” was changed to “opacity”. In order to maintain consistency throughout this thesis, the more up-to-date wording (opacity) is used, although I discuss the issue of terminology below.

Adjusting the opacity of points in scatterplots is an established technique used to address issues of overplotting or clutter [15, 93], in which scatterplots with large numbers of data points suffer from visibility issues caused by excessive point density. Lowering the opacity of all scatterplot points using alpha blending [43] addresses this, and makes data trends and distributions easier to see and interpret for the reader. Figure 4.2 demonstrates the impact of lowering global point opacity in a scatterplot with a very high number of data points.

Lowering opacity leads to a reduction of the contrast of isolated points with the background, and for regions with overlapping points, colour intensities are summed. The stimuli used in the experiments



Figure 4.3. The relationship between alpha values and rendered point opacity. Higher alpha values result in greater contrast between the foreground (scatterplot point) and background. When alpha = 0, the foreground is ignored and the background is rendered.

throughout this chapter had 128 small points, meaning the majority of points were clearly visible at all times. For this reason, the effects of point overlap were not taken into account when designing and analysing the experiments described here. Due to this, the approach to opacity described in this chapter would not be useful when dealing with much larger datasets where clutter becomes an issue.

The `ggplot2` [174] package (version 3.4.1) was used in R to create stimuli for this experiment. This package uses an alpha parameter to set point opacity. Alpha here refers to the level of linear interpolation [153] between foreground and background pixel values; alpha values of 0 (full transparency) and 1 (full opacity) result in no interpolation and rendering of either the background or foreground pixel values respectively. Alpha values between 0 and 1 correspond to different ratios of interpolation, and are illustrated in Figure 4.3.

Definitions of contrast, opacity, and transparency are fuzzy. Often, different works will use the terms interchangeably. As mentioned above, I initially elected to use the term “contrast”, given the fundamentality of contrast as a feature of human visual perception [56], however later reviewer comments prompted the change to “opacity”. Nevertheless, we can consider *opacity* as it is used here when pertaining to scatterplot points on a white background to be shorthand for “the contrast between foreground and background objects”, as visually, these concepts are the same. There are numerous psychophysical definitions of perceived contrast [182] based on what is being presented, for example, models that take into account visibility limits (CIELAB lightness), or contrast in periodic patterns such as sinusoidal gratings (Michelson’s contrast). The common thread running through these definitions is the use of a ratio between target and background luminances. The experiments described here take place online, with participants completing experiments on their personal laptop or desktop computers. Due to this, the experimenter has no control over the exact luminances of stimuli, only over the relative luminance between targets (scatterplot points) and backgrounds. Given my interest in relative differences in correlation perception averaged over a series of 180 single-plot trials, this lack of control over the exact nature of the stimuli was not problematic. It does mean that reporting

the exact luminance values would be pointless however, so where a value for opacity is referred to in this chapter and beyond, it is the alpha value specified by ggplot2.

4.4.2 Effects of Point Opacity on Correlation Estimation

Despite the popularity of adjusting opacity to address overplotting issues, little investigation had taken place into the effects of reducing point opacity on people's perceptions of correlation. In 2012, Rensink [131] found correlation perception to be invariant to changes in point opacity, although this work took place with a small sample ($N = 12$), and using bisection/JND methodologies (see Section 2.3.3 in Chapter 2).

Changing the contrast between a stimulus and its background (lowering its opacity) effectively reduces the strength of its signal. A likely consequence of this is greater levels of uncertainty in aspects of that stimulus, for example, the locations of points in scatterplots. Consequently, one might anticipate that increased uncertainty could lead to altered perceptions of correlation and/or the presence of greater levels of noise in correlation estimates due to effects on the perceived position of points within a scatterplot point cloud. While there is evidence [172] that perception of stimulus position becomes exponentially worse as contrast is reduced (as measured by vernier acuity tasks), this is only true for a narrow range of low contrast stimuli just above the detection threshold. For stimuli that feature higher contrasts between them and their backgrounds, vernier acuity appears robust to such changes. Nevertheless, there is evidence that other perceptual estimates become more uncertain with reduced contrast, such as speed perception [26]. With this in mind, I argue that the effects of stimulus opacity on perceived correlation in scatterplots warrants further investigation.

In 2022, Hong et al. [70] used point opacity and size to encode a third variable in trivariate scatterplots while asking participants to judge the average position of all the points displayed. It was found that participants' estimates of average point position were biased towards areas of larger or darker points; this was termed the *weighted average illusion*. Together with evidence that darker (more opaque) and larger points are more salient [61], this work suggested that there was the scope to use point opacity to systematically lower the salience of the points representing the widest parts of the probability distribution; if participants perceived a narrower distribution, one might expect this to be able to (at least partially) correct for the underestimation bias.

One way to correct for an underestimation of correlation in scatterplots would be to simply remove

outer data points until correlation perception is aligned with the actual correlation value. However, this would necessitate hiding data and thus changing the information presented to the viewer. An alternative approach is to manipulate the opacity of only some of the points; it would seem most sensible to do so for the points that are more extreme relative to the underlying regression line. In the present study these questions are explored in two online experiments with large sample sizes. In the first, the effects of point opacity over the entire scatterplot on correlation estimates is investigated. The second experiment examines how changing contrast as a function of distance to the regression line affects perceived correlation. To pre-empt the results, clear effects of both manipulations are found.

4.5 General Methods

The experiments described in this chapter share multiple aspects of their procedures. Both experiments were built using PsychoPy [121] and are hosted on Pavlovia.org. Both use 1-factor, 4-level designs. Ethical approval for both experiments was granted by the University of Manchester’s Computer Science Departmental Panel (Ref: 2022-14660-24397). In each experiment, participants were shown the respective Participant Information Sheet (henceforth PIS) and provided consent through key presses in response to consent statements. Participants were asked to provide their age and gender identity, after which they completed the 5-item Subjective Graph Literacy test described by Garcia-Retamero et al. [54] and discussed in Section 2.7 of Chapter 2. Early piloting with a graduate student in humanities suggested the potential for participants to be unfamiliar with the visual nature of different values of Pearson’s r . Participants were therefore shown examples of $r = 0.2, 0.5, 0.8$, and 0.95 (see Figure 4.4); a discussion of the effects of this training is provided in Section 4.8.1. Participants were given two practice trials to familiarise themselves with the response slider.

Each experimental trial was preceded by text that either told the participant:

- Please look at the following plot and use the slider to estimate the correlation ($n = 180$).
- Please IGNORE the correlation displayed and set the slider to 1 ($n = 3$) or 0 ($n = 3$).

The latter instructions were attention checks, and were formatted with red text to increase their visibility. Each experimental trial was preceded by a visual mask (see Figure 4.5) that was displayed for 2.5 seconds. Participants were instructed to make their judgements as quickly and accurately as

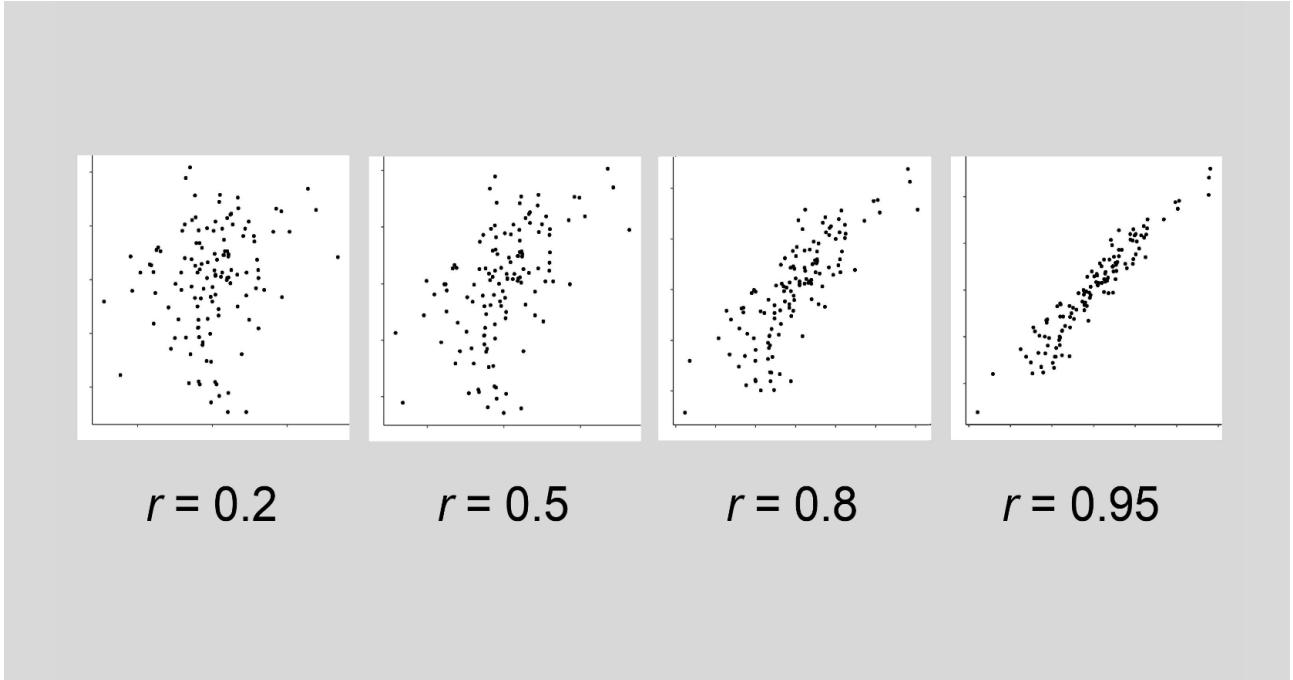


Figure 4.4. Participants viewed these plots for at least eight seconds before being allowed to continue to the practice trials.

possible, but there was no time limit per trial. Both experiments described here use a fully repeated-measures, within-participants design. All 150 participants saw all 180 experimental items, corresponding to $\sim 27,000$ individual judgements per experiment, in a fully randomised order.

4.5.1 Open Research

Both experiments were conducted according to principles of open and reproducible research [9]. All data and analysis code for the original paper are available on GitHub¹. This repository also includes a Docker implementation to reproduce the original computational environment the paper was written in. Experiment 1² and 2³ are hosted on Pavlovia.org, while the Open Science Framework hosts pre-registrations⁴. It is important to note at this point that Experiment 2 was conducted prior to Experiment 1; when the original paper was written, the order of presentation of the experiments was swapped to make the narrative more cohesive. I preserve this order in the present chapter.

¹https://github.com/gjstrain/contrast_and_scatterplots

²https://gitlab.pavlovia.org/Strain/exp_uniform_adjustments

³https://gitlab.pavlovia.org/Strain/exp_spatially_dependent

⁴Experiment 1 - <https://osf.io/tuexh>. Experiment 2 - <https://osf.io/6f5ev>

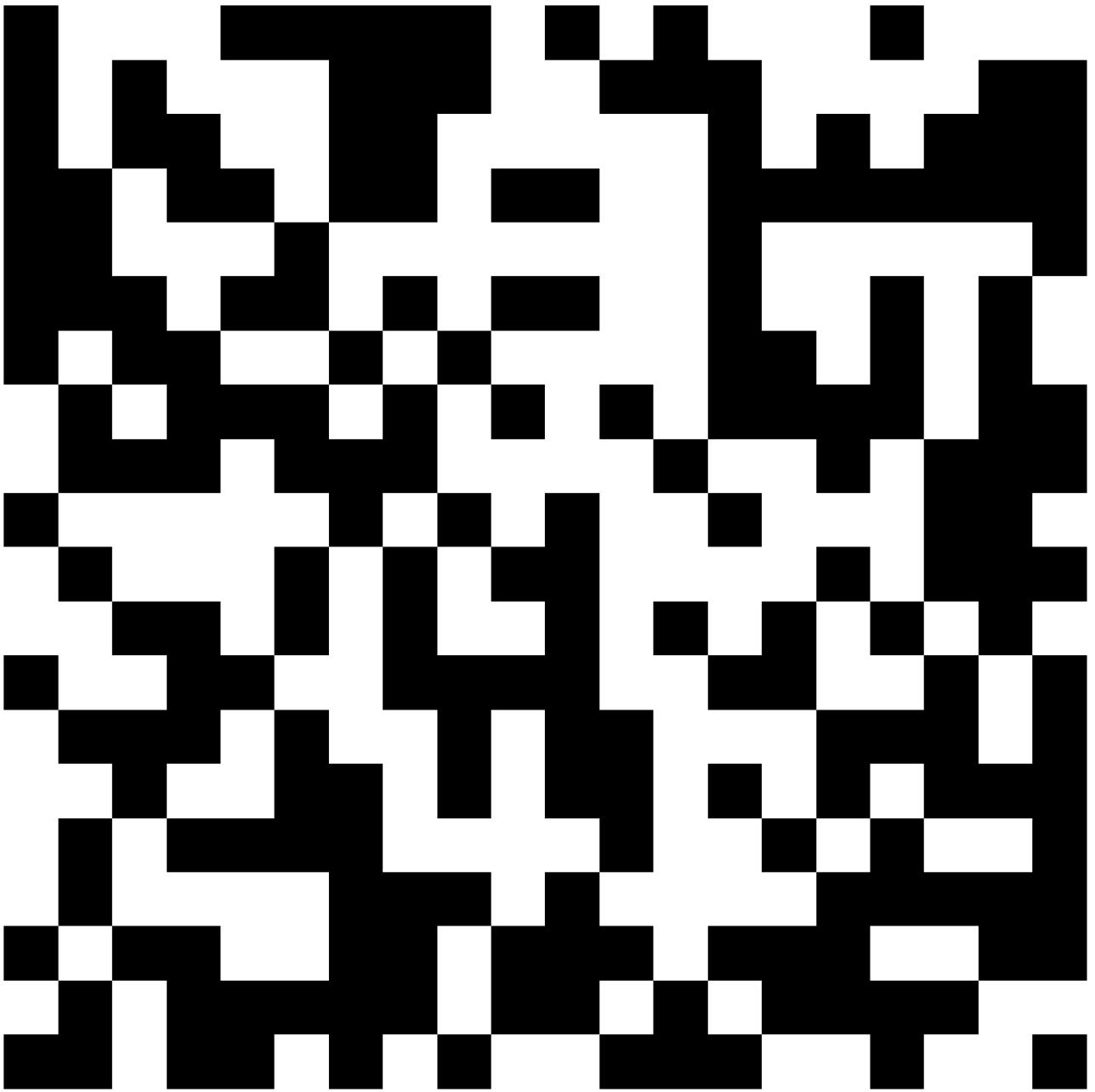


Figure 4.5. An example of a visual mask displayed for 2.5 seconds before each experimental trial.

4.6 Experiment 1: Uniform Opacity Adjustments

4.6.1 Introduction

Previous literature had described correlation perception as being resistant to changes in opacity [131, 133]. Findings from the pre-study described above provided evidence against this conclusion, so before proceeding with the fine-tuning of the immature point opacity decay function, I felt that gaining an understanding of how point opacity and correlation estimation interact more generally was important. Owing to the robust effects of altering stimulus opacity on perception described above [26, 172], it was hypothesised that:

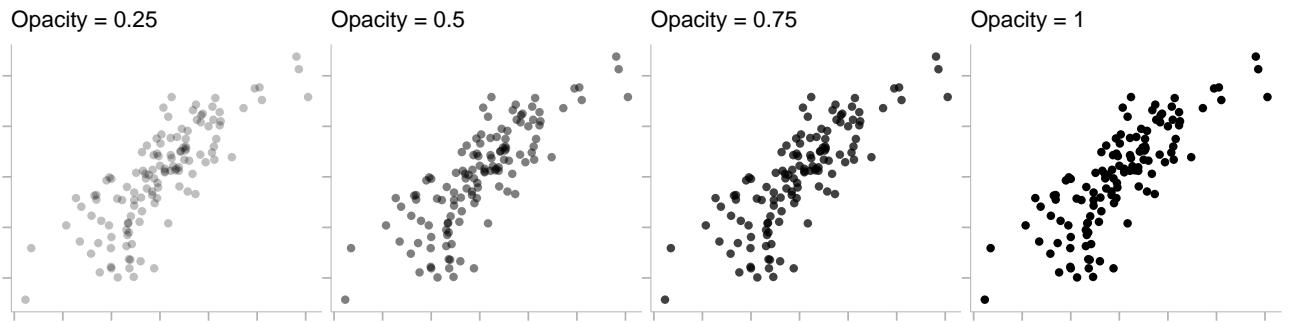


Figure 4.6. Examples of the stimuli used in Experiment 1, demonstrated with an r value of 0.8. Here, “opacity” refers to the alpha value used by ggplot2.

- H1: A greater spread of estimates of correlation for plots with lower global opacity compared to higher opacity plots will be observed.

4.6.2 Method

Participants

150 participants were recruited using the Prolific platform [129]. Normal or corrected-to-normal vision and English fluency were required. Participants who had completed the pilot study were prevented from participating. Data were collected from 158 participants. 8 failed more than 2 out of 6 attention check questions, and, as per the pre-registration, had their submissions rejected from the study. The data from the remaining 150 participants were included in the full analysis (76 male, 71 female, and 2 non-binary). Participants’ mean age was 28.29 ($SD = 8.59$). Mean graph literacy score was 21.79 ($SD = 4.47$). The mean time taken to complete the experiment was 33 minutes ($SD = 10$ minutes).

Design

For each of the 45 r values, there were four versions of each plot corresponding to the four levels of point opacity. Examples of each of these can be seen in Figure 4.6, demonstrated with an r value of 0.8.

4.6.3 Results

To investigate the effects of opacity condition on participants’ estimates of correlation, a linear mixed effects model was built whereby opacity condition is a predictor for the difference between objective

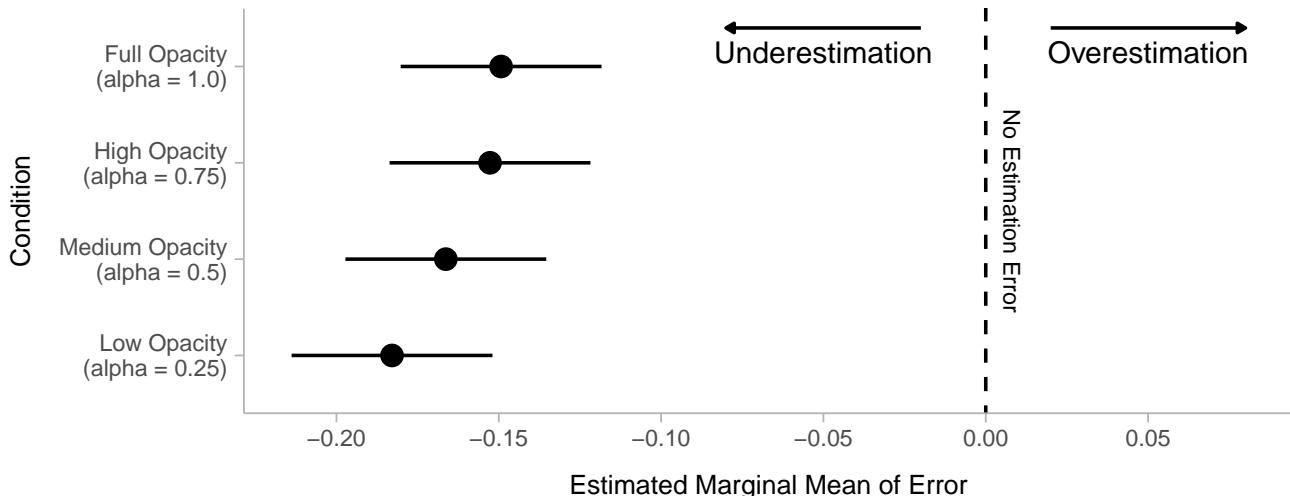


Figure 4.7. Estimated marginal means for the four conditions tested in Experiment 1. 95% confidence intervals are shown. The vertical dashed line represents no estimation error. The overestimation zone is included to facilitate comparison to later work.

Table 4.3. Contrasts between different levels of the opacity factor in Experiment 1.

Contrast	Statistics	
	Z ratio	p
Full Opacity (alpha = 1.0) vs High Opacity (alpha = 0.75)	-1.363	0.523
Full Opacity (alpha = 1.0) vs Medium Opacity (alpha = 0.5)	-6.809	<0.001
Full Opacity (alpha = 1.0) vs Low Opacity (alpha = 0.25)	-13.439	<0.001
High Opacity (alpha = 0.75) vs Medium Opacity (alpha = 0.5)	-5.443	<0.001
High Opacity (alpha = 0.75) vs Low Opacity (alpha = 0.25)	-12.071	<0.001
Medium Opacity (alpha = 0.5) vs Low Opacity (alpha = 0.25)	-6.631	<0.001

r values for each plot and participants' estimates of r . This model has random intercepts for items and participants. A likelihood ratio test revealed that the model including global opacity as a fixed effect explained significantly more variance than a null model ($\chi^2(3) = 223.13, p < .001$). Figure 4.7 shows the mean errors in correlation estimation for each opacity condition, along with 95% confidence intervals.

This effect was driven by significant differences between means of correlation estimation error between all conditions bar high and full opacity. Statistical tests for contrasts were performed using the `emmeans` package [87], and are shown in Table 4.3. To test whether the observed results could be explained by difference in participants' levels of graph literacy, an additional model was built. This model is identical to the experimental model, but also includes graph literacy as a fixed effect. Including graph literacy as a fixed effect explained no additional variance ($\chi^2(1) = .002, p = .962$), indicating that the differences observed in participants' correlation estimation performance were not as a result of differences in levels of graph literacy.

A function from the now archived `EMAtools` package [77] was used to calculate an approximation of

Table 4.4. Cohen's d effect sizes and summary statistics for levels of the opacity factor in Experiment 1. Each effect size is compared to the reference level, full contrast (alpha = 1).

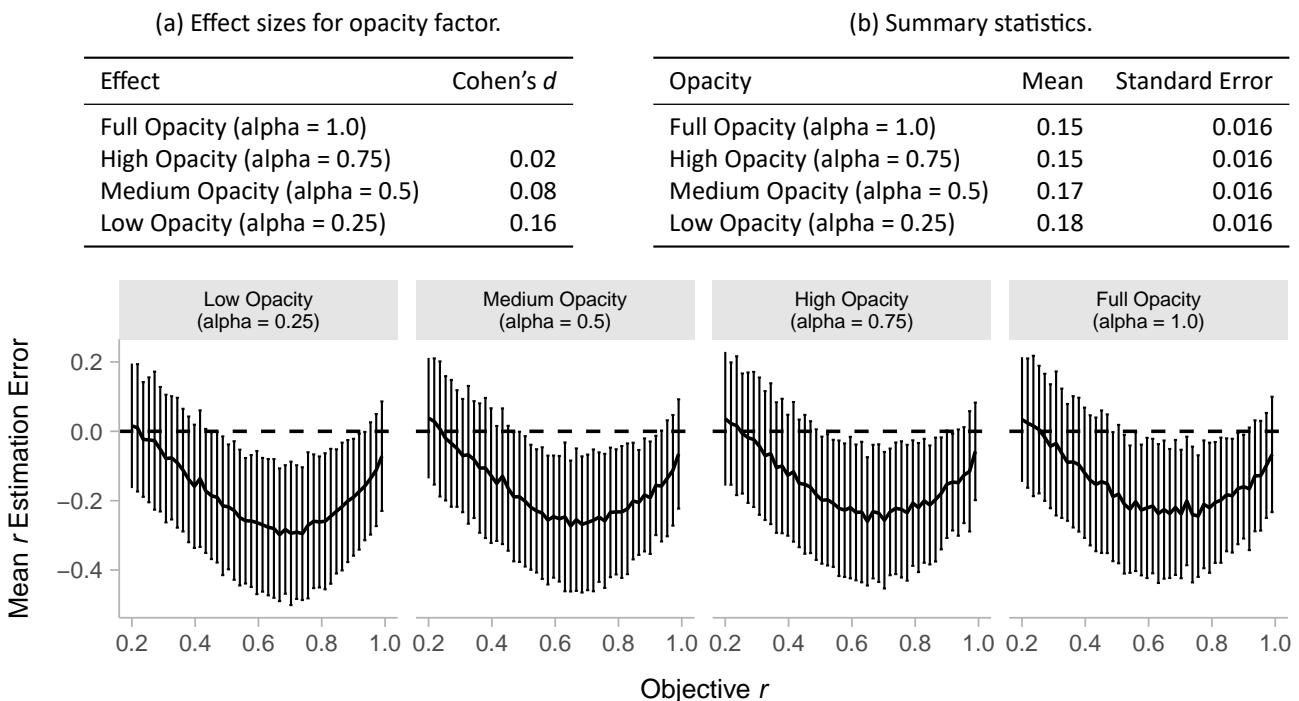


Figure 4.8. Participants' mean errors in correlation estimates grouped by factor and by r value. The dashed horizontal line represents perfect estimation. Participants were most accurate when presented with the plots featuring higher global point opacity.

Cohen's d between the reference level (full contrast, alpha = 1.0) and each other level of the opacity factor. These statistics can be seen in Table 4.4 (a) along with means and standard deviations (b). The largest effect size observed ($d \sim 0.16$) is between the low and full opacity conditions, and is small. This was unsurprising given the lack of previously reported effects on correlation perception of global point opacity [131]. Figure 4.8 shows how participants' estimates of correlation change with the objective r value in the plot. The line represent mean errors in correlation estimation, and standard deviations of error are provided as error bars. The dashed horizontal line represents hypothetical perfect estimation. As reported in previous literature (see Section ?? in Chapter 2), participants underestimated r in nearly all cases.

4.6.4 Discussion

The hypothesis, that there would be a greater spread of correlation estimates for plots with lower global opacity compared to those with higher global opacity, was not supported. As seen in Table 4.4 (b), standard errors for each opacity factor are identical to 3 decimal places. Participants' errors in correlation estimation were significantly greater when the opacity of all scatterplot points was lower compared when it was higher. This held true up until alpha was set to 0.75, implying a threshold

around this value past which there is little variation in the perception of opacity, at least as far as it is associated with correlation estimation. This lack of significant difference in correlation estimation between the two highest global opacity conditions is congruent with the logarithmic nature of contrast/brightness perception [42, 168]; despite there being equal linear distance between the opacity values used, the perceptual distance between them is minimal.

As mentioned previously, Rensink [131, 133] presents the only other account of experiments that directly test correlation perception as it pertains to the opacities of scatterplot points, and report no difference in either bias (error) or variability (spread) in correlation perception regarding point opacity manipulations. In comparison, the results observed here do report an effect. This effect may be explained by differences in experimental power, as it is a small effect, although I argue that methodological differences may have also played a small role. Given the small effect size (Cohen's $d = 0.16$), the small sample in Rensink (2014) [133] may have caused the experiment to be insufficiently powered. With the large sample size in the present work ($N = 150$), evidence for an effect has been found. The experimental methodology utilised here is more representative of the use of scatterplots in the wild, and is therefore more suited to informing design as opposed to investigating the mathematical relationship between real and perceived correlation. While the effect is small, it demonstrates definitively that differences in the opacities of scatterplot points *can* affect estimates of correlation in positively correlated scatterplots. From the results it is unclear why lowering global opacity causes greater errors in correlation estimation while causing no difference in spread.

I suggest that correlation perception functions similarly to speed perception [26] with regards to changes in the contrast between foreground targets (in this case scatterplot points) and the background; the greater spatial uncertainty brought on by reduced point opacity, while not eliciting greater spread in correlation estimates, might be responsible for the effects observed via an increase in the perceived width of the probability distribution displayed by the scatterplot

From the results it is clear that a scatterplot optimised for correlation perception should have contrast between the foreground (points) and background in a range corresponding to alpha values of between 0.75 and 1. That there are significant differences in correlation estimation between data-identical scatterplots with different global point opacities however, suggests that this effect may be leveraged to further improve participants' performances on a correlation estimation task.

4.7 Experiment 2: Spatially-Dependent Opacity Adjustments

4.7.1 Introduction

Experiment 1 found that point opacity in positively correlated scatterplots has an effect on the perception of correlation such that those scatterplots with higher levels of global point opacity are rated as being more strongly correlated. Given this finding, the question arises of whether additional changes in correlation perception may be observed as a function of the spatial arrangement of point opacity. Given the previously reported effects of changing scatterplot point opacity on other perceptual metrics (see Section 4.4.2), and with the findings from Experiment 1 in mind, it was hypothesised that:

- H1: the non-linear decay parameter in which point opacity falls with residual distance will result in lower mean errors in correlation estimation compared to linear decay and full global opacity conditions.
- H2: the use of the inverted non-linear decay parameter, in which point opacity becomes greater with residual distance, will result in higher mean errors in correlation estimation than for all other conditions.

4.7.2 Method

Participants

150 participants were recruited using the Prolific platform [129]. Normal to corrected-to-normal vision and English fluency were required. Participants who had completed the pilot study or Experiment 1 were prevented from participating. Data were collected from 158 participants. 8 failed more than 2 out of 6 attention check questions, and, as per the pre-registration, had their submissions rejected from the study. The data from the remaining 150 participants were included in the full analysis (77 male, 69 female, and 4 non-binary). Participants' mean age was 27.05 ($SD = 7.37$). Mean graph literacy score was 21.71 ($SD = 4.06$). The average time taken to complete the experiment was 33 minutes ($SD = 10$ minutes).

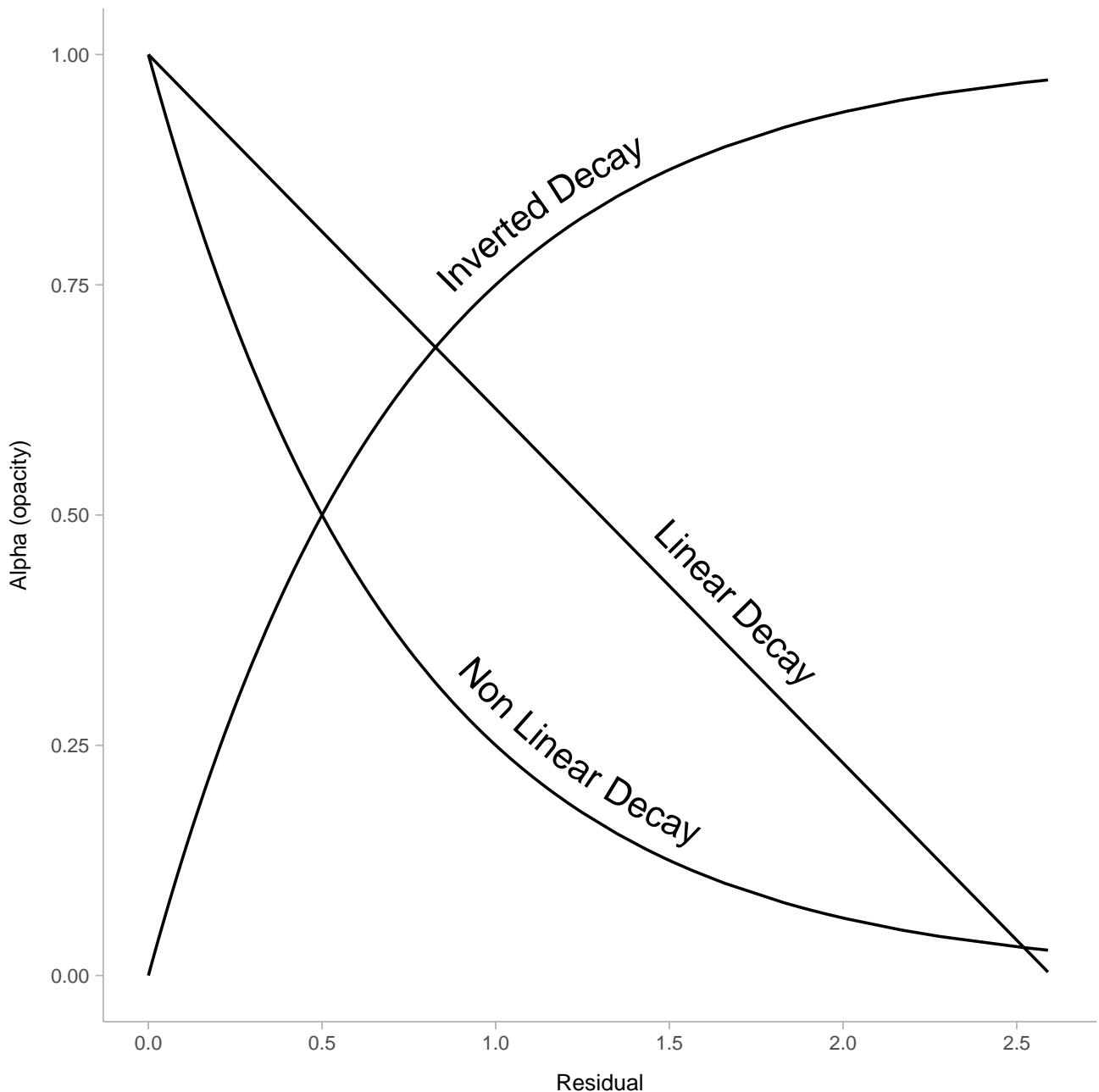


Figure 4.9. Using an r value of 0.2 to demonstrate the relationship between the size of a point's residual and the alpha value (opacity) rendered.

Design

For each of the 45 r values in Experiment 2, there were four versions of each plot corresponding to the three levels of point opacity decay function and the baseline global full opacity condition. Examples of each of these can be seen in Figure 4.10, demonstrated with an r value of 0.8. Given the shape of the underestimation curve found in previous work (see Figure 2.13, Chapter 2), intuition suggested employing a symmetrically opposing curve (see the non-linear decay curve in Figure 4.9) to relate point opacity to residuals.

Equation 4.1 was used to non-linearly map residuals to ggplot2 alpha values. 0.25 was chosen as

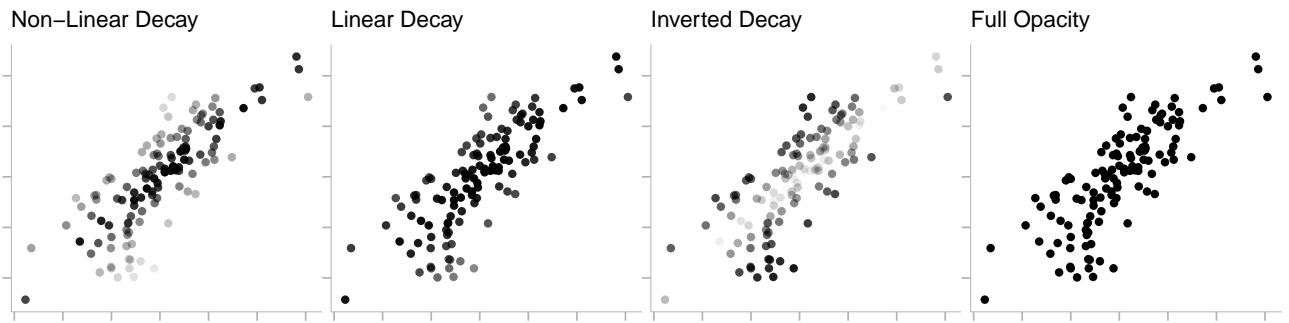


Figure 4.10. Examples of the stimuli used in Experiment 2, demonstrated with an r value of 0.8. Here, “opacity” refers to the alpha value used by ggplot2.

Table 4.5. Contrasts between different levels of opacity decay function in Experiment 2.

Contrast		Statistics	
		Z ratio	p
Full Opacity	Inverted Decay	-18.9	<0.001
Full Opacity	Linear Decay	1.6	0.405
Full Opacity	Non-Linear Decay	15.3	<0.001
Inverted Decay	Linear Decay	20.4	<0.001
Inverted Decay	Non-Linear Decay	34.2	<0.001
Linear Decay	Non-Linear Decay	13.7	<0.001

the value of b , as it was felt at the time that this rendered plots that maintained point visibility while also allowing a large enough point opacity range that, if an effect was present, it was likely to be found.

$$\text{point}_{\text{size}/\text{opacity}} = 1 - b^{\text{residual}} \quad (4.1)$$

4.7.3 Results

To investigate the effects of the opacity decay functions on participants’ estimates of correlation, a linear mixed effects model was built whereby decay function condition is a predictor for the difference between objective r values for each plot and participants’ estimates of r . This model has random intercepts for items and participants. A likelihood ratio test revealed that the model including opacity decay function as a fixed effect explained significantly more variance than a null model ($\chi^2(3) = 1,157.62, p < .001$). Figure 4.7 shows the mean errors in correlation estimation for each opacity decay function condition, along with 95% confidence intervals.

The effect seen in Experiment 2 was driven by significant differences in means of correlation estimation error between all levels of opacity decay function condition bar full opacity and linear decay.

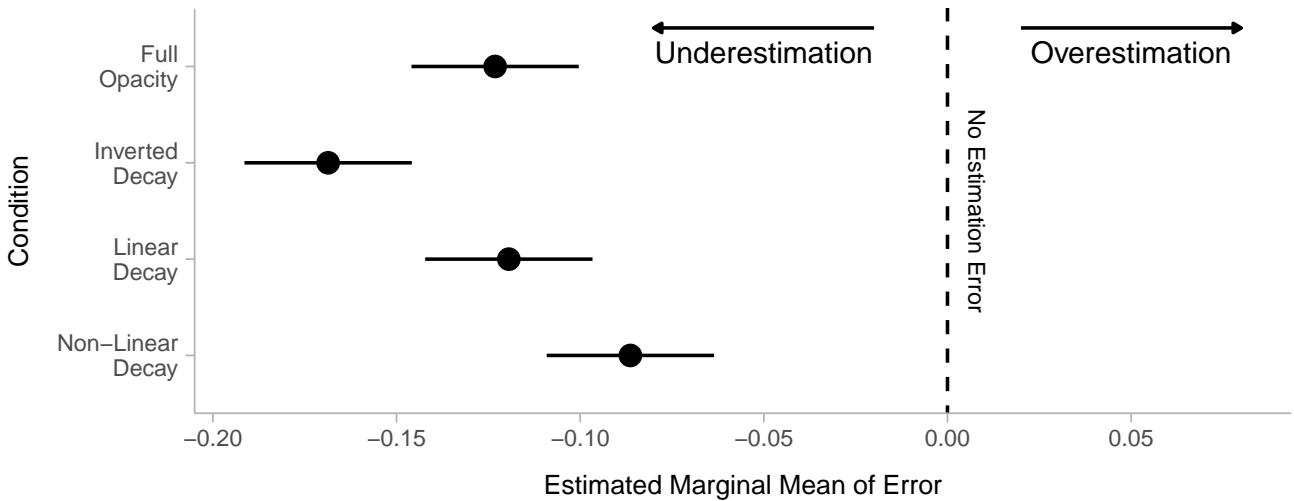


Figure 4.11. Estimated marginal means for the four conditions tested in Experiment 2. 95% confidence intervals are shown. The vertical dashed line represents no estimation error.

Table 4.6. Cohen's d effect sizes and summary statistics for the opacity decay function factor in Experiment 2. Each effect size is compared to the reference level, full contrast ($\alpha = 1$).

(a) Effect sizes for opacity decay factor.

Effect	Cohen's d
Full Opacity	
Inverted Decay	0.23
Linear Decay	-0.02
Non-Linear Decay	-0.19

(b) Summary statistics for opacity decay factor.

Opacity	Mean	Standard Error
Full Opacity	0.12	0.012
Inverted Decay	0.17	0.012
Linear Decay	0.12	0.012
Non-Linear Decay	0.09	0.012

Statistical testing for contrasts was performed using the `emmeans` package [87], and are shown in Table 4.5. To test whether the observed results could be explained by differences in graph literacy, a model including participants' graph literacy scores as a fixed effect was built. Including graph literacy as a fixed effect again explained no additional variance ($\chi^2(1) = .242, p = .623$).

Approximated Cohen's d effect sizes between the baseline (global full opacity) and each other condition can be seen in Table 4.6. The largest effect size observed (~0.23 between full opacity and inverted decay conditions) is small to moderate, and the effect size between the baseline and the non-linear decay condition (~0.19) is small. Figure 4.12 illustrates the effects of each manipulation on participants' correlation estimation performance separately for each value of r used. The dashed horizontal line represents perfect estimation, and standard deviations of estimation error are provided by way of error bars. Participants still underestimated correlation in all conditions, although the use of the non-linear decay function biased participants' estimates upwards to partially correct for the underestimation.

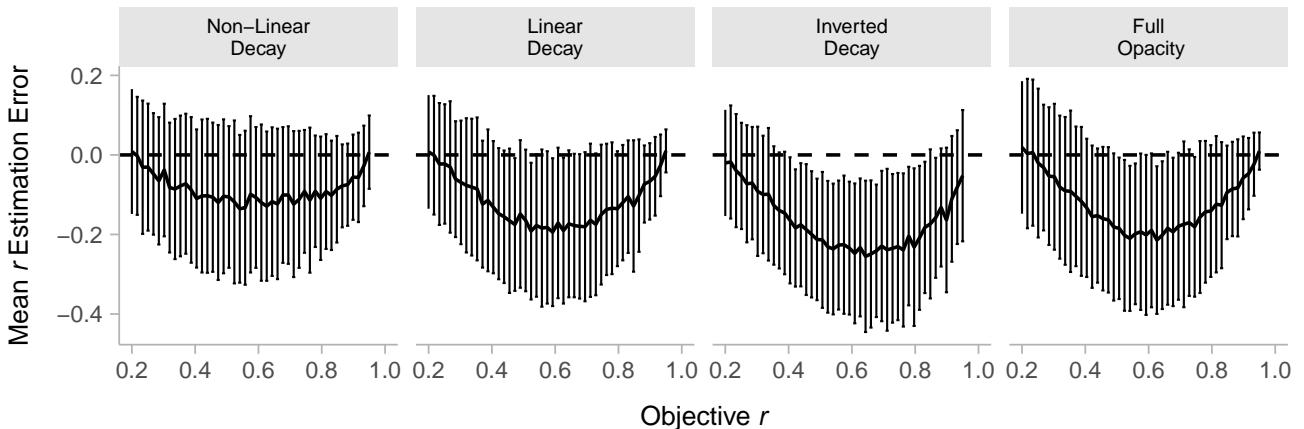


Figure 4.12. Participants' mean errors in correlation estimates grouped by factor and by r value. The dashed horizontal line represents perfect estimation. Participants were most accurate when presented with the plots featuring the non-linear opacity decay function. Error bars show standard deviations of estimates.

4.7.4 Discussion

Both hypotheses received support in Experiment 2. Participants' errors in correlation estimation were lowest when the non-linear decay function was used, and were highest when scatterplots employed the non-linear inverted decay function. There was no significant difference in correlation estimation errors between the linear decay function and full contrast conditions. This result was surprising, however on closer inspection of the scatterplots in the linear decay function condition, it is clear that the logarithmic nature of contrast perception [42, 168] means there was little perceptual distance between points with high opacity values ($\alpha > 0.75$). This resulted in no significant perceived differences between full opacity and linear decay function scatterplots. A similar threshold for high opacity values was found in Experiment 1. Selecting only lower r values, those with naturally higher residuals (arbitrarily $r < 0.6$), still results in no difference between correlation estimation errors for linear decay parameters and full opacity conditions ($\chi^2(1) = 0.09, p = .769$). Effect sizes were small, with the largest being between the baseline full opacity and inverted non-linear decay conditions. This suggests that it is easier to induce further bias in correlation estimates through a reduction in the salience of a point cloud's centre than it is to correct for the underestimation bias.

Looking at the standard deviations of correlation estimates plotted separately by opacity decay function and r value in Figure 4.12, it can be observed that as in Experiment 1 (see Figure 4.8), and aside from the inverted non-linear decay function condition, precision in r estimation increased with the objective r value. This finding corroborates previous work. In the inverted non-linear decay condition, as r approaches 1 (and point residuals accordingly approach 0), the opacity of points diminishes. Just as the standard deviation of correlation estimates was higher for the low global opacity condition

in Experiment 1, having lower opacity points at the high r end of the non-linear inverted condition in Experiment 2 resulted in fairly constant standard deviations across r values, as the usual reduction towards $r = 1$ was not observed.

The non-linear inverted opacity decay condition produced significantly lower estimates of correlation than all other conditions. This adds perspective to suggestions [180] that, among other visual features, the area of a hypothetical prediction ellipse [32, 180], a region used to predict new observations assuming a bivariate normal distribution (see Figure 4.13) is a better predictor of correlation estimation performance than the objective r value itself. In the inverted non-linear decay condition, the area of this ellipse remained the same, yet estimates of correlation differed significantly. These results suggest that the apparent density of the scatterplot point cloud also has effects on estimates of correlation. Prior work has found that more dense scatterplots are rated as having higher correlations [84, 133], although the effect found was weak. To explore this effect further, work investigating what people attend to when making correlation judgements must be completed. Eye-tracking offers an elegant solution to this problem, yet at the time of writing, had only been used for simpler scatterplot-related tasks, such as those asking participants to identify the number of, or distance between, points [109].

4.8 General Discussion

To summarise the results, evidence was found that changing the opacities of points in scatterplots can significantly alter participants' estimates of correlation, that lower global point opacity is associated with greater correlation underestimation error, that lowering point opacity as a function of residual error can partially correct for this underestimation, and that raising opacity as a function of residual error can increase the underestimation bias. These findings pave the way for using changes in point opacity to produce perceptually-optimised scatterplots that do not rely on data removal. As the focus of this work is on designing data visualisations that lay people are more easily able to interpret and understand, we used a large, representative sample, including people from a range of nationalities and educational backgrounds. This chapter demonstrates that a simple framework can be employed with these groups to gather high quality data, and, by design, produce conclusions that may generalise better to in-the-wild data visualisation usage.

In agreement with previous research [126, 131, 133–135], participants were more accurate and pre-

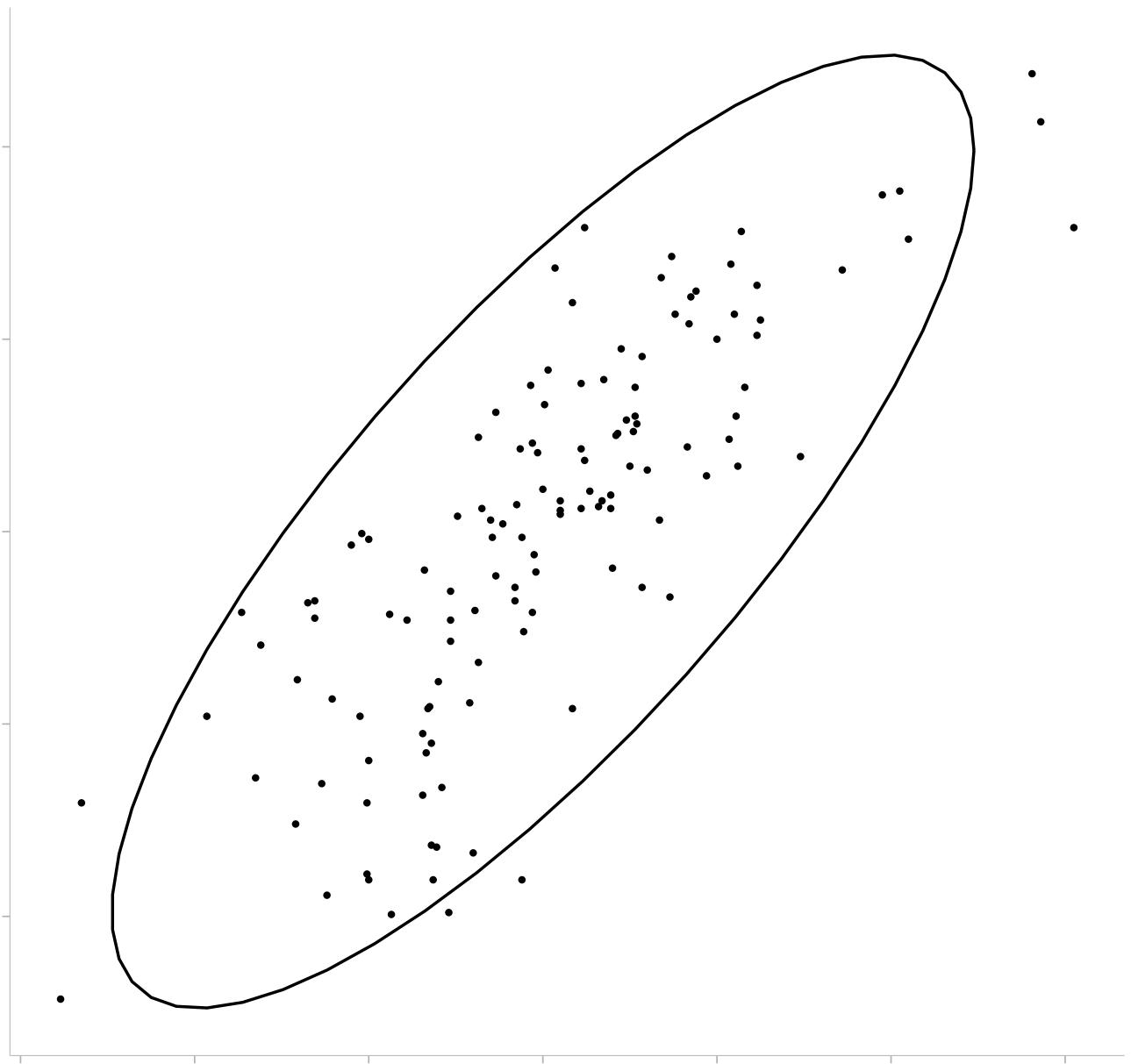


Figure 4.13. Plot showing a 95% prediction ellipse over a scatterplot with an r value of 0.8.

cise in their estimates of correlation when the r value shown in the scatterplot was higher. Figure 4.8 and Figure 4.12 plot objective r value against participants' errors in correlation estimation separately for each level of the respective independent variable. These plots illustrate, as in much previous work, the lower levels of precision and accuracy in correlation estimation that are seen for r values further from 0 or 1.

The results here contribute to a body of evidence that suggests participants are attending to the width of the probability distribution displayed in a scatterplot [32, 99, 134, 180] when making judgments of correlation. Further evidence is provided for the systematic underestimation bias, and a potential correction strategy is offered. This work does not attempt to redesign the scatterplot as a medium, but to provide a set of recommendations for designers based on the evidence; when designing positively correlated scatterplots to support correlation perception:

- Lowering the total opacity in a scatterplot can cause people to underestimate correlation compared to when contrast is maximal between the points and the plot background (when point opacity is high).
- The use of a non-linear opacity decay function, in which point opacity falls as a function of residual size, can be used to counteract the underestimation seen in correlation estimation in positively correlated scatterplots.

Scatterplots are widely used, and are often designed with a number of communicative concepts in mind. When one of these concepts is illustrating the degree of positive association between two variables, the findings presented suggest that designers should utilise the techniques described to give viewers the best chance of interpreting the correlation displayed as accurately as possible.

4.8.1 Training

Both experiments tested lay participants with varying levels of graph literacy. Due to concerns about participants' familiarity with scatterplots, each saw four scatterplots depicting correlations of $r = 0.2$, 0.5 , 0.8 , and 0.95 (see Figure 4.4) to familiarise themselves with the concept. To test if the patterns of results seen in correlation estimation could be attributed to this training, models were built that included the half of the session (first or second) as a predictor for participants' judgements of correlation. Comparing these models to the original revealed a significant effect in Experiment 1 ($\chi^2(1) = 7.60, p = 0.01$), but not Experiment 2 ($\chi^2(1) = 2.20, p = 0.14$). In Experiment 1, participants' errors in correlation estimation were higher in the second half of the experiment, suggesting that having more recently viewed the training material may have made participants more accurate. That this was not observed in Experiment 2 implies that the point opacity manipulation had a greater effect on estimates than any training effects.

4.8.2 Limitations

The results in Experiment 2 provide evidence that reducing the salience of points as they move further from the regression line can increase people's estimates of correlation, at least when plots like these are presented alongside conventional ones. Testing whether this phenomenon would exist with a plot in isolation would present a number of difficulties. As can be seen in Figure 4.8 and Figure 4.12, participants' estimates of correlation, especially between 0.2 and 0.7 , suffer from high variance. High

numbers of trials and participants ameliorate this to an extent, but this does prevent commentary on single plot judgements of correlation.

An important first step in the utilisation of point opacity changes to optimise perception of correlation in scatterplots has been made, however there remains much to do. Quantitative determination of whether 0.25 is indeed the most optimal value for b in equation 1, for example, is impossible from the present results. It may well be the case that changing the value of b as a function of the objective Pearson's r value could produce more accurate correlation estimation in participants; findings that participants were more accurate in correlation estimation when r was nearer 0 or 1 would suggest that the use of a decay parameter for these correlations is unnecessary.

The simplicity of the direct estimation task employed confers some limitations on the conclusions that can be drawn, although these limitations do not prevent the data gathered from being practically useful. The methods used to not allow for investigation of absolute correlation perception, as JND or bisection methodologies might. The finding that mean errors in judgements of correlation for full opacity conditions were different between Experiments 1 (0.149) and 2 (0.123) makes this limitation evident. In the experimental paradigm, participants indirectly made comparative correlation judgements, which may have resulted in this discrepancy. Nevertheless, the results found are promising with regards to design research.

4.8.3 Future Work

Future work may wish to investigate negative values of r , given findings that people may overestimate the correlation of negatively correlated scatterplots in a similar way [148]. The techniques presented here could be adapted to bias perceptions of correlation down and correct for this overestimation. The non-linear opacity decay function demonstrated here may be able to accomplish this.

The opacity decay function conditions in Experiment 2 used the vertical distance between a particular point and the regression line to set that point's opacity. Previous work [99] has suggested that the perpendicular distance between a point and the regression line may be a more accurate predictor of performance on correlation estimation tasks [32, 134, 180]. Future work exploring these manipulations further may wish to investigate whether there are differences between using perpendicular and vertical residual distance as bases for setting for opacity with regard to correlation estimation.

Finally, the most exciting avenue for related future work is the possibility of combining the techniques

developed and tested here with other novel scatterplot visualisation techniques that have a basis in the literature. Chapter 5 investigates the use of point size in place of opacity with the techniques described, while Chapter 7 combines the two to explore how these different modalities work together with regards to the estimation of positive correlation.

4.9 Conclusion

In a pair of experiments, I varied the opacity of point in scatterplots both uniformly (Experiment 1) and using functions relating point opacity to the size of a particular point's residual (Experiment 2). In Experiment 1, I showed that, in contradiction with previous work, changing the opacity of all points in a scatterplot can effect participants' performance on a correlation estimation task. In Experiment 2, I showed that varying point opacity as a function of a point's residual distance is able to significantly change participants' correlation estimates and partially correct for a long-standing underestimation bias.

Chapter 5

Adjusting the Sizes of Scatterplot Points Can Correct for a Historic Correlation Underestimation Bias

5.1 Abstract

Chapter 4 provided strong evidence for the effects of systematically varying the opacities of scatterplot points on participants' estimates of correlation in positively correlated scatterplots. Utilising the same function and experimental paradigm, I show in a single experiment that systematically varying the sizes of scatterplot points is able to bias participants' estimates of correlation to a greater degree than manipulations that only adjust point opacity. In a condition where point size decreases non-linearly as a function of residual distance, correlation estimation is significantly biased upwards to correct for the underestimation bias to a greater degree. I discuss the implications of these findings for the mechanisms behind both opacity and size adjustments in scatterplots in relation to correlation estimation, and recommend techniques for those who design with the estimation of positive correlation in mind.

5.2 Introduction

While I was successful at changing participants' perceptions of correlation in positively correlated scatterplots in Chapter 4, the extent to which these perceptions were changed was minimal. Figure

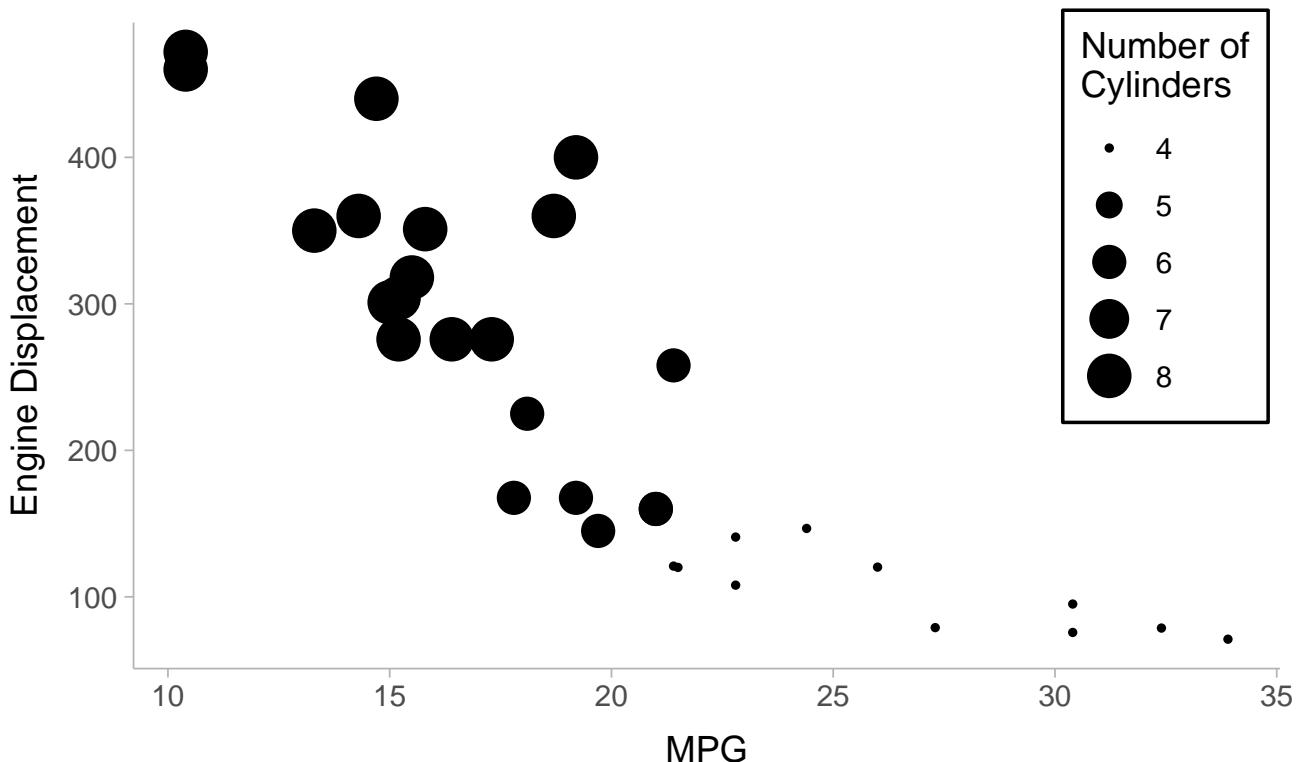


Figure 5.1. An example of a bubble chart. This plot compares car engine displacement (cubic inches) with fuel efficiency (miles per gallon). Additionally, the number of cylinders in the car’s engine are encoded with point size. One can see from this plot that vehicles with higher engine displacement and lower fuel efficiency tend to have a greater number of engine cylinders.

4.12 illustrates how participants’ mean errors in r estimation changed as a function of the subjective r value. Scatterplots employing non-linear opacity decay produced the most drastic changes in correlation estimation, however these changes were still small, with an effect size of Cohen’s $d = 0.19$. Recent evidence suggests that with regards to altering percepts in scatterplots, changes in point size may be more effective than changes in opacity. In a fully-reproducible, large sample ($N = 150$) study, I show that systematically altering point size using the same function is not only able to more effectively correct for the correlation underestimation bias, but is also able to alter the shape of the correlation estimation curve.

5.3 Related Work

5.3.1 Point Size and the Perception of Correlation in Scatterplots

Opacity adjustments in scatterplots have been used extensively to solve issues of overplotting and clutter in scatterplots [15, 93]. Figure 4.2 in Chapter 4 demonstrates this usage. Practicality also dictates that scatterplots visualising sufficiently large datasets inherently require their points to be

smaller to prevent obfuscation of the data they portray. Aside from being used to increase point visibility, point size changes in scatterplots have also been employed in the creation of bubble charts; here, the size of a scatterplot point is mapped to a third variable. Figure 5.1 demonstrates this class of scatterplot using the included `mtcars` dataset in `ggplot2`.

Despite the popularity of such charts, at the time of writing, very little investigation into how point size affects correlation perception in scatterplots had taken place. That which had been completed found bias and variability in correlation perception performance to be invariant to changes in point size [131, 133], however these studies were low-powered considering the small effect sizes found in the experiments described in Chapter 4. From the wider literature, there is evidence that larger points in scatterplots can bias judgements of mean point position to a greater degree than point opacity can [70], in what is termed the *weighted average illusion*. Outside of scatterplots and scatterplot-adjacent chart types, there is evidence that increased size can result in faster reaction times to peripherally presented stimuli [58], and there is evidence that larger stimuli are associated with lower levels of spatial certainty [2], but higher levels of salience [61]. The current experiment should therefore facilitate discrimination between these candidate drivers of the effects observed in a way that was not possible when manipulating opacity, as in that case effects of salience or spatial certainty would operate in the same direction.

5.4 Hypotheses

Based on the findings from Chapter 4, and on evidence from the wider literature described above, two hypotheses are made:

- H1: The non-linear decay parameter, in which scatterplots points become smaller as they move further from the regression line, will result in lower mean errors in correlation estimation than for linear decay and standard size conditions.
- H2: The use of an inverted non-linear decay parameter, in which scatterplot points become larger as they get further away from the regression line, will result in higher mean errors in correlation estimation than for all other conditions.

5.5 Method

5.5.1 Open Research

The experiment was conducted according to the principles of open and reproducible research. All data and code for the original paper are maintained in a GitHub repository¹. This repository also features an implementation of a Docker container that enables the full recreation of the computational environment the original paper was written in. The experiment itself is hosted on GitLab². The hypotheses and analysis plans were pre-registered with the Open Science Framework (OSF)³, and there were no deviations from them.

5.5.2 Stimuli

The creation of stimuli in this experiment follows the same general principles outlined in Section 3.2.4, Chapter 3, and was performed using `ggplot2` (version 3.4.4). As in Chapter 4, equation 5.1 was used to map point residuals to size values in the two non-linear decay conditions:

$$point_{size} = 1 - b^{residual} \quad (5.1)$$

Again, a value of $b = 0.25$ was used. As in Chapter 4, the use of this equation produces a curve around the identity line symmetrically opposing the underestimation curve found in previous work. Additionally, a constant of 0.2 was added to each raw size value, and a scaling factor of 4 was utilised; these adjustments resulted in the smallest points in the present experiment having a width of 12 pixels on a 1920×1080 pixel monitor, which is consistent with the point size used in the experiments described in Chapter 4. Examples of the stimuli used in this experiment can be seen in Figure 5.2.

5.5.3 Dot Pitch in Crowdsourced Experiments

When the experiments described in Chapter 4 took place, no method of obtaining dot pitch was implemented. Dot pitch is defined as the distance between the dots (sub-pixels) [25] that make up each

¹https://github.com/gjpstrain/size_and_scatterplots

²https://gitlab.pavlovia.org/Strain/exp_size_only

³<https://osf.io/k4gd8>

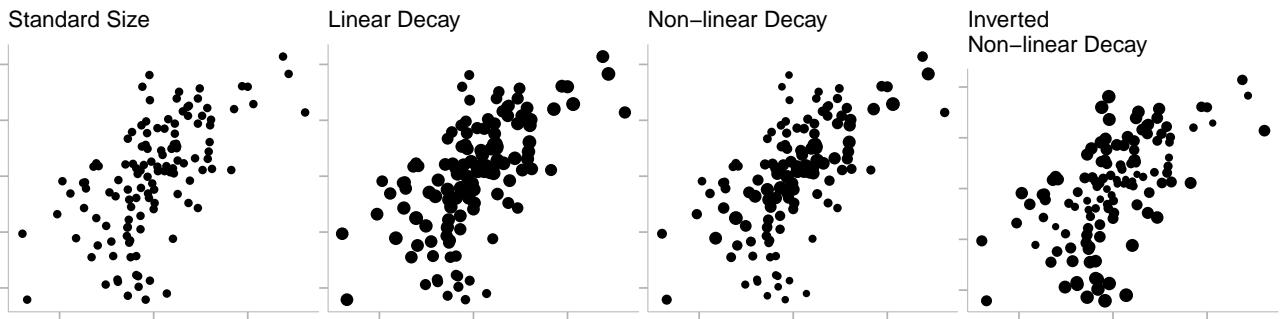


Figure 5.2. Examples of the stimuli used in Experiment 3, demonstrated with an r value of 0.8.

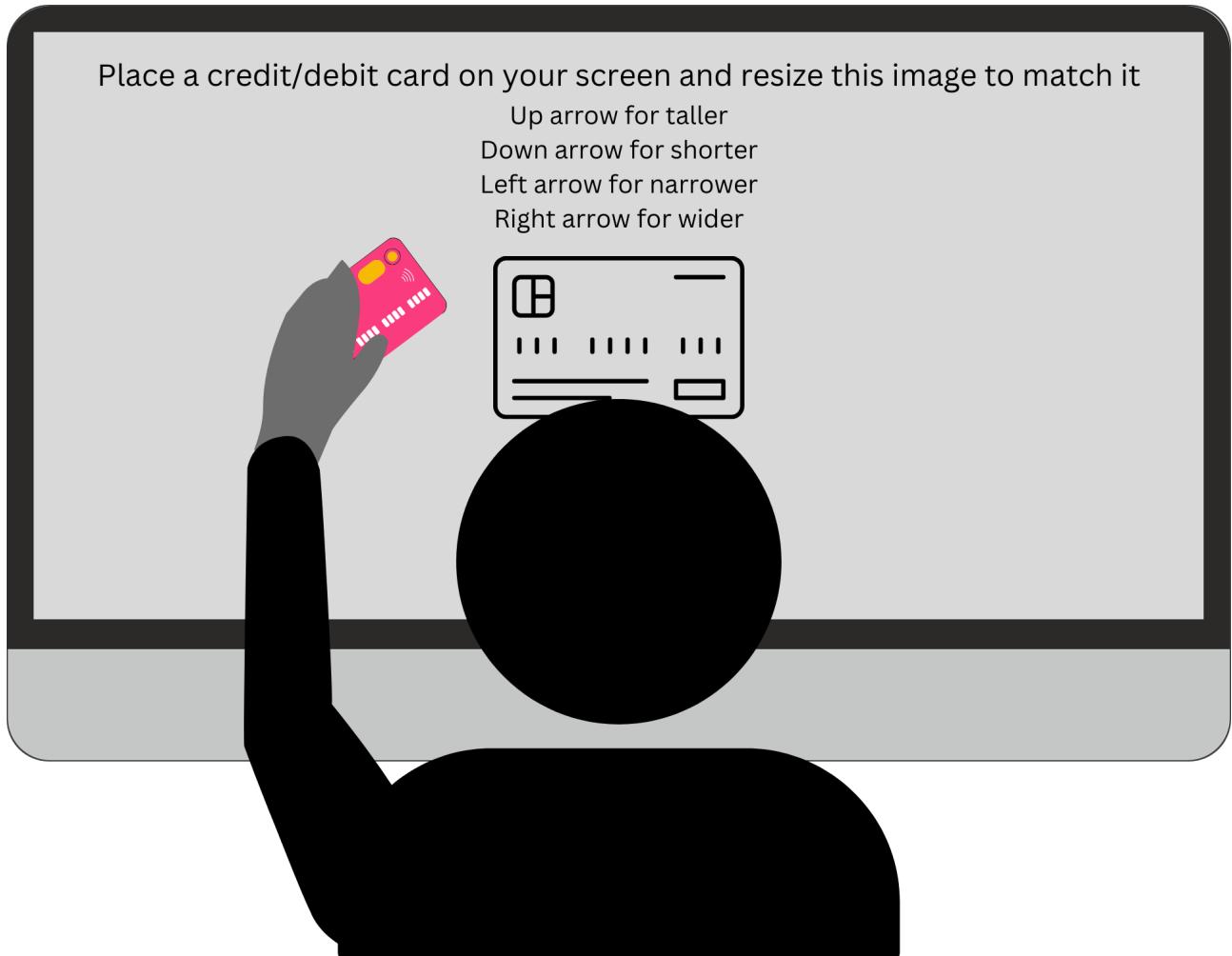


Figure 5.3. A mock-up of the screen scale [105] task used to infer the sizes of participants' monitors.

pixel. Calculating dot pitch is a requirement for the subsequent calculation of the physical on-screen sizes of the scatterplot points that participants saw. In the preamble to the current experiment, participants were asked to hold a standard size credit/debit/ID card up to the monitor, and then to resize a corresponding on-screen image until it matched the size of their physical card [105]. These cards have a universal standard size (ISO/IEC 7810 ID-1), which when combined with the monitor resolution information recorded by Psychopy, and assuming a widescreen 16:9 aspect ratio, allows for the inference of dot pitch and therefore the physical size of the points in the experiment. Mean dot pitch was 0.33mm ($SD = 0.06$), corresponding to a physical size on the screen of 4.32mm for the smallest

points displayed. Section 5.6 includes analysis that takes into account the physical on-screen sizes of scatterplot points.

5.5.4 Point Visibility Testing

It is key that the manipulations used do not remove (or appear to remove) data from scatterplots. Therefore, point visibility testing is included in this experiment prior to the experimental items. Participants were shown 6 scatterplots and were asked to enter in a text box how many points were being displayed. These points were the same size as the smallest points displayed in the experimental items. 5% of participants were correct on 5 out of 6 point visibility tests, while 95% were correct on 6 out of 6. It should be noted that those participants scoring 5/6 did not answer incorrectly, rather they did not answer at all for a particular question, which is suggestive of a mis-click or an initial misunderstanding of the task. Regardless, the results of this test indicate a sufficient level of point visibility.

5.5.5 Design

A fully repeated-measures, within-participants design was employed. Each participant saw and responded to each of the 180 scatterplots in a fully randomised order. There were four scatterplots for each of the 45 r values corresponding to the four levels of the size decay condition, examples of which are shown in Figure 5.2.

5.5.6 Procedure

Each participant viewed the PIS and provided consent through key presses in response to consent statements. Participants were asked to provide their age in a free text box, followed by their gender identity. Participants then completed the 5-item Subjective Graph Literacy test [54], followed by the screen scale and point visibility tasks described above. Participants were shown example of scatterplots depicting r values of 0.2, 0.5, 0.8, and 0.95. Section 5.6 contains discussion of the potential effects of this training. Following two practice trials, participants worked through the series of 180 experimental and six attention check trials in a randomised order. Visual masks preceded each plot.

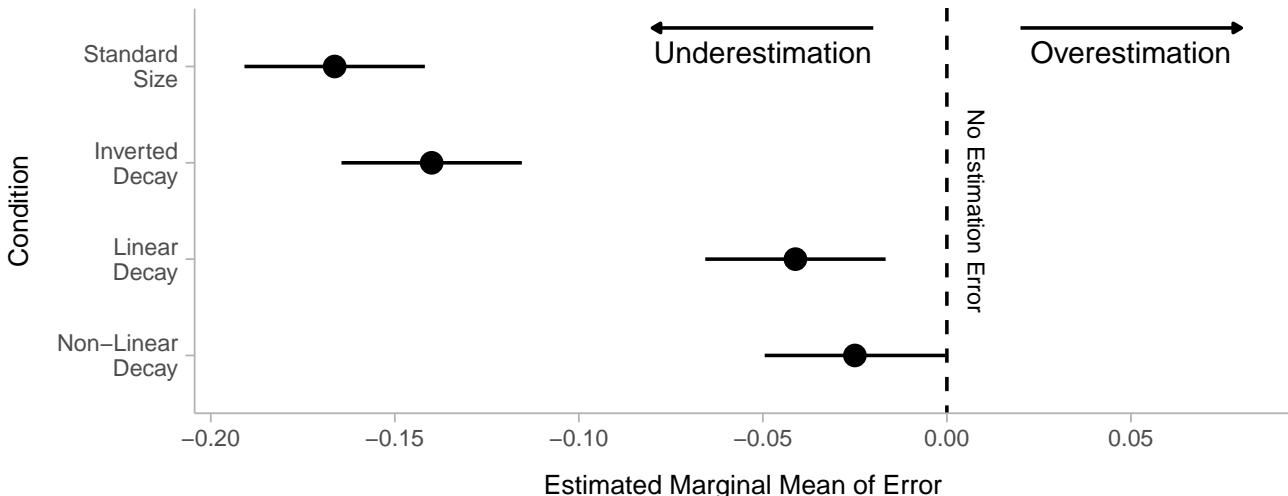


Figure 5.4. Estimated marginal means for the four conditions tested in Experiment 3. 95% confidence intervals are shown. The vertical dashed line represents no estimation error. The overestimation zone is included to facilitate comparison to later work.

5.5.7 Participants

150 participants were recruited using the Prolific platform [129]. Normal or corrected-to-normal vision and English fluency were required. Participants who had completed any of the experiments described in Chapter 4 were prevented from participating. Data were collected from 164 participants. 14 failed more than 2 out of 6 attention check questions, and, as per the pre-registration, had their submissions rejected from the study. The data from the remaining 150 participants were included in the full analysis (72 male, 75 female, and 3 non-binary). Participants' mean age was 29.56 ($SD = 8.54$). Mean graph literacy score was 21.77 ($SD = 4.29$). The average time taken to complete the experiment was 39 minutes ($SD = 14$ minutes).

5.6 Results

To investigate the effects of point size adjustments on participants' estimates of correlation, a linear mixed effects model was built whereby the point size condition is a predictor for the difference between objective r values for each plot and participants' estimates of r . This model has random intercepts for participants and items. A likelihood ratio test revealed that the model including size decay function as a fixed effect explained significantly more variance than the null model ($\chi^2(3) = 3,508.84, p < .001$). Figure 5.4 shows the mean errors in correlation estimation for each size decay condition, along with 95% confidence intervals.

Table 5.1. Contrasts between different levels of the size decay factor in Experiment 3.

Contrast		Statistics	
		Z ratio	p
Standard Size	Inverted Decay	9.3	<0.001
Standard Size	Linear Decay	44.4	<0.001
Standard Size	Non-Linear Decay	50.1	<0.001
Inverted Decay	Linear Decay	35.1	<0.001
Inverted Decay	Non-Linear Decay	40.8	<0.001
Linear Decay	Non-Linear Decay	5.7	<0.001

Table 5.2. Cohen's *d* effect sizes (left) and summary statistics (right) for levels of the size decay factor in Experiment 3. Each effect size is compared to the reference level, termed, "Standard Size".

(a) Effect sizes for size decay factor.		(b) Summary statistics for size decay factor.		
Effect	Cohen's <i>d</i>	Size	Mean of Error	Standard Error
Standard Size		Standard Size	0.17	0.013
Inverted Decay	-0.11	Inverted Decay	0.14	0.013
Linear Decay	-0.54	Linear Decay	0.04	0.013
Non-Linear Decay	-0.61	Non-Linear Decay	0.02	0.013

This effect was driven by significant differences between means of correlation estimation error between all conditions. Statistical tests for contrasts were performed using the `emmeans` package [87], and are shown in Table 5.1. To test whether the observed results could be explained by differences in participants' levels of graph literacy, an additional model was built. This model is identical to the experimental model, but also includes graph literacy as a fixed effect. Including graph literacy as a fixed effect explained no additional variance ($\chi^2(1) = 0.16$, $p = .690$), indicating that the differences observed in participants' correlation estimation performance were not as a result of differences in levels of graph literacy.

While participants performed well on the point visibility task, another facet of using a larger or smaller monitor with a lower or higher resolution could have affected estimates of correlation. Comparing a model including the dot pitch of participants' monitors to the experimental model revealed a significant main effect ($\chi^2(1) = 4.65$, $p = .031$). There was no interaction between size decay condition and dot pitch; a 0.1mm decrease in dot pitch resulted in correlation estimates decreasing by .03. Given the low range of dot pitches gathered from participants (0.13mm to 0.63mm), the effect is not substantial enough to warrant further discussion.

As in Chapter 4, an approximation of Cohen's *d* was calculated between the reference level (here labelled "Standard Size") and all other levels of the size decay factor. These statistics can be seen in Table 5.2 along with descriptive statistics for all levels. The largest observed effect size is between the standard size and non-linear decay conditions ($d = 0.64$), and is medium. This is a marked im-

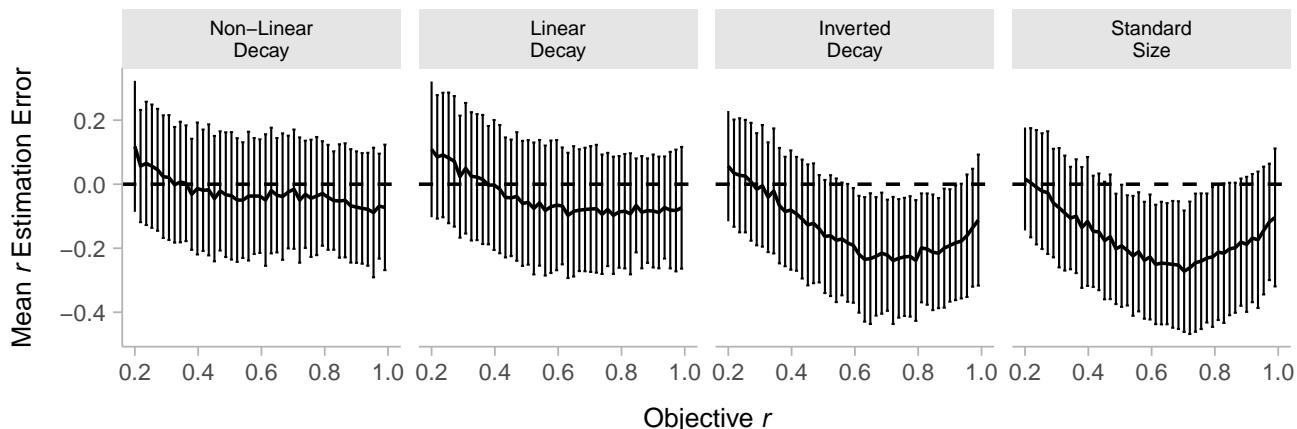


Figure 5.5. Participants' mean errors in correlation estimates grouped by condition and by r value. The dashed horizontal line represents perfect estimation. Participants were most accurate when presented with the plots featuring the non-linear size decay function. Error bars show standard deviations of estimates.

provement on the non-linear opacity decay condition from Experiment 2, Chapter 4, where the effect size observed was $d = 0.19$. Figure 5.5 plots the effects of each size decay manipulation separately for each r value used. Points represent means of estimation error for each r value, with standard deviations of error shown as error bars. The dashed horizontal line represents hypothetical perfect estimation. Expectedly, while participants still underestimated r in all conditions, the non-linear size decay condition provided the most promising correction to date; the average estimation error is approaching a flat line (see the left-most plot in Figure 5.5).

5.7 Discussion

Participants' errors in correlation estimation were significantly lower in the non-linear size decay condition (see Figure 5.5) compared to all other conditions. This finding provides support for the first hypothesis. Conversely, no support was found for the second hypothesis, that estimates would be least accurate in the inverted non-linear size decay condition. Errors in this condition were significantly higher than for the other two decay conditions, but were significantly lower than the errors that were observed for the standard size condition.

5.7.1 Increased Correlation Estimation Accuracy

The mean error in correlation estimation for the non-linear size decay condition in this experiment was .025, while for the equivalent opacity decay condition described in Experiment 2 (see Chapter 4) was .086. This finding, along with the significantly higher Cohen's d effect size reported in this ex-

periment compared to Experiment 2 for the non-linear decay conditions (0.86 vs. 0.19), provides evidence that point size is a stronger encoding channel for the manipulation of perceived correlation than point opacity. If these effects are being driven by the lower salience of more exterior points, the fact that a larger effect of point size has been reported is congruent with research showing clear influences of stimulus size on object salience and perceptual weighting [58, 61, 70]. The present results therefore provide support for point salience/perceptual weighting being a key driver of the effects observed; lower point salience brought on by reduced point size in the exterior of the scatterplot reduces the perceived width of the distribution of data points around the regression line, biasing estimates of correlation upwards and leading to a higher degree of accuracy. Other candidate mechanisms do exist; similar results would be expected if a feature-based attentional bias was responsible [70, 159]; the current methodology does not allow for distinguishing between these explanations, and it may be that both are partially responsible.

The lack of support for the second hypothesis is surprising, and suggests that point salience and perceived distributional width do not form the whole story. There is evidence that larger stimuli exhibit greater levels of spatial uncertainty [2], and it is possible that this uncertainty results in a perceptual under-weighting of the contribution of these points during correlation estimation. This is consistent with previous work [170, 171] suggesting that the brain may make robust statistical use of visuo-spatial information. These mechanisms act to downweight the influence of less reliable information (in this case the higher spatial uncertainty of larger exterior points) on subsequent perceptual estimates. In the present experiment, this resulted in participants making more accurate estimates of correlation in the inverted decay condition compared to the standard size condition. It was suggested in Chapter 4 that inverted opacity manipulations may be employed to correct for the *overestimation* of correlation observed with negatively correlation scatterplots [148]; findings here indicate that using an inverted size decay function in this way may not be appropriate.

5.7.2 Constant Correlation Estimation Precision

Unlike the experiments described in Chapter 4, in which standard deviations of correlation estimation errors generally became smaller as the actual r value increased, distributions of standard deviations here remained mostly constant. This was unexpected, as previous work [131, 133–135] routinely finds precision in r estimation to increase with the objective r value. This finding may be related to the nature of the stimuli in the current study. At high values of r there is a large amount of overlap

between points in the non-linear, non-linear inverted, and linear size decay conditions. This overlap may blur the percept and account for the absence of this effect, however cannot account for these findings with regards to the standard size condition. Aside from the inverted non-linear decay condition in Chapter 4, the finding that precision increased with r was robust. Its absence here is curious given that the standard size decay condition here is identical to the full opacity condition in that chapter. Relying on relative judgements means the interplay between scatterplots with different visual features must be accounted for within a particular experiment. The stimuli as r approaches 1 in the current study exhibit greater levels of visual variance than the stimuli in Experiments 1 and 2 (see Chapter 4), which may explain the lack of increased precision here. Further testing is required for a more concrete explanation.

Ultimately, my aim is to provide tools for the design of visualisations more suited for the tasks they are intended to support. When that task is the perception of positive correlation, the use of the non-linear size decay condition described here is recommended. For other scatterplot tasks, such as cluster separation or numerosity perception, or other chart types, the use of the size manipulation may in fact be a hindrance.

5.7.3 Training

Before beginning the experimental trials, participants viewed plots depicting $r = 0.2, 0.5, 0.8$, and 0.95 for a minimum of 8 seconds. Comparing a model including session half as a fixed effect with the origin experimental model revealed no significant effect ($\chi^2(1) = 1.06, p = 0.30$), suggesting that having more recently viewed the example plots did not have an effect on participants' performance.

5.7.4 Limitations

Despite confirming a method of obtaining dot pitch, no method of obtaining head-to-monitor distances is available. This, along with the comparative correlation judgements collected, prevents concrete psychophysical conclusions from being made. Instead, the experimental paradigm allows for findings that are rigorous to different viewing contexts and are of particular importance for the HCI and visualisation design audiences. It may be that a high level perceptual phenomenon is responsible for the effects seen here; investigating this is beyond the scope of the current study and does not negate the findings. There is also the potential for misinterpretation of the scatterplots presented in

the current study, especially given their similarity in form, but not purpose, to bubble charts.

5.7.5 Future Work

Evidence has been found for robust effects of varying point opacity and point size on correlation estimation in positively correlated scatterplots. There are different effects of changing these visual features, and these effects are also partially dependent on the objective r value in the scatterplot. Future work should investigate the combination of these visual manipulations as they pertain to correlation estimation.

Additionally, future qualitative work is required to investigate whether the modified scatterplots presented result in misinterpretation or a decrease in the levels of trust people place in data visualisations.

5.8 Conclusion

I conducted a single experiment, in which points on scatterplots had their sizes systematically changed according to their distance from the regression line. The equation relating point size to residual distance was identical to that used in Chapter 4, with the addition of a scaling factor and constant to provide parity between the smallest points in the previous experiment and those in the current. Additionally, I tested the visibility of the smallest points used to ensure that no data were functionally removed, as well as collecting the sizes and resolutions of participants' monitors. I found evidence that reducing the size of scatterplot points as a function of residual error using a non-linear equation not only produced significantly more accurate estimates of positive correlation, but also subtly changed the fundamental shape of the estimation curve in a way not previously reported. Unlike the inverted opacity decay function used in Chapter 4, the inverted size decay function here did not produce the least accurate estimates of correlation; these were found when participants viewed the "standard size" plot devoid of a size manipulation. Taken together, these results suggest a greater potential for manipulating correlation estimates when using point size compared to point opacity.

Chapter 6

Interactions of Opacity and Size Adjustments

6.1 Abstract

Chapter 4 and Chapter 5 each provide evidence for the effects of changing the opacities and sizes of scatterplot points on people's performance on a correlation estimation task. It is clear from the results of the experiments presented in those chapters, however, that the effects of changing point opacity and size on correlation estimation in positively correlated scatterplots are different, both with regards to the strength of the correction provided and the shape of the estimation curve produced. Mechanistically, however, point opacity and size may operate similarly by reducing the influence of more exterior points in scatterplots. To further investigate these mechanisms, and in the interest of providing a more complete description of the ways in which changing point opacity and size can bias correlation estimation, I present a single experiment study combining the point opacity and size manipulations from Chapter 4 and Chapter 5. In a condition where point opacity and size are reduced as a function of residual magnitude, correlation estimation is significantly biased upwards, a finding that suggests the effects of changing point opacity and size are not linearly additive.

6.2 Introduction

The work presented in Chapter 4 and Chapter 5 shows that point opacity and size adjustments can be used to bias perceptions of correlation in positively correlated scatterplots. These findings are in opposition to work finding both bias and variability in correlation perception to be invariant to both uniform and irregular changes in point opacities and sizes [131, 133]. Of course, when attempting to provide tools for visualisation designers to design *better* visualisations, more options are prefer-

able to few. In this spirit, then, I endeavoured to answer the next big question; what happens when point opacity and size manipulations are combined? Answering this question would not only allow for a deeper exploration of the potential mechanisms behind each decay function, but would further empower visualisation designers with the knowledge of how altering point opacities and sizes might affect people’s interpretations of the correlations displayed therein. In a fully-reproducible, large-sample ($N = 150$) study, I show that combining point opacity and size manipulations can produce more powerful effects on correlation estimation than either manipulation in isolation. Additionally, I comment on the impact that the adjustment of each visual feature may have on correlation estimation, and suggest future work to tune the effects seen in Chapter 4, Chapter 5, and here.

6.3 Related Work

Much of the related work presented here is covered in more detail in Chapter 2 and in Sections 4.4 and 5.3. For the reader’s convenience, and to properly contextualise the experimental methods and findings in the present chapter, this related work is reproduced in brief here.

6.3.1 Opacity and Contrast

Changing the opacities of points in scatterplots is standard practice when visualising very large datasets to address overplotting [93]. Figure 4.2 visualises how point opacity is often reduced when large numbers of points are present to preserve individual point discriminability. While work on the effects of point opacity on correlation perception in scatterplots has taken place, it is often contradictory. Earlier work found correlation perception invariant to changes in the opacities of points [131, 133], while the work I completed in Chapter 4 found clear, if small, effects. That work offered both point salience/perceptual weighting and spatial uncertainty as potential drivers for the effects found. Lower stimulus contrast, which for isolated stimuli is functionally identical to lower opacity, is associated with lower salience [61], can bias judgements of mean point position [70], increases error in positional judgements [172], and can result in greater uncertainty in speed perception [26]. Due to mechanistic accounts of both salience/perceptual weighting and spatial uncertainty predicting results in the same direction regarding opacity adjustments, the work I conducted in Chapter 4 was unable to distinguish between explanations rooted in point salience/perceptual weighting and spatial uncertainty as drivers for the effect found.

Micallef et al. [100] found that “merging, dark dots” support correlation estimation; despite only changing point opacity in a *uniform* manner, the sheer number of points used in that study results in scatterplots that appear similar to those that reduce opacity as a function of residual error (see Chapter 4). That this technique has been shown to produce more accurate correlation estimates as compared to unadjusted scatterplots may explain why the optimisation system employed in Micallef et al. [100] conferred benefits regarding correlation estimation.

6.3.2 Point Size

Again, for discriminability reasons, scatterplots dealing with larger datasets tend to have smaller individual points. Section 5.3.1 explores this, and related bubble charts, in greater detail. As with opacity, the work exploring the effects of point size on correlation perception in scatterplots is conflicted. Some finds correlation perception to be invariant to both global and irregular changes in point opacity [131, 133], while the work presented in Chapter 5 contradicts this, finding effects on correlation perception that go beyond that available through manipulation only of point opacity.

While the mechanistic driver of the effects of point opacity on correlation perception is unclear, the balance of evidence for point size points towards a point salience/perceptual weighting account. There is evidence that larger stimulus size is associated with lower levels of spatial certainty [2], but higher levels of salience [61], results which are supported by evidence that reaction times are slower to smaller stimuli [57, 116]. The predicted effects of spatial certainty and salience on correlation perception operate in the opposite direction to one another, lead to the conclusion in Chapter 5 that point salience is a more likely candidate mechanism. In an investigation into perceptions of global means in scatterplots, it has been found that perceptions are biased towards areas that contain larger points [70]. These findings form the theoretical basis behind the third hypothesis in the present work.

6.4 Hypotheses

A single experiment is present in this chapter based on the effects of adjusting point opacity and size on correlation estimation established in Chapter 4 and Chapter 5. Here, previously independently tested point opacity and size manipulations are tested in both typical orientation (point opacity/size is reduced with residual magnitude) and inverted orientation (point opacity/size is increased with

residual magnitude). Throughout this chapter, referral is made to *congruent* and *incongruent* conditions with respect to the combination of point opacity and size decay functions. *Congruent* conditions are those in which point opacity and size decay act in the same direction (typical or inverted), while for *incongruent* conditions, point opacity and size decay act against each other. Due to previous findings that non-linearly reducing point opacity or size as a function of residual magnitude can bias correlation estimates upwards, and that increasing point opacity and size can further bias estimates downwards, it is hypothesised that:

- H1: an increased reduction in correlation estimation error will be observed when congruent typical orientation decay functions are used.
- H2: the use of a congruent inverted function, such that point opacity and size are both increased with residual magnitude, will produce the least accurate estimates of correlation.

Owing to the finding from Chapter 5 that point size is a stronger channel for biasing correlation estimation than point opacity, it is also hypothesised that:

- H3: there will be a significant difference in correlation estimates between the two incongruent orientation conditions.

6.5 Method

6.5.1 Open Research

The experiment was conducted according to the principles of open and reproducible research [9]. All data and code for the original paper are maintained in a GitHub repository¹. This repository also features an implementation of a Docker container that enables the full recreation of the computational environment in which the original paper was written. The experiment itself is hosted on GitLab². The hypotheses and analysis plans were pre-registered with the Open Science Framework (OSF)³, and no deviations from them were made.

¹https://github.com/gjpstrain/size_opacity_and_scatterplots

²https://gitlab.pavlovia.org/Strain/size_and_opacity_additive_exp

³<https://osf.io/j32sk>

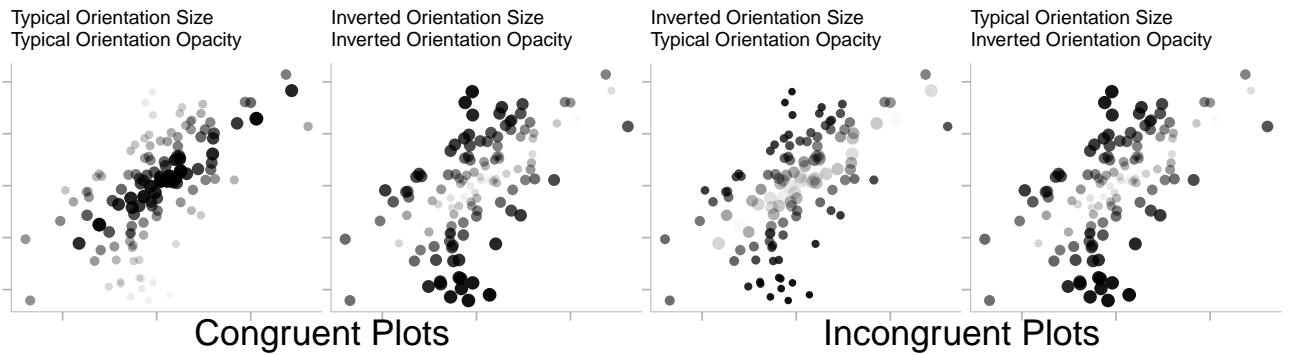


Figure 6.1. Examples of the stimuli used in Experiment 4, demonstrated with an r value of 0.8.

6.5.2 Stimuli

The creation of the stimuli in this experiment follows the same general principles outline in Section 3.2.4, Chapter 3. `ggplot2` (version 3.5.0) was used to create the stimuli. Again, equation 6.1 was used to map point residuals to opacity and size values:

$$point_{size/opacity} = 1 - b^{residual} \quad (6.1)$$

For the changes made to point size in this experiment, a constant of 0.2 was added to each raw value, along with a scaling factor of 4; as in Chapter 5, these adjustments resulted in the smallest points having a width of 12 pixels on a 1920x1080 pixel monitor, which is consistent with the point size used in Chapter 4 and the minimum point size used in Chapter 5. With regards to changing the opacities of points, an alpha = 0.2 floor was implemented, as informal piloting indicated low levels of visibility when very small points were especially transparent. In this experiment, point opacity or size manipulations in which a reduction with residual magnitude takes place are referred to as *typical orientation*, while those in which point opacity or size are increased with residual magnitude are referred to as *inverted orientation*. Additionally, as the current experiment only examines combinations of point opacity and size decay manipulations, the nature of these combinations are classified. When both point size and opacity decay operate in the same direction, that condition is referred to as *congruent*. When they operate in opposition to each other, those conditions are referred to as *incongruent*. Labelled examples of the stimuli used in this experiment can be viewed in Figure 6.1.

6.5.3 Point Visibility Testing

Discussions about the opacities and sizes of stimuli are difficult in the context of online, crowdsourced experiments. Unfortunately, it is not possible to exert much control over the types of device participants use beyond insisting on laptops or desktop computers. In particular, the varying physical sizes, resolutions, and dynamic ranges of participants' monitors can make commenting on the opacities and sizes of stimuli difficult. On the other hand, carrying out this kind of experimentation produces findings that are more resilient to different viewing contexts than traditional lab-based work. It is key that the manipulations employed here do not remove data; this includes removing data by rendering it invisible. As in Chapter 5, point visibility testing is included to address these concerns. Participants were shown scatterplots containing between 2 and 7 points; these points were the same size and opacity as the smallest and least opaque points used in the experimental stimuli. Participants were instructed to enter the number of points present for each plot in a textbox. Participants scored an average of 74.89% ($SD = 32.25\%$). Despite the use of the opacity floor and point size constant and scaling factor, some of the smallest, least opaque stimuli used were clearly not visible to participants. This was most likely due to low contrast between the foreground (scatterplot points) and the background, as Experiment 4, Chapter 5 found visibility mostly invariant to point size. In an idealised experimental setup, minimum point opacity and size would need to be calibrated on a per-monitor basis. Analysis including participants' performance on the point visibility task as a fixed effect is detailed in Section 6.6.

6.5.4 Dot Pitch

As in Chapter 5, a method for inferring the dot pitch of participants' monitors was included in this experiment [105]. Section 5.5.3 details precisely how this was accomplished. Mean dot pitch was 0.60mm ($SD = 0.09$), corresponding to a physical on-screen size of 7.80mm on a 1920×1080 pixel monitor for the smallest points displayed on a hypothetical 35.54×20.00 cm monitor. Analysis including dot pitch as a fixed effect is provided in Section 6.6.

6.5.5 Design

A fully repeated-measures, 2×2 factorial design was employed. Each participant saw each combination of opacity and size decay function scatterplots for a total of 180 experimental items. Participants viewed these experimental items, along with 6 attention check items, in a fully randomised order. The experiment is hosted on Pavlovia⁴.

6.5.6 Procedure

Participants viewed the PIS and provided consent through key presses in response to consent statements. Participants were asked to provide their age and gender identity. Participants completed the 5-item Subjective Graph Literacy test [54], followed by the screen scale and point visibility tasks described in Section 6.5. Following the completion of the pre-experimental tests, participants were briefly shown examples of scatterplots with correlations of 0.2, 0.5, 0.8, and 0.95. Section 6.6 contains a discussion of the potential effects of this training. Two practice trials were allowed before the experiment began. Participants worked through a series of 180 experimental and 6 attention check trials in a fully randomised order while being asked to use a slider (see Figure 3.1, Chapter 3) to estimate the correlation to two decimal places. Visual masks preceded each trial. The attention check trials explicitly asked participants to set the slider to 0 or 1.

6.5.7 Participants

150 participants were recruited using the Prolific platform [129]. Normal or corrected-to-normal vision and English fluency were required. Participants who had completed any of the experiments described in Chapter 4 or Chapter 5 were prevented from participating. Data were collected from 158 participants. 8 failed more than 2 out of 6 attention check questions, and, as per the pre-registration, had their submissions rejected from the study. The data from the remaining 150 participants were included in the full analysis (76 male, 73 female, and 1 non-binary). Participants' mean age was 30.65 ($SD = 8.64$). Mean graph literacy score was 22.49 ($SD = 3.55$). The average time taken to complete the experiment was 37 minutes ($SD = 12.3$ minutes).

⁴https://gitlab.pavlovia.org/Strain/size_and_opacity_additive_exp

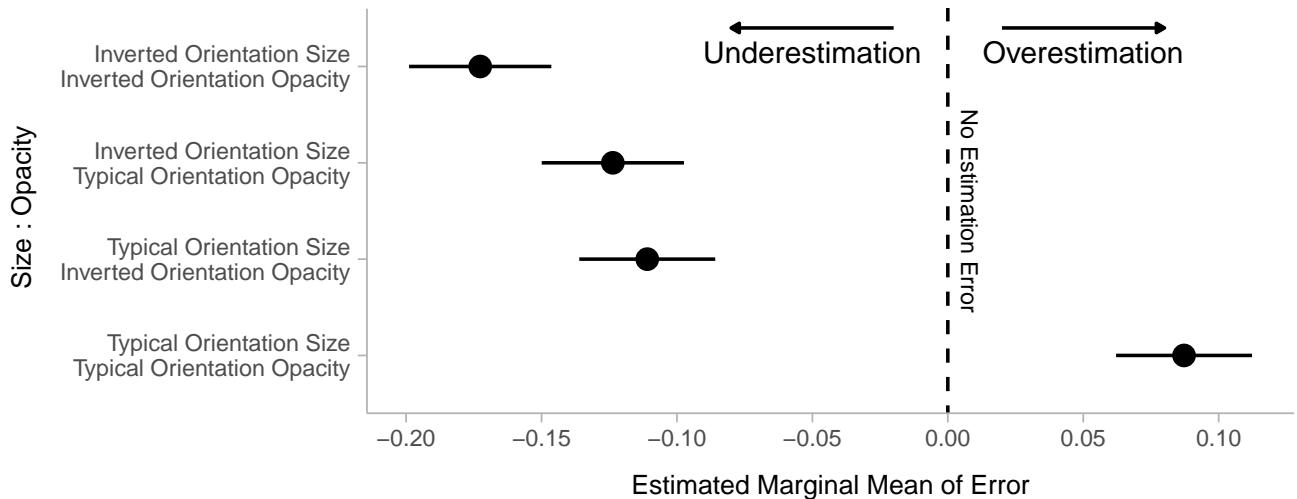


Figure 6.2. Estimated marginal means for the four conditions tested in Experiment 4. 95% confidence intervals are shown. The vertical dashed line represents no estimation error.

Table 6.1. Contrasts between different levels of the opacity and size decay factors in Experiment 4.

Contrast		Statistics	
		Z ratio	p
TO Size x IO Opacity	IO Size x IO Opacity	-10.945	<0.001
TO Size x IO Opacity	TO Size x TO Opacity	72.294	<0.001
TO Size x IO Opacity	IO Size x TO Opacity	-2.256	0.108
IO Size x IO Opacity	TO Size x TO Opacity	46.125	<0.001
IO Size x IO Opacity	IO Size x TO Opacity	17.838	<0.001
TO Size x TO Opacity	IO Size x TO Opacity	-37.438	<0.001

6.6 Results

To investigate the effects of combining point opacity and size decay functions on participants' estimates of correlation, a linear mixed effects model was built whereby the particular combination of point opacity and size decay function employed is a predictor for the difference between objective r values for each plot and participants' estimates of r . Deviation coding was used for each of the experimental factors, which allows comparison between means of r estimation error and the grand mean. This model has random intercepts for items and participants, and random slopes for participants with regards to the size decay factor. A likelihood ratio test revealed that the model including the opacity and size decay conditions as predictors explained significantly more variance than the null ($\chi^2(3) = 5,286.81, p < .001$). There were significant fixed effects of opacity and size decay function, as well as a significant interaction between the two. Figure 6.2 shows the mean errors in correlation estimation for each combination of conditions, along with 95% confidence intervals.

The effects found were driven by significant difference between means of correlation estimation error between all conditions besides that which compares the two incongruent decay conditions. Sta-

Table 6.2. Significances of fixed effects and the interaction between them. Semi-partial R^2 for each fixed effect and the interaction term is also displayed in lieu of effect sizes.

	Estimate	Standard Error	df	t-value	p	R^2
(Intercept)	0.08	0.013	103.32	6.27	<0.001	
Size Decay	-0.14	0.005	148.39	-25.77	<0.001	0.104
Opacity Decay	0.12	0.002	26327.21	63.71	<0.001	0.087
Size Decay x Opacity Decay	0.15	0.004	26327.13	38.47	<0.001	0.034

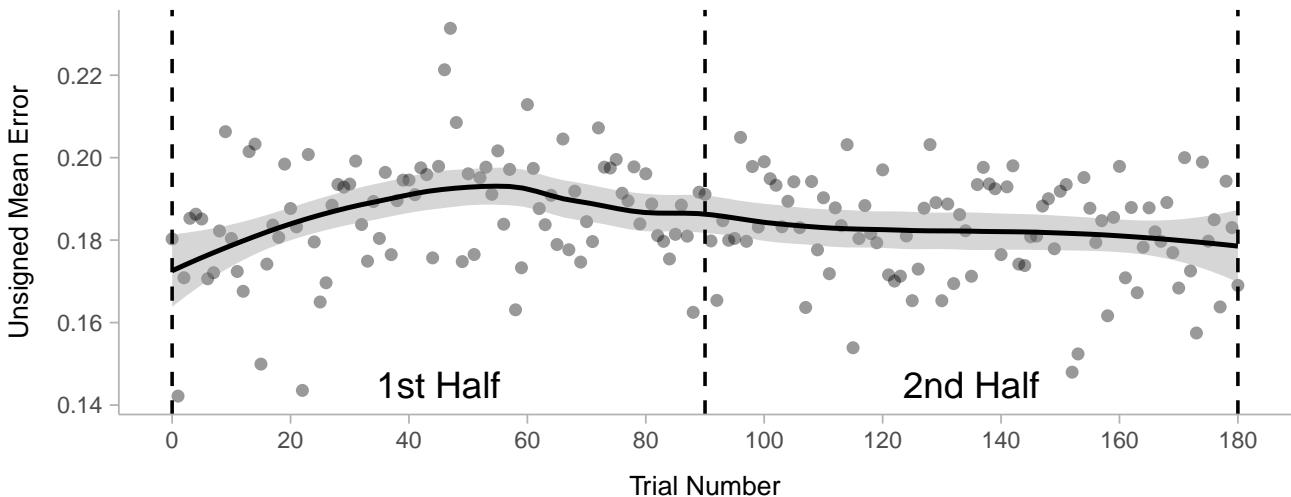


Figure 6.3. Comparing mean errors in correlation estimation by trial number. Points represent unsigned mean errors for each trial number. The plotted line is the locally estimated smoothed curve, with the ribbon representing standard errors.

Statistical testing for contrasts were performed using the `emmeans` package [87], and are provided in Table 6.1. Experiments 1, 2, and 3 all featured identical comparative baseline conditions. In the former two experiments, this was the full contrast condition (see Chapter 4), while in the latter, this was the standard size condition (see Chapter 5). In the current experiment, no baseline condition was used. Owing both to this and the use of a linear mixed effects model with an interaction term, the use of Cohen's d as a measure of effect size would be inappropriate. In its place, the amounts of variance in participants' errors in correlation estimation explained by each fixed effect term and the interaction term is represented as semi-partial R^2 [108]. These statistics were calculated using the `r2glmm` package (version 0.1.2) [71], and are presented along with model statistics in Table 6.2.

Models including participants' graph literacy, their performance on the point visibility task, the dot pitch of participants' monitors, and which half of the experiment a particular correlation judgement took place were built and compared with the experimental model. While no significant effects of graph literacy ($\chi^2(1) = 3.50, p = .061$), performance on the point visibility task ($\chi^2(1) = 1.29, p = .257$), or dot pitch ($\chi^2(1) = 1.52, p = .218$) were found, there was a significant effect of training ($\chi^2(1) = 23.78, p < .001$), with participants rating correlation .01 lower during the second half. This drop suggests that having more recently viewed the training plots may have increased participants' estimates of

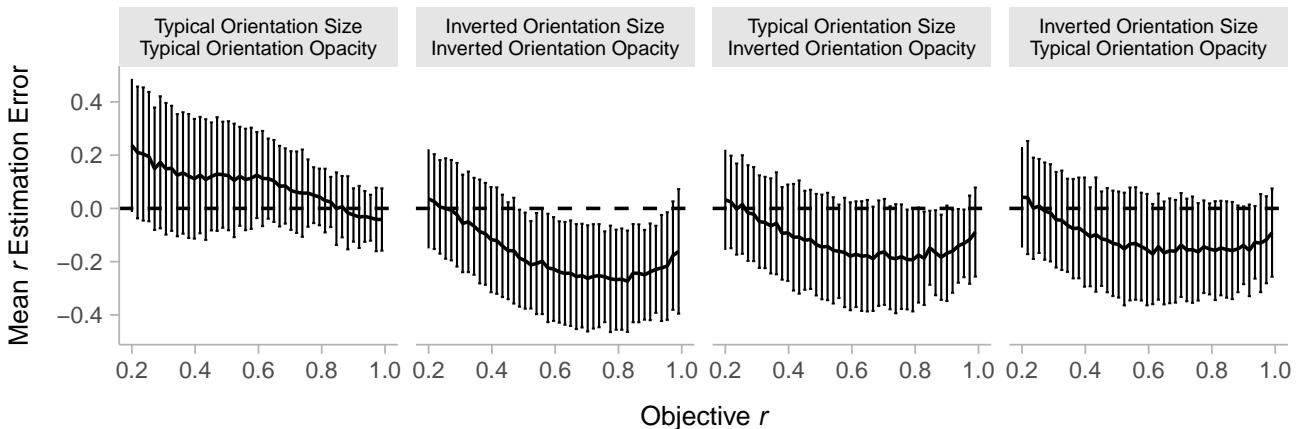


Figure 6.4. Participants' mean errors in correlation estimates grouped by factor and by r value. The dashed horizontal line represents perfect estimation. Participants were most accurate when presented with the plots in the congruent, typical orientation condition. Error bars show standard deviations of estimates.

correlation. To further analyse this variability, a model was built including trial number, allowing for the analysis of error purely as a function of when an experimental trial took place. A significant effect of trial number is also found ($\chi^2(1) = 29.31, p < .001$) on participants' correlation estimation errors. Figure 6.3 shows participants' unsigned mean errors in correlation estimation against trial number. Variability in error, as represented by the ribbon, stabilised quickly and remained stable for most the experiment, only widening again around trial number 170. The simplest explanation for this is that participants, knowing they were coming to the end of the experiment, became less vigilant and rushed their judgements more. Regardless of statistical significance, this effect is not large enough to warrant further investigation, at least as it pertains to correlation estimation in scatterplots.

6.7 Discussion

Hypothesis 1 received full support in this experiment. The combination of typical orientation opacity and size decay functions produced the most accurate estimates of correlation, although this also resulted in a marked over-correction and consequent overestimation for many values of r (see Figure 6.4). The second hypothesis also received support; the combination of inverted opacity and size decay functions produced the least accurate estimates of correlation. No support was found for the third hypothesis, that there would be a significant difference in correlation estimates between inverted orientation opacity/typical orientation size plots and typical orientation opacity/inverted orientation size plots. There was, however, a significant interaction term present, providing evidence that the combination of opacity and size decay functions is not additive in nature.

Further confirmatory evidence is found in favour of the phenomena reported in Chapter 4 and Chap-

ter 5. Namely, that while manipulations of both point opacity and size in scatterplots have significant effects on correlation estimation, the effect of changing point size is stronger, and that while manipulations such as those described in this thesis can influence estimates of positive correlation in either direction, typical orientation manipulations are more powerful than inverted ones. As expected, there is also an effect of congruency on the extent to which a manipulation can bias estimates of correlation; redundant encoding, such as that present here in congruent conditions, is known to support visual grouping and segmentation [111]. The findings presented here provide evidence that redundancy can be exploited to change perceptions of correlation.

Given the consistent finding throughout this thesis that point size is a stronger encoding channel for the purposes of altering perceptions of correlation compared to point opacity, the lack of support for the third hypothesis was unexpected. Tentatively, this may be a result of the non-additive nature of combining point opacity and size manipulations. Despite this, it was found that point size explained a greater proportion of the variance (.104) in the experimental model compared to point opacity.

Taking into account the work presented in this chapter, along with that described in Chapter 4 and Chapter 5, recommendations can be made for designers of correlation visualisations:

- When r is between approximately 0.3 and 0.75, and the scatterplot in question is intended solely for the communication of correlation, designers may wish to implement the non-linear size decay function, as findings have shown it to produce the most accurate correlation estimates in this range.
- Outside of this range, and with the same caveats in place, designers may wish to implement the opacity decay function described in Chapter 4; while its effect on correlation estimation is small, it does significantly increase estimation accuracy.
- There exists a combination of size and opacity decay functions that produces accurate correlation estimates while maintaining the increased r estimation precision expected with high r values. Finding this will require extensive future testing.

6.7.1 Combining Manipulations

Figure 6.2 and Figure 6.4 show how, on average, the combination of typical orientation opacity and size decay functions results in an overestimation of r for the majority of values. While not solving the underestimation problem directly, it demonstrates that with regards to using point opacity and

size manipulations to change estimates of correlation, there appear to be few limitations. If correlation estimation can be over-corrected, as in the typical orientation condition here, then there exists a *tuning* of the opacity and size decay parameters such that the degree of correction is appropriate; Section 6.7.6 explores the work that might be done to achieve this. The combination of inverted orientation opacity and size decay functions also had the expected effect, producing the lowest and least accurate estimates of correlation. Combining inverted manipulations did not, however, significantly change the shape of the estimation curve (see Figure 6.4). In addition to non-additive interaction, the effects observed operate differently depending on the direction of the change induced in perception. This finding may also explain the lack of support for the third hypothesis, that there would be a significant difference in estimation error between the two incongruent conditions. Despite the size channel being more powerful than opacity with regards to influencing correlation estimates, the fact that this power depends on the direction the function is set causes incongruent decay conditions to act against each other in unexpected ways. Indeed, the incongruent condition that used a typical orientation size decay function exhibited lower mean error than the one using inverted orientation size decay (see Figure 6.4), however in each case opacity decay appears to have blunted the power of the size decay function to the extent that the difference in errors is not statistically significant.

6.7.2 Estimation Precision

Previous work has been consistent regarding the finding that r estimation precision increases with the objective r value displayed in the scatterplot [38, 131, 133, 134]. The experiments carried out in Chapter 4 and Chapter 5 found that in some cases, precision in r estimation is constant across the range of r values investigated. For example, the use of a size decay function, whether linear or non-linear decay in typical or inverted directions, results in no change in r estimation precision (see Section 5.6, Chapter 5). When point opacity was altered in the same way, only an inverted decay function does not exhibit the conventional increase in r estimation precision with increasing objective r value. In the experiment described presently, precision in r estimation increased whenever a typical orientation opacity decay function was used. This may be part of the moderating effect of point opacity decay on the size decay function; the visual character of scatterplots with high r values that use the size decay function eliminates the usual increase in precision one would expect, however the introduction of the opacity decay function normalises this to the point where precision is restored.

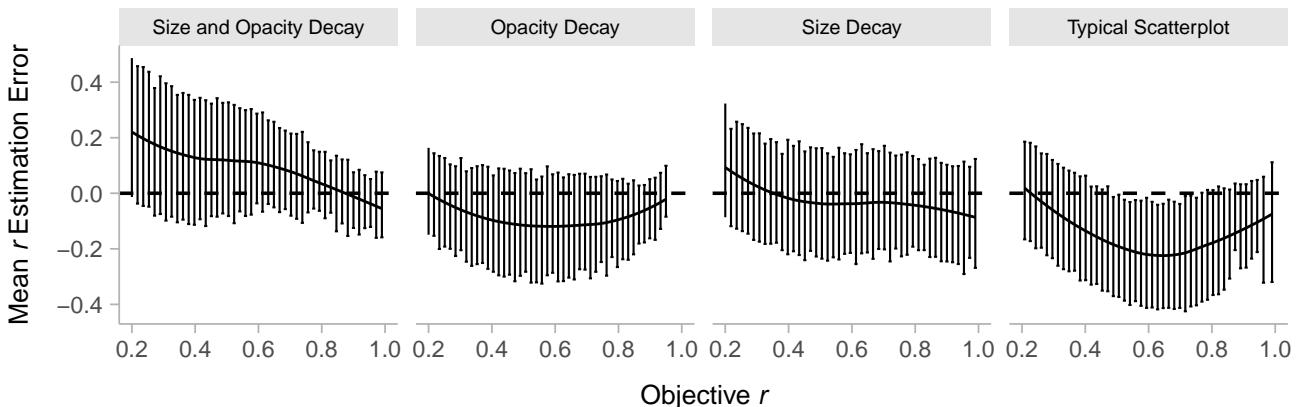


Figure 6.5. Plotting errors in r estimation against objective r values for opacity and size decay functions. On the left, opacity and size decay functions in combination in the typical orientation congruent condition from the current experiment. The plots in the centre show estimation error for opacity and size decay functions in isolation from previous chapters. The right-hand plot averages the comparative baseline (standard scatterplot) conditions from the previous two chapters.

6.7.3 Relative Contributions of Opacity and Size Decay

Incorporating the data gathered in Chapter 4 and Chapter 5 allows for the comparison of estimation curves for size decay and opacity decay both in isolation and combination. Figure 6.5 shows correlation estimation error curves in the present experiment (typical orientation congruent), and in the non-linear decay conditions for both opacity decay (Chapter 4) and size decay (see Chapter 5) conditions. The “no manipulations present” plot is averaged from the results of Experiments 1 to 3. Using opacity decay alone significantly changes the amplitude of the estimation curve, while leaving its shape intact; this can be seen by comparing opacity decay and standard scatterplot plots in Figure 6.5. Using size decay changes both the amplitude and the shape of the estimation curve. When opacity and size decay functions are combined, the shape of the curve is most similar to that observed when size decay is employed in isolation. This is in line with findings, both here and in previous work [70], that size is a more potent encoding channel for the manipulation of perceptual estimates derived from scatterplots. It would appear then that the addition of the opacity curve moderates the effect of size decay as a function of the objective r value itself, without affecting the general shape of the curve.

Using an opacity decay function in isolation has a small effect on correlation estimation. It does little to change the shape of the underestimation curve, but rather slightly biases r estimates upwards to partially correct for the underestimation observed with standard scatterplots. Importantly, it also preserves the increase in correlation estimation precision with r expected. Using the size decay function in isolation has a more dramatic effect. Size decay over-corrects at lower r values, leading to

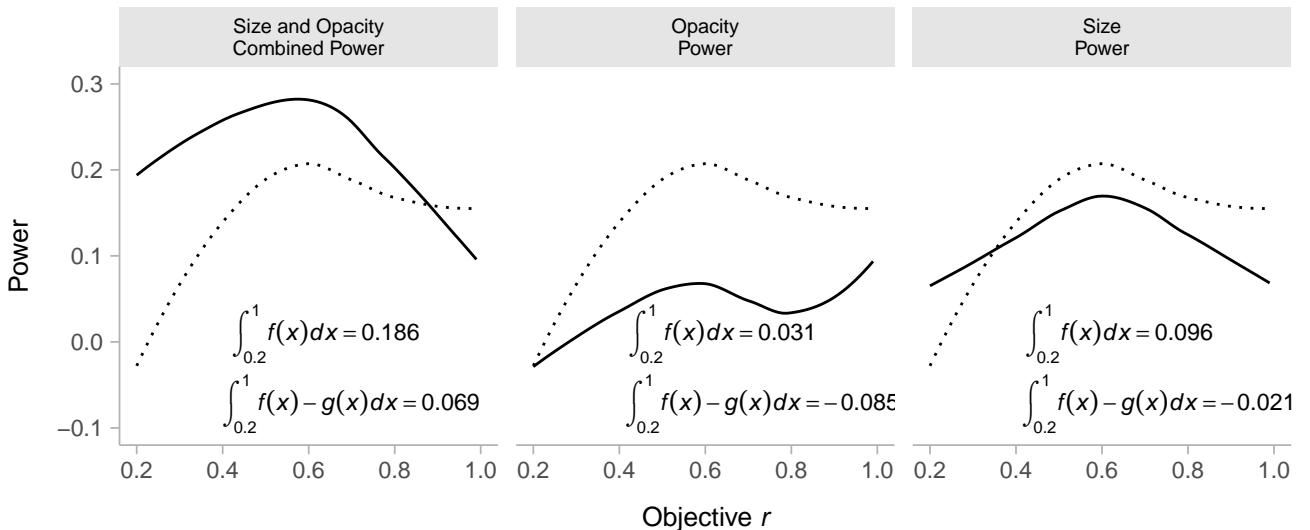


Figure 6.6. Power is the difference between what is observed when a decay function/combination of decay functions is used and what is observed when no manipulation is used. The dashed line represents the power that would be required to correct for the observed underestimation of correlation in scatterplots. The integral of each power curve over r is provided, as well as the difference between this integral and the integral of each required-power curve over r .

an overestimation effect; at higher values, underestimation still occurs. At mid-range values of r , however, the size decay function performs significantly better than all others. One option for tuning correlation estimation using the decay functions described in this thesis would therefore be to use size decay in isolation for mid-range r values (around 0.3 to 0.75), and to use opacity decay in isolation outside of this range. Combining the decay functions, however, allows for the exploitation of the power of the size decay function while maintaining the expected increase in r estimation precision that the opacity decay function confers. The simple combination used in the present experiment does not represent an ideal tuning, as participants overestimated r for the majority of values; the findings here do, however, confirm that there is the scope to bias r estimation to greater degrees compared to scatterplots that use point opacity or size decay functions in isolation. Precise values for the contributions of each encoding channel when they are used simultaneously would be needed to begin doing this work. To describe the effect that each decay function (and their combination) has on correlation estimates, new curves can be derived that compare estimates made with opacity and size decay to those made without.

This contribution is termed *power*, and is visualised in Figure 6.6. As can be seen in the right-hand plot, size decay in isolation provides the closest to the required level of correction, and combining point opacity and size decay results in gross overestimation. Figure 6.6 also includes the integral of each power curve over r as a measure of the total power of each curve. Subtracting the integral of the required power curve from the integral of the observed power curve allows a numerical value to be calculated for how far off each manipulation is.

6.7.4 Mechanisms

Findings in Chapter 4 and Chapter 5 made the case for opacity and size decay acting primarily through point salience and perceptual weighting, with the caveat that spatial certainty also plays a small part in the mechanism behind the effects of point size decay. The results in the present experiment are supportive of this notion, with the highest and lowest mean estimates being observed in the congruent typical and congruent inverted orientation conditions respectively. These findings also support dot density [181] and featured-based attention bias accounts [70, 159]. As all these mechanisms would produce similar results, making conclusions about the relative potential contributions of each is difficult. Nevertheless, on a higher level, the evidence generally points towards a probability distribution account [132, 134]. On a lower level, however, numerous candidate mechanisms exist. The results from the current experiment provide further evidence for a point salience/perceptual weighting account. Hong et al. [70] found that the inclusion of larger and more opaque scatterplot points was able to bias estimates of positional means, but that the relative contributions (weights) of these visual features with regards to perception change as a function of the ranges of opacities and sizes used. It is clear from this evidence and the present that the perceptual weightings of opacity and size are not the same.

6.7.5 Limitations

First, participants' performance on the point visibility task. This was poor, with an average of only 74.89%. It would seem that despite the implementation of the opacity floor and the size scaling factor and constant, many stimuli were simply not visible. While modelling indicates that this did not have a significant effect on errors of correlation estimation, for many participants it will have seemed as if data had been removed, violating the intended aims of the manipulations. Addressing this properly would require a by-participant calibration of the necessary minimum point opacity and size values to ensure visibility, as these values will change as a function of head-to-monitor distance and monitor characteristics.

While evidence has been found that combining point opacity and size decay is not additive, it is difficult to comment precisely on what proportions of the observed effects are a result of each manipulation. Previous work [70], as well as the work carried out here and in Chapter 5, suggest that point size has much more potential to change perceptual estimates, although the exact ways in which

of opacity and size manipulations would require future work.

Due to the extensive testing of “no manipulations present” conditions (*full opacity* in Chapter 4 and *standard size* in Chapter 5), a comparative baseline was not included in this work. At the time it was judged that the increased cost and experimental length was not worth the inclusion of further conditions beyond the four that were tested.

Channels such as point size, colour, opacity, and shape have been used in past work to encode variables beyond the standard two typically used in scatterplots [70, 150]. While this work focuses purely on correlation estimation, these techniques are likely to lead to incorrect interpretations when scatterplots are designed with other tasks in mind. Given evidence that size, shape, and colour are not entirely separable scatterplot features [150], if viewers assume that variations in point opacity/size correspond to additional encoded variables, confounds in interpretation may be introduced. If plots such as the ones presented here were to appear in the wild, however, it would be necessary to clarify that they were designed to aid in the rapid and intuitive interpretation of correlation (and *only* this). Irrespective of the potential for misinterpretation, strong baseline evidence for a perceptual effect of changing point size and opacity in scatterplots is provided that may be expanded on and further exploited in future work.

6.7.6 Future Work

The finding that combining point opacity and size manipulations significantly reduced point visibility suggests the potential for future work investigating the calibration of scatterplot visual features for a particular participant. Doing this would require a more dynamic experimental environment in which stimuli could be regenerated on-the-fly, but would allow researchers to test perceptions more accurately.

Experimental limitations meant that this work did not feature a comparative baseline condition, unlike in Chapter 4 and Chapter 5. Future work may wish to re-test all or some of the conditions described here in addition to a baseline, no manipulations condition.

There is evidence that viewers overestimate correlation in negatively correlated scatterplots [148]. Findings that correlation perception in negatively correlated scatterplots functions symmetrically to that of positively correlated scatterplots [59] suggest that the techniques implemented here may be used (in a symmetrical manner) to address the overestimation bias. Evidence that the influence

of size and opacity decay functions changes according to the direction they are operating in means experimental work with negatively correlated scatterplots would be required, and results may differ significantly from findings related to the underestimation of correlation in positively correlated scatterplots.

All the work in the present chapter, along with that in Chapter 4 and Chapter 5, uses the same equation (reproduced below) to relate point residuals to specific opacity and size values.

$$point_{size/opacity} = 1 - b^{residual} \quad (6.2)$$

Given the finding that the combination of point opacity and size decay is not additive, there are a multitude of parameters that may be adjusted and tested to explore the relative contributions of each to the effects seen. The value of b , which so far has only been $b = 0.25$, is one such parameter, and controls the severity of the fall-off in point opacity or size of the decay function in question. The opacity floor, size scaling factor, and size constant are other values that might be changed. Of course, the equation used is not exhaustive; future work may wish to investigate more complicated equations that link the objective r value to the decay condition.

Future experimental work may use the major axis through the probability ellipse instead of the regression line as a baseline to change point sizes and opacities; evidence that people often report the major axis when asked to visually estimate the regression line [35] suggests that this may produce a different pattern of results from those seen here. If changes in dot density are driving changes in correlation estimates, the congruent conditions here are an example of redundant encoding. Future work may explore using different channels to redundantly encode dot density, such as marker shape, orientation, or colour. Further testing of opacity and size manipulations in isolation and combination using different decay function parameters will allow researchers to build a more complete picture of how these visual features impact correlation estimation, and how we can exploit them to correct for well-known biases.

Finally, while point salience/perceptual weighting is put forward as the most likely driver of the effects observed, the data gathered here do not explain the differences in the shapes of the observed correlation estimation curves (see Figure 6.5). The context that a particular point manipulation is presented in, including the objective r value of the scatterplot, interacts with point opacity and size

adjustments in complex ways. Future work may wish to use the same decay conditions while fixing the objective r value to explore these interactions in greater detail.

6.8 Conclusion

In a single experiment that combined the point opacity manipulation from Chapter 4 and the point size manipulation from Chapter 5, evidence is provided that this combination is not additive in nature. In addition to opening up questions about the complexity of this interaction, this work shows that there is scope to bias correlation estimates significantly; the finding that the combination of non-linear opacity and size decay functions produces marked overestimation emphasises this. Exploring exactly how these decay functions interact with a view to tuning correlation perception is beyond the scope of this thesis; the foundation has, however, been laid.

Chapter 7

Visual Features Affecting Perceptual Estimates

Also Affect Beliefs About Correlations

7.1 Abstract

Chapter 4, Chapter 5, and Chapter 6 show through four experiments that point opacity and size changes can have powerful effects on participants' estimates of correlation in positively correlated scatterplots. In Chapter 4, global and spatially-dependent adjustments in point opacity were employed, and a small, but statistically significant level of correction for the underestimation of positive correlation was found. Spatially-dependent adjustment of point size, in which size is reduced as a function of residual error, was found in Chapter 5 to produce much stronger effects on estimates of correlation; the non-linear decay function used in that experiment produced higher levels of correction and resulted in highly accurate correlation estimates (see Figure 5.5, Chapter 5). In Chapter 6, these point opacity and size manipulations were combined. Their combination was found to produce stronger effects than would be expected if they were linearly additive. While my efforts at correcting for the underestimation bias were successful, my work has not yet attempted to investigate whether any of the techniques developed in this thesis may be used to change people's cognitions about data. Therefore, for my final experimental chapter, I show that scatterplot manipulations that are able to correct for a historical correlation underestimation bias are also able to induce stronger levels of belief change in viewers compared to conventional plots showing identical data. Through a pre-study and main experiment, I provide evidence that adjusting visual features in scatterplots can go beyond simple perceptual effects to influence beliefs about information from trusted news sources.

7.2 Introduction

Research consistently finds that the correlation displayed in positively correlated scatterplots is underestimated [16, 32, 35, 83, 84, 98, 134, 154]. This underestimation is particularly pronounced for Pearson's r values of $0.2 < r < 0.6$, and has been replicated extensively in the current thesis. If scatterplots were solely used for communication between experts, then the presence of this bias would not be especially problematic; those trained in statistics and data visualisation are more likely to be aware of, and make allowances for, their biases. Unfortunately, this is not the case; lay people are expected to be able to use and interpret data visualisations on an almost daily basis. It is therefore the duty of those who design visualisations to design with the naive, inexperienced viewer in mind. Doing so requires us to understand *how* visualisations work, and to gain an appreciation for the hidden processes that allow pictorial representations to convey more than words and numbers alone ever could.

In this thesis, Chapter 4, Chapter 5, and Chapter 6 demonstrate how changing the opacities and sizes of points in scatterplots is able to significantly alter participants' estimates of correlation in positively correlated scatterplots. Substantial progress has been made in correcting for the underestimation bias, however these efforts have only provided evidence about perceptual effects using a simple direct estimation paradigm. While successful, this work has not yet investigated whether, and to what extent, these techniques can influence cognition in the context of real-word data visualisations and the relatedness between variables.

Visualisation is a powerful tool. After all, if numerical data were sufficient for understanding, there would be no need to visualise beyond aesthetic preference. Pattern recognition, attention, and familiarity are aspects of human perception and cognition that can be exploited by visualisation designers to facilitate more efficient, enjoyable, and effective communication [45]. This, however, is a double-edged sword; poor design, be it malevolent or misguided, can cause distrust, confusion, and misunderstanding amongst viewers. It is for these reasons that belief change in scatterplots as a consequence of alternative designs is the next logical research direction for this project. Scatterplots, like many other data visualisations, have been submitted as evidence in court cases [16], and play key roles in organisational decision-making, including in healthcare [127]. It is reasonable to assume that data visualisations are used to make decisions that result in positive or negative outcomes with regard to health and policy more generally, especially given findings that in certain contexts, they

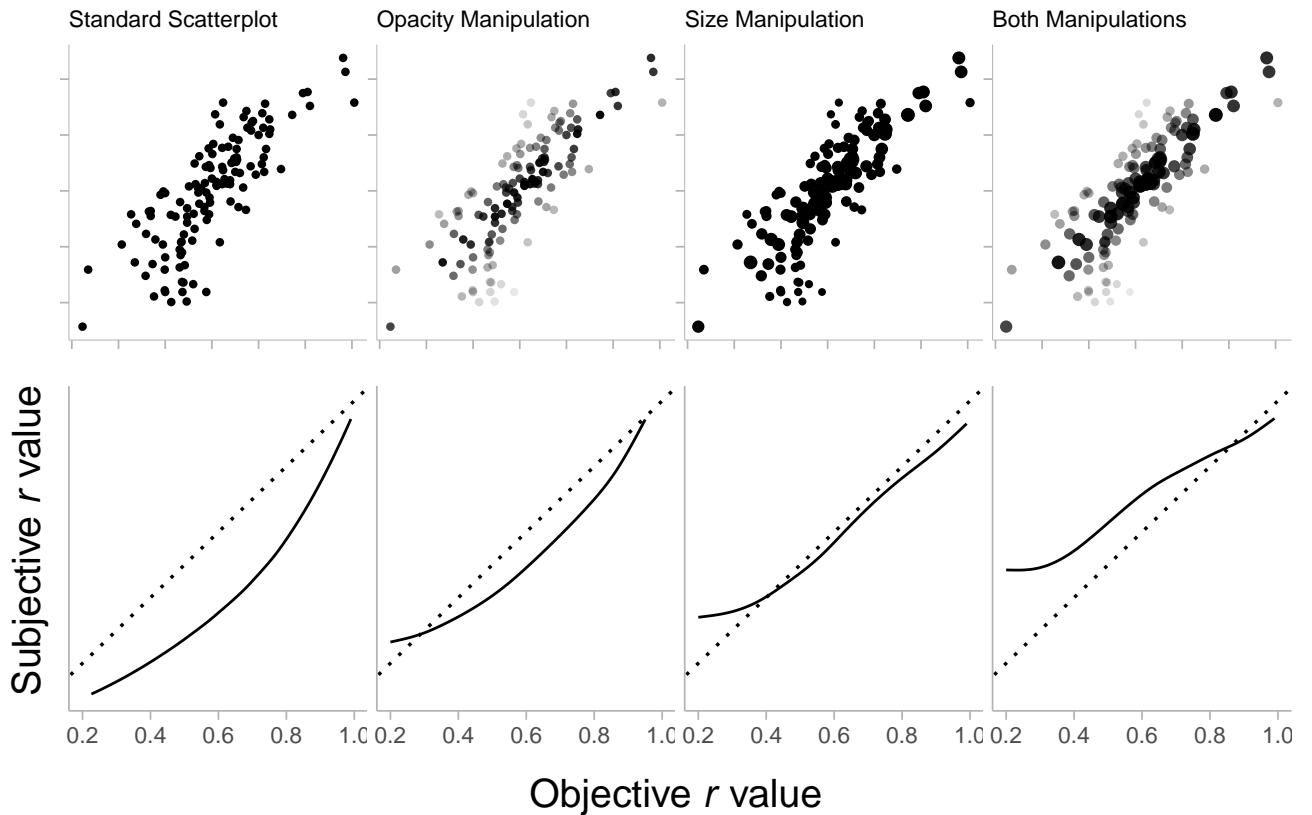


Figure 7.1. Top row: Examples of scatterplot manipulations from previous work using an r value of 0.6. Bottom row: the corresponding correlation estimation behaviour across values of r between 0.2 and 0.99. The dashed diagonal line represents hypothetically accurate estimation, while the solid line is what is observed when participants are asked to estimate correlation.

are more persuasive than textual information [119]. Studying the potential for new designs to alter beliefs about relatedness facilitates better visualisation techniques, but also effects understanding about how these designs might be used by malevolent actors with a view to inoculating those who engage with them. To this end, I present a two-experiment study. First, crowdsourcing is used to select part of the experimental stimuli. The propensity for previously established alternative scatterplot designs to alter beliefs about relatedness is then tested, taking into account the emotional content of the statement and the graph literacy and defensive confidence of participants.

7.3 Related Work

7.3.1 Scatterplots: Developments in This Thesis

In contradiction to previous work [133, 134], this thesis has found clear and powerful effects of systematically changing the opacities and sizes of scatterplot points on participants' estimates of correlation in positively correlated scatterplots. While Chapter 5, in which point sizes were lowered as

a function of residual distance, provided the best level of correction seen so far, Chapter 6 featured the most dramatic level of correction. In that that particular condition, both point opacity and size were lowered as a function of residual distance using equation 7.1:

$$point_{size/opacity} = 1 - b^{residual} \quad (7.1)$$

Figure 7.1 contains a summary of the point opacity and size manipulations from the previous three chapters in this thesis, along with their effects on performance on a correlation estimation task. Each manipulation specified employs the labelled non-linear decay function(s). The present experiment investigates the potential for alternative scatterplot designs to have effects on cognition. For this reason, and to facilitate comparison to the work carried out in Experiments 1 to 4, the same design protocols have been utilised here, including the number of points ($n = 128$), the value of b (0.25), and the size scaling factor, additional constant, and opacity floor. For the alternative scatterplot condition here, the manipulation which has been previously shown to cause the most dramatic change in participants' estimates of correlation was used: the combination of non-linear opacity and size decay functions described in Chapter 6 (see the right-hand pair of plots in Figure 7.1).

7.3.2 Perception & Cognition in Data Visualisation

Interacting with data visualisation is a complex process involving bottom-up and top-down mechanisms [45, 146, 179]. My previous work investigating alternative scatterplot designs has focused on perceptual factors and mechanisms; here the potential for top-down effects to bias participants is introduced. Recent work has established that scatterplots are able to induce different levels of belief change in viewers [74, 92]; this may depend on factors such as prior belief strength, attitudes, and the presence or absence of uncertainty visualisations. Accordingly, the pre-study is essential for isolating the variable of interest, the alternative scatterplot design. Data visualisation does not take place without context, and so the investigation of top-down effects is critical for providing designers with the tools to design visualisations that work as intended in the field. To this end, I present a two-experiment study investigating the propensity for established scatterplot visualisation techniques to bias participants' beliefs about the levels of relatedness between variables.

7.4 Open Research

All experiments in this chapter were conducted according to the principles of open and reproducible research [9]. All experimental code, materials, and instructions are hosted on GitLab for the pretest¹ and for the main test as a pair of separate experiments²³. The original paper is maintained as a GitHub repository⁴. This repository contains all code, analysis, and visualisation. The repository also contains instructions for building a Docker container to recreate the computational environment the paper was written in. Pre-registrations for both the pre-test⁵ and the main experiment⁶ are hosted on the Open Science Framework (OSF), and any deviations are noted where relevant in this chapter.

7.5 Pre-Study: Investigating Beliefs About Relatedness Statements

The goal of the present study is to investigate the extent to which a novel scatterplot design, incorporating the findings of Chapter 4, Chapter 5, and Chapter 6, is able to alter participants' beliefs about correlations. Due to the targeting of lay populations, and my previous experience with lay participants failing to understand the term "correlation" (although not the concept), I elected to operationalise correlation as "strength of relatedness". There is evidence that belief change can be affected by prior beliefs and attitudes [92, 179], and that emotion, including the content of a visualisation [60, 124] and the emotional state of a participant [164] can have perceptual effects on participants and their performance. I was unable to find resources for correlative statements that included ratings for belief strength and emotional valence, so elected to create my own. To control for these factors as much as possible, the pre-study was run with the intent of finding a correlative statement that was matched on emotional valence and level of belief strength. As opposed to manually creating a list of candidate statements, I used the ChatGPT4 Large Language Model (LLM) [115]. On the 9th April, 2024, ChatGPT was asked:

"Generate 100 statements that describe the correlation between two variables, such as: "X is associated with a higher level of Y" or "As X increases, Y increases". Try to match all the statements on emotionality."

¹https://gitlab.pavlovia.org/Strain/beliefs_scatterplots_pretest

²https://gitlab.pavlovia.org/Strain/atypical_scatterplots_main_t

³https://gitlab.pavlovia.org/Strain/atypical_scatterplots_main_a

⁴https://github.com/gjpstrain/beliefs_alternative_scatterplots

⁵<https://osf.io/xuf4d>

⁶<https://osf.io/anmez>

The full list of statements can be found in Appendix X. Myself and a co-author rated each statement on emotional valence and belief about strength of relatedness using Likert scales from 1 to 7. Both statement emotional valence and strength of relatedness were anchored at points 1 and 7: *Very Negative* and *Very Positive* for the former, and *Not Related At All* and *Strongly Related* for the latter. All other points were unlabelled. The `irr` (version 0.84.1 [53]) package was used to calculate a quadratic weighted Cohen's Kappa between the raters, which penalises larger magnitude disagreements more harshly. Following each statement, a pair of Likert scales were presented labelled "Statement Emotionality" and "Strength of Relatedness". We agreed above chance for both emotional valence ($\kappa = 0.49, p < .001$) and strength of relatedness ($\kappa = 0.51, p < .001$), indicating moderate levels of agreement in both cases [33, 44]. Together, we selected strongly and weakly correlated statements with the highest levels of absolute agreement, resulting in 14 strongly and 11 weakly correlated statements. These statements are reproduced in Appendix X2.

7.5.1 Hypotheses

In an attempt to control for the potential effects of belief about strength of relatedness and emotional valence in the main study, the 25 candidate statements selected by myself and a co-author were then tested with a representative UK population in order to ascertain consensus. It was hypothesised that:

- H1: there will be a significant difference in the average ratings of emotional valence between statements.⁷
- H2: there will be a significant difference in the average ratings of strength of relatedness between statements.

7.5.2 Method

Design

The pre-study featured a within-subjects design. Each participant saw all 25 survey items (see Appendix A), along with the six attention check items, in a fully randomised order.

⁷The pre-registration for this hypothesis refers to "emotionality", as did an earlier draft of this paper. In response to reviewer comments, I clarify that it is really emotional valence that is being tested, and therefore this wording is used here.

Procedure

Participants viewed the PIS and were asked to provide through key presses in response to consent statements. They were prompted to provide their age in a free text box and their gender identity. Participants were told that they would be asked to read statements about the relatedness between a pair of variables, after which they would be asked to answer some questions. To familiarise themselves with the sliders used to collect responses, they were asked to complete a practice trial in response to the statement: “As participation in online experiments increases, society becomes happier”.

Participants

100 participants were recruited using the Prolific platform [129]. English fluency and UK residency were required for participation, as the main experiment relied on familiarity with data visualisations from a popular British news source. In addition to 25 experimental items, six attention check items were included that instructed participants to ignore the statement and provide specific answers to the Likert scale sliders. No participants failed more than 2 out of 6 attention check items, and therefore data from all 100 were included in the full analysis (52 male and 48 female). Participants’ mean age was 41.1 ($SD = 12.3$). The average time taken to complete the survey was 14.2 minutes ($SD = 2.9$ minutes).

7.5.3 Results

As before, the `irr` package (version 0.84.1 [53]) was used to measure interrater agreement on statement emotional valence and strength of relatedness for the 25 experimental items. This analysis revealed that participants agreed above chance on statement emotional valence ($\kappa = 0.07, p < .001$) and strength of relatedness ($\kappa = 0.06, p < .001$).

7.5.4 Selecting Statements for the Main Experiment

Statements representing neutral emotional valence were selected to control for the potential effects of statement emotionality in the main experiment. Statements with average emotionality ratings between 3 and 5 are statements 2, 10, 22, 16, and 23, which can be seen in Table 7.1. To ascertain which statements represent the greatest consensus, standard deviations of ratings for statement

Table 7.1. Statements with neutral average emotional valence ratings.

Item	Statement
2	As caffeine consumption increases, so does the average heart rate.
10	Higher sugar consumption is associated with an increased risk of dental cavities.
16	As the amount of sleep decreases, the risk of obesity increases.
22	Higher consumption of spicy foods is associated with a lower risk of certain types of cancer.
23	Greater adherence to a Mediterranean diet is linked to a lower risk of neurodegenerative diseases.

emotional valence and strength of relatedness were summed. Due to concerns about experimental power, and in line with evidence that propensity for belief change is highest when prior beliefs are not strongly held [92, 179], at this point the decision was made to test only the statement corresponding to weak beliefs about the strength of relatedness between the variables in question. Statement number 22 was therefore selected: “Higher consumption of spicy foods is associated with a lower risk of certain types of cancer”, however the wording was modified such that the variables (food consumption and cancer risk) are positively correlated. While the work carried out in Chapter 4, Chapter 5, and Chapter 6 show that opacity and size adjustments in scatterplots can affect estimates of positive correlation, no work regarding the effects of these manipulations in negatively correlated scatterplots has been completed.

7.5.5 Discussion

Fleiss' Kappa values for interrater agreement on both statement emotional valence and strength of correlation scales are low ($\kappa = 0.07$ and $\kappa = 0.06$ respectively), however do exceed that which would be expected by chance. In light of this, decisions regarding which statement to use are not based on the values of Fleiss' Kappa observed, but rather on the standard deviations of ratings across all raters. Statement emotionality and strength of relatedness are tested with participants in the main study and included as fixed effects as part of the analyses.

7.6 Main Experiment: Alternative Scatterplot Designs and Beliefs

The statement selected exhibits the lowest average level of belief about strength of relatedness and the 2nd highest level of consensus. Modified for directionality, the statement reads:

“Higher consumption of plain (non-spicy) foods is associated with a higher risk of certain types of cancer.”

To maximise the likelihood of finding an effect of viewing alternative scatterplots, the stimuli were designed based on a popular British news source and the data were falsely credited as being supplied by the British National Health Service (NHS). Participants were told that the news source had requested their identity be obscured. They were debriefed that this was not the case, and in fact the data were false, at the end of the experiment. Based on evidence that beliefs can change after viewing visualisations [74, 92], and that scatterplots employing point opacity and size manipulations described in Section 7.3 are able to affect perceptual estimates, the following hypotheses were made:

7.6.1 Hypotheses

- H1: there will be a significant difference between ratings of strength of relatedness made before and after participants viewed scatterplots in either the standard or alternative conditions.
- H2: this difference will be greatest when participants are exposed to scatterplots in the alternative scatterplot condition.

Exploratory investigations also took place taking into account participants' scores on a defensive confidence test, their scores on a graph literacy test, and each participant's rating of the emotional valence of the correlative statement used. Analysis including each of these factors can be found in Section 7.6.3, and their inclusion is justified below.

Defensive Confidence

In line with evidence that those who are more confident in their ability to defend their own positions are more susceptible to having those positions changed [3], participant's defensive confidence was measured using Albarracín and Mitchell's [3] 12-item scale. This scale is replicated from previous work in Appendix B, and has been utilised more recently [92] to explore the potential for attitude change specifically with regard to correlations in scatterplots. Participants provide answers to the 12 scale items using a 5-point Likert scale anchored at points 1 (*not at all characteristic of me*) and 5 (*extremely characteristic of me*), with all other points being unlabelled.

Graph Literacy

No effect of graph literacy was found in Experiments 1 to 4 (see Chapter 4, Chapter 5, and Chapter 6). Despite this, the scale was included here due to the higher predicted cognitive load of the current tasks, as there is evidence that graph literacy may affect performance on more cognitively demanding visualisation tasks [24, 113]. Additionally, the graph literacy test used [54] is extremely short; in the present study, this took participants an average of 27 seconds ($SD = 16$ seconds).

Emotionality

The emotional content of a visualisation and the emotional state of a participant may have cognitive and perceptual effects on performance in visualisation tasks [60, 124, 164]; this was the primary motivation behind performing the pre-study. Nevertheless, it is not guaranteed that each participant considers the emotional content of the correlative statement to be the same. To account for these individual differences, ratings of emotional valence are also collected during the main study.

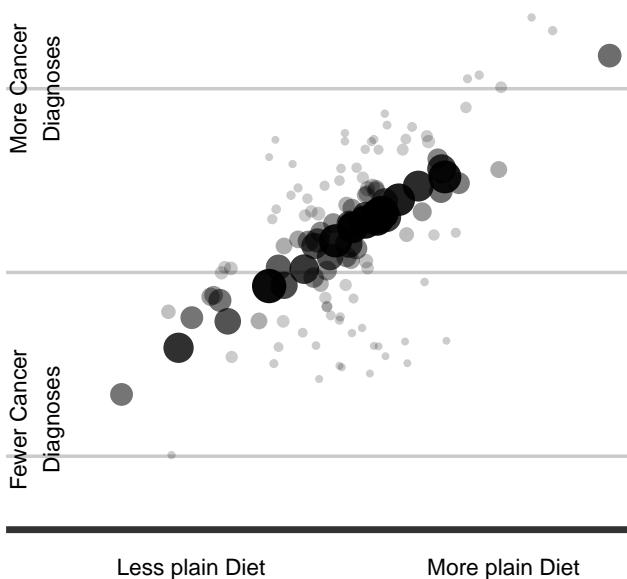
7.6.2 Method

Stimuli

After selecting a correlative statement describing a weak relationship and with a high level of consensus between participants, the `ggplot2` package (version 3.5.1) [174] was used to create the stimuli for the main experiment. As the statement was rated as describing a low level of relatedness, scatterplots describing a strong relationship ($0.6 < r < 0.99$) were used with the intent of inducing belief change. Plots in the alternative scatterplot condition were created using a combination of non-linear opacity and size decay, as this particular condition was shown to bias correlation estimates to a greater degree than either point opacity or size decay alone (see Experiment 4 in Chapter 6). 45 r values uniformly distributed between 0.6 and 0.99 were used to create 45 scatterplots for each condition. Examples of stimuli using an r value of 0.6 for both the standard and alternative scatterplot conditions can be seen in Figure 7.2.

Spicy Foods

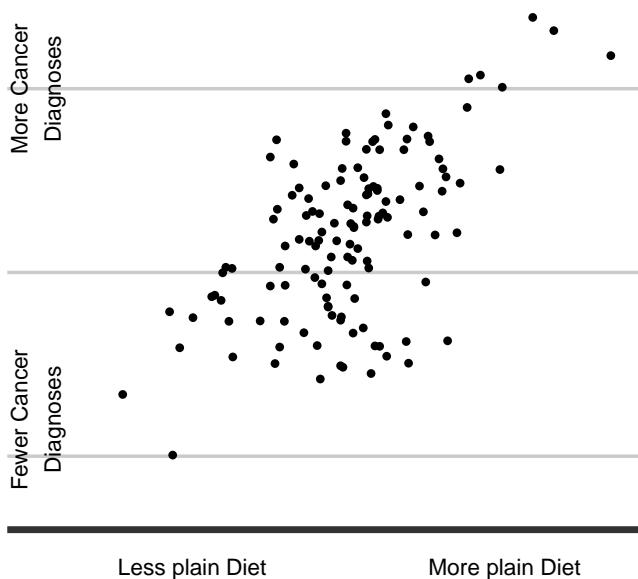
Higher consumption of plain (non-spicy) foods is associated with a higher risk of certain types of cancer.



Source: NHS England
Atypical Scatterplot

Spicy Foods

Higher consumption of plain (non-spicy) foods is associated with a higher risk of certain types of cancer.



Source: NHS England
Typical Scatterplot

Figure 7.2. Examples of the experimental stimuli for Experiment 5 using an r value of 0.6. Group A saw the alternative scatterplot presented on the left, while group B saw the typical design on the right. The labels below the plots are included for the reader's convenience, and were not a part of the experimental stimuli.

Design

Unlike all previous experiments, a between-participants design was employed here. Each participant was randomly assigned either to group A, in which they viewed standard scatterplots, or group B, in which they viewed alternative scatterplots designed deliberately to elicit higher levels of belief change. Participants saw all 45 experimental items for their group, along with 4 attention check items, in a fully randomised order. The dependent variable was the level of belief change induced by viewing the scatterplot visualisations, so participants were tested on how strongly related they believed the variables described by the correlative statement were both **before** and **after** viewing the experimental items.

Procedure

Participants viewed the PIS and provided consent through key presses in response to displayed consent statements. Participants were then asked to provide their age and gender identity. Following this, participants completed the 5-item Subjective Graph Literacy scale as in previous experiments [54], and Albarracín and Mitchell's [3] 12-item defensive confidence scale . To give legitimacy to

the data visualisations with the hope of maximising any potential belief change, participants were told that the graphs they would see were taken from a well-known British news source, but that the identity of this source had been obscured at their request. To promote engagement with the visualisations, participants were instructed to use a slider to estimate the correlation displayed in each scatterplot; no hypotheses were made based on these data, and therefore they were not analysed further. Following instructions, which included textual descriptions of scatterplots and Pearson's r , participants were given two practice trials; these trials took the form of a typical "standard scatterplot" trial from Experiment 1. Participants were then asked to indicate their beliefs about emotional valence and strength of relatedness described in the chosen correlative statement; these data were captured using Likert scales identical to those described previously. After completing 45 experimental trials, participants were then asked again, using the same Likert scales, to indicate their beliefs about emotional valence and strength of relatedness. Interspersed among the experimental items were four attention check trials which explicitly asked participants to set the slider to 0 or 1.

Participants

150 participants were recruited using the Prolific platform [129]. Normal or corrected-to-normal vision and English fluency were required. As in the pre-study, UK residency was required of participants, as the experiment relied on familiarity with the visual style of a British news source. Participants who took part in the pre-study, or in any of the experiments described in Chapter 4, Chapter 5, or Chapter 6 were prevented from completing this experiment. Data were collected from 77 participants in each condition. 2 participants failed more than 2 out of 4 attention check questions for each condition, meaning their data were excluded per pre-registration stipulations. Data from the remaining 150 participants were included in the full analysis (73 male, 73 female, and 4 non-binary). Participants' mean age was 39.3 ($SD = 11.5$). Participants' mean graph literacy score was 21.3 ($SD = 4.3$) out of 30, their mean defensive confidence score was 43 ($SD = 6.8$) out of 60, and their mean rating of statement emotional valence was 2.9 ($SD = 1.3$) on a 7-point Likert scale. On average, participants took 14.2 minutes to complete the experiment ($SD = 6.41$).

7.6.3 Results

Likert scales capture whether one rating is higher or lower than another, however they do not quantify the differences between levels of rating. Metric modelling assuming equal levels of difference

Table 7.2. Statistics for the significant main effect of rating time. Odds ratio and the equivalent Cohen's *d* value is also supplied.

	Estimate	Standard Error	Z-value	<i>p</i>	Odds Ratio	Cohen's <i>d</i>
Rating Time	3.77	0.049	76.63	<0.001	43.2	2.08

Table 7.3. Statistics for the significant main effect of rating time and the significant interaction between rating time and condition on the difference between pre- and post-scatterplot viewing ratings for standard and alternative plots. Odds ratios and equivalent Cohen's *d* effect sizes are also shown.

	Estimate	Standard Error	Z-value	<i>p</i>	Odds Ratio	Cohen's <i>d</i>
Rating Time	4.15	0.063	66.34	<0.001	63.33	2.29
Condition	0.49	0.390	1.25	0.211	1.63	0.27
Rating Time <i>x</i> Condition	-0.72	0.071	-10.22	<0.001	2.06	0.40

between ratings, such as linear regression, is therefore inappropriate [89]. In light of this, the **ordinal** package (version 2023.12-4.1 [29]) was used to build cumulative link mixed effects models to analyse Likert scale data⁸. As in previous chapters, the **buildmer** (version 2.11 [169]) package is used to automate the selection of the random effects structure (see Section 3.3.3 for further details). Odds ratios and equivalent Cohen's *d* effect size values are calculated using the **effectsize** package (version 1.0.0 [13]).

To test the first hypothesis, that ratings of strength of relatedness would be different before and after participants viewed experimental items, a model is built whereby the rating of strength of relatedness the participant made is predicted by whether it was made **before** or **after** viewing the experimental items. The first hypothesis was supported; there was a significant difference in ratings of strength of relatedness made before and after participants viewed the experimental plots. A likelihood ratio test revealed that the model including time of rating as a predictor explained significantly more variance than the null ($\chi^2(1) = 8,046.95, p < .001$). This model has random intercepts for participants. Statistical testing providing support for this hypothesis is shown in Table 7.2. Figure 7.3 shows means and dot plots for ratings of strength of relatedness made before and after viewing scatterplots in either the standard or alternative condition.

The second hypothesis, that the difference between ratings of strength of relatedness made before and after participants viewed the experimental plots would be greater when they were assigned to the alternative scatterplot condition, also received support. Treatment coding was used for each of the experimental factors of rating time (pre- or post-) and scatterplot condition, which facilitates direct comparisons between means of ratings made before and after plot viewing. A cumulative

⁸The linked pre-registration (see Section 7.4) specifies linear mixed effects models. This was an oversight; conclusions are identical when using said models.

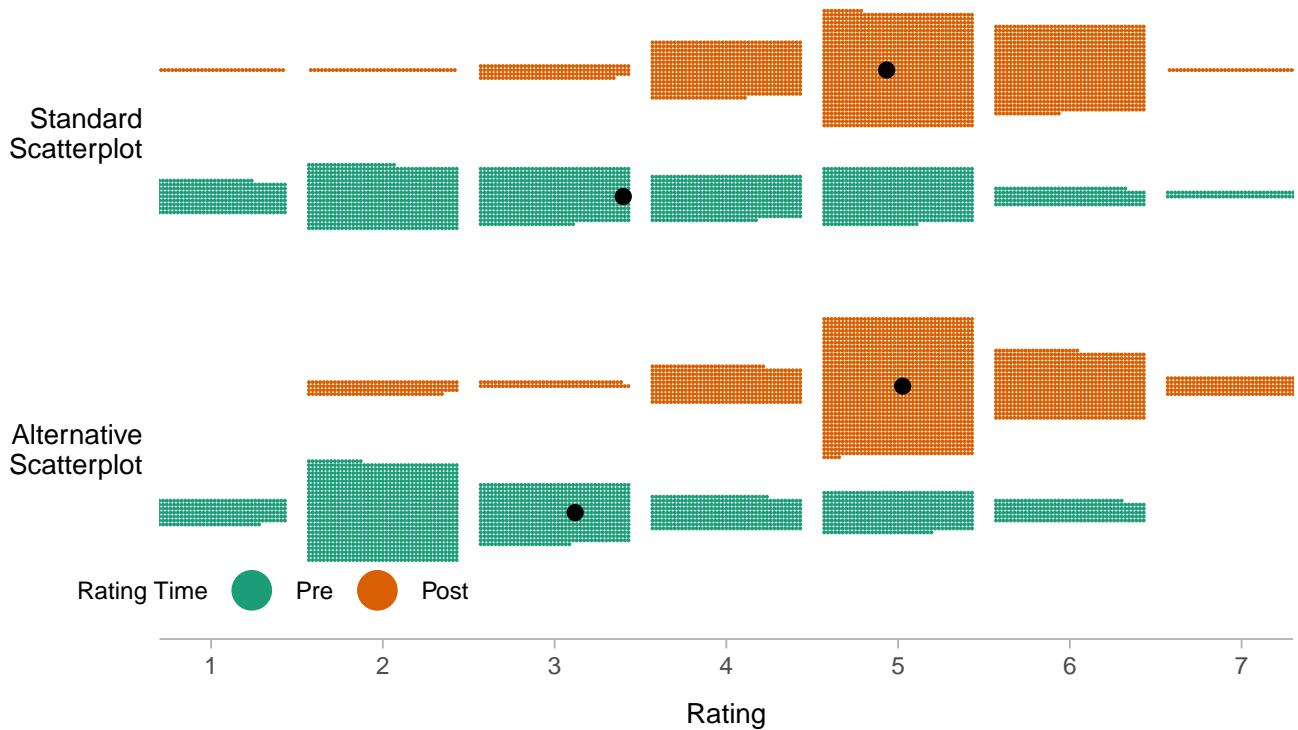


Figure 7.3. Dot plots for pre- and post-plot viewing ratings of strength of relatedness for standard and alternative scatterplot conditions. Mean ratings are also shown as points.

link mixed effects model, whereby the rating of strength of relatedness the participant made was predicted by the condition they were assigned to *and* the time they made the rating was built. A likelihood ratio test revealed that the model including condition and rating time as predictors explained significantly more variance than the null ($F(3) = 8,151.94, p < .001$). This model had random intercepts for participants. There was a main effect of rating time, no main effect of condition, and an interaction between rating time and condition. Test statistics, along with odds ratios and equivalent Cohen's d effect sizes can be seen in Table 7.3. The estimate for the interaction corresponds to the difference-in-difference between ratings made pre- and post-viewing for standard and alternative scatterplots. This difference-in-difference is visualised in Figure 7.4; the difference in ratings between pre- and post-plot viewing times is greater for the participants who were exposed to the alternative scatterplot condition.

Additional Analyses

Effects were also found of participants' scores on the defensive confidence test ($F(4) = 69.73, p < .001$), participants' scores on the graph literacy test ($F(4) = 42.66, p < .001$), and of how emotionally valent participants rated the chosen correlative statement before beginning the block of trials ($F(4) = 43.51, p < .001$). The interactions between the main effect and graph literacy, defensive confidence, and statement emotional valence are discussed in Section 7.6.4.

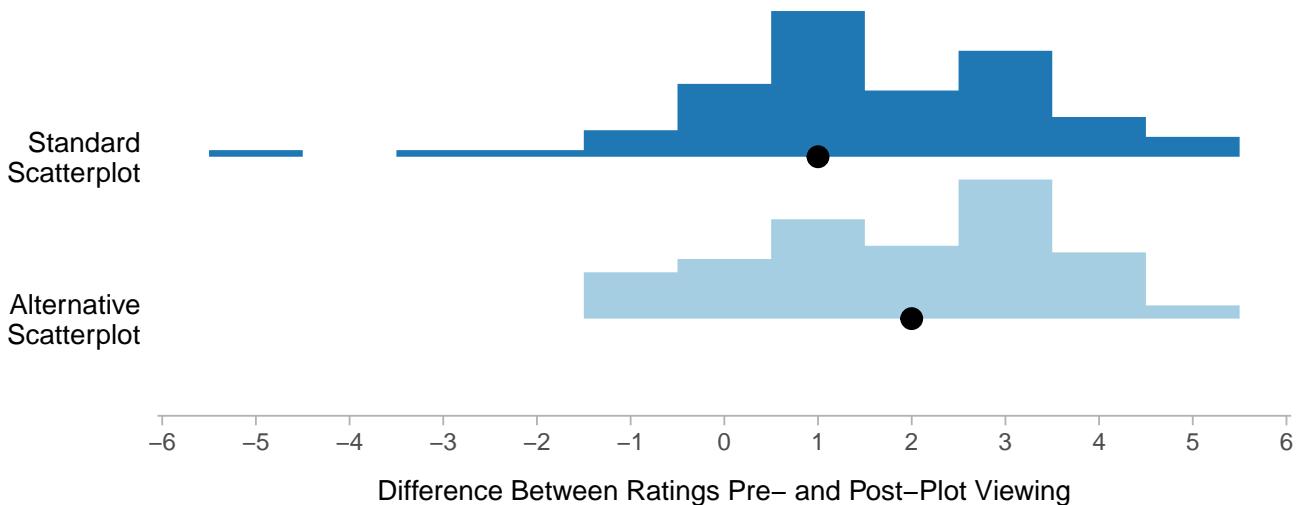


Figure 7.4. Histograms illustrating the magnitudes of the difference between pre- and post-plot viewing ratings of strength of relatedness for standard and alternative scatterplots. Median values are plotted as points.

7.6.4 Discussion

Both hypotheses were supported by the results of the main experiment. Participants reliably updated their beliefs after viewing scatterplots, and the difference between pre- and post-viewing beliefs was greater for those participants who viewed scatterplots in the alternative condition. These results suggest that the perceptual effects found in Chapter 4, Chapter 5, and Chapter 6 can be extended into a higher level cognitive space to change people’s beliefs about the strength of relatedness between a pair of variables. These findings are encouraging for data visualisation designers who wish to design scatterplots such that correlation perception more closely matches the underlying statistics, however further work is required before developing guidelines for the use of alternative scatterplot designs with regards to producing more persuasive visualisations.

Graph Literacy, Defensive Confidence, and Statement Emotional Valence

Mean differences in pre- and post-plot viewing ratings of strength of relatedness by Subjective Graph Literacy score can be seen in Figure 7.5. Generally, participants with higher scores on a graph literacy test experienced smaller changes in their ratings of strength of relatedness. This is in line with previous work suggesting that those with higher levels of graph or visualisation literacy show better performance in inference tasks related to visualisations [24], are more capable of describing effects that visualisations aim to communicate [146], and can preferentially attend to relevant features of visualisations to a greater degree [112], than those with lower levels of graph literacy. In the present study, evidence is provided that those with greater levels of graph literacy are *less susceptible* to

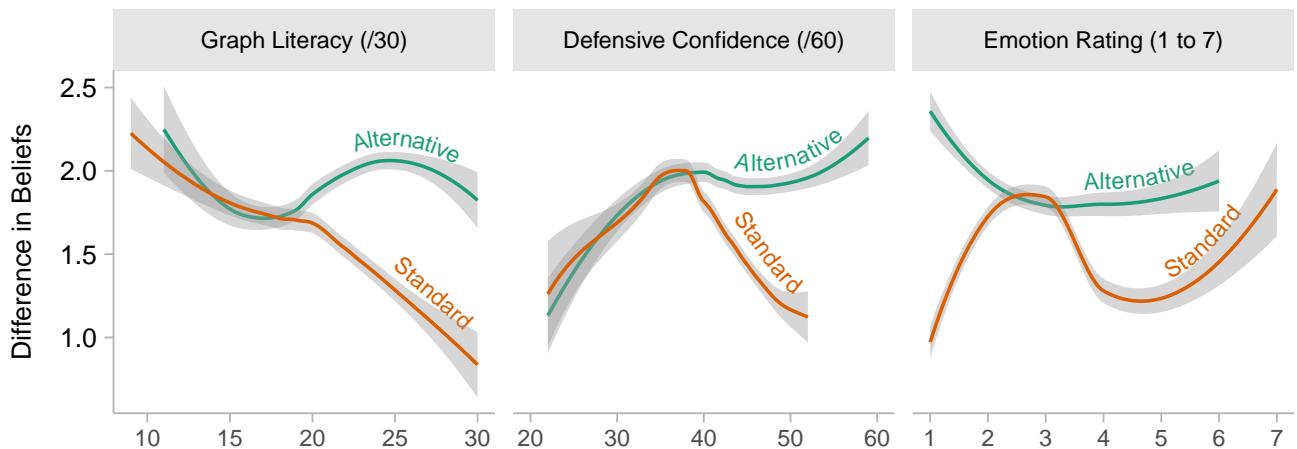


Figure 7.5. Illustrating how differences in beliefs about strength of relatedness change as a function of participants' scores on the graph literacy test (left), their scores on the defensive confidence test (centre), and their ratings of statement emotional valence (right). Locally smoothed curves with 95% CI ribbons are shown separately for standard and alternative scatterplot viewing conditions. Lower ratings of Difference in Beliefs (*y* axis) corresponds to lower levels of belief change between pre- and post-scatterplot viewing times.

having their beliefs changed by visualisations. The use of the alternative plot manipulation largely removes this effect, suggesting that there is less systematic reliance on graph literacy when participants are faced with an unfamiliar data visualisation.

An opposing pattern of results is observed when examining the effects of defensive confidence on participants' propensity for belief change. Generally, participants with higher scores on the defensive confidence test experienced greater levels of belief change. This is in line with evidence that those who are more confident in their ability to defend their own beliefs are more liable to having those beliefs changed in light of evidence [3]. This effect has previously been explained as being due to those with a greater degree of confidence in their own ability to defend their ideas engaging with information with lower levels of attention to the fact it opposes their beliefs. The present study provides additional evidence in favour of this phenomenon. While the general pattern of results is expected based on previous work, the interaction present between defensive confidence and scatterplot condition is novel. It would appear that despite following the normal pattern of results for low to moderate levels of defensive confidence, those participants who viewed the standard scatterplots experienced a drop in belief change as defensive confidence increased past $\sim 36/60$. It may be that the unfamiliar nature of the alternative scatterplots was protective against an unexpected behaviour whereby very high levels of defensive confidence decrease susceptibility to belief change.

The effect of statement emotional valence on belief change is also illustrated in Figure 7.5. There is a broad research space regarding emotionality and data visualisation [82], and it is clear from previous work that emotion may affect perception, cognition, and behaviour [60, 124, 164] pertaining to

data visualisation. Harrison et al. [60] found that participants who were positively primed performed better on a low-level visual judgement task compared to those who were negatively primed. Comparison of this work to the current is difficult, as *success* is hard to define in the present experimental paradigm.

Further experimental work is required to provide more comprehensive explanations for the interactive effects of graph literacy, defensive confidence, and statement emotionality as they pertain to belief change after scatterplot viewing.

7.7 General Discussion

The most parsimonious explanation for the results observed in the present study is as follows; things that *look* more related will be *judged* as being more related, and are therefore *more* able to change beliefs about the levels of relatedness between variables. Given the frequent real-world usage of scatterplots, and the role of data visualisations in decision-making, it is particularly important to test empirically whether perceptual effects may be extended into a cognitive space to influence beliefs. Doing so is a necessary step in broadening the data visualisation design space and bringing novel designs closer towards use cases with the potential for real-world consequences while maintaining a strong foundation of experimental evidence. Having controlled as far as possible for factors such as emotional content, the consensus on how related the variables in question were, and the general design (bar the points themselves) of the scatterplot, I can conclude with strong evidence that alternative scatterplot design was responsible for increasing the level of belief change amongst participants.

An alternative (although not competing) explanation for the results have seen here comes from recent work on the incorporation of uncertainty visualisations in scatterplots. Karduni et al. [74] found in 2020 that visualisations that encode uncertainty produce lower levels of belief change compared to those that do not. The alternative scatterplot design employed in the main experiment can be thought of as masking some of the uncertainty inherent in a scatterplot point cloud by reducing the salience of the most exterior points.

Previous work has provided support for the idea that it is the shape of the point cloud, more specifically, the width of the probability distribution it represents, that drives correlation perception in scat-

terplots. If this mechanism were valid, the observed results would be expected. These results are broadly consequential. For data visualisation designers, they provide strong evidence that utilising the alternative scatterplot designs described in this thesis can affect beliefs about levels of relatedness without requiring the removal of data. For researchers, these results pave the way for work in a number of directions, which is discussed in Section 7.8.

7.8 Future Work

Because alternative scatterplot designs have not been tested before with regard to belief change, the current study was designed with the intention of capturing effects, should they exist. To this end, the design was simple; multiple correlative statements were not investigated, nor was the propensity for strongly held beliefs to be changed or the effect of topics with strong or polarised emotional components. A simple, blunt measure of belief about relatedness was utilised, and only one of the alternative scatterplot visualisations described in this thesis was used. Each of these components deserves study, and each is ripe for future work to investigate the contributions of each factor to the effects we have seen here.

Section 7.6.4 describes the effects that graph literacy, defensive confidence, and participants' ratings of the emotional valence of the correlative statement have on the propensity for belief change. Future work may wish to investigate these factors, along with others that affect perceptions of correlation, such as educational background or spatial abilities [162]. Xiong et al. [179] describe how correlation estimation may differ according to the context the data are presented in; this could be extended to instead investigate statements with differing emotional contents and how alternative scatterplot designs might interact with emotional valence. Similarly, selecting matched participant groups with low or high graph literacy or defensive confidence would facilitate understanding of how alternative designs may be employed to cater for people with different levels of experience, or who differ in terms of their faith in their own ideas and abilities.

Previous works investigating beliefs with regard to correlation estimation have made distinctions between beliefs and attitudes [92, 179]. No such distinction was made here due to the novel utilisation of alternative designs. Markant et al. [92] found that while beliefs about correlations changed in participants as a result of interaction with scatterplots, attitudes did not. Future work may wish to investigate whether this finding would persist with scatterplots utilising the alternative designs de-

scribed here. Finally, while changing perceptions, beliefs, and attitudes are promising early steps, changing people's behaviours would be the real test of the power of alternative visualisation techniques; while this may be difficult to study, future work should investigate whether what has been found here and throughout this thesis may be used to induce behaviour change.

7.9 Limitations

The commitment to finding an effect, should one exist, is also the biggest limitation of the present chapter. The exploratory nature of the work means I cannot comment specifically on how different forms of size and opacity manipulation in scatterplots may change beliefs in different ways, although addressing this using the framework presented here would be simple to accomplish. To date, there has been no qualitative work performed on alternative scatterplot designs such as those utilised here; it may be that any perceptual or cognitive benefits are outweighed by distrust or unfamiliarity with novel designs. The dependent variable in the main experiment was represented to participants as a simple, blunt, 7-point Likert scale. While I argue that this is not particularly problematic given that the intention was to find an effect, future work may wish to use techniques that provide further scope for analysis, such as the graphical elicitation method developed by Karduni et al. [74, 75].

7.10 Conclusion

In a single, final experiment testing whether previously established perceptual techniques could be extended into a cognitive space to influence participants' beliefs about the level of relatedness between variables, evidence is provided that using a combination of non-linear, typical orientation point opacity and size decay functions is able to change beliefs to a greater extent than data-identical scatterplots that do not use these techniques. Scatterplots using these techniques were presented as news items, and featured a variable pair that had been selected by the target population as describing a weakly held, emotionally neutral correlation. Participants who viewed such scatterplots experienced greater levels of belief change compared to participants who only viewed standard scatterplots. Additionally, interaction effects were found of a number of participant characteristics. These results suggest that visualisation techniques that have previously been employed to improve perception amongst participants are deserving of study with regards to their potential to change beliefs.

Chapter 8

Conclusion

Changing the opacities and sizes of point in positively correlated scatterplots has clear effects on the estimation of correlation. This thesis set out to investigate whether visual manipulations of scatterplots based on aspects of human perception might be able to change estimates of correlation, and, in particular, whether these visual manipulations could be used to correct for a historic underestimation bias observed since the 1960s.

8.1 Main Findings

Each published experimental chapter in this work builds on the previous and provides a unique contribution to knowledge about how changing visual features may affect thoughts and perceptions about correlations.

In Chapter 4, the opacities of scatterplot points were altered while participants were tested on a correlation estimation task. The two published experiments in this chapter were partially based on an earlier pilot study that ultimately remains unpublished due to data quality issues (see 4.2 for a full account of this). In the first experiment in this chapter, the opacities of scatterplot points were changed in a uniform manner; this was done with the intent of discovering any general effects of point opacity on correlation estimation. Evidence was found for an increase in underestimation error when global point opacity was reduced. This result provided evidence that, in disagreement with previous work, changing the opacities of scatterplot points could be used to alter estimates of the correlation therein. Based on these findings, data from the pilot study, and evidence that the arrangement of points in scatterplots may be a good proxy for correlation judgements, experiment 2 varied point opacity using functions related to each point's distance from a regression line. In this experiment,

evidence was found that reducing the opacity of a point as it moved further from the regression line could bias participants' estimates of correlation upwards to partially correct for the underestimation bias. While this effect was small, it showed that estimates of correlation could be changed to correct for bias *without* removing any data from the plot.

Chapter 5 expanded these findings to a second visual modality (point size). Building on findings from Chapter 4 and evidence that the size of a stimulus may have stronger effects on perception than its opacity, a single experiment took the functions relating opacity and residual distance in the previous chapter and applied them to point size. The effect on correlation estimation observed here was much stronger than that observed in Chapter 4. Additionally, several changes in the nature of the estimation curve were observed in this experiment. Firstly, while the shape of the estimation curve when using point opacity manipulations was identical to that when using no visual adjustments, the use of size decay manipulations such that point size decreased with residual error resulted in an estimation curve with a visibly altered shape. Secondly, in previous work, and in all experiments undertaken in Chapter 4, precision in correlation estimation increases as the objective r value itself increases; this behaviour was not observed when employing manipulations that alter point size. The non-linear decay condition in this experiment remains the most promising condition with regards to correcting for the underestimation bias.

In a final implementation of the same experimental paradigm, Chapter 6 combined point opacity and size manipulations from the two previous chapters to further explore the potential for biasing estimates of correlation and the potential mechanisms behind the effects observed. Using point opacity and size manipulations together in both congruous (same direction) and incongruous (opposing directions) combinations, this single experiment showed that employing both manipulations was able to bias correlation estimates further than either manipulation in isolation, resulting in a marked over-estimation being introduced. This combination is not, however, linearly additive, suggesting different mechanisms of action behind the effects of each visual manipulation on correlation estimation. This experiment provided evidence that with regards to using point opacity and size manipulations to bias correlation estimates, there are few limits. Figure 8.1 summarises the mean estimation errors from all 4 experiments described in Chapter 4, Chapter 5, and Chapter 6.

Extending these findings into a cognitive space, Chapter 7 utilised the congruous typical orientation size and opacity decay condition from experiment 4, contextualised as part of news items, to explore whether the perceptual effects found in the previous three chapters might also be able to

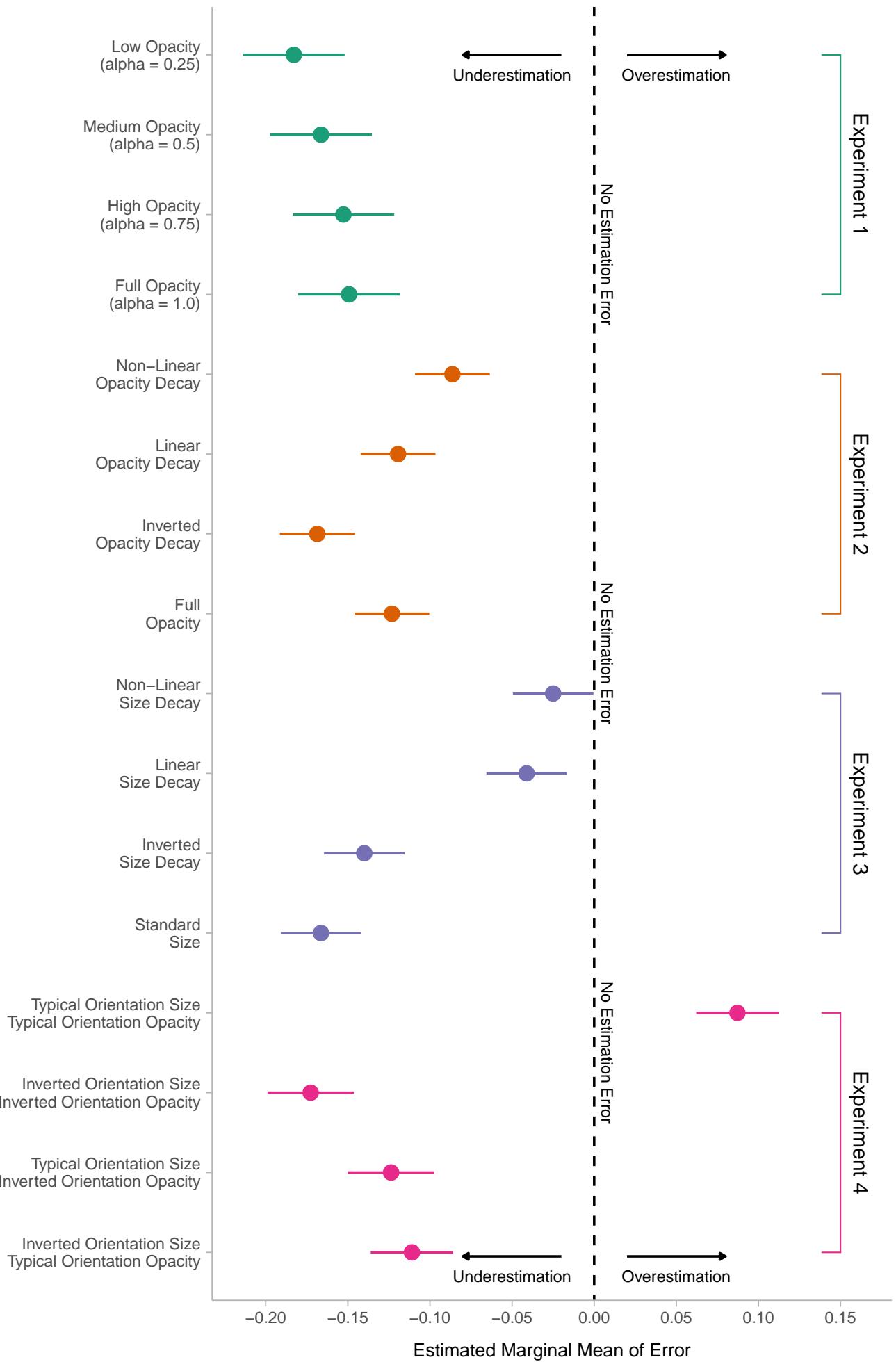


Figure 8.1. Estimated Marginal Mean errors in correlation estimation for experiments 1 to 4. This figure is an amalgamation of Figures 4.7, 4.11, 5.4, and 6.2. 95% confidence intervals are shown as error bars. The vertical dashed line represents no estimation error.

bias people's beliefs about the levels of relatedness between variable pairs. In this experiment, people's beliefs about a pair of variables were tested before and after viewing a series of data-identical scatterplots with either the congruous typical orientation opacity and size decay condition, or with no opacity or size decay present. While beliefs about relatedness were changed in each condition, they changed to a greater degree when participants viewed the scatterplots with opacity and size decay present. This finding provided evidence that the perceptual effects of systematically altering the opacities and sizes of scatterplot points can be extended into a cognitive space, and are able to change what people believe about the levels of relatedness between variable pairs.

8.2 Relationship to Prior Work

Overall, the work presented in this thesis contributes to wider evidence that simple changes in visual features can have perceptual [70] and cognitive [74] effects on those who view such visualisations. The finding that changing the opacities and sizes of scatterplot points can affect viewers' estimates of correlation is in direct contradiction to earlier work [131, 133], which found no effects of changing point opacities and sizes in different ways. The novelty of the findings in this thesis necessarily means that commenting on the potential mechanisms behind them is difficult. Taking the findings from Chapter 4, Chapter 5, and Chapter 6 together, the balance of evidence suggests that decreasing the opacity or size of more exterior scatterplot points is able to bias estimates of correlation upwards by reducing the influence of those points when viewers make their estimates. The results presented in Chapter 7 provide evidence that cognitions about data visualisations can also be altered by small changes in visual presentation. This is in line with a large body of work demonstrating that small changes in visual presentation - often of the kind that may be considered only as aesthetic preference by designers - can lead to large changes in perceptions and the conclusions that viewers of visualisations draw; see Fygenson et al./ 2023 [51] for examples with bar chart spacing and arrangement, Moritz et al., 2023 [104] for examples with line graphs, and Bradley et al., 2025 [19] for examples with axis limits and truncation. Franconeri et al. (2021 [45]) and Szafir (2018 [160]) both provide excellent wider reviews on how poor design can lead to visualisation-driven miscommunication and bias.

8.3 Reproducibility

As detailed in Chapter 3, this project has been conducted according to the principles of open and reproducible science [9]. Doing so effectively facilitates future replication and authentication of my published results, in addition to enabling others to learn from where I have succeeded and failed. Materials are provided throughout for the re-creation of each individual experimental paper, including Docker containers to reproduce the specific computational environments used. This thesis also features an all-in-one Docker implementation, details of which can be found in the repository associated with this paper. Owing to the accessibility issues present with the PDF format [81], this thesis is also hosted as a website¹.

8.4 Contributions

This thesis addresses a long-standing bias in the interpretation of correlation in positively correlated scatterplots by drawing on influences from perceptual psychology and data visualisation design. A thorough exploration of the effects of two salient visual features (opacity and size) on correlation estimation was carried out over a series of well-powered experiments in Chapter 4, Chapter 5, and Chapter 6. Furthermore, it was shown that the effects of point opacity and size with regards to the estimation of positive correlation could be extended into a cognitive space to change peoples' beliefs about the levels of relatedness between a pair of variables.

The most important contribution this thesis makes is to the visualisation design and HCI audiences; secondarily, I add to a growing body of work that, taken together, may one day be able to reveal precisely how viewers' interpret correlation from scatterplots.

Additionally, I hope to have provided an example of a PhD project conducted from the ground up in an open and reproducible manner. A PhD project is as much a story as it is a piece of research, and through acknowledgement of the successes and failures of the work I have done, along with the employment of a host of reproducibility-enabling tools, I hope to contribute an honest account of what one PhD project may look like.

¹placeholder

8.5 Implications

8.5.1 For Design

It is clear from the results presented in this thesis that designers must be aware of the potential effects on perception and cognition of aesthetic choices in data visualisation design. This is particularly salient given that point opacity changes and point size changes are both common techniques in scatterplot design. The former is often used to reduce overplotting issues, while the latter is used to create trivariate bubble charts. Given that many data visualisations are designed with a range of communicative intentions, designers may be inadvertently sacrificing effectiveness in one modality (correlation perception), for clarity in another (reduction in overplotting, for example); while this trade-off may be acceptable, it is important for designers to be aware of the effects their designs have on people's thoughts and feelings about the data stories they are being told.

On the other hand, this work also provides marked opportunities. Where appropriate, using some form of the point opacity and/or size manipulations described in this thesis could enhance the communicative efficacy of positively correlated scatterplots. I would not recommend this until further work has been done on the potential for misinterpretation from scatterplots such as those described here (see Section 8.7).

8.5.2 For Society

Given the ubiquity of scatterplots and the fundamental nature of correlation as a mathematical concept, the findings here have the potential to change the way that people engage with visualisations depicting relatedness. This is especially pertinent given the findings detailed in Chapter 7 that suggest that the techniques described here are also able to bias viewers' beliefs about correlations. Whether or not such techniques would be appropriate is a question that requires further work to answer.

Additionally, I hope that this project is demonstrative of the need for the re-examination of common data visualisations from the ground up. In an age where everything is measured, and the complexity of information increases year on year, data visualisations are crucial for the communication of difficult topics to the general public. Many of the most popular, widely-used data visualisations are not designed to be accurately interpreted by lay people; given that this is the group that represents the

greatest viewership, and that has the most to gain from engaging with data visualisations, more design must take place that acknowledges and addresses the ways in which perceptions and cognitions around data visualisations are biased. Designing in this way requires humility; as designers we must recognise that biases exist in common data visualisations and work thoroughly to address them.

8.6 Limitations

For Chapter 4, Chapter 5, and Chapter 6, the shared experimental method confers the largest limitation in this thesis. As opposed to previous work that employed more psychophysical designs, such as correlation discrimination or magnitude estimation via bisection tasks [134, 135], experiments 1 to 4 in this thesis utilised a direct estimation paradigm more akin to much older correlation perception work [16, 32, 35, 83, 84, 98, 154]. While this technique supplies robust conclusions for the HCI and design audiences, it means that making claims about the precise mathematical relationship between perceived and actual correlation is difficult based on the data gathered. Similarly, the correlation judgements that participants made in these experiments must be considered in context; it would not be expected that people may view 180 data visualisations in sequence during the course of normal, day-to-day visualisation viewing.

All experiments presented in this thesis were conducted entirely online. Doing so allowed for the rapid collection of large amounts of high quality data, however necessarily limits the amount of additional data that can be collected. A method for determining the dot pitch of participants' monitors was employed throughout and included in statistical modelling paradigms. Generally, this explained little to no variance in participants' responses. Online experimentation does not allow for the determination of other factors that may explain variance, such as head-to-monitor distance or monitor specifications that may influence the visual quality of scatterplots with altered point opacities or sizes.

8.7 Future Directions

There are numerous avenues for future work based on this thesis, many of which are relatively simple extensions of the techniques described. Firstly, all experiments in this thesis took place using positively correlated scatterplots. There is evidence that, similarly to how viewers *underestimate* correlation in positively correlated scatterplots, they may also *overestimate* correlation in negatively

correlated scatterplots [148]. Uniquely among correlation visualisations, the visual forms of positively and negatively correlated scatterplots are symmetrical [59], suggesting that the techniques modifying point opacity and size described in this thesis may be used (in a symmetrical manner) to address the overestimation bias. Doing so would require a relatively simple replication of a subset of the experiments described here using negatively correlated scatterplots.

This thesis represents a large amount of quantitative data on how viewers perceive correlation in scatterplots, and how their beliefs about relatedness may be changed following scatterplot viewing. Before recommendations can be firmly made about the potential use of these techniques, however, qualitative work should take place investigating users' thoughts and feelings about novel visualisations that have been designed based on perceptual metrics. It may be the case that any potential advantages of the novel scatterplots described herein are outweighed by distrust or unfamiliarity with novel designs.

Finally, to further investigate precisely *why* the techniques described in this thesis work, lab-based experiments should be conducted. While more costly and time-consuming, this would allow for far more control over the precise experimental set-up, and would additionally allow for the employment of eye-tracking to understand how changing point opacities and sizes may influence perceptions of correlation.

8.8 Closing Remarks

The design of data visualisations should be based, at least partially, on the ways in which those data visualisations are perceived by the people that use them. The opacities and sizes of points in positively correlated scatterplots have clear, strong effects on the perceptions of correlation based on them. The extent to which viewers update their beliefs following scatterplot-viewing can also depend on the opacities and sizes of scatterplot points. These findings highlight that small changes in the visual features of common data visualisations may produce large changes in the perceptions and cognitions of viewers.

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Appendices

Appendix A

First appendix

A.1 Section in Appendix