

# Friday 01/17/2025: Theorems

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CSCI 246: Discrete Structures

Textbook reference: Sec. 4, Scheinerman

# Quiz

Replace each ? with a checkmark ✓ if the combination of truth values for propositions A and B is *possible* under the given logical connective.

Replace it with a ✗ if the combination is *impossible*.

Propositions		Logical Connectives		
A	B	if A then B	if B then A	A if and only if B
T	T	?	?	?
T	F	?	?	?
F	T	?	?	?
F	F	?	?	?

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T	T	?	?	?
T	F	?	?	?
F	T	?	?	?
F	F	?	?	?

See whiteboard for solution.

# Propositional logic

Original propositions		New propositions		
A	B	if A then B	if B then A	A if and only if B
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

- The column headings show 3 new propositions, formed from the original propositions by **logical connectives**.
- The first two columns combined with one remaining column gives the **truth table** for that logical connective.
- Each logical connective can be thought of as a **function** or mapping  $\{T, F\} \times \{T, F\} \rightarrow \{T, F\}$ .
- There are other such functions (**and**, **or**, **xor**, etc.), some of which were discussed in the text.
- The study of how to combine and change propositions under logical connectives to form more complex propositions is called **propositional logic**.

## Group work!

Students are randomly assigned into groups of 3 on the next slide.

Each group gets  $\frac{1}{2}$  of a white board.

If the  $\frac{1}{2}$  white board is inconvenient, feel free to write on a window!

Group 1: timothy.true,conner.reed1,connor.mizner  
Group 2: jacob.ruiz1,evan.barth,evan.schoening  
Group 3: matthew.nagel,connor.graville,adam.wyszynski  
Group 4: lynsey.read,connor.yetter,ryan.barrett2  
Group 5: caitlin.hermanson,james.brubaker,peter.buckley1  
Group 6: derek.price4,alexander.goetz,jacob.ketola  
Group 7: tristan.nogacki,jeremiah.mackey,michael.oswald  
Group 8: nicholas.harrington1,aaron.loomis,joseph.windmann  
Group 9: samuel.rollins,zeke.baumann,samuel.hemmen  
Group 10: erik.moore3,colter.huber,devon.maurer  
Group 11: jonas.zeiler,luke.donaldson1,carver.wambold  
Group 12: jett.girard,carsten.brooks,justice.mosso  
Group 13: luka.derry,nolan.scott1,owen.obrien  
Group 14: anthony.mann,samuel.mosier,blake.leone  
Group 15: yebin.wallace,peyton.trigg,emmeri.grooms  
Group 16: julia.larsen,tyler.broesel,sarah.perolat  
Group 17: bridger.voss,jack.fry,micaylyn.parker  
Group 18: jacob.shepherd1,ethan.johnson18,joseph.triem  
Group 19: cameron.wittrock,lucas.jones6,jada.zorn  
Group 20: reid.pickert,delaney.rubb,alexander.knutson  
Group 21: griffin.short,jakob.kominsky,john.fotheringham  
Group 22: mason.barnocky,william.elder1,kaden.price  
Group 23: pendleton.johnston,joseph.mergenthaler

## Group exercises

1. It is a common mistake to confuse the following two statements (i) If A, then B and (ii) If B, then A. Find two conditions A and B such that statement (i) is true but statement (ii) is false. Then find two conditions A and B such that both statements are true.
2. Two propositions are considered *equivalent* if they have the same truth table values. Show that the biconditional  $A \iff B$  is equivalent to  $(A \implies B)$  and  $(B \implies A)$ .
3. Consider these two statements: (i) If A, then B, (ii) If (not B), then (not A). Under what circumstances are these statements true? When are they false? Explain whether these statements are identical or not. [Note: (ii) is called the **contrapositive** of (i).]
4. (Challenge problem, from philosopher Norman Swartz.) Is the following statement true or false, and why? *A's-being-a-necessary-condition-for-B is both a necessary and sufficient condition for B's-being-a-sufficient-condition-for-A.*

## Question 1: Solution

$A \implies B$  but  $B \not\implies A$ :

$A =$  I lived in Los Angeles

$B =$  I lived in California.

$A \iff B$ :

$A =$  Valentine's Day is this month

$B =$  This month is February.



## Question 2: Solution

The truth table for the “and” operator (also written  $\wedge$ ) is given by

Original propositions		New propositions
X	Y	$X \wedge Y$
T	T	T
T	F	F
F	T	F
F	F	F

Now we apply the  $\wedge$  operator to the results of the  $\Rightarrow$  and  $\Leftarrow$  operators.

Orig. props.		New props.		
A	B	$\overbrace{A \Rightarrow B}^X$	$\overbrace{B \Rightarrow A}^Y$	$\overbrace{(A \Rightarrow B) \wedge (B \Rightarrow A)}^{X \wedge Y}$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Note that  $(A \Rightarrow B) \wedge (B \Rightarrow A)$  gives the same results as  $A \Leftrightarrow B$  as on Slide 3.

## Question 2: Solution

The truth table for the “and” operator (also written  $\wedge$ ) is given by

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X	Y	$X \wedge Y$
T	T	T
T	F	F
F	T	F
F	F	F

Now we apply the  $\wedge$  operator to the results of the  $\implies$  and  $\impliedby$  operators.

Orig. props.		New props.		
A	B	$\overbrace{A \implies B}^X$	$\overbrace{B \implies A}^Y$	$\overbrace{(A \implies B) \wedge (B \implies A)}^{X \wedge Y}$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Note that  $(A \implies B) \wedge (B \implies A)$  gives the same results as  $A \iff B$  as on Slide 3.

So what? This guides us towards how we'll prove  $A \iff B$  in Sec. 5.

## Question 3: Solution

Recall from Slide 3 that the truth table for the  $\implies$  operator is given by

X	Y	If X, then Y $X \implies Y$
T	T	T
T	F	F
F	T	T
F	F	T

Now we apply the  $\implies$  operator to the results of the "not" operator (also written  $\neg$ ).

Orig. props.		New props.		
A	B	$\neg A$	$\neg B$	$\neg B \implies \neg A$
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

Note that  $\neg B \implies \neg A$  gives the same results as  $A \implies B$  as on Slide 3.

## Question 3: Solution

Recall from Slide 3 that the truth table for the  $\implies$  operator is given by

X	Y	If X, then Y $X \implies Y$
T	T	T
T	F	F
F	T	T
F	F	T

Now we apply the  $\implies$  operator to the results of the "not" operator (also written  $\neg$ ).

Orig. props.		New props.		
A	B	$\neg A$	$\neg B$	$\neg B \implies \neg A$
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

Note that  $\neg B \implies \neg A$  gives the same results as  $A \implies B$  as on Slide 3.

Remark: We've shown that a proposition is logically equivalent to its contrapositive. So what? Sometimes it's easier to verify the contrapositive version.

## Question 4: Solution

The simplest way to see this is as follows:

- A's-being-a-necessary-condition-for-B can be expressed as  $B \implies A$ .
- B's-being-a-sufficient-condition-for-A can be expressed as  $B \implies A$ .
- In other words, both propositions are the same:  $B \implies A$ . And a proposition is always necessary and sufficient for itself.