Wednesday 01/22/2025: Proofs

CSCI 246: Discrete Structures

Textbook reference: Sec. 5, Scheinerman

Announcements before today's quiz

- **Sheet of paper**: Please bring your own sheet of paper to class each day for quizzes if possible. However, if you don't have any, you are welcome to take a blank sheet of paper from the stack in the front of the room.
- **Rules for quizzes**: For all quizzes in the course, you should use only paper and pencil. Please close your computers and textbooks, and put away your cellphones.

Reading Quiz

Quiz Question

Prove that the the sum of two even integers is even.

Use the appropriate proof template from the textbook.

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Definition 3.1 (**Even**)

An integer is called *even* provided it is divisible by two.

Definition 3.2 (**Divisible**)

Let a and b be integers. We say that a is *divisible* by b provided there is an integer c such that $bc = a$. We also say that b *divides* a, or b is a *factor* of a, or b is a *divisor* of a. The notation for this is $b|a$.

Solution Sketch

Proposition. The sum of two even integers is even.

Proof.

Annotation	**Main Text**
Convert Prop. to "if-then" form	We show that if x and y are even integers, then $x + y$ is even.
State "if"	Let x and y be even integers
Unravel defs.	Then by Defs. 3.1 and 3.2, there exist integers a, b such that $x = 2a$ and $y = 2b$.
*** The glue ***	What goes here?!?!
Unravel defs.	So there is an integer c such that $x + y = 2c$.
State "then"	Hence, $x + y$ is even.

Solution

Proposition. The sum of two even integers is even.

Proof.

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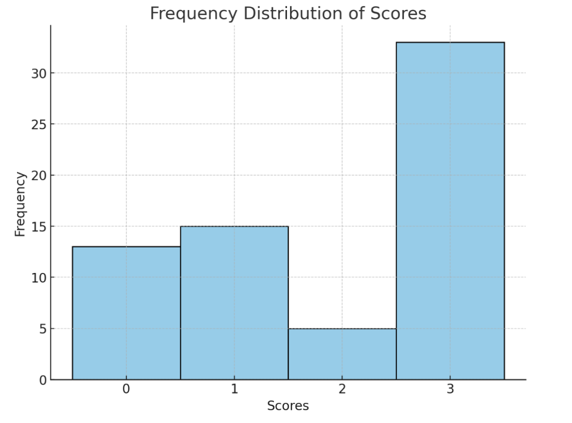
Convert Prop. to "if-then" form	We show that if x and y are even integers, then $x + y$ is even.
State "if"	Let x and y be even integers
Unravel defs.	Then by Defs. 3.1 and 3.2, there exist integers a, b such that $x = 2a$ and $y = 2b$.
*** The glue ***	Hence, $x + y = 2a + 2b = 2(a + b)$.
Unravel defs.	So there is an integer $c = a + b$ such that $x + y = 2c$.
State "then"	Hence, $x + y$ is even.

General announcements

- **Course repo**: Can navigate to syllabus (updated with TA/tutoring hours) and slides (e.g. group exercises with solutions). Both are mutable. Url is in Brightspace if you forget.
- **Brightspace email question**: Sometimes will send a message (e.g. survey response) in Brightspace email. Is that something you see?
- **Roster Logistics**: Drew Bolster please come see me after class.
- **Friday's problems quiz**: Although the group exercises are done collaboratively in groups of 3 people, the "problems quizzes": on Fridays will be taken by individuals. It will be combined with the reading quiz and can be done the same sheet of paper.

Sec 4. Reading Quiz Scores



What to conclude if your score was lower than you wanted?

- **Growth mindset**: ✓ Abilities are malleable and capable of improvement with effort. (e.g. "I need to change my reading strategy.")
- **Fixed mindset**: ✗ Abilities are fixed and unchangeable. (e.g. "I'm not smart/a math person/a good test taker.")

Review solutions to Sec. 4 group exercises

Sec. 5 group work!

Students are randomly assigned into groups of 3 on the next slide.

Each group gets $\frac{1}{2}$ of a white board.

If the $\frac{1}{2}$ white board is inconvenient, feel free to write on a window!

Group 1: bridger.voss,connor.mizner,nolan.scott1
Group 2: john.fotheringham,william.elder1,justice.mosso
Group 3: ethan.johnson18,michael.oswald,lynsey.read
Group 4: aaron.loomis,conner.reed1,luke.donaldson1
Group 5: griffin.short,joseph.triem,caitlin.hermanson
Group 6: joseph.mergenthaler,reid.pickert,yebin.wallace
Group 7: erik.moore3,samuel.rollins,james.brubaker
Group 8: jacob.shepherd1,connor.graville,connor.yetter
Group 9: zeke.baumann,ryan.barrett2,jada.zorn
Group 10: peyton.trigg,jakob.kominsky,jonas.zeiler
Group 11: jett.girard,jacob.ketola,carver.wambold
Group 12: emmeri.grooms,nicholas.harrington1,lucas.jones6
Group 13: blake.leone,tyler.broesel,sarah.periolat
Group 14: luka.derry,anthony.mann,pendleton.johnston
Group 15: peter.buckley1,jack.fry,cameron.wittrock
Group 16: samuel.hemmen,jacob.ruiz1,derek.price4
Group 17: jeremiah.mackey,matthew.nagel,devon.maurer
Group 18: micaylyn.parker,samuel.mosier,owen.obrien
Group 19: mason.barnocky,alexander.goetz,carsten.brooks
Group 20: adam.wyszynski,timothy.true,joseph.windmann
Group 21: evan.barth,alexander.knutson,tristan.nogacki
Group 22: julia.larsen,evan.schoening,colter.huber
Group 23: delaney.rubb,kaden.price

Group exercises

1. Prove that the square of an odd integer is odd.
2. Prove that the difference between consecutive perfect squares is odd.
3. Let x be an integer. Prove that $0|x$ if and only if $x = 0$.
4. Prove that an integer is odd if and only if it is the sum of two consecutive integers.

Group exercise #1: Solution

Proposition. The square of an odd integer is odd.

Proof.

Annotation	**Main Text**
Convert Prop. to "if-then" form	We show that if x is an odd integer, then x^2 is odd.
State "if"	Let x be an odd integer.
Unravel defs.	Then by definition of *odd*, there is an integer a such that $x = 2a + 1$.
*** The glue ***	So $x^2 = (2a+1)(2a+1) = 4a^2 + 4a + 1 = 2(2a^2 + 2a) + 1$.
Unravel defs.	So there is an integer b (where $b = 2a^2 + 2a$) such that $x^2 = 2b + 1$.
State "then"	Hence, x^2 is odd.

Group exercise #1: Solution

Proposition. The square of an odd integer is odd.

Proof.

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Unravel defs.	So there is an integer b (where $b = 2a^2 + 2a$) such that $x^2 = 2b + 1$.
State "then"	Hence, x^2 is odd.

Remark. You do not need to provide the annotations or colors in your own proofs. I am using them here in the solution to highlight the formulaic structure of an if-then proof.

Group exercise #2: Solution

Proposition. The difference between consecutive perfect squares is odd.

Proof.

Annotation	**Main Text**
Convert Prop. to "if-then" form	We show that if x and y are consecutive perfect squares, then $x - y$ is odd.
State "if"	Let x and y be consecutive perfect squares
Unravel defs.	Then $x = (z+1)^2$ and $y = z^2$ where z is an integer.
*** The glue ***	So $x - y = (z+1)^2 - z^2 = (z^2+2z+1) - z^2 = 2z+1$.
Unravel defs.	So there is an integer b (where $b = z$) such that $x - y = 2b + 1$.
State "then"	Hence, $x - y$ is odd.

Group exercise #2: Solution

Proposition. The difference between consecutive perfect squares is odd.

Proof.

Annotation	**Main Text**
Convert Prop. to "if-then" form	We show that if x and y are consecutive perfect squares, then $x - y$ is odd.
State "if"	Let x and y be consecutive perfect squares
Unravel defs.	Then $x = (z+1)^2$ and $y = z^2$ where z is an integer.
*** The glue ***	So $x - y = (z+1)^2 - z^2 = (z^2 + 2z + 1) - z^2 = 2z + 1$.
Unravel defs.	So there is an integer b (where $b = z$) such that $x - y = 2b + 1$.
State "then"	Hence, $x - y$ is odd.

Remark. You do not need to provide the annotations or colors in your own proofs. I am using them here in the solution to highlight the formulaic structure of an if-then proof.

Group exercise #3: Solution

Proposition. Let x be an integer. Prove that $0|x$ if and only if $x = 0$.

Proof. We decompose the *if-and-only-if* statement into two *if-then* statements.

(a) We show that if $0|x$, then $x = 0$.

Let x be an integer such that $0|x$.

Then by definition of *divisible*, there is an integer a such that $0 \cdot a = x$.

But $0 \cdot a = 0$.

Hence $x = 0$.

(b) We show that if $x = 0$, then $0|x$.

Let $x = 0$.

Let a be any integer. (For example, take $a = 7$.) Then $a \cdot 0 = 0$.

Hence, there is an integer a such that $0 \cdot a = x$.

Hence, $0|x$.

Group exercise #3: Solution

Proposition. Let x be an integer. Prove that $0|x$ if and only if $x = 0$.

Proof. We decompose the *if-and-only-if* statement into two *if-then* statements.

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Hence $x = 0$.

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Let $x = 0$.

Let a be any integer. (For example, take $a = 7$.) Then $a \cdot 0 = 0$.

Hence, there is an integer a such that $0 \cdot a = x$.

Hence, $0|x$.

Remark. An *if-and-only-if* proof consists of two *if-then* proofs. Each uses the same *if-then* template (and same color-scheme) as in Group Exercises #1 and #2. Note that some green rows were skipped (as there was no definition to unravel for $x = 0$).

Group exercise #4: Solution

Proposition. An integer is odd if and only if it is the sum of two consecutive integers.

Proof. We decompose the *if-and-only-if* statement into two *if-then* statements.

(a) We show that if x is the sum of two consecutive integers, then x is an odd integer.

Let x be the sum of two consecutive odd integers.

So there is an integer a such that $x = a + (a + 1)$.

So $x = 2a + 1$

Hence, there is an integer a such that $x = 2a + 1$.

Hence, x is an odd integer.

(b) We show that if x is an odd integer, then x is the sum of two consecutive integers.

Let x be an odd integer.

Then by definition of *odd*, there is an integer a such that $x = 2a + 1$.

So we have $x = 2a + 1 = a + (a + 1)$.

Hence x is the sum of two consecutive integers.

Group exercise #4: Solution

Proposition. An integer is odd if and only if it is the sum of two consecutive integers.

Proof. We decompose the *if-and-only-if* statement into two *if-then* statements.

(a) We show that if x is the sum of two consecutive integers, then x is an odd integer.

Let x be the sum of two consecutive odd integers.

So there is an integer a such that $x = a + (a + 1)$.

So $x = 2a + 1$

Hence, there is an integer a such that $x = 2a + 1$.

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(b) We show that if x is an odd integer, then x is the sum of two consecutive integers.

Let x be an odd integer.

Then by definition of *odd*, there is an integer a such that $x = 2a + 1$.

So we have $x = 2a + 1 = a + (a + 1)$.

Hence x is the sum of two consecutive integers.

Remark. An *if-and-only-if* proof consists of two *if-then* proofs. Each uses the same *if-then* template (and same color-scheme) as in Group Exercises #1 and #2.