04/11/2025: Subgraphs

CSCI 246: Discrete Structures

Textbook reference: Sec 48, Scheinerman

Graded Quiz Pickup

Quizzes are in the front of the room, grouped into four bins (A-G, H-L, M-R, S-Z) by last name. The quizzes are upside down with your last name on the back. Come find yours before, during, or after class. Only turn the quiz over if it's yours.

Today's Agenda

- Problems and reading quiz (15 mins)
- Mini-lecture ($\approx$ 10 mins)
- Group exercises ($\approx$ 20 mins)

Feedback on Wednesday's Quiz

Reading Quiz Scores

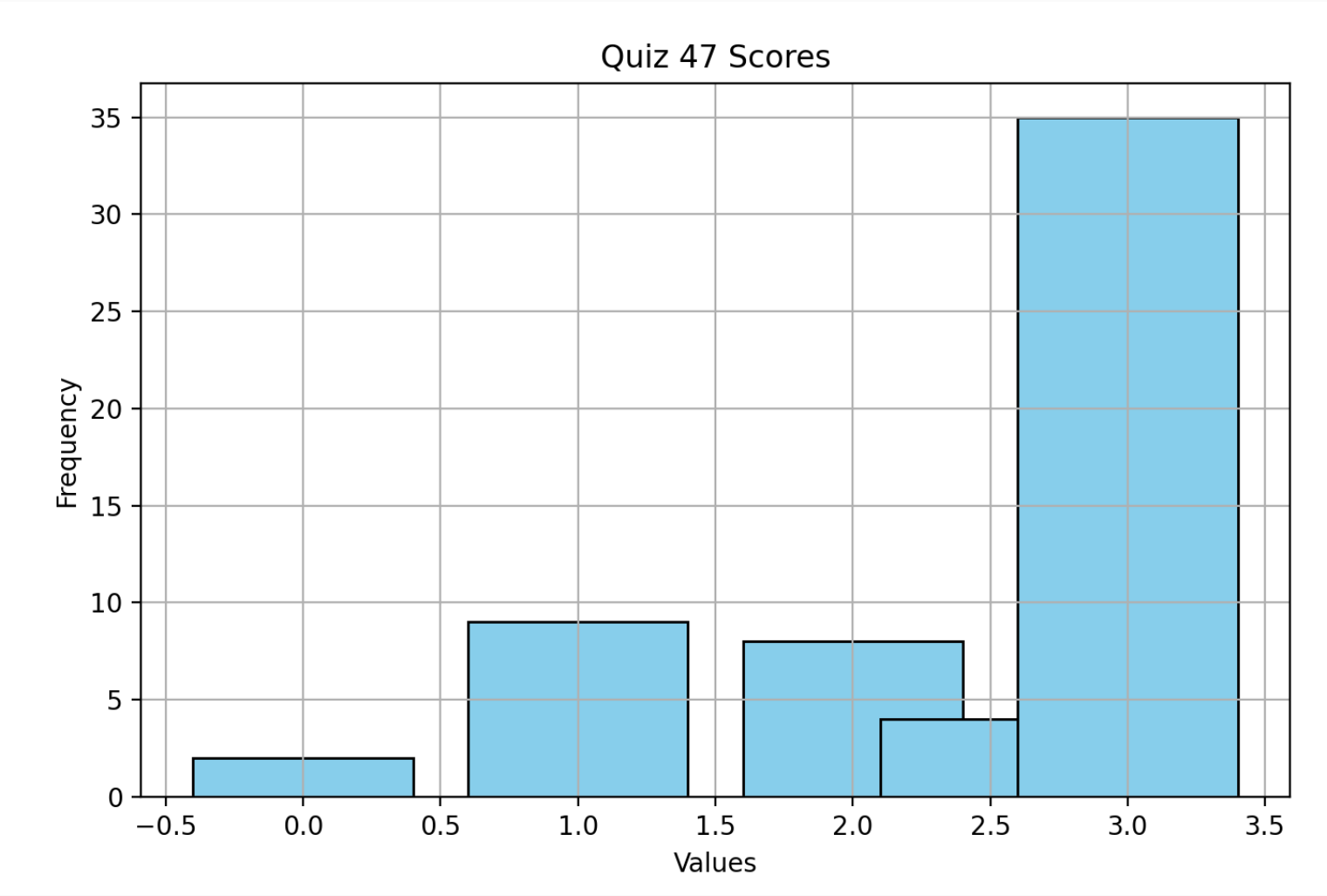


Figure 1: Median Score = 3/3 (100%)

Today's quiz

Problems Quiz (recurrence, big O notation, algorithm efficiency)

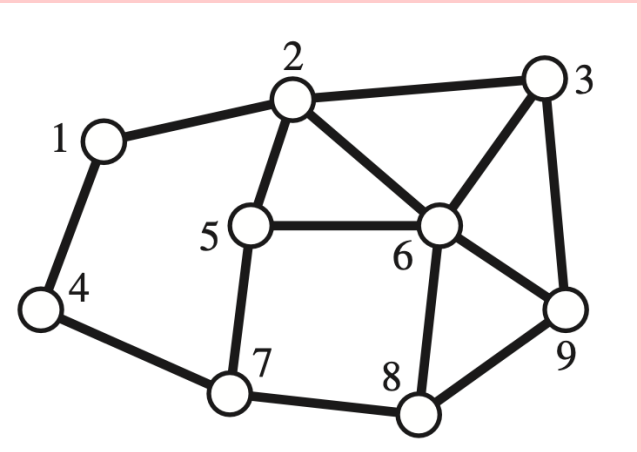
1. Solve the recurrence relation $a_n = 4a_{n-1} - 4a_{n-2}$ with initial conditions $a_0 = 5$ and $a_1 = 1$ to give an explicit formula for a_n.
2. Let $a_n = 3n^2 + 7$. Prove that $a_n = \Theta(n^2)$.

Reference material about second-order recurrences

To solve a recurrence of the form $a_n = s_1 a_{n-1} + s_2 a_{n-2}$, solve the quadratic formula $x^2 - s_1 x - s_2 = 0$ to find roots r_1 and r_2. If $r_1 \neq r_2$, then $a_n = c_1 r_1^n + c_2 r_2^n$. If $r_1 = r_2 \triangleq r$, then $a_n = c_1 r^n + c_2 n r^n$. Then find c_1 and c_2.

Reading Quiz (Subgraphs)

Name one clique and one independent set from the graph below.



Thoughts On Subgraphs

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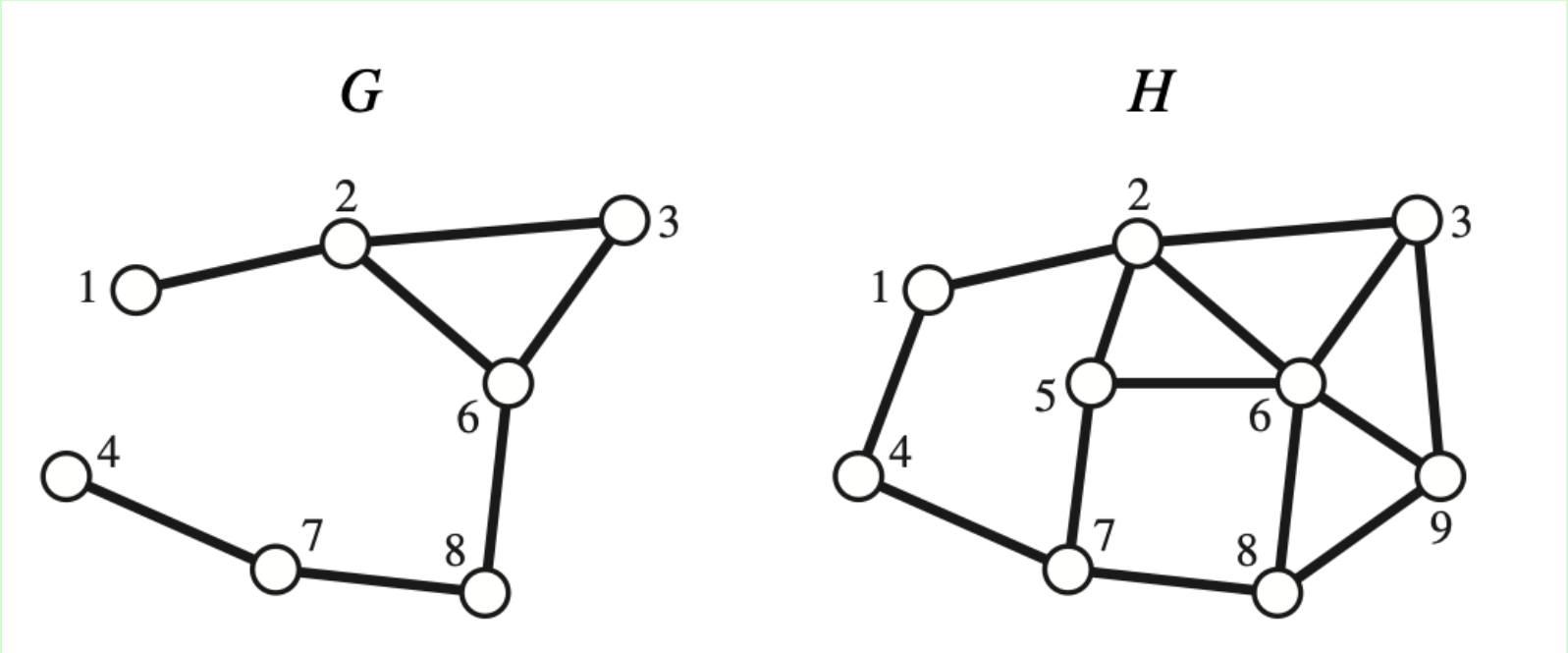
Example: G is a **subgraph** of H



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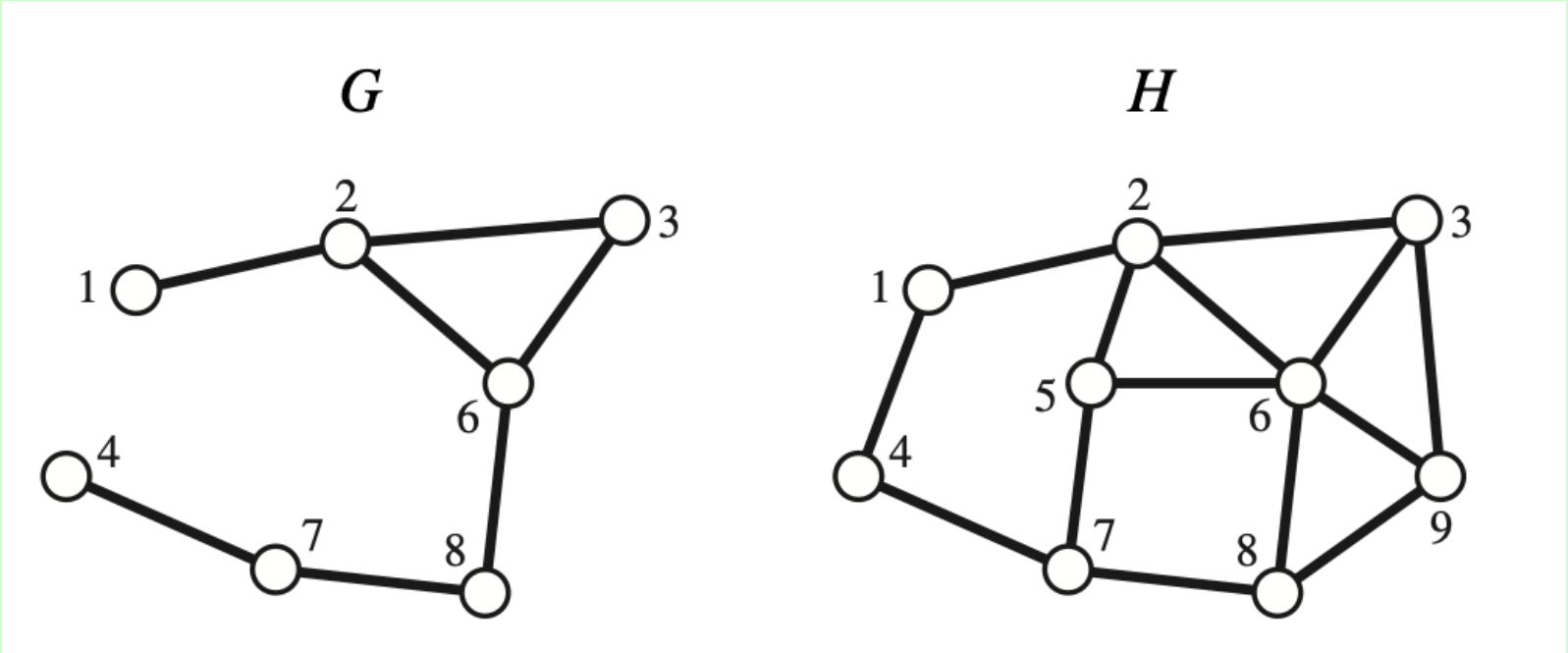

Solution. Possibly remove some vertices (and edges that touch them, so that you still have a graph). Then possibly remove some additional edges.

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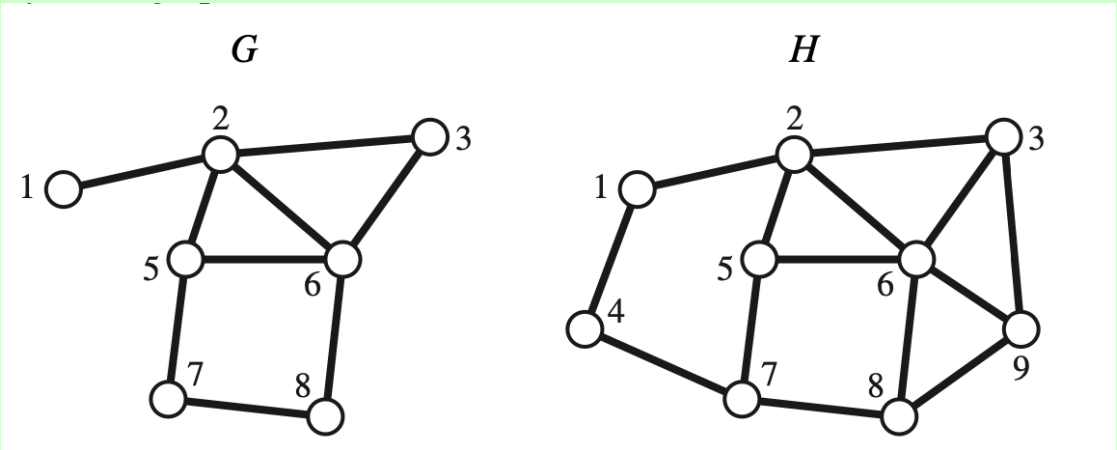

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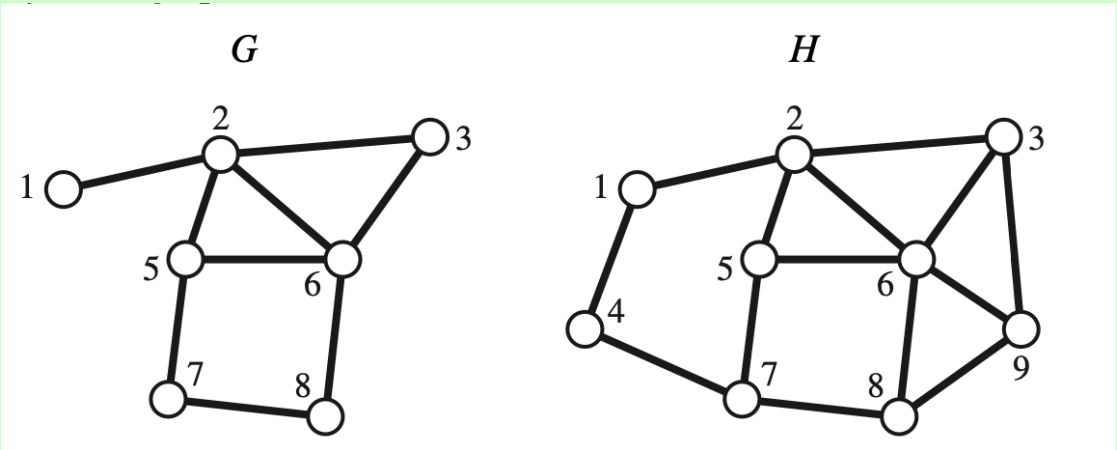

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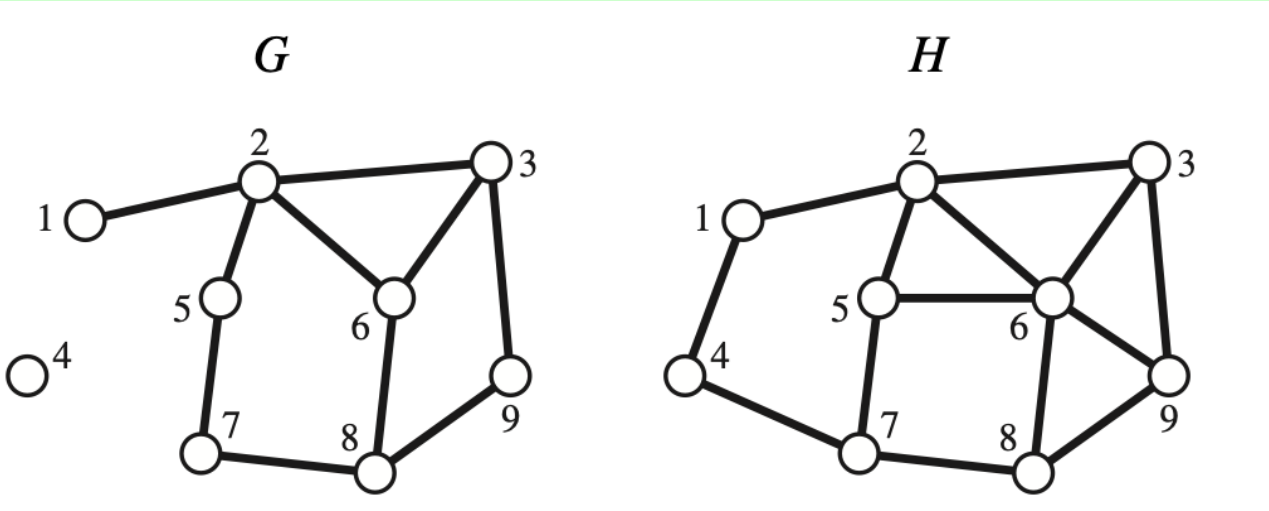

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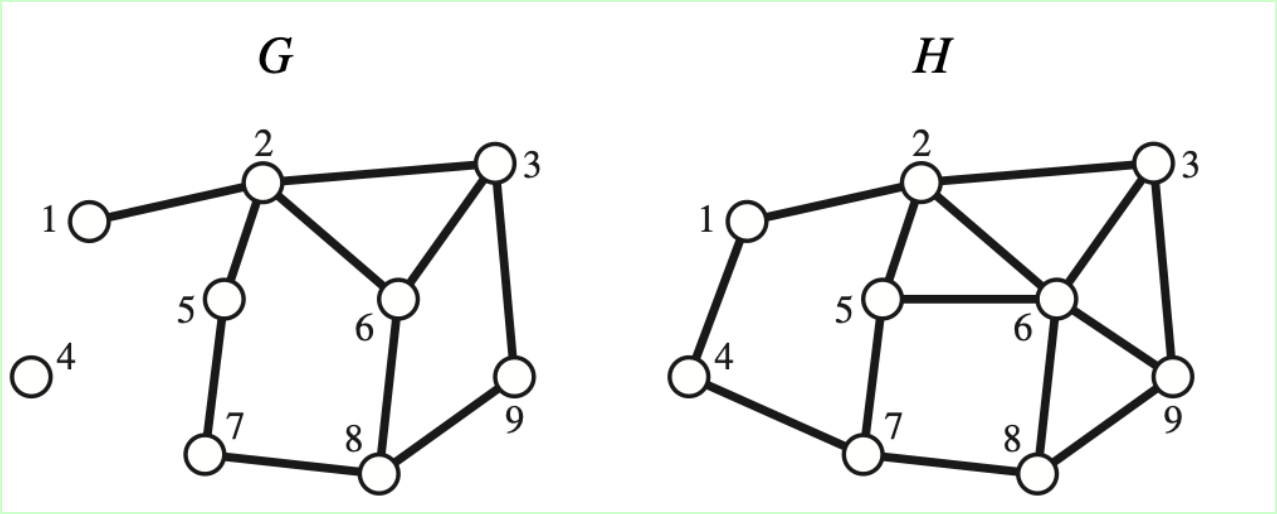

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Group exercises

aaron.loomis: 18
adam.wyszynski: 4
alexander.knutson: 2
anthony.mann: 3
blake.leone: 8
bridger.voss: 5
caitlin.hermanson: 3
cameron.wittrock: 20
carsten.brooks: 5
carver.wambold: 10
colter.huber: 6
conner.reed1: 12
connor.mizner: 11
connor.yetter: 20
derek.price4: 6
devon.maurer: 8
emmeri.grooms: 1
erik.moore3: 12
ethan.johnson18: 10
evan.barth: 4
evan.schoening: 19
griffin.short: 9
jack.fry: 8
jacob.ketola: 6
jacob.shepherd1: 14
jada.zorn: 1
jakob.kominsky: 14
james.brubaker: 19
jeremiah.mackey: 14
jett.girard: 16
john.fotheringham: 4
jonas.zeiler: 1
joseph.mergenthaler: 15
joseph.triem: 9
julia.larsen: 13
justice.mosso: 18
kaden.price: 11
lucas.jones6: 2
luka.derry: 16
luke.donaldson1: 9
lynsey.read: 16
mason.barnocky: 20
matthew.nagel: 2
micaylyn.parker: 7
michael.oswald: 19
nolan.scott1: 13
owen.obrien: 18
pendleton.johnston: 13
peter.buckley1: 17
reid.pickert: 3
ryan.barrett2: 15
samuel.hemmen: 10
samuel.mosier: 11
samuel.rollins: 16
sarah.periolat: 5
timothy.true: 17
tristan.nogacki: 12
tyler.broesel: 7
william.elder1: 15
yebin.wallace: 7
zeke.baumann: 17

Group exercises

1. Let G be the graph in the top figure. Draw pictures of the following subgraphs: (a) $G - 1$, (b) $G - \{5, 6\}$, (c) $G[\{2, 4, 6\}]$.
2. Which of the various properties of relations does the is-a-subgraph-of relation exhibit? Is it reflexive? Symmetric? Transitive?
3. Let G be a complete graph with n vertices. (a) How many spanning subgraphs does G have? (b) How many induced subgraphs does G have?
4. Let G and H be the two graphs in the bottom figure. Please find $\alpha(G), \omega(G), \alpha(H), \omega(H)$. (Recall that $\alpha(\cdot)$ is the size of a largest independent set and $\omega(\cdot)$ is the size of a largest clique.)