

02/05/2025: Lists

CSCI 246: Discrete Structures

Textbook reference: Sec. 8, Scheinerman

Graded Quiz Pickup

Quizzes are in the front of the room, grouped into four bins (A-G, H-L, M-R, S-Z) by last name. The quizzes are upside down with your last name on the back. Come find yours before, during, or after class. Only turn the quiz over if it's yours.

Announcements

- Friday's problems quiz will cover **Boolean Algebra** and **Induction**.
- The handwritten scores on Friday's quizzes were presented out of five points TOTAL (2.5 points per problem), but the histograms I showed last class were out of five points per each question.

Today's Agenda

- Reading quiz (5 mins)
- Mini-lecture (\approx 20 mins)
 - Review induction
- Group exercises (\approx 20 mins)

Today's Quiz

Logistics Alert

Please write your last name on the back of the page.

Reading Quiz (Lists)

Five students in our class are randomly selected to form an emergency rock band. The rock band will have a singer, a guitarist, a bassist, a drummer, and a keyboardist. How many different rock bands can be formed in this way?

Notes: (1) No student plays more than one instrument (and singing counts as an instrument). (2) If different students play different instruments, it counts as a "different band". (3) Our class has 67 students.

Monday's Reading Quiz on Induction

Scoring rubric

Description	E.C.
Correct and well-written.	+20%
Good work but some mathematical or writing errors that need addressing.	+15%
Some good intuition (about induction), but there is at least one serious flaw.	+5%
I don't understand this, but I see that you did work on it.	+0%
No work is evident.	+0%

Review Induction Group Exercises.

Group 1: bridger.voss,jakob.kominsky,pendleton.johnston
Group 2: lucas.jones6,anthony.mann,jacob.ketola
Group 3: michael.oswald,lynsey.read,john.fotheringham
Group 4: connor.yetter,ryan.barrett2,jonas.zeiler
Group 5: sarah.perolat,jacob.ruiz1,william.elder1
Group 6: joseph.mergenthaler,matthew.nagel,aaron.loomis
Group 7: owen.obrien,kaden.price,erik.moore3
Group 8: ethan.johnson18,micaylyn.parker,luke.donaldson1
Group 9: jeremiah.mackey,tyler.broesel,joseph.triem
Group 10: timothy.true,caitlin.hermanson,jack.fry
Group 11: samuel.rollins,nolan.scott1,justice.mosso
Group 12: james.brubaker,delaney.rubb,emmeri.grooms
Group 13: samuel.hemmen,blake.leone,mason.barnocky
Group 14: colter.huber,connor.mizner,yebin.wallace
Group 15: conner.reed1,devon.maurer,derek.price4
Group 16: griffin.short,adam.wyszynski,carsten.brooks
Group 17: carver.wambold,alexander.knutson,tristan.nogacki
Group 18: evan.schoening,cameron.wittrock,samuel.mosier
Group 19: zeke.baumann,reid.pickert,luka.derry
Group 20: julia.larsen,jett.girard,alexander.goetz
Group 21: connor.graville,jacob.shepherd1,jada.zorn
Group 22: evan.barth,peyton.trigg,peter.buckley1

Group exercises: Lists

1. I want to create two playlists on my cell phone by downloading from a collection of 500 songs. One playlist is called "Exercise" and the other is called "Relaxing". I want 20 different songs on each list. How many different ways can I load songs onto my phone if I allow a song to be on both playlists? And how many different ways can I load the songs if I want the two lists to have no overlap?
2. A padlock has digits 0 through 9 arranged in a circle on its face. A combination for this padlock is four digits long. Because of the internal mechanics of the lock, no pair of consecutive numbers in the combination can be the same or one place apart on the face. For example, 0-2-7-1 is a valid combination, but neither 0-4-4-7 (repeated digit 4) nor 3-0-9-5 (adjacent digits 0-9) are permitted. How many combinations are possible?
3. Four cards are drawn from a standard deck of 52 cards. In how many ways can this be done if the cards are all of different values (e.g., no two 5s or two jacks) and all of different suits? (For this problem, the order in which the cards are drawn matters, so drawing $A\spadesuit - K\heartsuit - 3\diamondsuit - 6\clubsuit$ is not the same as drawing $6\clubsuit - K\heartsuit - 3\diamondsuit - A\spadesuit$, even though the same cards are selected.)
4. Let n be a positive integer. Prove that $n^2 = (n)_2 + n$ in two different ways: (1) algebraically, and (2) by list counting.



To derive the solutions, it may help to apply the theorems below.

Multiplication Principle (Theorem 8.2 from Scheinerman)

Consider the two-element lists for which there are n choices for the first element, and for each choice of the first element there are m choices for the second element. Then the number of such lists is nm .

Theorem 8.6 from Scheinerman

The number of lists of length k whose elements are chosen from a pool of n elements

$$= \begin{cases} n^k, & \text{if repetitions are permitted} \\ (n)_k, & \text{if repetitions are forbidden} \end{cases}$$

Solution to #1

Problem. I want to create two playlists on my cell phone by downloading from a collection of 500 songs. One playlist is called "Exercise" and the other is called "Relaxing". I want 20 different songs on each list. How many different ways can I load songs onto my phone if I allow a song to be on both playlists? And how many different ways can I load the songs if I want the two lists to have no overlap?

Solution.

- a) By Theorem 8.6 of Scheinerman, there are $(500)_{20} = 500 \cdot 499 \cdot 498 \cdots 481$ ways to construct either the Exercise playlist, and likewise for the Relaxing playlist. So applying the multiplication principle to the two types of playlists, the total number of ways to load the songs is $(500)_{20} \cdot (500)_{20}$ (which can be written as $((500)_{20})^2$).
- b) By Theorem 8.6 of Scheinerman, the solution is $(500)_{40} = 500 \cdot 499 \cdot 498 \cdots 461$. Alternatively, one could argue that there are $(500)_{20}$ ways to construct the first playlist and $(480)_{20}$ ways to construct the second playlist; hence, by the multiplication principle the total number of ways to load the songs is $(500)_{20} \cdot (480)_{20}$. Note that these two answers are equal; that is $(500)_{40} = (500)_{20} \cdot (480)_{20}$.

Solution to #2











Problem. A padlock has digits 0 through 9 arranged in a circle on its face. A combination for this padlock is four digits long. Because of the internal mechanics of the lock, no pair of consecutive numbers in the combination can be the same or one place apart on the face. For example, 0-2-7-1 is a valid combination, but neither 0-4-4-7 (repeated digit 4) nor 3-0-9-5 (adjacent digits 0-9) are permitted. How many combinations are possible?

Solution. By the multiplication principle, the total number of solutions is $10 \cdot 7 \cdot 7 \cdot 7$.

Solution to #3

Problem. Four cards are drawn from a standard deck of 52 cards. In how many ways can this be done if the cards are all of different values (e.g., no two 5s or two jacks) and all of different suits? (For this problem, the order in which the cards are drawn matters, so drawing $A\spadesuit - K\heartsuit - 3\diamondsuit - 6\clubsuit$ is not the same as drawing $6\clubsuit - K\heartsuit - 3\diamondsuit - A\spadesuit$, even though the same cards are selected.)

Solution. The number of ways this can be done is $13 \cdot 4 \cdot 12 \cdot 3 \cdot 11 \cdot 2 \cdot 10 \cdot 1$. By the multiplication principle, the number of choices for the first card is $C_1 \triangleq 13 \cdot 4$, since there are 13 values and 4 suits available. (See the table below.) After that choice, we cross out one value (column) and one suit (row). So by the multiplication principle again, the number of choices for the second card is $C_2 \triangleq 12 \cdot 3$. Continuing in this manner, the number of choices for the third card is $C_3 \triangleq 11 \cdot 2$ and for the fourth card is $C_4 \triangleq 10 \cdot 1$. Then, applying the multiplication principle again, the number of ways to draw all four cards is $C_1 \cdot C_2 \cdot C_3 \cdot C_4$.

Suits	Values												
	A	2	3	4	5	6	7	8	9	10	J	Q	K
	(A, )	(2, )	(3, )					...					
	(A, )						...						
	(A, )						...						
	(A, )						...						

Solution to #4

Problem. Let n be a positive integer. Prove that $n^2 = (n)_2 + n$ in two different ways: (1) algebraically, and (2) by list counting.

Solution

1. By applying the definition of the falling factorial and doing simple algebra, we see that $(n)_2 + n = n(n-1) + n = n^2 - n + n = n^2$.
2. We can think of $(n)_2 = n(n-1)$ as the number of lists in a chart with n rows and $n-1$ columns. Now suppose we add another column. This gives us n additional lists, making the total number of lists $(n)_2 + n$. But the new chart now has n rows and n columns, so it contains $n \cdot n = n^2$ lists.