02/07/2025: Factorial

CSCI 246: Discrete Structures

Textbook reference: Sec. 9, Scheinerman

Graded Quiz Pickup

Quizzes are in the front of the room, grouped into four bins (A-G, H-L, M-R, S-Z) by last name. The quizzes are upside down with your last name on the back. Come find yours before, during, or after class. Only turn the quiz over if it's yours.

Announcements

- Wednesday's reading quiz was graded out of 1 point. 45/60 students who took the quiz scored 100%.
- Note: The reading quizzes are equally weighted. The denominator is typically chosen for convenience.

Today's Agenda

- Reading and problems quiz (25 mins)
- Mini-lecture ($\approx$ 10 mins)
- Group exercises ($\approx$ 15 mins)

Today's Quiz

Reminder: Please write your last name on the back of your page.

Reading Quiz (Factorial)

Evaluate $0!$. Explain your answer. (Only one explanation is needed.)

Problems Quiz (Boolean Algebra & Induction)

1. The **Fibonacci sequence** is the sequence given by the following recursive rules:
$$f_0 = 0, \qquad f_1 = 1, \qquad f_{n+2} = f_n + f_{n+1}$$
Show by induction that $f_0 + \cdots + f_n = f_{n+2} - 1$ in the Fibonacci sequence.
2. Is the following a tautology or a contradiction?
$$(X \vee Y) \vee (X \vee \neg Y)$$
Justify your answer using either a truth table or the properties of Boolean operators.
3. (Extra Credit.) Provide a second justification for #2.
4. (Extra Credit.) When does $(X \implies X) \implies X$? Justify your answer.

Solution to #4

Problem. When does $(X \implies X) \implies X$? Justify your answer.

Solution. In a group exercise on Boolean Algebra, we showed that $(X \implies X)$ is a tautology (i.e., it is always `TRUE`). Hence, the question reduces to when does `TRUE` $\implies X$?

Now recall the truth table for implication. Looking only at the cases where the hypothesis (the proposition in the "if" statement) is true, we have

W	X	$W \implies X$
T	T	T
T	F	F

Hence, `TRUE` $\implies X$ only when X is true.

We conclude that $(X \implies X) \implies X$ only when X is true.

Review List Group Exercises.

Group 1: adam.wyszynski,blake.leone,tristan.nogacki
Group 2: john.fotheringham,jacob.ruiz1,samuel.mosier
Group 3: connor.graville,nolan.scott1,jack.fry
Group 4: jonas.zeiler,peter.buckley1,timothy.true
Group 5: reid.pickert,james.brubaker,pendleton.johnston
Group 6: connor.yetter,lucas.jones6,cameron.wittrock
Group 7: conner.reed1,colter.huber,kaden.price
Group 8: anthony.mann,joseph.triem,jacob.shepherd1
Group 9: aaron.loomis,carver.wambold,peyton.trigg
Group 10: zeke.baumann,jakob.kominsky,emmeri.grooms
Group 11: luke.donaldson1,ethan.johnson18,derek.price4
Group 12: jacob.ketola,luka.derry,samuel.hemmen
Group 13: joseph.mergenthaler,connor.mizner,evan.barth
Group 14: matthew.nagel,alexander.knutson,lynsey.read
Group 15: michael.oswald,griffin.short,erik.moore3
Group 16: justice.mosso,owen.obrien,mason.barnocky
Group 17: yebin.wallace,evan.schoening,tyler.broesel
Group 18: william.elder1,caitlin.hermanson,jett.girard
Group 19: devon.maurer,sarah.periolat,julia.larsen
Group 20: samuel.rollins,bridger.voss,jada.zorn
Group 21: ryan.barrett2,carsten.brooks,micaylyn.parker
Group 22: delaney.rubb,jeremiah.mackey,alexander.goetz

Group exercises: Factorial

1. Your friend Sandy has six different books of classical literature, eight different *romantasy* (i.e. romance fantasy) books, and five different self-help books.
   a) In how many different ways can Sandy's books be arranged on a bookshelf?
   b) In how many different ways can Sandy's books be arranged on the bookshelf if all books of the same category are grouped together?
2. Evaluate $\frac{100!}{98!}$ without calculating 100! or 98!.
3. Calculate the following products:
   a) $\prod_{k=1}^{n} \frac{k+1}{k}$. (Reduce this product to a simpler expression.)
   b) $\prod_{k=1}^{n}(2k-1)$. (This product has a simple English-language description. What is it?)

Solution to group exercise #1

Problem. Your friend Sandy has six different books of classical literature, eight different *romantasy* (i.e. romance fantasy) books, and five different self-help books.

a) In how many different ways can Sandy's books be arranged on a bookshelf?

b) In how many different ways can Sandy's books be arranged on the bookshelf if all books of the same category are grouped together?

Solution.

a) By Theorem 8.6b from Scheinerman, there are $(6 + 8 + 5)! = 19!$ different ways to arrange the books.

b) By Theorem 8.6b from Scheinerman, there are $6!$ ways to arrange the classical literature books, $8!$ ways to arrange the romantasy books, and $5!$ ways to arrange the self-help books. So by the multiplication principle, there are $6!8!5!$ ways to arrange the books, if we assume a particular ordering of the categories (e.g. classical $\rightarrow$ romantasy $\rightarrow$ self-help). Now by Theorem 8.6b from Scheinerman, there are $3!$ ways to arrange the categories. Hence, by the multiplication principle again, there are $3!6!8!5!$ ways to arrange the books overall.

Solution to group exercise #2

Problem. Evaluate $\frac{100!}{98!}$ without calculating 100! or 98!.

Solution.

$$\frac{100!}{98!} = \frac{100 \cdot 99 \cdot 98!}{98!} = \frac{100 \cdot 99 \cdot \cancel{98!}}{\cancel{98!}} = 100 \cdot 99 = 9900.$$

Solution to group exercise #3

Problem. Calculate the following products:

a) $\prod_{k=1}^{n} \frac{k+1}{k}$. (Reduce this product to a simpler expression.)
b) $\prod_{k=1}^{n}(2k-1)$. (This product has a simple English-language description. What is it?)

Solution.

a)

$$\begin{aligned}\prod_{k=1}^{n} \frac{k+1}{k} &= \frac{2}{1} \cdot \frac{3}{2} \cdots \frac{n}{n-1} \cdot \frac{n+1}{n} \\ &= \frac{\cancel{2}}{1} \cdot \frac{\cancel{3}}{\cancel{2}} \cdots \frac{\cancel{n}}{\cancel{n-1}} \cdot \frac{n+1}{\cancel{n}} \\ &= n+1\end{aligned}$$

b)

$$\begin{aligned}\prod_{k=1}^{n}(2k-1) &= (2\cdot 1-1)\cdot(2\cdot 2-1)\cdot(2\cdot 3-1)\cdots(2\cdot n-1) \\ &= 1\cdot 3\cdot 5\cdots 2n-1\end{aligned}$$

This is the product the first n odd natural numbers. (The k-th factor is the k-th odd natural number.)