04/09/2025: Graph Theory Fundamentals

CSCI 246: Discrete Structures

Textbook reference: Sec 47, Scheinerman

Graded Quiz Pickup

Quizzes are in the front of the room, grouped into four bins (A-G, H-L, M-R, S-Z) by last name. The quizzes are upside down with your last name on the back. Come find yours before, during, or after class. Only turn the quiz over if it's yours.

Today's Agenda

- Reading quiz (5 mins)
- Review problems quizzes (15 mins)
- Mini-lecture ($\approx$ 10 mins)
- Group exercises ($\approx$ 15 mins)

Feedback on Monday's Quiz

Reading Quiz Scores

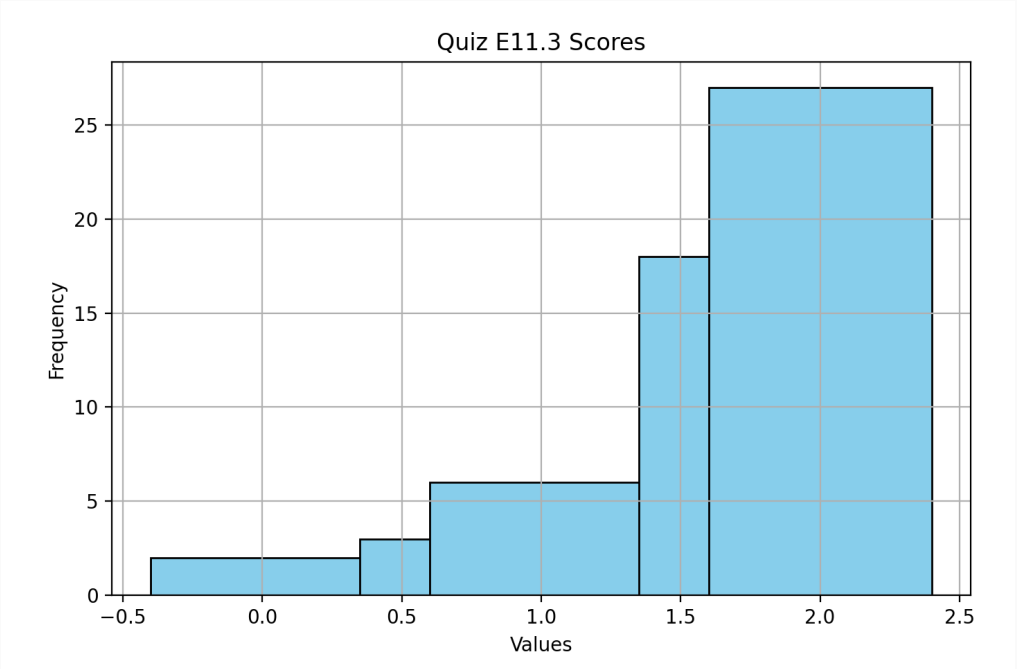


Figure 1: Median Score = 1.75/2 (87.5%)

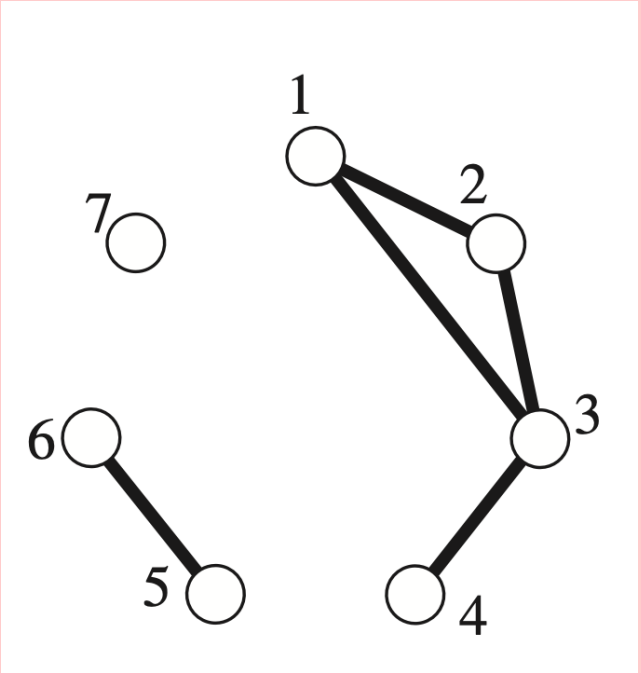
Grading Rubric:

1. (1 point) Needed to give reasonable answer to number of elementary operations (for the WHOLE algorithm snippet).
2. (1 point) Stating the order (ideally as Big Theta, but Big O was accepted)

Today's reading quiz

Reading Quiz (Graph Theory Fundamentals)

1. What is the degree of vertex 1 in the graph below?



2. Let $G = (V, E)$. The sum of the degree of vertices in G is *how many* times the number of edges? That is, if we write

$$\sum_{v \in V} d(v) = C|E|,$$

what is C?

3. Give a brief explanation for your answer to number 2.

Review for Friday's Problems Quiz

Thoughts On Graph Theory Fundamentals

Graphs as abstractions: The Bridge of Königsberg.

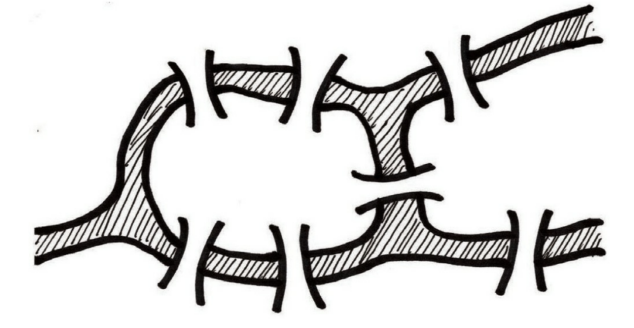


Figure 2: Bridge of Königsberg.

Claim. There was a pub on the center island, with challenges and late-night attempts to "walk the bridges," to make a tour of the town crossing every bridge exactly once. Despite the accompanying boasts, rarely reproducible in the sober morning, this was a difficult task. Most who attempted found that they had missed a bridge or that they crossed a bridge twice.

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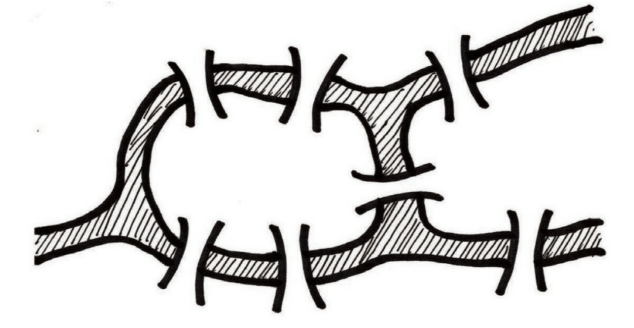

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Question. Can you find a tour of the town that traverses each of the seven bridges exactly once?

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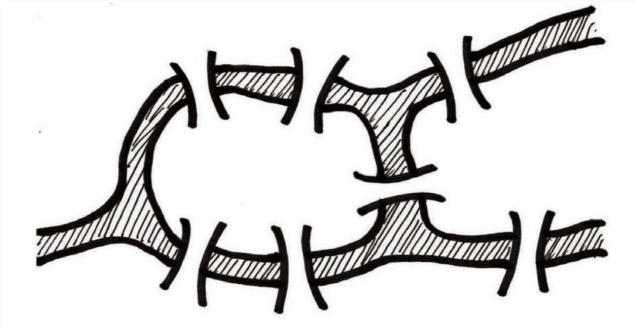

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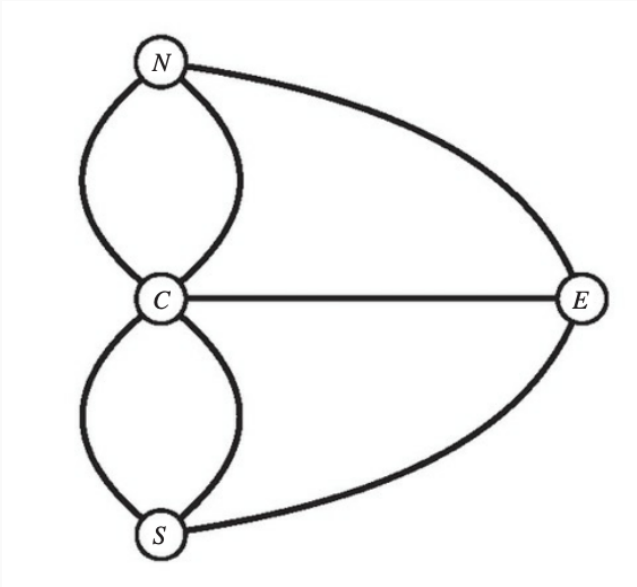

Figure 3: Graph representation

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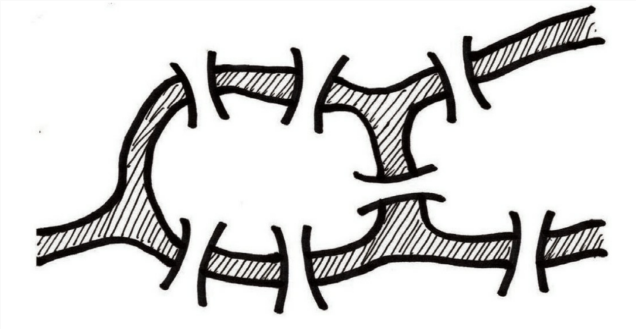

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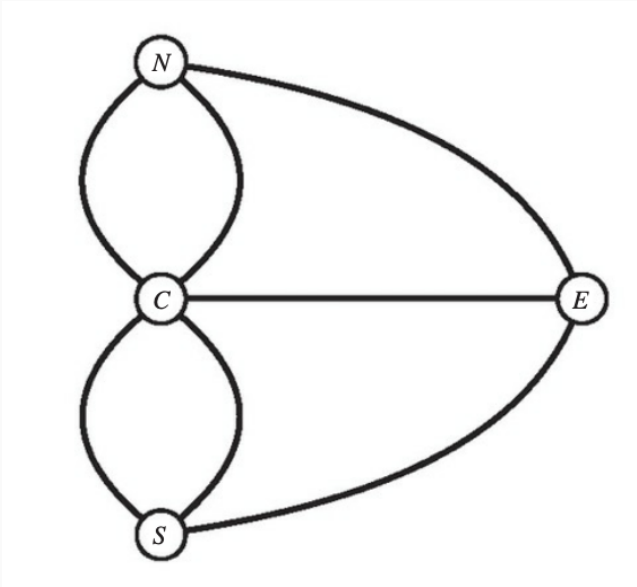

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Click here for short video.

Group exercises

aaron.loomis: 14
adam.wyszynski: 9
alexander.goetz: 11
alexander.knutson: 7
anthony.mann: 1
blake.leone: 20
bridger.voss: 20
caitlin.hermanson: 3
cameron.wittrock: 6
carsten.brooks: 2
carver.wambold: 17
colter.huber: 2
conner.reed1: 15
connor.mizner: 17
connor.yetter: 4
derek.price4: 18
devon.maurer: 19
emmeri.grooms: 7
erik.moore3: 8
ethan.johnson18: 13
evan.barth: 10
evan.schoening: 15
griffin.short: 14
jack.fry: 10
jacob.ketola: 12
jacob.shepherd1: 16
jada.zorn: 13
jakob.kominsky: 11
james.brubaker: 14
jeremiah.mackey: 19
jett.girard: 3
john.fotheringham: 3
jonas.zeiler: 21
joseph.mergenthaler: 10
joseph.triem: 8
julia.larsen: 8
justice.mosso: 6
kaden.price: 5
lucas.jones6: 9
luka.derry: 12
luke.donaldson1: 4
lynsey.read: 13
mason.barnocky: 11
matthew.nagel: 6
micaylyn.parker: 1
michael.oswald: 21
nolan.scott1: 5
owen.obrien: 7
pendleton.johnston: 1
peter.buckley1: 9
reid.pickert: 19
ryan.barrett2: 18
samuel.hemmen: 2
samuel.mosier: 16
samuel.rollins: 5
sarah.periolat: 4
timothy.true: 12
tristan.nogacki: 20
tyler.broesel: 18
william.elder1: 16
yebin.wallace: 17
zeke.baumann: 15

Group exercises

1. Write the graph in the top figure as a pair of sets (V, E).
2. Draw a picture of the graph below $\Big(\{a, b, c, d, e\}, \Big\{\{a, b\}, \{a, c\}, \{a, d\}, \{b, e\}, \{c, d\}\Big\}\Big)$.
3. Construct a graph for which the is-adjacent relation, $\sim$, is transitive.
4. How many edges are in K_n, a complete graph with n vertices?
5. How many different graphs can be formed with vertex set $V = \{1, 2, 3, \ldots, n\}$?
6. Prove that in every graph, the number of vertices with odd degree is even. (For example, the graph from the reading quiz, reproduced in the bottom figure, has exactly four vertices of odd degree.)