Assignment 2

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1 Problem 1

1.1 (a)

$\mathbf{x} = [x_1, x_2, ..., x_n]^T, \mathbf{y} = [y_1, y_2, ..., y_n]^T$, $\mathbf{i} = [1, ..., 1]^T$

- $\phi_1(r) = r^2$, we can rewrite this problem to :

$$\min_{a,b} \|\mathbf{y} - a\mathbf{i} * \mathbf{x} - b\mathbf{i}\|_2^2 \tag{1}$$

$$s.t. a \in R, b \in R \tag{2}$$

  y - a**i*****x**-b**i** can be viewed as an affine of a, b and $\|\|\|_2^2$ is convex, so this is a convex question.

- $\phi_1(r) = |r|$, we can rewrite this problem to :

$$\min_{a,b} \|\mathbf{y} - a\mathbf{i} * \mathbf{x} - b\mathbf{i}\| \tag{3}$$

$$s.t. a \in R, b \in R \tag{4}$$

  y - a**i*****x**-b**i** can be viewed as an affine of a, b and $\|\|\|$ is convex, so this is a convex question.

- $\phi_1(r) = log(1 + r^2)$, we can rewrite this problem to :

$$\min_{a,b} log(\mathbf{y} - a\mathbf{i} * \mathbf{x} - b\mathbf{i})^2 \tag{5}$$

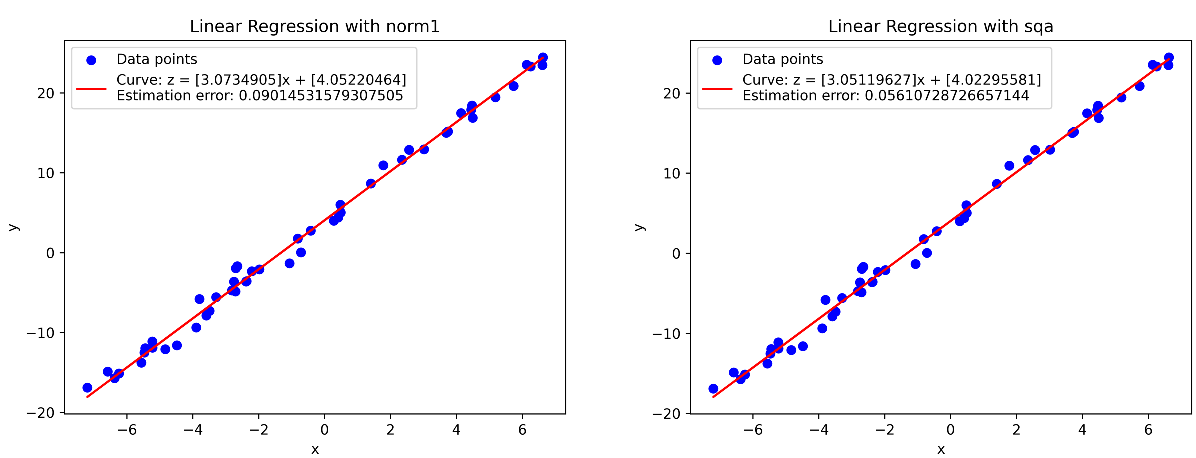
$$s.t. a \in R, b \in R \tag{6}$$

  y - a**i*****x**-b**i** can be viewed as an affine of a, b and $log(1 + x^2)$ is not convex, so this is not a convex question.

1.2 (b)

Based on problem (a), we have $\phi_1(r)$ and $\phi_2(r)$ are convex problems and here are the answers.

- $\phi_1(r) = r^2$, $\phi_2(r) = \|r\|$ with $dataset1$. So based on the estimation error, $\phi_1(r) = r^2$ can minimizes the error better.

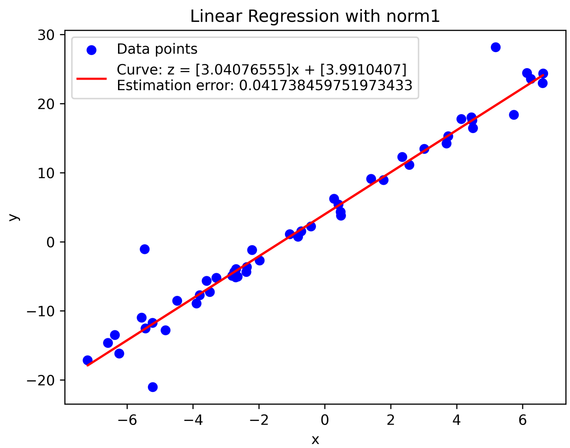


(a) norm1 penalty

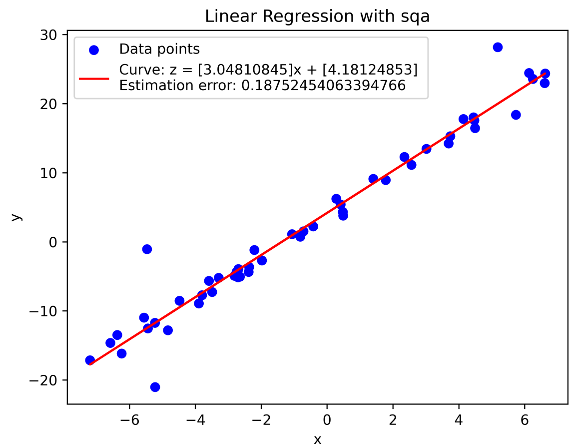
(b) square penalty

Figure 1: Comparison for square penalty and norm1 penalty for dataset1

- $\phi_1(r) = r^2$, $\phi_2(r) = \|r\|$ with *dataset*2. So based on the estimation error, $\phi_1(r) = \|\|$ can minimizes the error better.



(a) norm1 penalty



(b) square penalty

Figure 2: Comparison for square penalty and norm1 penalty for dataset2

2 Problem 2

2.1 (a)

- $J_{track} = \|\mathbf{y} - \mathbf{y}_{des}\|_1$: for any given t, y_{des} can actually be viewed as an affine of $\mathbf{u}$ and $\|\|_1$ can be viewed as linear problem, so it's LP;
- $J_{track} = \|\mathbf{y} - \mathbf{y}_{des}\|_2$: same as above, and $\|\|_2$ can be viewed as quadratic, so it's QP;
- $J_{track} = \|\mathbf{y} - \mathbf{y}_{des}\|_\infty$: can be viewed as $\max \|\mathbf{y} - \mathbf{y}_{des}\|$, which is a linear function, so it's LP;

By comparing their output sequences with the desired output sequence, we find that they are very similar, indicating that all three methods perform exceptionally well in solving this problem.

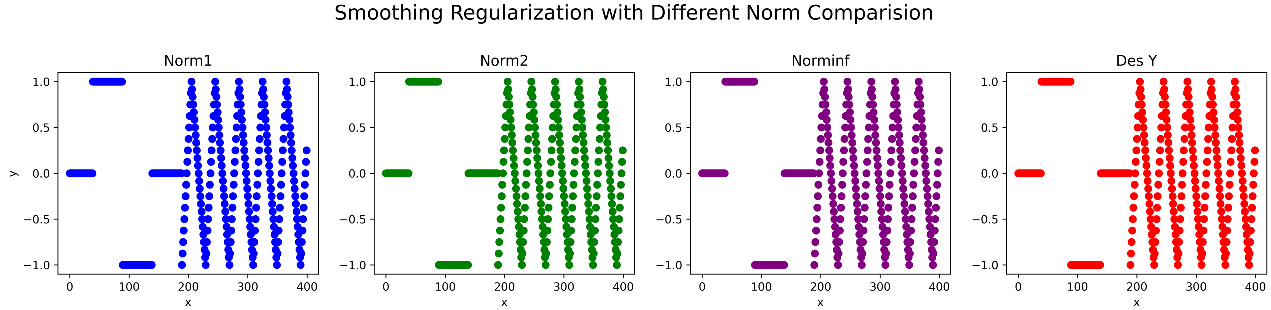


Figure 3: Comparison of Different Norm Penalty with Desired Output

And by doing so, we can actually rewrite this problem as

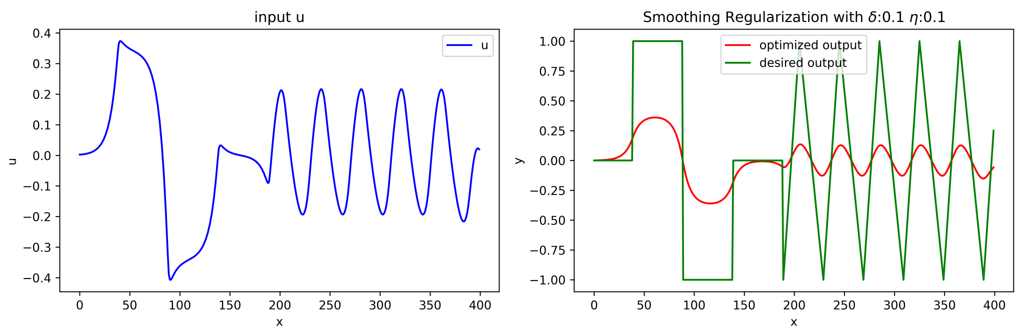
$$cm@u = y_{des} \tag{7}$$

which is a LP problem indicates u should be the same for different objective function.

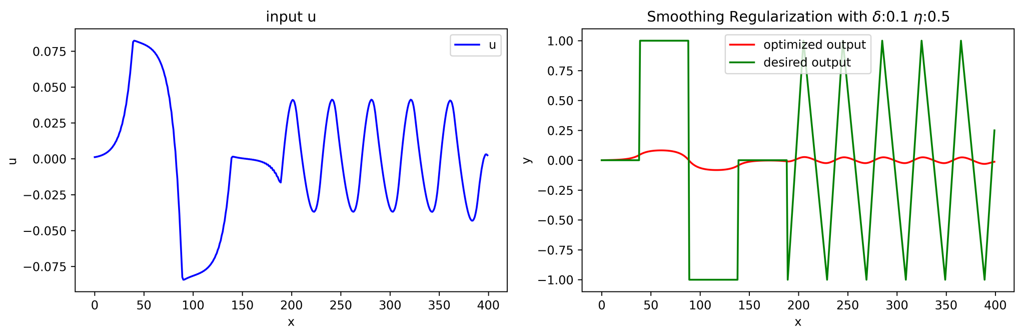
2.2 (b)

- Since each part of the J is convex, the linear combination should also be convex.
- I tried for different settings [δ=0.1, η=0.1], [δ=0.1, η=0.5], [δ=0.5, η=0.1], [δ=0.5, η=0.5]

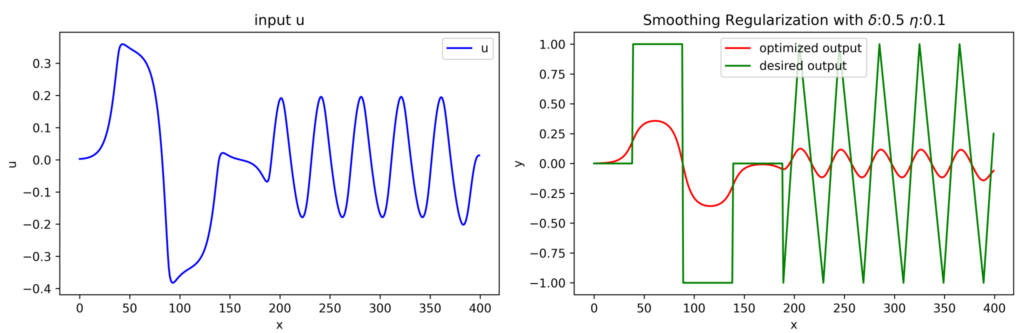
- There are two conclusions we can get from Fig.4
  - Comparing Fig.3 and Fig.4a the adding two penalties can decrease the performance and larger the two coefficients are, larger the loss will be.
  - Comparing Fig.4a, Fig.4b and Fig.4c, we can find that the change of η can affect the performance more compared to δ, which indicates $\|\mathbf{u}\|_2^2$ gives hard penalty compared to $\sum_{t=0}^{N-1}(u(t+1)-u(t))^2$



(a) δ=0.1, η=0.1



(b) δ=0.1, η=0.5



(c) δ=0.5, η=0.1

Figure 4: Tradeoff Penalty