Summary

Detail can be found at https://github.com/gjunjie/stats_practice

1. Fat Tail Analysis (`fat_tail.ipynb`)

Objective

Analyze the fat-tailed nature of SPX (S&P 500) returns and compare different distribution models to assess tail risk.

Key Findings

Risk Metrics (99% Confidence Level)

Method	Metric	Value
Empirical	VaR 99%	-3.42%
Empirical	ES 99%	-4.92%
Normal	VaR 99%	-2.81%
Normal	ES 99%	-3.22%
Student-t	VaR 99%	-3.49%
Student-t	ES 99%	-5.78%

Key Insight: The Normal distribution significantly underestimates tail risk:

- **VaR gap**: Normal underestimates by 0.62% compared to empirical
- **ES gap**: Normal underestimates by 1.70% compared to empirical

Distribution Fitting Results

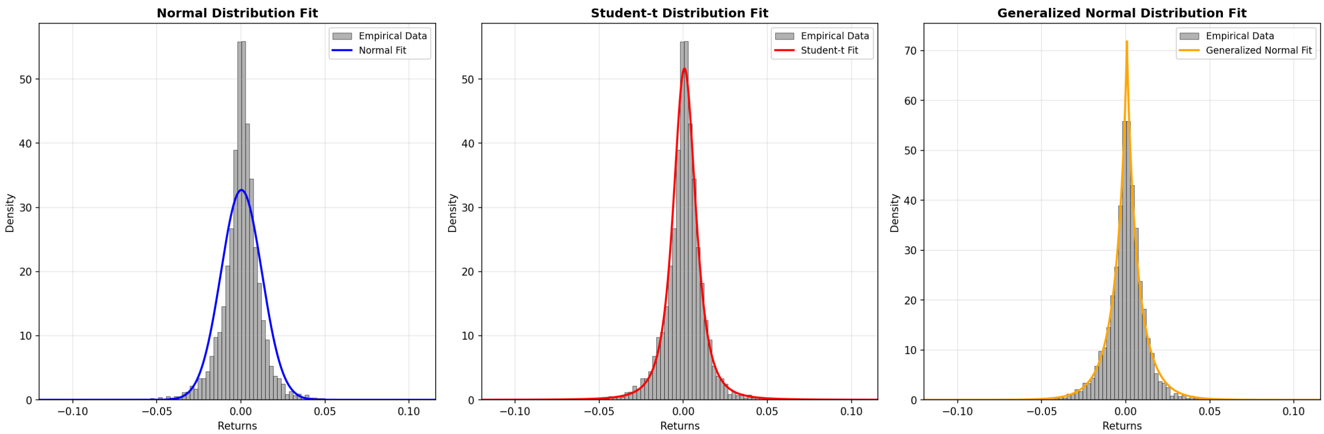
Distribution	Parameters	AIC	KS Statistic	KS p-value
Student-t	df = 2.65	-41,005.64	0.0176	0.0348
Generalized Normal	β = 0.87	-41,003.85	0.0179	0.0297
Normal	μ = 0.000314, σ = 0.012193	-39,102.22	0.0923	0.0000

Conclusion:

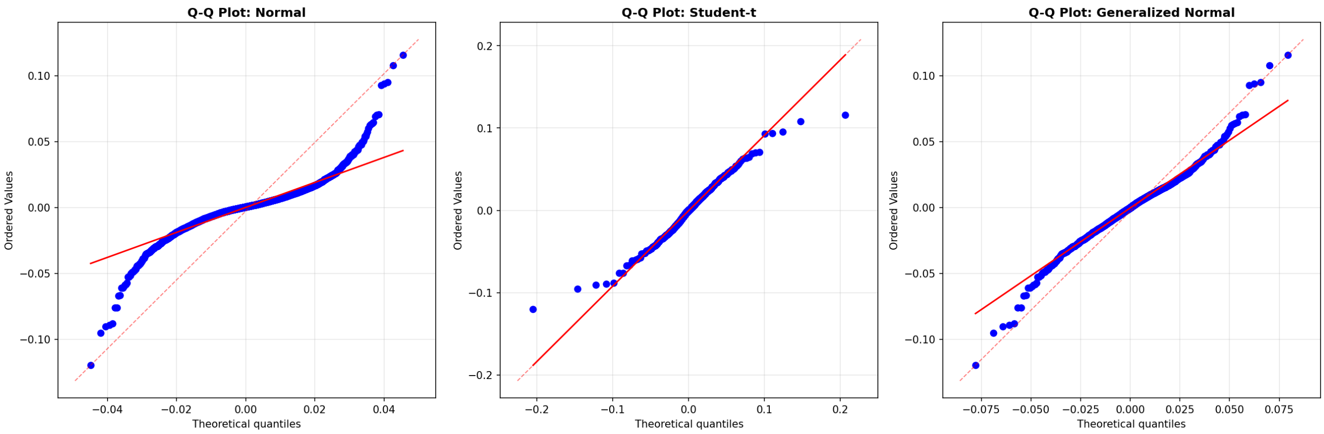
- **Student-t distribution** (df = 2.65) provides the best overall fit with the lowest AIC
- **Normal distribution** fails to adequately model extreme losses (KS p-value = 0.0000)

Visualizations

1. **Histogram with Fitted PDFs**: The Student-t and Generalized Normal distributions better capture the fat tails of the empirical data.



1. **Q-Q Plots**: Three Q-Q plots comparing empirical quantiles against theoretical quantiles for each distribution. The Student-t and Generalized Normal show better alignment with the diagonal reference line, especially in the tails.



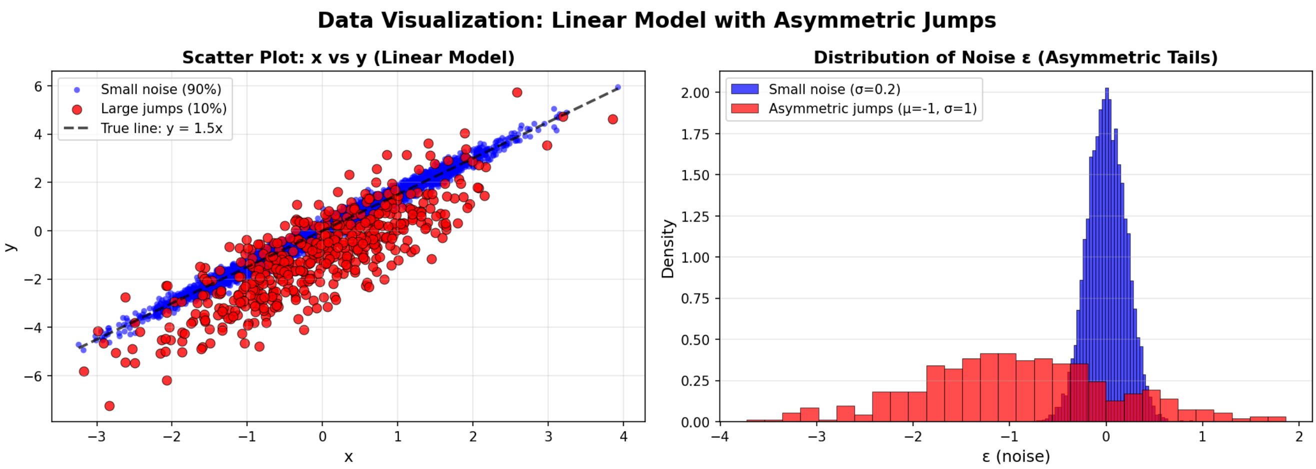
2. Loss Function Comparison (`loss_function.ipynb`)

Objective

Compare the performance of different loss functions (MSE/OLS, Huber, MAE) on data with asymmetric jumps and outliers.

Data Generation

- **Model**: $y = 1.5x + \varepsilon$
- **90% of the time**: Small Gaussian noise $N(0, 0.2^2)$
- **10% of the time**: Large asymmetric jump $N(-1, 1^2)$
- **Sample size**: 5,000 observations



Model Performance

Model	$\hat{\beta}$ (coefficient)	Intercept	MSE (all data)	MSE (normal only)
OLS (MSE)	1.5082	-0.0901	0.2146	0.0485
Huber Loss	1.5016	-0.0170	0.2199	0.0403
MAE	1.5016	-0.0170	0.2199	0.0403

Key Insights

1. **Overall MSE**: OLS achieves the lowest MSE (0.2146) when including all data points, as it directly optimizes for this metric.
2. **Normal Day Performance**: Huber and MAE losses achieve **lower MSE on typical data points** (0.0403 vs 0.0485), demonstrating better robustness to outliers.
3. **Coefficient Estimation**:
   - OLS coefficient (1.5082) is slightly biased away from the true value (1.5) due to outliers
   - Huber and MAE recover the true coefficient more accurately (1.5016)

Conclusion: Robust loss functions (Huber, MAE) are preferred when outliers are present but accurate predictions for typical observations are more important than overall MSE minimization.

3. Multiple Testing Problem (`multiple_test.ipynb`)

Objective

Demonstrate how multiple testing increases the probability of observing spurious high Sharpe ratios when all strategies are pure noise.

Simulation Setup

- **Time period**: T = 1,260 days
- **Daily volatility**: σ = 0.01
- **Number of simulations**: M = 10,000
- **Test scenarios**: 1 test vs 50 tests

Results

Scenario	P(Sharpe ≥ 1.8)
1 test	0.0% (0 out of 10,000)
50 tests	0.17% (17 out of 10,000)

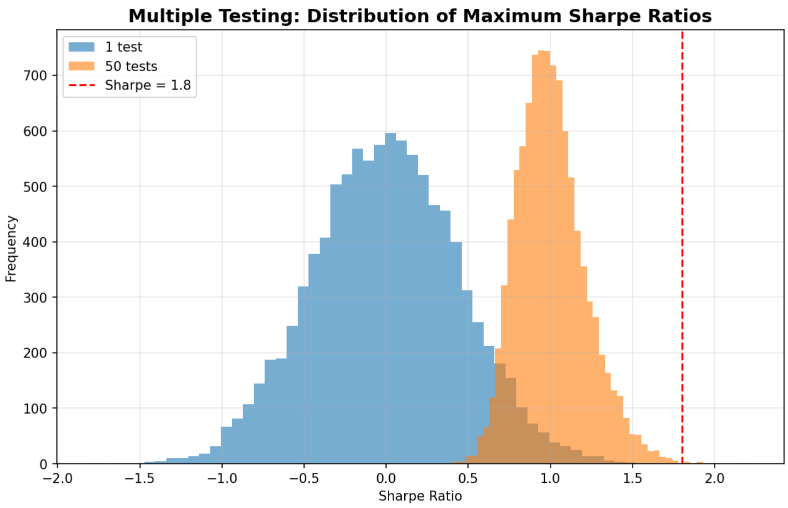
Key Insight

When testing multiple strategies simultaneously, the probability of observing spurious high Sharpe ratios increases dramatically, even when all strategies are pure noise (null hypothesis is true).

Conclusion: Multiple testing significantly increases the probability of false discoveries. Proper multiple testing corrections (e.g., Bonferroni, FDR) are essential when evaluating multiple strategies.

Visualization

Histogram Comparison: Overlapping histograms showing the distribution of maximum Sharpe ratios for 1 test vs 50 tests. A vertical red dashed line marks Sharpe = 1.8. The distribution for 50 tests shows a longer right tail, indicating higher probability of extreme values.



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4. Principal Component Analysis (`pca.ipynb`)

Objective

Perform PCA on synthetic implied volatility (IV) surfaces to identify the main factors driving volatility structure.

Data Structure

- **Surface matrix**: 100 assets × 50 strikes (log-moneyness from -0.3 to 0.3)
- **Basis functions**: Level, skew, curvature, and fourth-order terms
- **Noise**: Small Gaussian noise added to each surface

PCA Results

Explained Variance

Principal Component	Explained Variance	Cumulative Variance
PC1	57.5%	57.5%

Principal Component	Explained Variance	Cumulative Variance
PC2	30.2%	87.7%
PC3	6.3%	**94.0%**
PC4	0.3%	94.4%
PC5	0.3%	94.7%

Key Finding: The first 3 principal components explain **94.0%** of the total variance, indicating that IV surfaces can be effectively represented by just three factors.

Trading Applications

1. Relative-Value (Cross-Sectional) Strategies

- Compare PCA scores across assets
- Identify mispricings (e.g., skew too steep vs peers)
- Trade relative misalignments in volatility structure

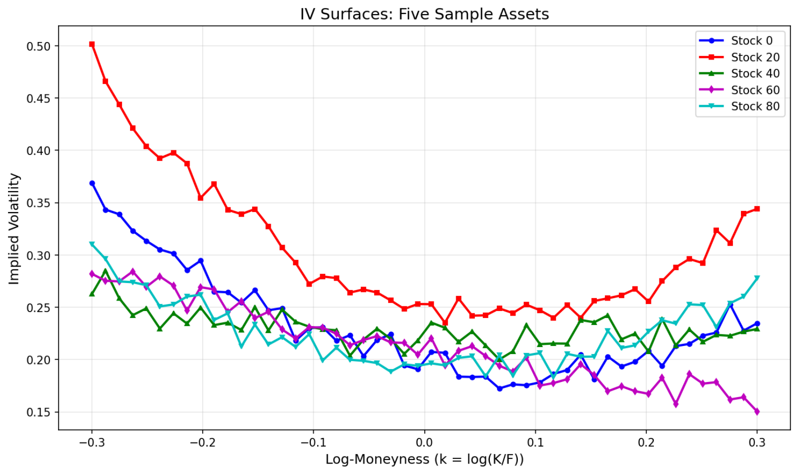
2. Time-Series Strategies

- Model PCA scores as state variables
- Trade mean reversion in factor loadings
- Identify and trade regime shifts in volatility structure

Visualizations

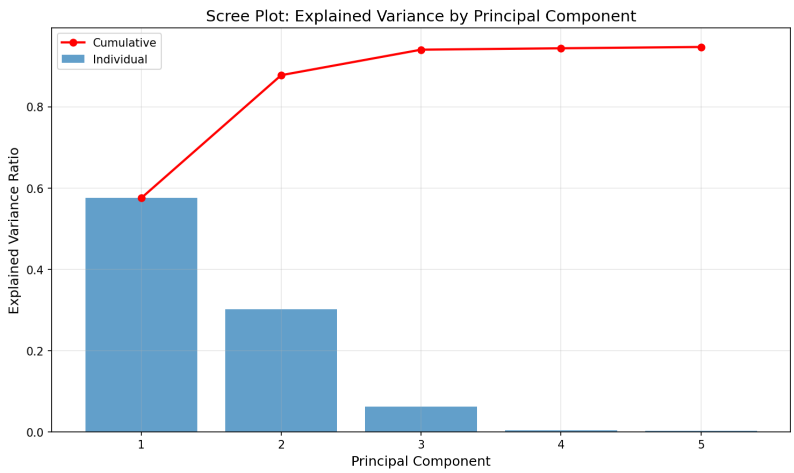
1. Sample IV Surfaces:

Plot showing five sample IV surfaces across different assets, demonstrating the variation in volatility structure.



2. Scree Plot:

Bar chart showing individual explained variance for the first 5 PCs, with a cumulative line overlay.



3. Principal Component Loadings:

Plot of the first 3 PC loadings as functions of log-moneyness (k).

- **PC1**: Likely captures the overall level
- **PC2**: Likely captures skew (asymmetric structure)

• **PC3**: Likely captures curvature (smile shape)