Robot Path Planning Using Newton-Raphson method Based on RRT-GD

Junxiang Ge, Fuchun Sun, Chunfang Liu[1]

Abstract— **We proposed a new method on 7-arm redundant manipulator for obstacle-avoiding path planning. As we all know, RRT (rapidly-exploring random tree) algorithm performs well when doing obstacle-avoiding works, but sometimes it is not fast enough and less goal-direction. In this paper, we use RRT method in the configuration space(C-space), concentrating on both the end actuators status and the joints movements. In order to reduce the time for path planning, i.e., reduce the steps when doing RRT, we change RRT algorithm to be goal-direction (we call it RRT-GD), to be adapted to our common works. With this improve, we can see in this paper that result could be reached in less than 10 steps usually, while the original RRT algorithm needs often more than 100 steps doing the same task. In addition, we apply Newton-Raphson method to do inverse kinematics optimization, thus we can focus on both the status space(S-space) and C-Space. In the end, we use quintic polynomial to smooth the planning path.**

I. Introduction

As we all know, RRT (Rapidly-exploring random tree) is rapidly used in obstacle-avoiding path planning for redundant manipulator. It constructs a graph of obstacle free points in feasible space based on random sampling, in which we search the feasible path from the initial point to goal point. As a result of this, there are some great properties with RRT method compared to other obstacle-avoiding method:

- RRT method attends to explore unknown space.
- The graph constructed by RRT will gradually fill up the feasible space if the exploring times is enough.
- RRT method don't need precise modeling of the obstacles, instead, it only needs the information if or not a point is obstacle free through doing collision detect, thus ending up with the high efficiency of obstacle free planning.

Though it has these advantages, RRT will still perform low planning success rate when there is a great deal of obstacle surround, or the free degree of manipulator is high. In this way, Kuffner proposed a bidirectional RRT algorithm, bi-RRT, also called RRT-connect. Within this method, two trees were built based on the initial point and goal point separately. When expanding each time, the tree of initial point will extend to the other tree, or the contrary way, until the two trees meets. For it has goal heuristic, it improves the success rate of path planning.

In spite of these advantages of RRT and its modified algorithm, we still find it defective sometimes in doing our common works. As mentioned before, RRT attends to explore unknown space, so that it could exploring the whole free space. But we just dont need to explore the whole space when doing simply goal-reaching works. In this way, all we need to do is to find a feasible way from initial point to goal point without obstacle collision, i.e., we only need to explore a part of the whole space include initial point, goal point, and some obstacle. Thus we proposed RRT-GD (RRT with goal-directionality), using the train of thought above. As we can see later in this paper, by doing this way, we can get the result rapidly, usually ten times faster than the normal RRT method.

As we doing RRT in C-space (configuration space), we need to compute the joint angles of each point in the path tree, i.e. inverse kinematics optimization. There are also a lot of methods to do inverse kinematics optimization, e.g., GP (gradient projection method), WLN (weighted least norm method), extended Jacobi method, etc. In this paper, we adopt Newton-Raphson method to do this inverse kinematics optimization work. Compared to other methods, Newton-Raphson method performs faster and more accurate. Provided the result can get by Newton-Raphson method, it would take only less than 10 iteration times (usually 5 or 6 times), with bias less than micro-meters.

After we get the path from initial point to goal point, with detailed status information (pose & joint angles), we then use quintic polynomial to smooth the path [9]. We show our results in the experiment result part of this paper, and then make comparison with the other methods mentioned above.

We organize this paper according to our experiment process, to show our results more fluently. First, we discuss the basic mathematics knowledge and algorithm we use, include RRT and its modified method in more detail. Then, we introduce Newton-Raphson method in inverse kinematics by using the normal RRT method. After that, we present quintic polynomial method in smoothing path. Later, in order to accelerate the planning time and doing works real-time, we propose RRT-GD method, which press more close to our work in common use.

II. Experiment Environment

All the results we show is based on a redundant robot arm. Before introduce our algorithm, we can have a look at the basic environment.

A. Robot Model & Its Relevant Variables (Heading 2)

We use a 7-arm redundant robot manipulator to test the results in our experiment. We use the DH method (Denavit and Har- tenberg method) to model our robot arm, with the

[1]State Key Lab of Intelligent Technology and Systems, Tsinghua National Laboratory for Information Science and Technology (TNList), Department of Computer Science and Technology, Tsinghua University. gejx14@mails.tsinghua.edu.cn, fcsun@tsinghua.edu.cn

model axis built obeying D-H parameter method as the Fig.1 show.

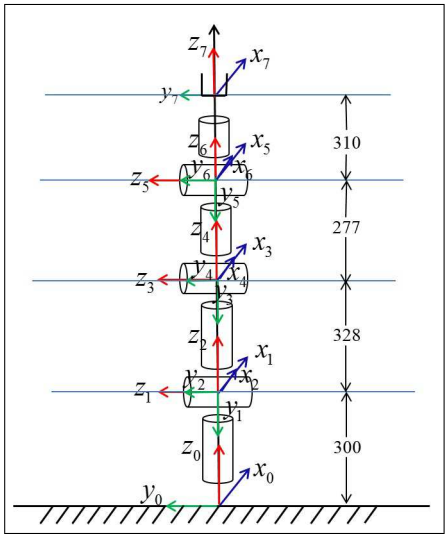


Fig. 1. Robot axis coordinates, obeying D-H parameter method, numbers in the graph is in milimeter.

As a result of this axis coordinates, we get the DH parameters as follow (show in table 1) :

TABLE I

DH Parameters of robot. (θ : joint angle; d : connecting rod skew; a : connecting rod; α : tortuosity angle of connecting rod.)

Joint Number	θ_i	d_i(mm)	a_i(mm)	$\alpha_i(^\circ)$	area of $\theta_i(^\circ)$
1	q_1	300	0	-90	-180∼180
2	q_2	0	0	90	-90∼90
3	q_3	328	0	-90	-180∼180
4	q_4	0	0	90	-120∼120
5	q_5	277	0	-90	-180∼180
6	q_6	0	0	90	-120∼120
7	q_7	310	0	0	-180∼180

All our later works are based on table 1 and fig.1, including experiments and simulation. As we can see in fig.1, the joint angles from q_1 to q_7 are the free variables and the others are fixed within our experiments. So when we write rotation and translation matrix in homogeneous way, which is a 4×4 matrix, we can get the transformation equation from current coordinate (x_i, y_i, z_i) to the next coordinate (x_{i+1}, y_{i+1}, z_{i+1}) as below:

$$^{i-1}T_i = A_i = \begin{array}{l} Rot(z,\theta_i) \cdot Trans(0,0,d_i) \cdot \\ Trans(a_i,0,0) \cdot Rot(x,\alpha_i). \end{array} \quad (1)$$

Among (1), Rot(z, θ) means homogeneous transformation matrix when rotate the current coordinate θ degree around z-axis, and Trans(x, y, z) means homogeneous transformation matrix when transform the current cordiante with the vector $\vec{v} = (x, y, z)$.

Then, we can get the transformation matrix from base coordinate (x_0, y_0, z_0) to the end manipulator coordinate (x_7, y_7, z_7) as follow:

$$M = {}^0T_1 \, {}^1T_2 \ldots {}^6T_7 \quad (2)$$

B. Status Space & Configuration Space

Status Space is expressed with 7 parameters from q_1 to q_7, with the expression like this:

$$S = (q_1, q_2, \ \ldots \ , q_7) \quad (3)$$

As for the configuration space, i.e., pose space, we use Euler angles that obeys Euler Z-X-Z transformation rules to express the azimuth angles, which displays like $\gamma = (\psi, \theta, \phi)$. Position writes like $P = (x, y, z)$. Get the position and Euler angles together, we define the expression of pose like this:

$$X = (P, \gamma) = (x, y, z, \psi, \theta, \phi) \quad (4)$$

See that S is expressed with 7 free variables while X with 6, which tells the redundancy of our system, on which our work is based.

C. Forward Kinematics & Jacobian Matrix

As we can see in (2) & (3), M is decided by the current state S, thus the current pose X is also decided. When we transform X into homogeneous matrix defined as $H(X)$ and write M as $M(S)$ (means M is decided by S), we get the connection between S and X:

$$H(X) = M(S) \quad (5)$$

That is what our forward kinematics based on. Knowing the current S, we can get $M(S)$ from (2), then use (5) to get X. The detail operation from H(X) to X will not be discussed here, which can be easily found in primary textbook of robot kinematics. Combineg the process of turning $H(X)$ to X in M, we can finally get forward kinematics as below:

$$X = M(S) \quad (6)$$

The letter M in (6) is not the same meaning with (2) & (5), which combining the process from $H(X)$ to X. We can see M here in (6) as just a function with independent variables S and dependent variables X.

D. Inverse Kinematics & the Extend Inverse Jocobian Matrix

Inverse kinematics comes with the question: How could we get S when we know X? Since the system is reduntant, there may not be only one answer for S. There are methods from X to S directly using fixed angle, but we do inverse kinematics with the help of Jacobian matrix instead.

Looking back at (6), we write M in partial form:

$$X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \end{bmatrix} = \begin{bmatrix} M_1(q_1, q_2, \ldots, q_7) \\ M_2(q_1, q_2, \ldots, q_7) \\ M_3(q_1, q_2, \ldots, q_7) \\ M_4(q_1, q_2, \ldots, q_7) \\ M_5(q_1, q_2, \ldots, q_7) \\ M_6(q_1, q_2, \ldots, q_7) \end{bmatrix} \tag{7}$$

The Jacobian Matrix is defined as (8):

$$\begin{aligned} J &= (\tfrac{\partial M}{\partial q_1}, \tfrac{\partial M}{\partial q_2}, \ \ldots \ , \tfrac{\partial M}{\partial q_7}) \\ &= \begin{bmatrix} \frac{\partial M_1}{\partial q_1}, & \frac{\partial M_1}{\partial q_2}, & \cdots & \frac{\partial M_1}{\partial q_7} \\ \frac{\partial M_2}{\partial q_1}, & \frac{\partial M_2}{\partial q_2}, & \cdots & \frac{\partial M_2}{\partial q_7} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial M_6}{\partial q_1}, & \frac{\partial M_6}{\partial q_2}, & \cdots & \frac{\partial M_6}{\partial q_7} \end{bmatrix} \end{aligned} \tag{8}$$

In our expremental environment, the Jacobian matrix should be a 6×7 matrix for 6 pose variables and 7 joint angle variables, but which is not the same with all redundant manipulator. Then we get the relation between X and S in differential way:

$$\dot{X} = J\dot{S} = J(\dot{q_1}, \dot{q_2}, \ldots, \dot{q_7}) \tag{9}$$

We can't use the direct inverse matrix of J to do inverse kinematics, but can do it with the generalized inverse matrix of J (J^+) defined like this:

$$J^+ = J^T(JJ^T)^{-1} \tag{10}$$

In generalized form, we have:

$$\dot{S} = J^+\dot{X} \tag{11}$$

$$S - S_0 = \Delta S = J^+\Delta X = J^+(X - X_0) \tag{12}$$

In this way, when knowing the initial pose X_0, very near pose X, and the initial S_0, we can compute the status S relative to X according to (11)(12), which is our inverse kinematics based on, both for MLG and Newton-Raphson method we will introduce later.

III. RRT WITH EXTENDED STEPS

Since we use modified RRT to do obstacle-avoiding path planning, we can have a look at the initial RRT method to better understand the results. In this chapter, we introduce RRT and relevant algorithm in a more detail way.

A. Pre-define before RRT

We define the relevant collection and functions below of RRT before looking at its sudocode. Remind the basic problem RRT do with first: how can we run the manipulator from the initial position (knowing X_0 & S_0) to the goal position (only knowing pose of end position, taken down as X_g) without collision into the obstacle in the space? The following collection are defined to express RRT in convenience:

- A_0: whole pose space which can be reached by robot manipulator in configuration space.
- A_{free}: free space belong to A_0, i.e., $A_{free} \in A_0$, and there is no obstacle in A_{free}.
- A_{obs}: space of obstacle, i.e., $A_{obs} = A_0 \backslash A_{free}$.
- $B_0, B_{free}, Bobs$: these are the corresponding collection with A, but in the status space. The subscript takes the same meaning as in A.

Below define the main functions used in RRT. Before we focus on these functions, we should remind that RRT method will build a tree when it runs, we called it RRT tree, writen as tree T, with the initial point X_0 for pose and S_0 for status as the first tree point. We note that: $T = (V, E)$, while V is for all points and E is the relation between these points (father point and its child). Note that T is a tree, so each point has no more than one child.

- sample: generate a random point X_{ram} in pose format in A_{free}.
- distance: compute the distance of two points in A_0. Within our work, this distance is defined as the Euclid distance.
- nearest_neighbor: given the point X, this function return the nearest point X_{near} in tree T.
- steer: given the random point X_{ram} and X_{near}, this function return a new point X_{new} in A_{free}, whihc is nearer to X_{ram} than X_{near}.
- collisionTest: given point X, return 1 for obstacle free and 0 for obstacle collision.

B. RRT and its relative functions

Then we can have a look at the basic algorithm of RRT. We write it in pseudocode show in Fig.2:

```
RRT(S_0, A_0)
1. V ← {S_0}; E ← ∅; i ← 0;
2. while i < N do
3. {
4.     T ← (V, E);
5.     X_rand ← sample(i); i ← i + 1;
6.     T ← extend(T, X_rand);
7. }

extend(T, X_rand)
1. X_near = nearest_neighbor(T, X_rand);
2. X_new = steer(X_near, X_rand);
3. if X_new and collisionTest(X_new)
4.     V ← V ∪ {X_new};
5.     E ← E ∪ {(X_near, X_new)};
5.     return success;
6. else
7.     return fail;
```

Fig. 2. pseudocode of RRT algorithm.

Though simple as it is, RRT work well with obstacle-avoiding task. Here, we focus on the detail of the basic functions specially, for they will be related later in this paper. As defined before, we use Euclid distance but make a little change, named rrtDistance, that is defined bellow with (13):

$$rrtDistance(X_1, X_2) = \alpha||P_1 - P_2|| + (1-\alpha)||\gamma_1 - \gamma_2||,\ 0 < \alpha < 1. \quad (13)$$

Since we concentrate more on the accuracy of position, we mainly choose $\alpha > 0.5$, like $\alpha = 0.8$, which is use in our program. The sample function generate a point in A_0, we use X_{min} and X_{max} to limit area of A_0. Then the random point X_{ram} that sample function produce will be like (14):

$$X_{ram} = X_{min} + t \times (X_{max} - X_{min}), 0 < t < 1; \quad (14)$$

When t is randomly produce in interval (0, 1), sample method would produce all the points of A_0. For steer function, a new point X_{new} will be produce depending on X_{near} and X_{ram}, which works like (15):

$$X_{new} = X_{near} + s(X_{ram} - X_{near})/||X_{ram} - X_{near}||,\ 0 < s < ||X_{ram} - X_{near}||. \quad (15)$$

We call s in (15) the step-length. In RRT algorithm, each step when steer function run, the RRT tree T will attend to explore forward s distance in A_0. Cause X_{rand} will be anywhere in the whole space, it is easy to infer that the direction tree T extends is all-dimension, i.e., no certain direction, but different direction with different time, reminded that is important cause we will discuss it later.

C. modified RRT algorithm

Though RRT can explore the whole space A_0 to get a obstacle free path to reach the goal point, it still has some shortage when it actually run. Since RRT extends only one step (with step-length s) further, it seems slow sometimes when A_0 is big enough. On the other point, RRT tree are built beginning with the initial point, leaving out the goal point in the tree-built process. Still, there are also other insufficients with RRT, which results in many modified RRT method. Here, we present bi-RRT as an example. The detail bi-RRT algorithm can be seen in [12]. We just take it in brief here.

As the shortage we tell, bi-RRT solve it by extend more steps until in collision with obstacle or reach the random point X_{ram}. And bi-RRT maintain two tree in its memory, one starting extend function from the initial point and the other from the goal point. The two tree extend to each other each tiem when extend method run until they meet in the space, which is the reason the name 'bi-RRT' come from. With such modification, bi-RRT take in count of the intial point and goal point together with a faster extending speed.

In our exprement, we just take the more steps extending thought into our code. Our multi-extend algorithm is like in Fig.3:

The function '$surpass(X_{new}, X_{rand})$' detect that if X_{new} is near enough to X_{rand}. The 'true' return for this function means the two points are close enough, then 'multi_extend' method will be stop, and the 'false' return will continue the extending process. With these change, if the extend direction is correct, we could get faster to the goal point, but simutaneously, we would also extend further to a bad direction, which would take up more time if this situation happens frequently. We solve this problem with RRT-GD (RRT with goal-direction), which will be related in the later paragraph. But within our process now, we have two extend functions: extend and multi-extend, and we can think of the inverse kinematics optimization, which will be discuss first.

```
multi_extend(T, X_rand)
1. [X_new, result] = extend(T, X_rand);
2. if result = fail or collisionTest(X_new) = 0
3.     return fail;
4. while result = success
5. {
6.     [X_new, result] = extend(X_new, X_rand);
7.     if collisionTest(X_new) = 0 or
                surpass(X_new, X_rand)
8.         break;
9. }
10. return success.
```

Fig. 3. multi-step extend pseudocode

IV. Inverse Kinematics with Newton-Raphson method

We have related some inverse kinematics method before, e.g., GP (gradient projection method), WLN (weighted least norm method), MLG (method of most likely gradient)[6]. Within our program, we take Newton-Raphson method into account, for its high accuracy and speed.

A. Newton-Raphson method

We reminded back to our inverse kinematics problem: we know S_0 (and so X_0 can be compute) and X_g, how could we compute S_g? In Newton-Raphson method, which is an iteration method, each time we just go straight from the current point to goal point. We can see the iteration equation in (16):

$$S_{n+1} = S_n + J^+ \times minus(X_g, X_n). \quad (16)$$

The current status and pose are written as S_n and X_n, and J^+ is also relative to the current point, which can be compute in (10). Then when X_n is close enough to X_g, we get the answer S_n as result S_g. Note that though the function name called 'minus', it dost just not simple minus as $X_g - X_n$, because the azimuth doesn't supplement the minus operation. We should use the Rotate-By-1-Axis method to get the corret answer, since the combination of fix-axis rotation of rigid body can be seen as one-time fix-axis.

After this work, we take down the Newton-Raphson method as Fig. 4:

On Fig.4, the 1st and 6th line use (6) and (10) to compute it. Here we use N (we set it to 10 in program) to limit the iteration time, for the fact that Newton-Raphson method would always get the correct answer in less than 10 iteration times with accuracy of micrometer, otherwise it would fail with bias greater than ϵ, i.e., more iteration times have no use cause it can't improve the accuracy.

$Newton\_Raphson(S_0, X_g)$
1. $compute\ X_0\ and\ J^+; i = 0; times = 0;$
2. $while\ \epsilon < rrtDistance(X_i, X_g)\ and\ times < N$
3. {
4. $\quad S_{i+1} = S_i + J^+ \times minus(X_g, X_i);$
5. $\quad i = i + 1; times = times + 1;$
6. $\quad compute\ X_i\ and\ J^+;$
7. }

Fig. 4. multi-step extend pseudocode

B. Comparation with MLG

At the beginning, we have complement the MLG method before using Newton-Raphson method for inverse-optimization. Here, we introduce MLG method and compare it with Newton-Raphson method, to see some great properties of Newton-Raphson.

MLG method is also a iteration method, using the gradient of current point as the iteration direction, the detail of it can be seen in [3]. We just make comparation between MLG and Newton-Raphson method.

Time, iteration times, accuracy compare figure locating here.

We do comparation between these two method using 4 groups of experiments. These tests start with the same initial point q_0, and so the same with X_0, and go to different end point appointed by X_g. Note that the iteration times of MLG method can be a input variable which can be assigned before MLG algorithm runs. So we take 5, 10, and 20 iteration times as examples.

As table 2 shows, in these groups of exprements, we make sense that:

- The time spent by Newton-Rapshon method is almost the same with 5-iteration-times MLG method, sometimes even less than it.
- Newton-Raphson method uses less than 10 iteration times ending up with bias no more than $10^{-6}m$, but MLG method, no matter using 5, 10, or 20 iteration times, always ending up with deviration error higher than $10^{-3}m$, spending time in the same order of Newton-Raphson method.
- Newton-Raphson method adapt to a wider range of 'dist' change, i.e., it is suitified to more tasks when doing inverse kinematics. When 'dist' becomes bigger, Newton-Raphson works better than MLG method, that can be find in group 4, with Newton-Raphson method successful and MLG failed. Here, we limit the result bias to $0.1m$, i.e., when the accuracy is bigger than this number, we think it failed.
- When Newton-Raphson method fail in doing inverse kinematics, MLG method may always can't work well, because they both use the gradient of current point. Minded that whether the result fail or success doesn't depend on 'dist' parameter, it is decided by many reasons. Sometimes a small 'dist' even like $0.02m$ may also result in fail both for Newton-Raphson method and MLG method.

One point which should be reminded is that we implement MLG method according to [3]. There exists some limitaion condition with the end trajectory, e.g., the velocity of end manipulator can't be 0, which is set to greater than 0.05 in our program. So more iteration times may not makes the more accuracy in MLG method. But the conclusion that MLG method always ends up with accuracy of $10^{-3}m \sim 10^{-2}m$ when it successes is right, which has been tested by many times of experiments.

V. Quintic Polynomial with path smoothing

There comes another question after doing Newton-Raphson method: knowing S_0 and S_g, how could we move the manipulor? The most direct answer might be simply adapt the joint angle to goal angle, however, we can't know the trajectory of the end manipulator in this way, and it might be also not the shortest distance in the thought of end manipulator. Thus, we need to smooth the path from S_0 crossing a little distance to S_g.

As mentioned before, MLG method can do the path smoothing in its iteration, i.e., when finishing compute S_g, it also finishes planning the path from S_0 to S_g, with a designing trajectory of then end manipulator, as for a straight line. But the same thing cannot apply to Newton-Raphson, for its variant iteration times and variant distance within one iteration. So we need to do path planning separately. The quintic polynomial thus has been use in our program, the principle of whihc can be seen in [9]. We use its basic conclusion. Supposed we know S_0 and S_g, and their distance ($rrtDistance(S_0, S_g) = s$, see in (15)) fixed, we need to plan the medium path in 1 second (of course it can be changed by timing a factor). Then in quintic polynomial method, we plan the path as in (17):

$$S(t) = a_5t^5 + a_4t^4 + a_3t^3 + a_2t^2 + a_1t + a_0;$$
$$with: \begin{cases} a_5 = & 6(S_g - S_0); \\ a_4 = & -15(S_g - S_0); \\ a_3 = & 10(S_g - S_0); \\ a_2 = & 0; \\ a_1 = & 0; \\ a_0 = & S_0; \end{cases} \tag{17}$$

Using (17), we can get a smooth path. When $t = 0s$, $S(t)$ equals to S_0, that is the initial point, and when $t = 1s$, $S(t)$ equals to S_g, that is the goal point. And we can also compute the speed of path giving by (17), just by doing differential operation. What we can deduce from (17) is that the speed of end manipulator when in initial point or goal point is 0. We could also change the coefficients from a_0 to a_5 according to [9] to fit different uses. These method works well on our 7-arm robot manipulator when doing exprements.

VI. RRT-GD to adapt RRT to goal-direction

When finishing the work above, we can already finishing any work in the work space in theory. But just as the problem we have mentioned in chapter 3, both RRT and its modified

TABLE II

Comparation between RRT and NEWTON-RAPHSON *method.* $S_0 = [0.7854; 0.5236; 0; 0.5236; 0; 0.5236; 0](rad)$; $X_0 = [0.5045; 0.5045; 0.7223; 2.3562; 1.5708; -1.5708]$; $dist = rrtDistance(X_0, X_g)$, *all their units are meters*(m).'$\surd$' *means successful.* ' $\times$' *means result failed, i.e., bias (accuracy) comes out more than* $0.1m$.

No.	task(m)	Method	iteration times	time spent($10^{-3}s$)	accuracy(m)	result
1	$X_g = [\ 0.50; 0.45; 0.72; 2.35; 1.57; -1.57\ ]$ $dist = 0.045.$	N-R	5	6.15	1.28×10^{-9}	$\surd$
		MLG	5	5.70	6.50×10^{-3}	$\surd$
			10	6.52	2.64×10^{-3}	$\surd$
			20	11.76	1.25×10^{-3}	$\surd$
2	$X_g = [\ 0.5; 0.48; 0.72; 2.35; 1.55; -1.55\ ]$ $dist = 0.025.$	N-R	4	2.38	2.34×10^{-7}	$\surd$
		MLG	5	3.95	9.74×10^{-2}	$\surd$
			10	6.60	9.56×10^{-2}	$\surd$
			20	12.77	9.39×10^{-2}	$\surd$
3	$X_g = [\ 0.44; 0.44; 0.68; 2.30; 1.57; -1.57\ ]$ $dist = 0.092.$	N-R	7	3.87	5.87×10^{-8}	$\surd$
		MLG	5	3.94	1.65×10^{-2}	$\surd$
			10	6.64	1.73×10^{-2}	$\surd$
			20	12.20	1.78×10^{-2}	$\surd$
4	$X_g = [\ 0.45; 0.55; 0.60; 2.00; 1.57; -1.57\ ]$ $dist = 0.184.$	N-R	9	5.70	5.53×10^{-10}	$\surd$
		MLG	5	4.46	1.37×10^{-1}	$\times$
			10	7.31	1.22×10^{-1}	$\times$
			20	11.65	1.12×10^{-1}	$\times$

method have the great opportunity to explore unuseful space which we don't need in our daily task, i.e., it lacks goal-direction. In order to solve this problem, taking the normal task in our experiment circumstance into account, we turn our sights to the 'sample' function in RRT. Simple to find that if we just don't use a whole space sample method, but limit it to a useful space, we can decide the extend direction of RRT tree T, that is the kernel principle within our modified RRT method called RRT-GD.

Originally, 'sample' method is random in the whole work space, and thus then extend direction is all-around. The sketch map can be seen in Fig. 5. When we use a space containning the goal point X_0, called sample space A_{sam}, which can simply be a sphere with X_0 centering (of course other shapes also works, e.g., cuboid), the extending direction can be limit to goal-direction, as Fig.5 shows.

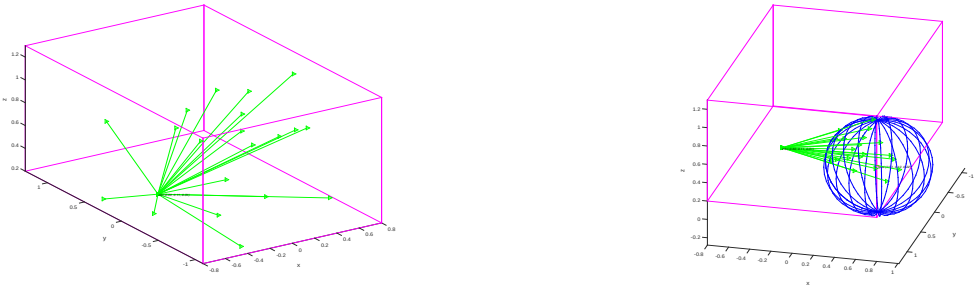
In our program, we implement sample method with both sphere mode and cuboid mode, both work well with our daily task. We call the space given by 'sample' function the sample space A_{sam}. We can see the comparation between RRT and RRT-GD within iteration times and time property in table 3.

We take the same initial point as S_0 in table 3, and let the end manipulator go to different goal points. The 'extending times' in table 3 is the total times of successful extending when finding the exact path to goal point, and so 'failed extending times' means the total failed times. The altogether iteration is their sum. For RRT-GD here, we use a sphere whose radius is $0.5m$ with goal point centering to do 'sample' method. One point that should be reminded is that the result below is one time among many tests, which is the mid-value within these tests. From Table III and Fig. 5, we can learn that:

- RRT-GD uses much less time and iteration times compared to RRT when doing the same work. As we can see, RRT-GD can attends to be 100 times faster, speeding up the algorithm remarkably.
- RRT-GD can almost to be realtime path planning in these tasks, with planning time less than $0.5s$, that is great when comparing to the traditional RRT method.
- RRT-GD can do the task that RRT cannot, as see in task 3 of table 3. Here, we set the iteration times limitation to 10000, surpassing which we think the algorithm failed. In task 3, we do RRT method for 10 times but none of them succeed, while RRT-GD could always succeed.
- Within our experiments, RRT-GD could succeed as long as RRT method succeed, i.e., the correctness of RRT-GD can be assured.
- Since RRT-GD uses less iteration times, so the planning path gived by RRT-GD since to be more directly, i.e., less zig-zag on the path, which can be seen in Fig. 6.

Note that we just return one successful result in table 3 when doing these tasks. Of course, we could also set iteration times to a fix number, then use RRT-GD method to get multiple paths and get the best as output. When doing these way, says iteration times limitation to 10000, RRT-GD can always get more paths than RRT method.

One more to say, RRT-GD is a modification of RRT algorithm. It changes the 'sample' method of RRT algorithm. We can think of the two peaks of RRT-GD. One situation is when sample space fill up the whole working space, i.e., $A_{sam} = A_0$. At this time, RRT-GD algorithm attends to be RRT method, i.e., they are the same when running. Another situation is that when sample space shrinks to only one point, i.e., $A_{sam} = X_{goal}$, Then the situation becomes a straight line path planning from the initial point to goal point. In fact, since we can't give the whole space an accuracy limitation, RRT method itself thus can't ensure giving a path even the path exists, i.e., it isn't even correct in theory. So RRT-GD is an important ideology which proves to be more pratical, press closer to the real circumstance and daily common tasks.



(a) Extend direction of RRT: all-direction.

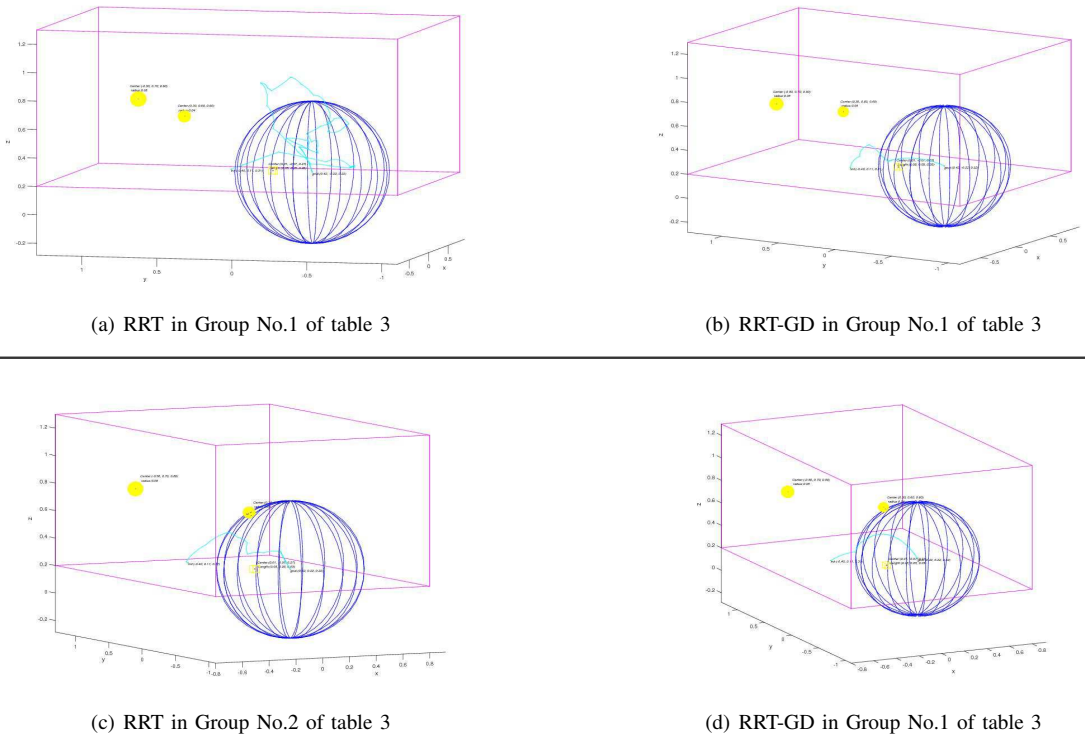
(b) Extend direction of RRT-GD: goal-direction.

Fig. 5. Extend direction of RRT and RRT-GD. Pink cuboid is for the whole working space. Blue sphere is for sample space.

TABLE III

Comparation between RRT *and* RRT-GD *method.* $S_0 = [-0.2618; -0.2618; 0; -1.3090; 0; -1.3962; 0](rad)$; $X_0 = [0.4011; 0.1075; 0.3115; -1.8326; 2.9671; 1.5708]$; $dist = rrtDistance(X_0, X_g)$, *all their units are meters*(m).‘ $\times'$ *means result failed.*

No.	task(m)	Method	extending times	failed extending times	time spent(s)
1	$X_g = \begin{bmatrix} 0.42; -0.22; 0.22; -1.83; 2.97; -1.57 \end{bmatrix}$ $dist = 0.7108.$	RRT	808	8401	108.20
		RRT-GD	9	27	0.44
2	$X_g = \begin{bmatrix} 0.42; 0.22; 0.22; -1.83; 2.80; -1.50 \end{bmatrix}$ $dist = 0.6668.$	RRT	506	4250	47.39
		RRT-GD	3	1	0.16
3	$X_g = \begin{bmatrix} 0.51; 0.12; 0.22; -1.73; 2.90; -1.57 \end{bmatrix}$ $dist = 0.7326;$	RRT	$\times$		
		RRT-GD	4	6	0.219



(a) RRT in Group No.1 of table 3

(b) RRT-GD in Group No.1 of table 3

(c) RRT in Group No.2 of table 3

(d) RRT-GD in Group No.1 of table 3

Fig. 6. Result comparation between RRT and RRT-GD. The blue sphere is the sample method sphere we use with goal point centering in red. The range of pink cuboid is our whole work space. The yellow sphere or cuboid is obstacle in the free space. The crayon line is the planning path starting from initial point to goal point.

VII. CONCLUSIONS

In this paer, we introduce a new method in obstacle-avoiding path planning, with modified RRT method doing obstacle-avoiding, Newton-Raphson doing inverse kinematics, and quintic polynomial doing path smoothing. After that,

we find the less goal-direction with RRT and its modified algorithm, so we implement RRT-GD to solve this problem. With RRT-GD, we can accomplish daily works faster and more efficient, usually 10 100times faster than the usual RRT method because of its goal-direction property. Note that when the sample space of RRT-GD is not full of the whole working space, RRT-GD may not give a right planning path when in RRT in theory, i.e., it is not a usual method. However, RRT-GD works well in practice, as we can see in TABLE III and Fig. 6, for that in our common working space, there are usually not so many obstacles and the size of each obstacle are usually small enough for RRT-GD to work on.

REFERENCES

[1] A Methodology for Intelligent Path Planning, Suman Chakravorty, John L. Junkins. Department of Aerospace Engineering, Texas A&M University Young, College Station.

[2] Kinect-based Service Robot for Desktop Cleaning, Zheng Han, Meng Gao, ShiJiaZhuang Tiedao University.

[3] Planning Method Considering the kinetic characteristics of the end effector of a redundant manipulator, Wenbing Huang, Fuchun Sun, Huaping Liu, Qinghua Daxue Xuebao/journal of Tsinghua University, 2014,54(12):1544-1548.

[4] Incremental kinesthetic teaching of motion primitives using the motion refinement tube, Dongheui Lee, Chritian Ott, Autonomous Robots, 2011, 31(2-3):115-131.

[5] New Heuristic Algorithms for Efficient Hierarchical Path Planning, David Zhu and Jean-Claude Latombe, IEEE Transactions on Robotics & Automation, 1991.7(1):9-20.

[6] A Subdivision Algorithm in Configuration for Findpath with Rotation, Rodney A. Brooks, Tomas Lozano-Perez, Systems Man & Cybemetics IEEE Transactions on, 1985, SMC-15(2):244-233.

[7] Motion Planning and Skill Transfer of Anthropomorphic Arms Based on Movement PrimitivesXilun Ding, Cheng Fang, International Conference on Mechatronics & Automation, 2012:303-310.

[8] Algorithm Based on Analytical Method and Genetic Algorithm for Inverse Kinematics of Redundant Manipulator, Yun Dong, Tao Yang, Wen Li, Computer Simulation, 2012.

[9] Trajectory Planning for a Robotic Ping-pong Player, Guowei Zhang, Bin Li, Huaibing Zhen, Haili Gong, Cong Wang, Chinese Journal of Scientific Instrument, Vol.32 No.6, Jun 2011.

[10] Virtual Reality in the Loop Providing an Interface for an Intelligent Rule Learning and Planning System, Jurgen Rossmann, Christian Schlette, Nils Wantia, IEEE International Conference on Robotics & Automation, 2013.

[11] Research on Kinematics and Obstacle Avoidance Path Planning for Redundant Manipulator, Yin Bin, Shi Shicai, Classified Index:TP242.2, U.D.C:681.5.