Computer Graphics

Discussion 7

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Part I: Phong shading

Lighting formulas

Light equation

- The two shaders must put together some equation to come up with pixel brightness
- We can find out what sort of equation we can make by looking at what the inputs of the shader are - what information is available?

Light equation starts with just all this info

- Elements to be drawn
  - Per vertex in each element:
    - Positions - all of them
    - Normals (the full set of perpendicular vectors for all points)
    - Colors if you want some per vertex
    - Texture coords

- Per whole draw call:
  - Matrices
  - Flags
    - Does this shape consult a texture image or not?
    - Color this shape solid? Set color = normals?
    - Specialized program just for this shape? etc.
  - Light position (and specify if point or vector)
  - Material properties of shape - how chalky/shiny its interaction with light source is

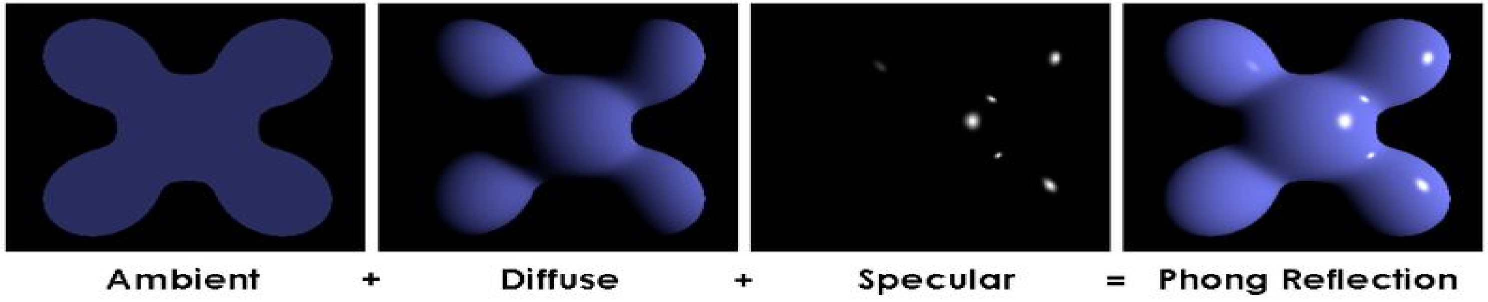
Light equation starts with just all this info

- Normal is the most important input to light equation

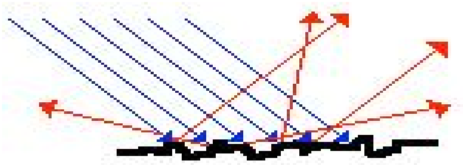
- Intermediate calculations: Compute eye, L

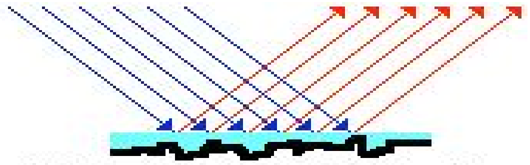
- Our equation choice: Phong model

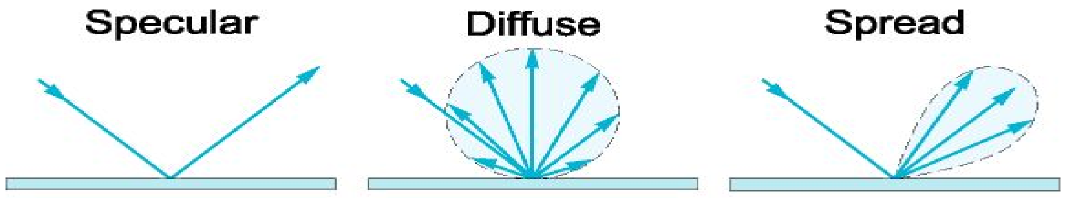
Components of light



Combining components of light


A dry asphalt roadway diffuses incident light.

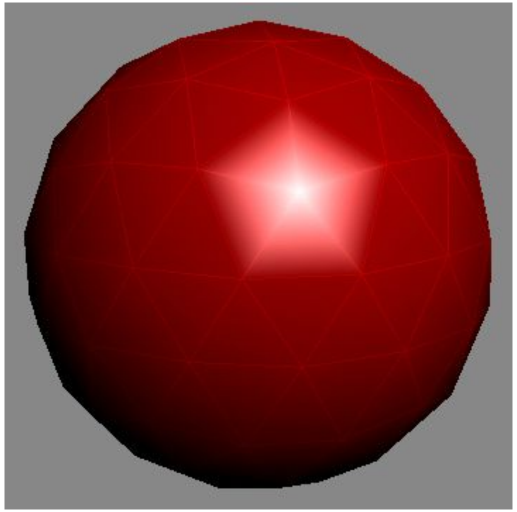

When wet, water fills in the crevices, resulting in specular reflection and a glare.

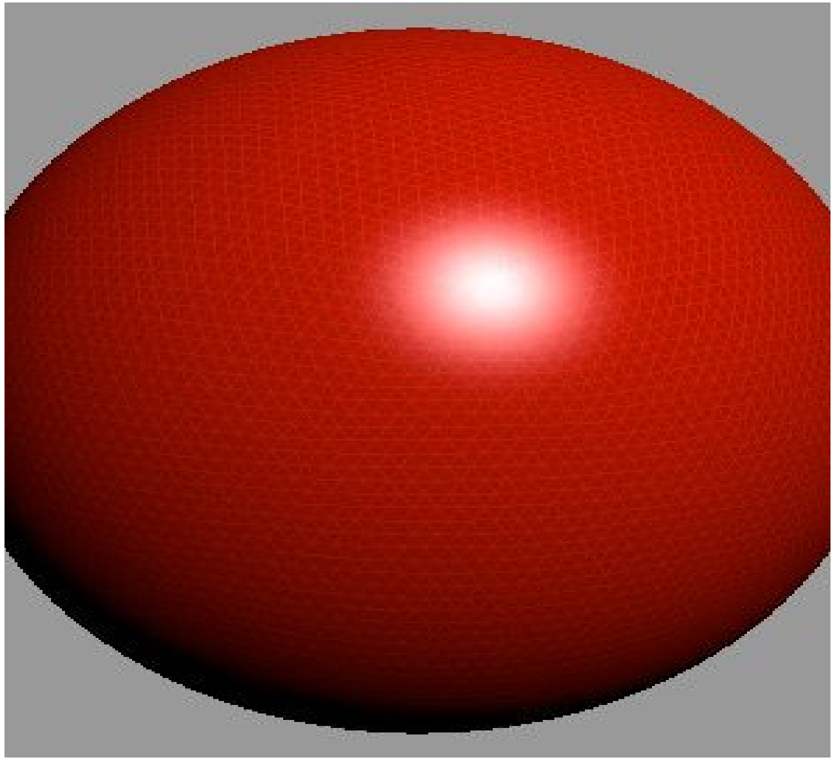


Specular, diffuse, and spread reflection from a surface.

Gouraud shading

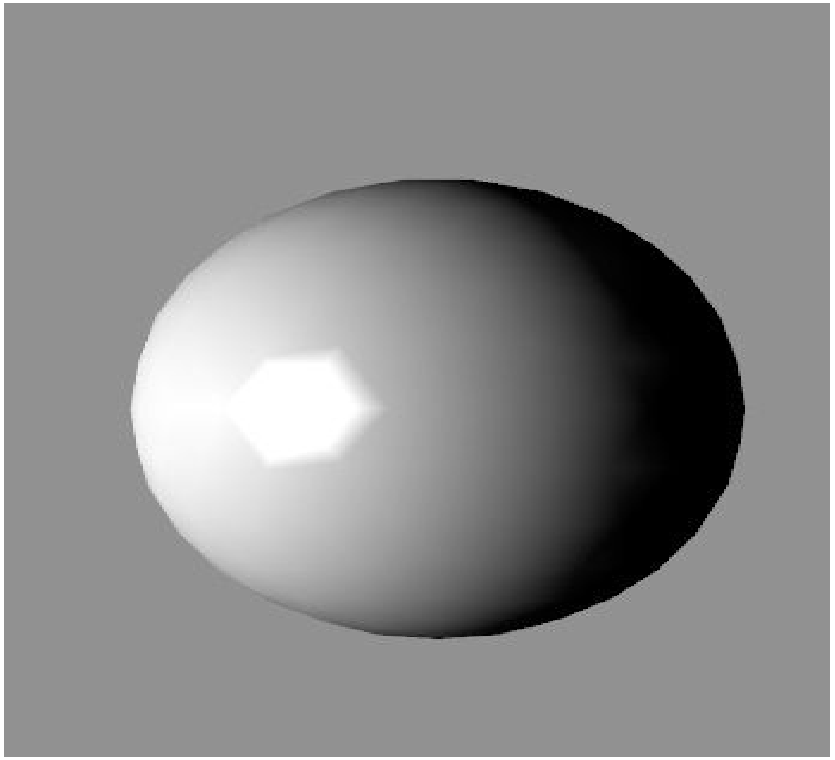
- Simplest method: Just assign brightnesses per vertex and interpolate
- The specular highlight performs poorly at edges
- Only solution - make edges matter less by increasing polygon count





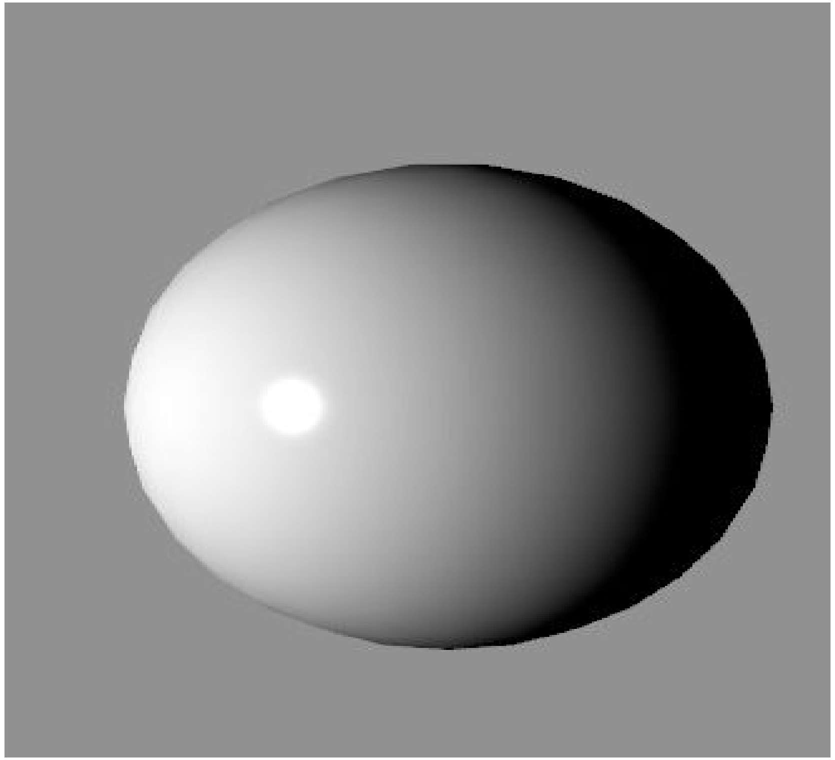
Light equation

- Gouraud shading
  - Diffuse and specular components calculated at every point and added together
  - Linearly interpolate the brightnesses



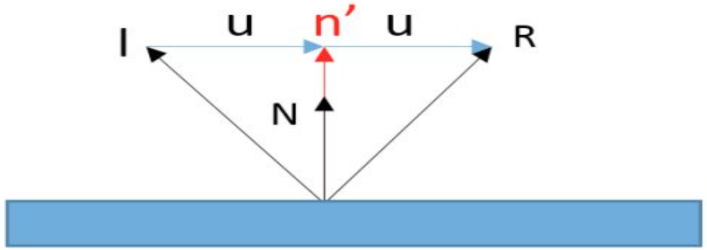
Light equation

- Phong shading
  - Linearly interpolate the <u>normals</u> across each triangle
  - Only when you get to an actual pixel do you calculate the specular and diffuse brightnesses
  - At <u>every pixel</u>, a much finer scale than Gouraud



Light equation

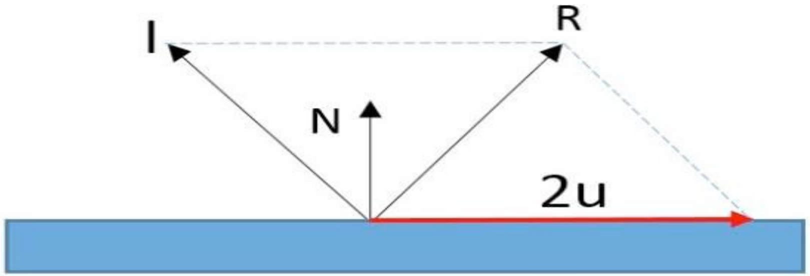
- Calculating R, the (non-physical, made-up) reflection of the point light source



The $\vec{n'}$ is the projection of $\vec{I}$ on $\vec{N}$

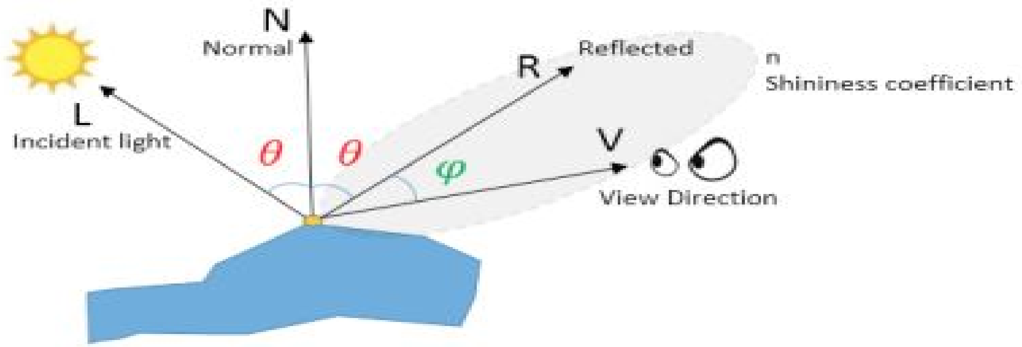
$\vec{n'} = (\vec{N} \cdot \vec{I})\,\vec{N}$, with $\|\vec{N}\|^2=1$

$\vec{u} = \vec{n'} - \vec{I}$



$$\vec{R} = \vec{I} + 2\vec{u} = \vec{I} + 2(\vec{n'} - \vec{I})$$
$$\vec{R} = 2(\vec{N} \cdot \vec{I})\,\vec{N} - \vec{I}$$

Light equation



I = emissive + ambient + diffuse + specular

emissive = k_e

ambient = $k_a * ambientColor$

diffuse = $k_d * lightColor * \cos(\theta)$

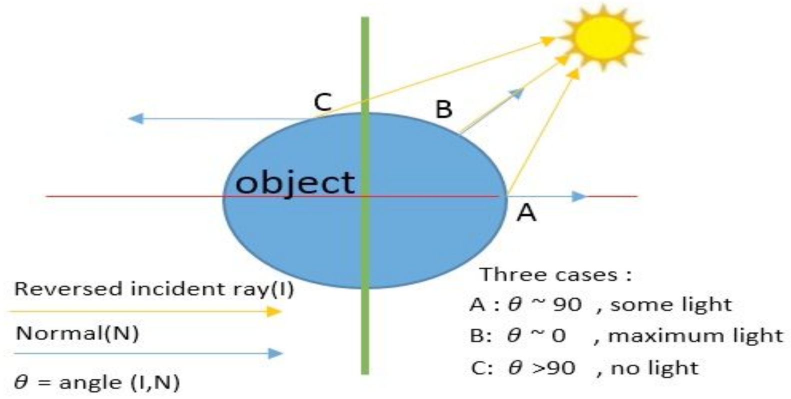
= $k_d * lightColor$ * max(0,N · L)

specular = $k_s * lightColor * \cos(\varphi)^n$

= $k_s * lightColor$ * max(0,R · V) ^{n}

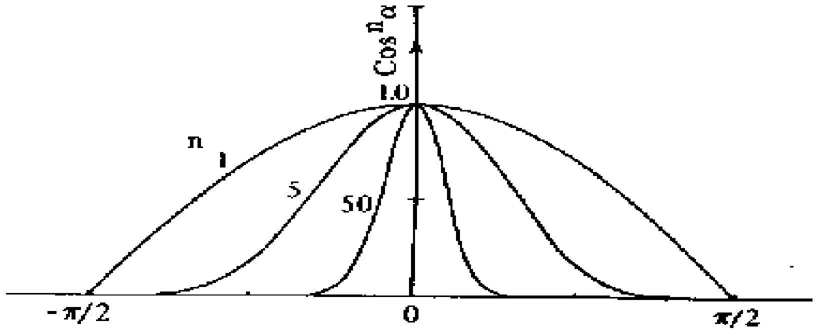
Lambert’s law

“The amount of reflected light is proportional with the cosine (dot product) of the angle between the normal and incident vector”

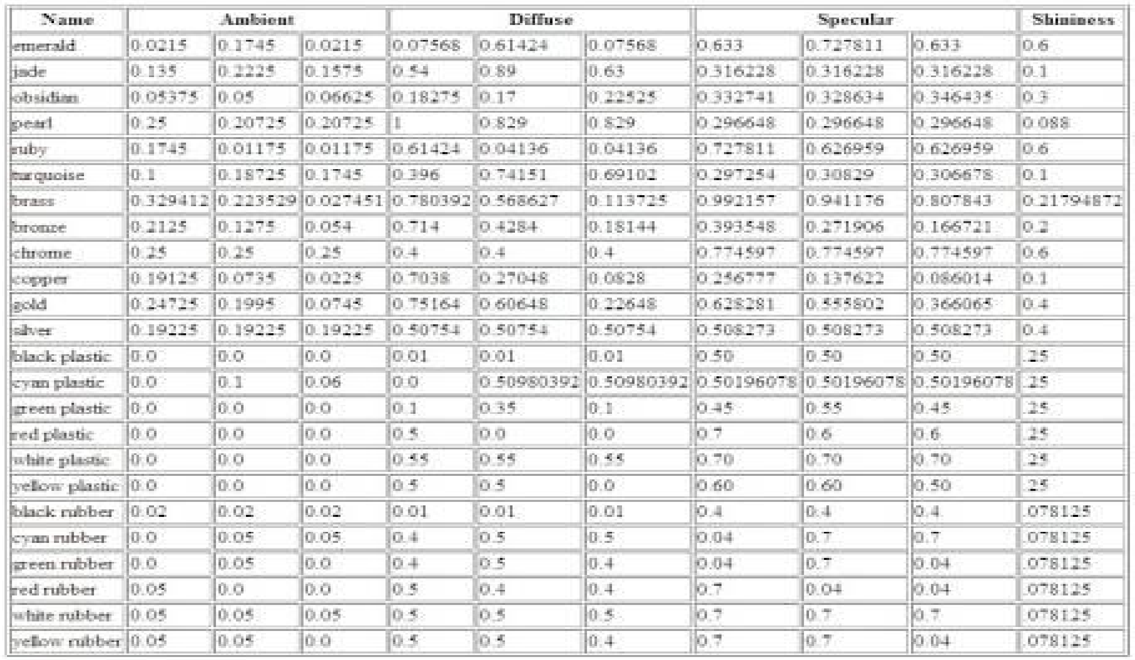

C
B
A
object
Reversed incident ray(I)
Normal(N)
θ = angle (I,N)
Three cases :
A : θ ~ 90 , some light
B: θ ~ 0 , maximum light
C: θ >90 , no light

Specular term - Smoothness exponent effect

- Exponentiating a function that has values < 1 draws those values closer to zero
- Higher exponent = smaller region where point light’s reflection is considered “aligned” with the viewer.
- Smaller shiny spot



Material properties - coefficients

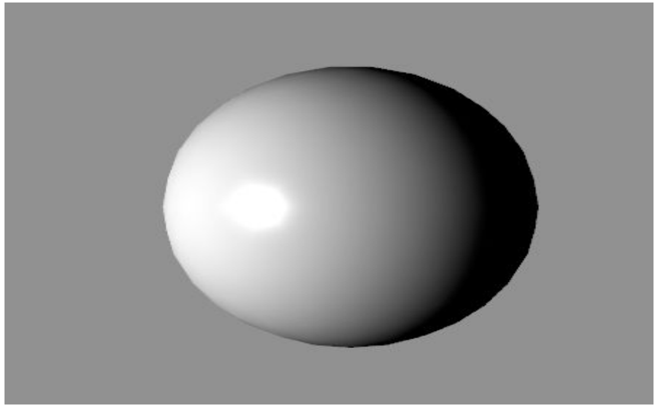


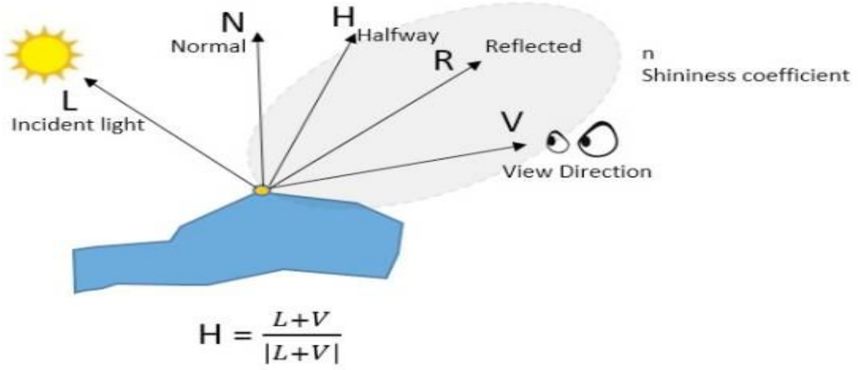
Name	Ambient			Diffuse			Specular			Shininess
emerald	0.0215	0.1745	0.0215	0.07568	0.61424	0.07568	0.633	0.727811	0.633	0.6
jade	0.135	0.2225	0.1575	0.54	0.89	0.63	0.316228	0.316228	0.316228	0.1
obsidian	0.05375	0.05	0.06625	0.18275	0.17	0.22525	0.332741	0.328634	0.346435	0.3
pearl	0.25	0.20725	0.20725	1	0.829	0.829	0.296648	0.296648	0.296648	0.088
ruby	0.1745	0.01175	0.01175	0.61424	0.04136	0.04136	0.727811	0.626959	0.626959	0.6
turquoise	0.1	0.18725	0.1745	0.396	0.74151	0.69102	0.297254	0.30829	0.306678	0.1
brass	0.329412	0.223529	0.027451	0.780392	0.568627	0.113725	0.992157	0.941176	0.807843	0.21794872
bronze	0.2125	0.1275	0.054	0.714	0.4284	0.18144	0.393548	0.271906	0.166721	0.2
chrome	0.25	0.25	0.25	0.4	0.4	0.4	0.774597	0.774597	0.774597	0.6
copper	0.19125	0.0735	0.0225	0.7038	0.27048	0.0828	0.256777	0.137622	0.086014	0.1
gold	0.24725	0.1995	0.0745	0.75164	0.60648	0.22648	0.628281	0.555802	0.366065	0.4
silver	0.19225	0.19225	0.19225	0.50754	0.50754	0.50754	0.508273	0.508273	0.508273	0.4
black plastic	0.0	0.0	0.0	0.01	0.01	0.01	0.50	0.50	0.50	.25
cyan plastic	0.0	0.1	0.06	0.0	0.50980392	0.50980392	0.50196078	0.50196078	0.50196078	.25
green plastic	0.0	0.0	0.0	0.1	0.35	0.1	0.45	0.55	0.45	.25
red plastic	0.0	0.0	0.0	0.5	0.0	0.0	0.7	0.6	0.6	.25
white plastic	0.0	0.0	0.0	0.55	0.55	0.55	0.70	0.70	0.70	.25
yellow plastic	0.0	0.0	0.0	0.5	0.5	0.0	0.60	0.60	0.50	.25
black rubber	0.02	0.02	0.02	0.01	0.01	0.01	0.4	0.4	0.4	.078125
cyan rubber	0.0	0.05	0.05	0.4	0.5	0.5	0.04	0.7	0.7	.078125
green rubber	0.0	0.05	0.0	0.4	0.5	0.4	0.04	0.7	0.04	.078125
red rubber	0.05	0.0	0.0	0.5	0.4	0.4	0.7	0.04	0.04	.078125
white rubber	0.05	0.05	0.05	0.5	0.5	0.5	0.7	0.7	0.7	.078125
yellow rubber	0.05	0.05	0.0	0.5	0.5	0.4	0.7	0.7	0.04	.078125

Multiply the shininess by 128!

Phong-Blinn

- Combine V and L, the two constants in the scene, into one vector
- Given H = halfway between V and L, Use (H dot N) instead of (R dot V)
- If directional light, you can compute H once per frame per light source and it's the same everywhere in the scene - no dependence on normal, just viewer and light
- Re-use it instead of re-calcuating in shader - shader only has to dot H with each N - cheap
- Also behaves better at glancing angles

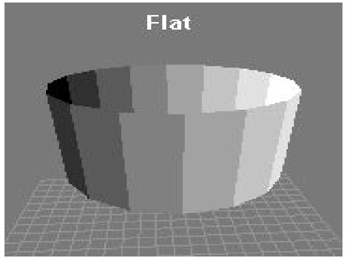


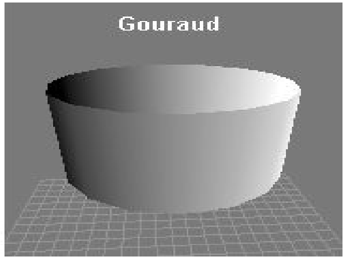


$$H = \frac{L+V}{|L+V|}$$

Light equation

- How many times to do the lighting equation?
  - Once per triangle - flat
  - Once per point - gouraud
  - Once per pixel - smooth / "phong shading"

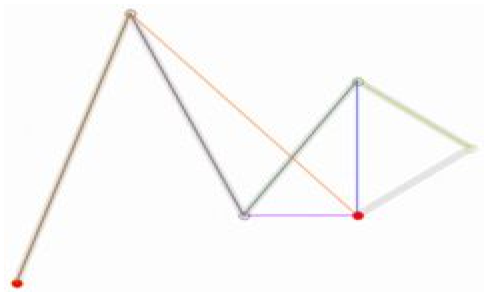




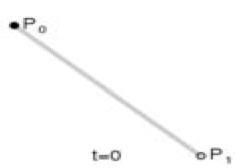
Part II: Using certain advanced topics

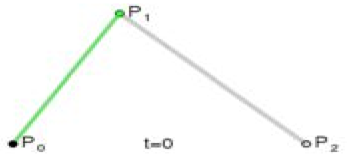
- Today: Smooth curves that join places in your scene

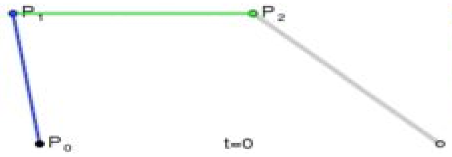
Curves

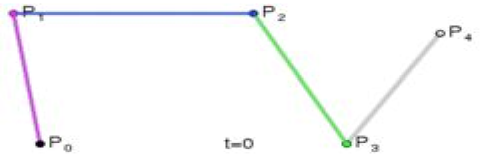


- They use convex combinations to interpolate.
  - $a\underline{\mathbf{x}} + (1 - a)\underline{\mathbf{y}}$ where $a > 0$
- But use more than two points – interpolate some of them, then interpolate those interpolations until you have a single point!





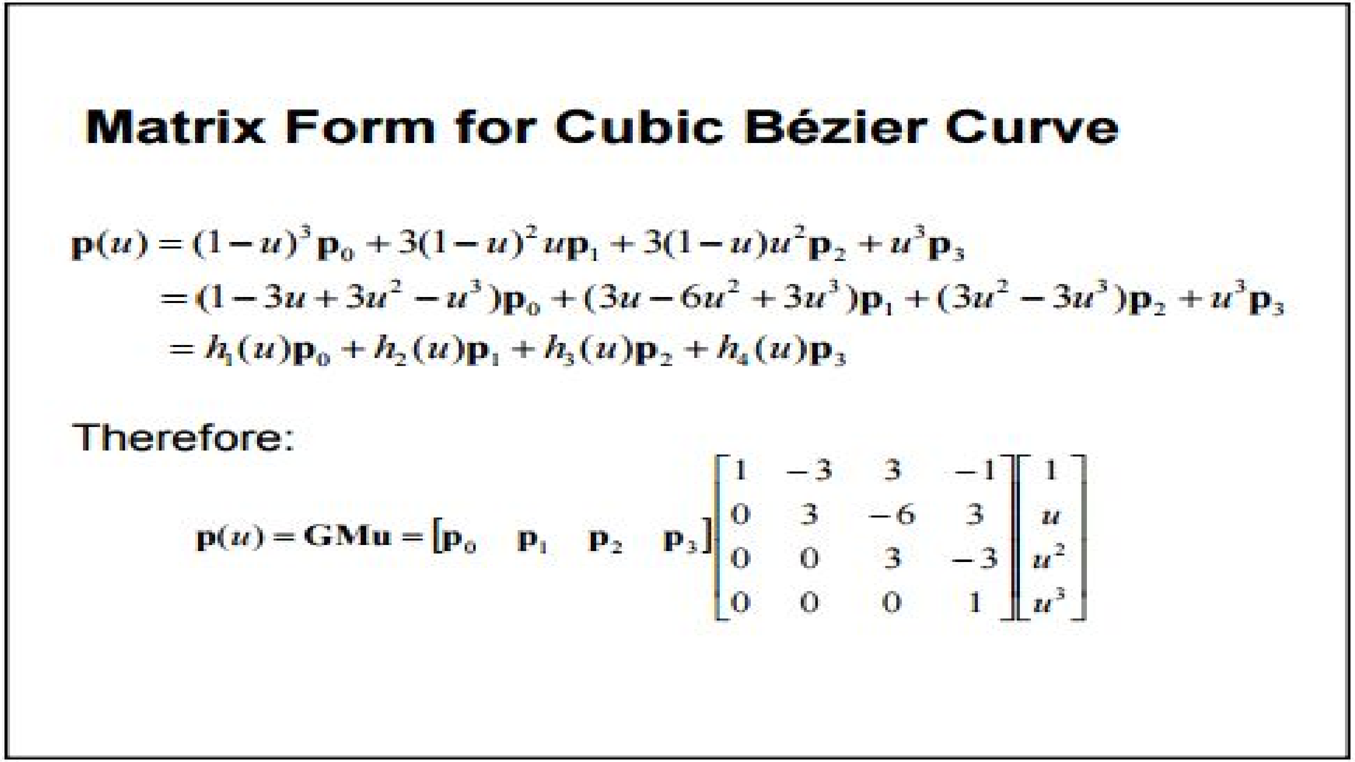




Bezier Curves

- Interpolation formulas can be nested into one another, producing a system of four equations to calculate each of [x,y,z,1].
- Multiply out the nested expressions.
- There's lots of terms now; group them by powers of the parameter (t^3,t^2,t,constant) and factor those out
- Group the four points (four vec4's) constraining the curve out as well:

- What’s left is the matrix of constants for any cubic Bezier curve:



Matrix Form for Cubic Bézier Curve

$$\begin{aligned}\mathbf{p}(u) &= (1-u)^3\mathbf{p}_0 + 3(1-u)^2u\mathbf{p}_1 + 3(1-u)u^2\mathbf{p}_2 + u^3\mathbf{p}_3 \\ &= (1-3u+3u^2-u^3)\mathbf{p}_0 + (3u-6u^2+3u^3)\mathbf{p}_1 + (3u^2-3u^3)\mathbf{p}_2 + u^3\mathbf{p}_3 \\ &= h_1(u)\mathbf{p}_0 + h_2(u)\mathbf{p}_1 + h_3(u)\mathbf{p}_2 + h_4(u)\mathbf{p}_3\end{aligned}$$

Therefore:

$$\mathbf{p}(u) = \mathbf{GMu} = \begin{bmatrix}\mathbf{p}_0 & \mathbf{p}_1 & \mathbf{p}_2 & \mathbf{p}_3\end{bmatrix}\begin{bmatrix}1 & -3 & 3 & -1 \\ 0 & 3 & -6 & 3 \\ 0 & 0 & 3 & -3 \\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 \\ u \\ u^2 \\ u^3\end{bmatrix}$$

Bezier Curves

- One disadvantage - two of our control points only bias, rather than touch, our curve - so specification is imprecise
- Another disadvantage - it’s unclear how we smoothly would chain several of these piecewise cubic functions into one long “spline” curve, so that the function stays C1 continuous across sections

Hermite Spline

- “Hermite curve” - Constrain the curve using two points and their two derivatives (tangents), rather than four points.
- Grouping out the terms as we did before (including our input points & tangents) yields a different matrix of coefficients than the Bezier one gave us.

Hermite Spline

- Solving for the unknowns gives:

$$\begin{aligned} a &= -2x_1 + 2x_0 + x_1' + x_0' \\ b &= 3x_1 - 3x_0 - x_1' - 2x_0' \\ c &= x_0' \\ d &= x_0 \end{aligned}$$

- Rearranging gives:

$$\begin{aligned} x = x_1&(-2t^3 + 3t^2) \\ &+ x_0(2t^3 - 3t^2 + 1) \\ &+ \mathrm{x}_1'(t^3 - t^2) \\ &+ x_0'(t^3 - 2t^2 + t) \end{aligned}$$

or

$$x = \begin{bmatrix} x_1 & x_0 & x_1' & x_0' \end{bmatrix} \begin{bmatrix} -2 & 3 & 0 & 0 \\ 2 & -3 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 1 & -2 & 1 & 0 \end{bmatrix} \begin{bmatrix} t^3 \\ t^2 \\ t \\ 1 \end{bmatrix}$$

Hermite Curves

- Still yields piecewise cubic sections, but...
- As we end one section and begin another, we can simply force both the point and the tangent to match, creating **<u>C1 continuity</u>**
- Relaxing things to only match tangent direction (not magnitude) means **<u>G1 continuity</u>**
- “Paths” tool in Photoshop / GIMP lets you draw G1 continuous splines using mouse clicks

Other Curves

- Other formulations of cubic curve sections exist
- There’s one that lets you define four points (like Bezier curves) but this time the curve touches all four control points
- Now we can build a longer spline out of only specifying points that touch the spline curve
- Suppose we advance a window of four points along a longer array of spline points, and build a matrix of each group of four to interpolate between the middle two points
- Each section interpolated this way will flow with C1 continuity into the next one, just like with Hermite sections, only now we don’t even have to specify tangents or think about derivatives.