Zadaće 3 i 4

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1)

$$G = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

a)

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{K_2 \downarrow\uparrow K_3} \sim \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{K_3 \downarrow\uparrow K_4} \sim$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right] \quad \underbrace{}_{I} \quad P$$

b)

$$H = \left[P^T \mid I_{n-k}\right]$$

$$H = \left[\underbrace{1\ 1\ 0}_{P^T} \mid \underbrace{1}_{I_{n-k}}\right]$$

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{array}{r} 1001 \ R_1 \\ +\ 1001 \ R_1 \\ \hline 0000 \end{array} \qquad \begin{array}{r} 1001 \ R_1 \\ +\ 0101 \ R_3 \\ \hline 1100 \end{array} \qquad \begin{array}{r} 1001 \ R_1 \\ +\ 0010 \ R_3 \\ \hline 1011 \end{array} \qquad \begin{array}{r} 0101 \\ +\ 0010 \\ \hline 0111 \end{array}$$

$$C = \{0000, 1001, 0101, 0010, 1100, 1011, 0111\}$$

b) a + C

$0000 + C = \{000, 1001, 0101, 0010, 1100, 1011, 0111\}$

$1000 + C = \{1000, 0001, 1101, 1010, 0100, 0011, 1111\}$

$0100 + C = \{0100, 1101, 0001, 0110, 1000, 111; 0011\}$

$0010 + C = \{0010, 1011, 0111, 0000, 1110, 1001, 0101\}$

0000	1001	0101	0010	1100	1011	0111
1000	0001	1101	1010	0100	0011	1111
0100	1101	0001	0110	1000	1111	0011
0010	1011	0111	0000	1110	1001	0101

d)

$H = [1101]$

$$H^T = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

$$[0000] \cdot \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} = [0] \qquad [0100] \cdot \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} = [1]$$

$$[1000] \cdot \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} = [1] \qquad [0010] \cdot \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} = [0]$$

$r = 1111$

$1+1+0+1$

$$s(1111) = [1111] \cdot \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} = [1]$$

$r - e = 1111 - 1000 = 0111$

$r - e = 1111 - 0100 = 1011$

c) $C = \{0000, 101, 0101, 0010, 1100, 1011, 0111\}$

$$[\quad]_{1\times4} \cdot G_{4\times5} = [\quad]_{1\times5}$$

$$G = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}_{4\times5}$$

$\vec{a}\,[10101]$ $\vec{b}\,[01011]$

$\vec{c}\,[00100]$ $\vec{d}\,[00010]$

$$\alpha \cdot \vec{a} + \beta \cdot \vec{b} \cdot \gamma + \vec{c} + \delta \cdot \vec{d} = 0$$

$$\alpha[10101] + \beta \cdot [01011] + \gamma \cdot [00100] + \delta \cdot [00010] = 0$$

$$\begin{cases} 1\cdot\alpha + 0\beta + 0\gamma + 0\cdot\delta = 0 \\ 0\cdot\alpha + 1\beta + 0\gamma + 0\cdot\delta = 0 \\ 1\cdot\alpha + 0\beta + 1\gamma + 0\cdot\delta = 0 \\ 0\cdot\alpha + 1\beta + 0\gamma + 0\cdot\delta = 0 \\ 1\cdot\alpha + 1\beta + 0\gamma + 0\cdot\delta = 0 \end{cases} \qquad \begin{cases} \alpha = 0 \\ \beta = 0 \\ \alpha + \gamma = 0 \rightarrow \gamma = 0 \quad \text{L.P.V} \\ \beta + \gamma = 0 \rightarrow \delta = 0 \end{cases}$$

$$[0000]_{1\times4} \cdot \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}_{4\times5} = [00000]_{1\times5}$$

$$[1001]_{1\times4} \cdot [- \ | \ | \ -]_{4\times5} = [10111]_{1\times5}$$

$$[0101]_{1\times4} [- \ | \ | \ -]_{4\times5} = [01001]_{1\times5}$$

$$[1100]_{1\times4} \cdot [- \ | \ | \ -]_{4\times5} = [11110]_{1\times5}$$

$$[0\ 1\ 1\ 1]_{1\times 4} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} = [0\ 1\ 1\ 0\ 1]_{4\times 5}$$

$$C = \{0000,\ \underset{x^0\,x^1\,x^2\,x^3\,x^4}{10111},\ 01001,\ 00100,\ 11110,\ 10011,\ 01101\}$$

$$C' = \{0,\ 1+x^2+x^3+x^4,\ x+x^4,\ x^2,\ 1+x+x^2+x^3,\ 1+x^3+x^4,\ x+x^2+x^4\}$$

(3) $r = 3$

$r = [11\ 10] \to 4$

$K = 2^r - 1 - r$

$4 = 2^r - 1 - r$

$2^r = 4 + 1 + r$

$2^r = 5 + 3$

$2^r = 8$

$\boxed{r = 3}$ $\{000,\ 001,\ 010,\ 0100,\ 011,\ 110,\ 101,\ 111\}$

$4 + 3 = 7$

1	1	1	$a_4 = 1$	0	$r_2 = 0$	$r_1 = 0$
P_6	P_5	P_4	P_3	P_2	P_1	

$r_1: P_3 \; D_5 \; D_7$ $\quad 1+3+5+7=16$

$r_1: 0 \; 1 \; 1$ $\quad 0+1+1=2$

$r_1 = 0$

$r_2: D_3 \; D_6 \; D_7$ $\quad 2+3+6+7=18$

$r_2: 0 \; 1 \; 1$ $\quad 0+1+1=2$

$r_2 = 0$

$r_4: D_5 \; D_6 \; D_7$ $\quad 4+5+6+7=22$

$r_4 \; 1 \; 1 \; 1$ $\quad 1+1+1=3$

$r_4 = 1$

1111000

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(4) $r=3$

$\{000, 100, 010, 001, 110, 011, 101, 111\}$

$m = [0011] \Rightarrow 4$

$4 + 3 = 7$

$K = 2^r - 1 - r$

$4 = 2^r - 1 - r$

$-2^r = -4 - 1 - r \quad / \cdot (-1)$

$2^r = 4 + 1 + r$

$2^r = 8$

$r = 3$

0	0	1	$r_4=1$	1	$r_2=1$	$r_1=0$
P_7	P_6	P_5	P_4	P_3	P_2	P_1

$r_1: P_3 \; D_5 \; P_7$ $\quad 1+3+5+7=16$

$r_1: 1 \; 1 \; 0$ $\quad 1+1+0=2$

$r_1 = 0$

$r_2: D_3 \; D_6 \; D_7$ $\quad 2+3+6+7=18$

$r_2: 1 \; 0 \; 0$ $\quad 1+0+0=1$

$r_2 = 1$

$r_4 = D_5 \; D_6 \; D_7$ $\quad 4+5+6+7=22$

$r_4: 1 \; 0 \; 0$ $\quad 1+0+0=1$

$r=1$

6 $r = [0\ 0\ 1\ 1\ 1\ 0\ 1]$

$\quad\ D_7\ D_6\ D_5\ r_4\ D_3\ r_2\ r_1$

$r_1 = 1 \quad D_3\ D_5\ D_7 \qquad 1 + 3 + 5 + 7 = 16$

$r_1 = 1 \quad 1\ \ 1\ \ 0 \qquad 1 + 1 + 1 + 0 = 3$ Tek Galim

$r_2 = 0 \quad D_3\ D_6\ D_7 \qquad 2 + 3 + 6 + 7 = 18$

$0 \quad 1\ \ 0\ \ 0 \qquad 0 + 1 + 0 + 0 = 1$ Tek Galim

$r_4 = 1 \quad D_5\ D_6\ D_7 \qquad 4 + 5 + 6 + 7 = 22$

$1 \quad 1\ \ 0\ \ 0 \qquad 1 + 1 + 0 + 0 = 2$ çift

$D_3 = 1 \rightarrow D_5 = 0 \qquad m = [0\ 0\ 1\ 1\ \cancel{0}\ 1]$

$m = [0\ 0\ 0\ 1]$

(2)

$$\begin{array}{ccccc} m = [& 0 & 1 & 1 & 1] \\ & x^0 & x^1 & x^2 & x^3 \end{array}$$

$g(x) = x^4 + x + 1$

$m(x) = x + x^2 + x^3$

$x^3 \cdot m(x) = x^3 \cdot (x^3 + x^2 + x) = x^6 + x^5 + x^4$

$$\begin{array}{l} x^6 + x^5 + x^4 : x^4 + x + 1 = x^2 + x + 1 \\ \underline{x^6 + x^3 + x^2} \\ \quad x^5 + x^4 + x^3 + x^2 \\ \quad \underline{x^5 + x^2 + x} \\ \quad\quad x^4 + x^3 + x \\ \quad\quad \underline{x^4 + x + 1} \\ \quad\quad\quad x^3 + 1 = r(x) \end{array}$$

$C(x) = x^3 \cdot m(x) + P(x)$

$= x^3 \cdot (x + x^2 + x^3) + x^3 + 1$

$= x^4 + x^5 + x^6 + x^3 + 1$

$= x^6 + x^5 + x^4 + x^3 + 1$

$= c(x)$

x^6	x^5	x^4	x^3	x^2	x^1	x^0
1	1	1	1	0	0	1

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$$v = \begin{array}{c} \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix} \\ \begin{matrix} x^8 & x^7 & x^6 & x^5 & x^4 & x^3 & x^2 & x^1 & x^0 \end{matrix} \end{array}$$

$$g(x) = x^5 + x^1 + 1 + 1$$

$$\cancel{x^8} + x^7 + x^6 + \cancel{x^3} + x^2 + 1 : x^5 + x^2 + x + 1 = x^3 + x^2 + x + 1$$

$$\cancel{x^8} + x^5 + x^4 + \cancel{x^3} \quad \downarrow$$

$$\cancel{x^7} + x^6 + x^5 + \cancel{x^4} + \cancel{x^2}$$

$$\cancel{x^7} + \cancel{x^4} + x^3 + \cancel{x^2}$$

$$\cancel{x^6} + x^5 + \cancel{x^3} + 1$$

$$\cancel{x^6} + \cancel{x^3} + x^2 + x$$

$$x^5 + x^2 + x + 1$$

$$x^5 + x^2 + x + 1$$

$$00$$