Advanced Topics

Labor and Finance

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Unemployment

- Main motivation for search and matching models
  $\rightarrow$ There are always people who want to work but cannot find jobs (involuntary unemployment)
- The big question: why don't wages adjust so that the market clears (like in other markets)?
  $\rightarrow$ We look for a model that explains this fact with rational individuals
- In a search and matching model, a worker will receive random job offers from the firms. If the offer is higher than the worker's reservation wage, the offer is accepted.

Model-job seeking worker's problem

- Every period, the worker receives a job offer w from a distribution $F(w)$. If she accepts, she will keep the offer forever. If she rejects, she gets unemployment insurance b.
- Worker maximize $\mathbb{E}\sum_{t=0}^{\infty}\beta^t x_t$, where x is w or b
  $\rightarrow$ Value of accepting the offer $W(w) = w + \beta W(w) \Rightarrow W(w) = w/(1-\beta)$.
  $\rightarrow$ Value of rejecting the offer $U = b + \beta\int_0^{\infty} max\{U, W(w)\}dF(w)$
- There is a wage level, w_R, that makes the worker indifferent between accepting and rejecting the offer (reservation wage): $W(w_R) = U$
  $\rightarrow \frac{w_R}{1-\beta} = b + \frac{\beta}{1-\beta}\int_0^{\infty} max\{w_R, w\}dF(w) \Rightarrow w_R = b + \frac{\beta}{1-\beta}\int_{w_R}^{\infty}(w - w_R)dF(w)$
  $\rightarrow$ w_R has to be higher than b. The better the possible wage offer, the higher w_R. High discount factor reduces w_R.

Model-job seeking worker's problem

- The same specification in continuous time (r is discount parameter, α hazard rate or probability of a new job offer)
  $\rightarrow rW(w) = w$
  $\rightarrow U = b + \alpha \int_0^\infty max\{0, W(w) - U\} dF(w)$
  $\rightarrow w_R = b + \frac{\alpha}{r} \int_{w_R}^\infty (w - w_R) dF(w)$
- Let's introduce labor turnover
  $\rightarrow$ A worker may lose her job with probability λ. Then,
  $\rightarrow rW(w) = w + \lambda(U - W(w))$
- The value of unemployment is the same. Also, the argument for reservation does not change. Therefore, we have
  $\rightarrow w_R = b + \frac{\alpha}{r+\lambda} \int_{w_R}^\infty (w - w_R) dF(w)$
  $\rightarrow$ Reservation wage decreases as λ increases: higher job loss risk makes the worker less picky!

Diamond Mortensen Pissarides Model

- Employees and employers search each other to find a match. Since search is time-consuming, we have unemployment
  $\rightarrow$ Note that the labor market is a two-sided market. Many markets in finance are also two-sided
- Notation$\rightarrow$ E: employed, U: unemployed, V: vacant jobs, F: filled jobs, C: cost of maintaining a job
- An employed worker produces A ($A > C$) and earns wage w. Worker's utility is w if employed, 0 otherwise. Firm's profit is $A - w - C$ for a filled job and $-C$ for a vacant job

Diamond Mortensen Pissarides Model

Matching function

- $M = M(U, V) = KU^{\beta}V^{\gamma}$
  $\rightarrow$ This function combines searching decisions of the firms and workers together, without modeling them explicitly
  $\rightarrow$ The more unemployed workers and the more vacant jobs, the easier it is to find a match
  $\rightarrow$ If $\beta + \gamma < 1$, there is crowding effect, vice versa
- We assume exogenous job destruction with the ratio b
- Then, the dynamics of the number of employed workers: $\dot{E} = M(U, V) - bE$
- We can easily define two other ratios
  $\rightarrow$ The rate unemployed workers find jobs $a = \frac{M(U,V)}{U}$
  $\rightarrow$ The rate vacant jobs are filled $\alpha = \frac{M(U,V)}{V}$

Diamond Mortensen Pissarides Model

Value functions

1. Value of being employed $rV_E = w - b(V_E - V_U)$
2. Value of being unemployed $rV_U = a(V_E - V_U)$
3. Value of vacant job $rV_V = -C + \alpha(V_F - V_V)$
4. Value of filled job $rV_F = A - w - C - b(V_F - V_V)$

We assume that workers and firms have the same bargaining power,leading to the same rents
$\rightarrow V_e - V_U = V_F - V_V$

Diamond Mortensen Pissarides Model

Solve for w

- $V_E - V_U = \frac{w}{a+b+r}$
- $V_F - V_V = \frac{A-w}{a+b+r}$
- In the equilibrium, the two equations above must be equal to each other, if workers and firms have the same bargaining power. Hence,
  $\rightarrow w = \frac{(a+b+r)A}{a++\alpha+2b+2r}$
  $\rightarrow$ Wage depends on a and α. Their relative size to each other determines which side gets a larger share.

Diamond Mortensen Pissarides Model

- On steady state, E is constant (the economy has a constant unemployment rate).
  $\rightarrow$ Then, $M(U, V) = bE$
  $\rightarrow$ We can use this to find a and α (Recall $a = \frac{M(U,V)}{U}$ and $\alpha = \frac{M(U,V)}{V}$)
- $a = \frac{bE}{U} = \frac{bE}{\bar{L}-E}$
- We need to express V in terms of E to express α in E
- $KU^{\beta}V^{\gamma} = bE \Rightarrow V = (\frac{bE}{K(\bar{L}-E)^{\beta}})^{\frac{1}{\gamma}} \Rightarrow \alpha = K^{\frac{1}{\gamma}}(bE)^{\frac{\gamma-1}{\gamma}}(\bar{L} - E)^{\frac{\beta}{\gamma}}$
- a is increasing in E since the more people are employed, the smaller is the number of people competing for the new job vacancies and the easier it for an unemployed worker to find a job
- α is decreasing in E since the same argument makes it harder for a firm to fill a vacancy

Diamond Mortensen Pissarides Model

- Notice that firms will create vacancies until doing so generates positive value. Thus, in equilibrium, the value of a vacant job is zero.
- $rV_V = -C + \alpha\frac{A-w}{a+b+r} = -C + \frac{\alpha(E)}{a(E)+\alpha(E)+2b+2r}A = 0$
  $\rightarrow$ V_V is decreasing in E
  $\rightarrow$ When E is zero, the firm gets all the surplus and the value of vacancy is $A - C$. When the economy is at full employment ($E = \bar{L}$), it is impossible to fill the vacancy and the value of vacancy is $-C$

Diamond Mortensen Pissarides Model

What happens when there is a negative demand shock (a drop in A)?

1. Vacancy has a lower value. Thus, firms create fewer vacancies, increasing unemployment.
   → Note the effect via the matching function: When the unemployment rate is high, the matching rate is low. This discourages firms from posting new vacancies because these vacancies are likely to be unfilled for a long time.
   → Lower vacancies discourage people from searching for a job, which discourages firms from opening vacancies.
2. Wage also decreases, but less than one-to-one.
   → Lower employment also affects the value of vacancies in a positive way.

- This unemployment type is called frictional unemployment: it occurs because it takes time to match heterogeneous workers to heterogeneous jobs. Can this mechanism explain long-term unemployment?

Diamond Mortensen Pissarides Model

Efficiency wages

- Main idea: Worker's productivity depends on effort and the effort depends on wage. Therefore, a firm may prefer a higher wage than the market equilibrium wage.
  $\rightarrow$ This can generate long-term unemployment
- Firm's profit $\pi = Y - wL$, where $Y = F(eL)$ and $e = e(w)$
  $\rightarrow$ Effort is increasing in wage
- Firm's problem $Max_{L,w}\ F(e(w)L) - wL$
- $\frac{\partial \pi}{\partial L} = F'e - w = 0$, $\frac{\partial \pi}{\partial w} = F'Le'(w) - L = 0$
  $\rightarrow \frac{we'(w)}{e(w)} = 1 \Rightarrow$ This is called efficiency wage condition: The elasticity of effort wrt wage is 1
- If the firm's optimal labor demand is higher than labor supply, firm does not reduce the wage, leading to unemployment.

Diamond Mortensen Pissarides Model

Wages and effort

- Why do firms want to pay higher wages?
  $\rightarrow$ Consider the following moral hazard problem: your employer wants to pay you according to your effort, but she cannot observe your effort. She can only observe your production. In this case, efficient wages can solve this problem.
- Introduce effort to worker's utility: $u_t : w_t - e_t$ if employed, 0 otherwise. Assume that effort can take two values $\bar{e}$, or 0. Thus, the worker has three states: employed and exerting effort, employed and shirking, and unemployed.
- Again, b is prob. of losing the job, a is prob. of finding a new job, and q is prob. being caught when shirking
- We have three value functions for the worker for each state
  $\rightarrow$ Effort: $\rho V_E = (w - \bar{e}) + b(V_U - V_E)$
  $\rightarrow$ Shirking: $\rho V_S = w + (b + q)(V_U - V_S)$
  $\rightarrow$ Unemployed: $\rho V_U = a(V_E - V_U)$

Diamond Mortensen Pissarides Model

- Firm's problem $\pi_t = F(\bar{e}E_t) - w_t[E_t + S_t]$, where E is of effort exerting workers, S is of shirking workers
- Workers exert effort if $V_S \leq V_E$. Cheapest way is $V_S = V_E$, which implies
  $\rightarrow (w - \bar{e}) + b(V_U - V_E) = w + (b + q)(V_U - V_S)$
  $\Rightarrow V_E - V_U = \frac{\bar{e}}{q} > 0$
- Firms let workers get the rents$\rightarrow$ being employed is strictly better than being unemployed
- We can find this wage by using the value functions and the difference above
  $\rightarrow w = \bar{e} + (\rho + a + b)\frac{\bar{e}}{q}$
  $\rightarrow$ Wage compensates for the effort and adds an extra that depends on the monitoring technology. If monitoring is weak, workers are more likely to shirk. Thus, the firm needs to pay a higher wage to avoid shirking.

Diamond Mortensen Pissarides Model

- In equilibrium, again, the unemployment rate is constant
  $\rightarrow$ Number of workers losing jobs must equal the number of workers finding a new job
  $\rightarrow$ Job finding rate a is exogenous to workers but determined in equilibrium
- N: of firms, L: working workers, $\bar{L}$: total labor supply
  $\rightarrow a(\bar{L} - NL) = bNL \Rightarrow a + b = \frac{b\bar{L}}{\bar{L} - NL}$
  $\rightarrow$ LHS: Number of unemployed workers times job finding probability
  $\rightarrow$ RHS: Number of employed workers who lose their jobs
  $\rightarrow w = \bar{e} + (\rho + \frac{b\bar{L}}{\bar{L} - NL})\frac{\bar{e}}{q}$
  $\rightarrow$ All workers exert effort with this wage. Note that the higher the unemployment rate, the lower the wage and the higher the monitoring tech, the lower the wage.

- L in equilibrium comes from firm's maximization problem
  $\rightarrow$ $Max_{L,w}\ F(e(w)L) - wL$
  $\rightarrow$ FOC wrt L: $\bar{e}F'(\bar{e}L) = w$
- Since firms optimally leave rents to workers, wage will not go down to market-clearing levels, creating unemployed workers.
- In this case, this unemployment is inefficient since marginal product of labor exceeds the marginal cost of effort. The best allocation is everybody to be employed and to exert effort. Yet, due to informational problems, this cannot be implemented.

Diamond Mortensen Pissarides Model

- Unionisation is an important aspect of labor economics. It has been decreasing in the US, but still highly relevant in Europe.
- Unionisation gives a higher bargaining power to the workers who are a part of the union, increasing the wages. Yet, the outside workers cannot bargain for a lower wage, hence are left unemployed.
  $\rightarrow$ The unemployment rate is higher in Europe, and a popular scapegoat is unionisation (or stricter labor market protection that drive the wages higher).

Example: Donaldson, Piacentino, Thakor

Household Debt Overhang and Unemployment

JASON RODERICK DONALDSON, GIORGIA PIACENTINO, and ANJAN THAKOR*

ABSTRACT

We use a labor-search model to explain why the worst employment slumps often follow expansions of household debt. We find that households protected by limited liability suffer from a household-debt-overhang problem that leads them to require high wages to work. Firms respond by posting high wages but few vacancies. This *vacancy posting effect* implies that high household debt leads to high unemployment. Even though households borrow from banks via bilaterally optimal contracts, the equilibrium level of household debt is inefficiently high due to a *household-debt externality*. We analyze the role that a financial regulator can play in mitigating this externality.

Donaldson, Piacentino, Thakor: Model

- Three-agent, two-period model
- Households: Consumes at t=1, and needs liquidity at=0. Thus, it has to borrow at t=0. At=1, it matches with a firm and produces. Working is costly: c.
- Firms: At t=1, firm posts a vacancy with a cost k. If a match occurs, firm produces y, $y > c + k$.
- Banks: It lends to HH at t=0.

Donaldson, Piacentino, Thakor: Model

Labor market: one-shot random search model

- The number of HH is fixed. The number of firms is determined by endogenous entry (job posting).
- q: The ratio of searching HHs to firms posting vacancies
  $\rightarrow$ It determines the probability that HHs and firms are matched
  $\rightarrow$ The higher the q, the harder it is for a HH to be matched with a firm and the easier for a firm to be matched with a HH.
- $\alpha(q)$: Each HH is matched with a firm with this probability. $q\alpha(q)$: each firm is matched with a HH with this probability.
  $\rightarrow \alpha(q) : \frac{a}{\sqrt{q}} \Rightarrow \alpha'(q) < 0$, and $(q\alpha(q))' > 0$

Contracts

- After a match, the firm and HH negotiate over a wage, w. The labor contract determines how the surplus will be split between these two.
  $\rightarrow$ A random-proposer bargaining protocol: with 0.5 probability, the firm makes a take it or leave it to offer to HH, and with 0.5 probability, vice versa.
- Debt contract: HH borrows B from a bank at t=0, and makes a repayment, $R(w)$, if it receives w at t=1. HH is protected by limited liability: $R(w) \leq w$
- At t=0, HH negotiates a debt contract with the Bank. At t=1, the firm posts vacancies, and the firm and HH may match. Then, they negotiate over wages and HH makes a decision to work or not.
- We look for a debt contract, a labor contract given R, HH's decision to work given w and R, firm's decision to entry given q.

Donaldson, Piacentino, Thakor: Results

- Either firm or HH proposes a wage
  $\rightarrow$ Firm proposes $max\ y - w_f$ s. t. $w_h - R(w_f) - c \geq 0$ (HH's participation constraint)
  $\rightarrow$ HH proposes $max\ w_h - R(w_h) - c$ s. t. $y - w_h \geq 0$ (firm's participation constraint)
- What about the debt contract? At t=0, HH and the bank know that HH will be employed by α probability. The wage will be w_f with prob. 0.5, and w_h with prob. 0.5.
  $\rightarrow$ Note that the agents are small $\Rightarrow$ They don't consider the effect of their actions on employment

Donaldson, Piacentino, Thakor: Results

- Debt contract R solves
  $\max \alpha\left(\frac{1}{2}(w_f - R(w_f) - c) + \frac{1}{2}(w_h - R(w_h) - c)\right)$ s.t.
  $w_f \in argmax\{y - w | w - R(w) - c \geq 0\}$
  $w_h \in argmax\{w - R(w) - c | y - w \geq 0\}$
  $\alpha\mathbb{E}[R(w)] \geq B$ (Banks should break even)
  $R(w) \leq w$ and weakly increasing (HH has limited liability)
- $R(w) = min\{B/\alpha, w\}$ solves this problem where the debt's face value is F, $F = B/\alpha$
- Intuition: HH knows that it will not have any surplus when the firm proposes the wage, w_f. If HH lowers $R(w_f)$, the difference will be captured by the firm. Thus, HH wants to set $R(w_f)$ as high as possible to satisfy the bank's participation constraint.
  $\rightarrow$ HH captures the surplus only when it proposes the wage, w_h. To increase its surplus, HH offers the lowest possible $R(w_h)$.
  $\rightarrow$ Optimal contract that minimizes $R(w_h)$ and maximizes $R(w_f)$ is a flat contract since $R(w)$ is monotonic, which is a debt contract.

Donaldson, Piacentino, Thakor: Results

- Equilibrium wages are $w_f = F + c$, $w_h = y$
  $\rightarrow \bar{w} := \mathbb{E} = \frac{y+F+c}{2}$
  $\rightarrow$ The expected wage is increasing in F! The more indebted HH is, the more of its wage goes to the bank, and the more the firm has to compensate HH for working.

Vacancy posting

- q: queue length, $\alpha(q)$: employment rate, $q\alpha(q)$: prob. that firms match wit HH, k: vacancy posting cost
- Expected payoff from posting vacancy: $q\alpha(q)(y - \bar{w})$.
  $\rightarrow$ Firm posts vacancy if $q\alpha(q)(y - \bar{w}) \geq k$
- We already know that $\bar{w} = \frac{y+F+c}{2}$ and $\alpha(q) : \frac{a}{\sqrt{q}}$
  $\rightarrow q = \left(\frac{2k}{a(-F-c)}\right)^2$
  $\rightarrow \alpha(q) = \frac{a^2(y-F-c)}{2k}$
- Firms post fewer vacancies when HH debt is high, leading to lower employment!

Donaldson, Piacentino, Thakor: Results

Equilibrium

- So far we define F in terms of α and α in terms of F
- We can find the equilibrium by using $F(\alpha(F)) = F$
  $\rightarrow$ Bank's break even condition $\alpha F = B$
  $\rightarrow$ Plug in α
- $B = \frac{a^2}{2k}(y - F - c)F$
  $\rightarrow$ Two equilibria: Bank's belief about future employment is self-fulfilling. When bank believes that employment will be high, it demands a low face value of debt and employment becomes high, vice versa.
  $\rightarrow$ Externality: Banks take α given. Yet, lending decreases employment, which increases the default rate of all loans. Banks do not consider the negative effect of their lending on other banks and HHs via labor market.