Pricing of risk in credit and equity index options
A role for option order flow?

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Agenda

Motivation

Credit indexes and options

Option risk factors and factor-mimicking portfolios

Returns on options and factors

Performance of equity-based factor pricing models

Credit option residual factor and order flow

Pricing of Credit and Equity Claims

- Structural model of default risk views bonds (CDSs) and equity as contingent claims on the **same** underlying firm value (Black-Scholes-Merton).
- **First-generation** structural models **underpredict level** of credit spreads when calibrated to low historical default rates:

$\rightarrow$ The **credit spread puzzle**
(Jones, Mason, and Rosenfeld (1984), Huang and Huang (2003))

- **Second-generation** structural models calibrated to match equity risk premia and equity option implied volatilities **match fairly well level** of credit spreads
(Cremers, Driessen, and Maenhout (2008), Chen, Collin-Dufresne, and Goldstein (2009), Du, Elkamhi, and Ericsson (2019))

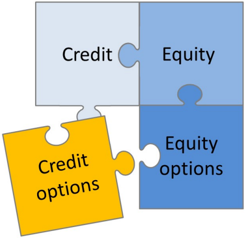
$\rightarrow$ "a good deal of **integration** between corporate bond and options markets" (Culp, Nozawa, and Veronesi (2018)).

Credit and equity returns

- **Mixed evidence** when looking at corporate bond **returns**:
  - Common factors in credit spread changes unexplained by standard structural model factors
    (Collin-Dufresne, Goldstein, and Martin (2001), He, Khorrami, and Song (2020))
  - Equity factor bond betas do not explain cross-section of bond returns
    (Fama and French (1993), Choi and Kim (2018), Dickerson, Mueller, Robotti (2023))
  - CDS and bond returns seem integrated with equity returns
    (Ericsson, Jacobs, and Oviedo (2009), Koijen, Lustig, and Van Nieuwerburgh (2017))

Credit Volatility

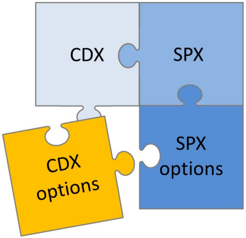
- Since 2012 very active market in options on credit indexes
- Recent papers focus on the relative pricing of credit and equity index options:



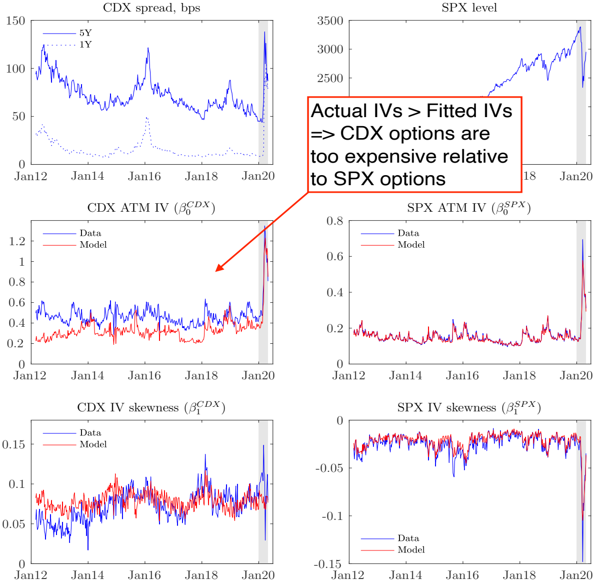
→ C-D, Junge, Trolle (2024) and Doshi, Ericsson, Fournier, Seo (2024) reach different conclusion.

CDJT (2024)

- Propose structural (Stochastic Volatility Jump Diffusion) model for consistent pricing of credit and equity index options
- Calibrate parameters to
  - CDX term structure
  - SPX level
  - SPX implied volatility surface
- ...and then price CDX options out-of-sample



CDJT (2024)


CDX spread, bps
150
100
50
0
5Y
1Y
Jan12
Jan14
Jan16
Jan18
Jan20
SPX level
3500
3000
2500
18
Jan20
Actual IVs > Fitted IVs
=> CDX options are
too expensive relative
to SPX options
CDX ATM IV (β_0^{CDX})
1.2
1
0.8
0.6
0.4
0.2
0
Data
Model
Jan12
Jan14
Jan16
Jan18
Jan20
SPX ATM IV (β_0^{SPX})
0.8
0.6
0.4
0.2
0
Data
Model
Jan12
Jan14
Jan16
Jan18
Jan20
CDX IV skewness (β_1^{CDX})
0.15
0.1
0.05
0
Data
Model
Jan12
Jan14
Jan16
Jan18
Jan20
SPX IV skewness (β_1^{SPX})
0
-0.05
-0.1
-0.15
Data
Model
Jan12
Jan14
Jan16
Jan18
Jan20

DEFS (2024)

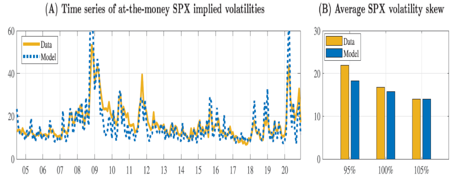


Fig. 4. SPX implied volatilities. The figure compares the Black-Scholes-implied volatilities for SPX options in the data and in the model. Panel A plots the time series of at-the-money SPX implied volatilities in the data (yellow solid line) and in the model (blue dotted line). Panel B presents the average SPX implied volatilities with moneyness of 95, 100, and 105% in the data (yellow bars) and in the model (blue bars). The data frequency is monthly, where the data and model values are sampled at the end of each month. The implied volatilities are expressed in percentages. The sample period is from June 2004 to November 2020.

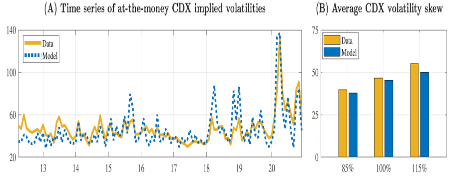
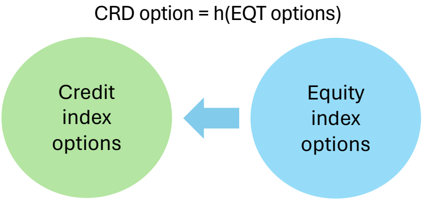


Fig. 6. CDX implied volatilities. The figure compares the Black-implied volatilities for CDX options in the data and in the model. Panel A plots the time series of at-the-money CDX implied volatilities in the data (yellow solid line) and in the model (blue dotted line). Panel B presents the average CDX implied volatilities with moneyness of 85, 100, and 115% in the data (yellow bars) and in the model (blue bars). The data frequency is monthly, where the data and model values are sampled at the end of each month. The implied volatilities are expressed in percentages. The sample period is from March 2012 to November 2020.

- Differences in models, calibration, and data. How to reconcile?

This paper

- Model-free empirical analysis
- Focus on tradable portfolio returns and risk premia instead of derivative pricing
- More Data: IG and HY, U.S. and Europe



Q? Can equity-based factor model price credit index options?

A! No!

Q? Can credit option order-flow risk help explain credit residual and alpha?

A! Yes!

Related literature

- Determinants of credit spread changes:
  Collin-Dufresne, Goldstein, and Martin (2001), Hull, Nelken, and White (2004), Ericsson, Jacobs, and Oviedo (2009), Cremers, Driessen, Maenhout, and Weinbaum (2008)

- Factors in corporate bond and equity returns:
  Fama and French (1993), Israel, Palhares, and Richardson (2018), Ericsson, Jacobs, and Oviedo (2009), Koijen, Lustig, and Van Nieuwerburgh (2017), Lewis (2019)), Dickerson, Mueller, Robotti (2023)

- Factor Models for option returns:
  Jones (2006), Buchner and Kelly (2022), Horenstein, Vasquez, and Xiao (2023), and Fournier, Jacobs, and Orlowski (2024).

Related literature

- Bond and equity market segmentation:
  Kapadia and Pu (2012), Bao and Pan (2013), Driessen and Van Zundert (2017)
- Demand based option pricing:
  Bollen and Whaley (2004) and Gârleanu, Pedersen, Poteshman (2009), Chen, Joslin, Ni (2019), Dew-Becker and Giglio (2023)
- The relative pricing of tranche swaps and SPX options:
  Coval, Jurek, and Stafford (2009), Collin-Dufresne, Goldstein, and Yang (2012), Seo and Wachter (2018)
- The out-of-the-money put premium:
  Bates (2001), Pan (2002)
- CDX swaption pricing and credit volatility risk-premium:
  White (2014), Chen, Doshi, Seo (2020), Amman, Moerke (2019)

Agenda

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Agenda

Index CDSs

- Index CDS: Default protection on a portfolio of reference entities
- North America:
  - CDX.IG: Default protection on 125 IG names
  - CDX.HY: Default protection on 100 HY names
- Europe:
  - iTraxx Europe: Default protection on 125 IG names
  - iTraxx Xover: Default protection on 75 HY names
- Maturities up to 10Y, with 5Y the most liquid
- Every 6 months (in March and September), a new series of an index is launched
  - Set of index constituents revised according to rating and liquidity criteria
  - On-the-run index most liquid

Interpretation: Corporate bond index

- Highly standardized in terms of payment dates and coupons
  - 100 bps for IG index
  - 500 bps for HY index
- Upfront cost, U_t, when entering into index CDS contract
- Price quotation:

$$S_t = 1 - U_t$$

- Interpretation: Price of synthetic corporate bond index
  - Buy risk-free floating-rate note with face value of one
  - Sell protection on credit index with notional value of one.
- Value at $t + \Delta$ in case of 1 default:

$$S_{t+\Delta} = \frac{N_t - 1}{N_t} \tilde{S}_{t+\Delta} + \frac{1}{N_t}(1 - \ell),$$

  - N_t is number of index constituents
  - $1 - \ell$ is the recovery rate of the defaulted name
  - $\tilde{S}_{t+\Delta}$ is the price of the refreshed credit index at time $t + \Delta$ without the defaulted name.

Credit index options

- Option to buy/sell index protection at a future date at an agreed upfront cost
- Main features:
  - Underlying: 5Y, on-the-run index
  - European style
  - Expiration on the 3rd Wednesday of the month
  - Wide strike range
  - 1-3 month options most liquid
  - Basis for recently-launched credit VIX:

Credit VIX®: A New Tool for Measuring and Managing Credit Risk

Interpretation: Option on corporate bond index

- Payoff at T of call option (payer swaption) bought at time t with unit notional is

$$\left(\frac{N_T}{N_t} \tilde{U}_T + \underbrace{\frac{1}{N_t} \sum_{i=1}^{N_t - N_T} \ell_i}_{\text{front-end protection}} - K^U \right)^+$$

- Equivalent to put option on corporate bond index

$$\left(K^S - S_T \right)^+$$

  - $K^S = 1 - K^U$ is the strike in price terms
  - S_T is time-T value of the bond index that prevailed at time t

$$S_T = \frac{N_T}{N_t} \tilde{S}_T + \frac{1}{N_t} \sum_{i=1}^{N_t - N_T} (1 - \ell_i).$$

Implied volatility

- Forward price of the credit index

$$F_{t,T} = \bar{\mathbb{E}}_t \left[S_T \right]$$

  - Inferred from put-call parity using the entire cross-section of options.
- Implied volatility assumes that forward price follows a GBM
- Moneyness

$$m = \frac{\log\left(\frac{K^S}{F_{t,T}}\right)}{\sigma^{ATM}\sqrt{T-t}}$$

  - Measures the number of standard deviations that an option is in or out of the money given log-normally distributed prices.

Agenda

P&L attribution

- Option i is held from t to $t + \Delta t$
- Consider bivariate stochastic volatility model

$$(S,\sigma) \text{ or } (S,I),$$

  where I is short-term ATM implied volatility
- Option P&L:

$$\begin{aligned}
\Delta P^i &\approx \frac{\partial P^i}{\partial t}\Delta t + P^i_S \Delta S + P^i_I \Delta I + \frac{1}{2} P^i_{SS} (\Delta S)^2 + \frac{1}{2} P^i_{II} (\Delta I)^2 + P^i_{IS} \Delta S \Delta I \\
&= \frac{\partial P^i}{\partial t}\Delta t + P^i_S S I R^S + P^i_I I R^I + \frac{1}{2} P^i_{SS} S^2 I^2 (R^S)^2 + \frac{1}{2} P^i_{II} I^2 (R^I)^2 + P^i_{IS} S I^2 R^S R^I.
\end{aligned}$$

where $R^S = \frac{1}{I}\frac{\Delta S}{S}$ and $R^I = \frac{\Delta I}{I}$

- Infer excess return on delta-hedged option...

Risk factors

- Dollar excess return

$$\Delta V^i - rV^i\Delta t \approx$$
$$\frac{1}{2}\underbrace{P^i_{SS}S^2I^2}_{\gamma^i}\Big((R^S)^2 - \bar{\mathbb{E}}[(R^S)^2]\Big) + \underbrace{P^i_I I}_{\nu^i}\Big(R^I - \bar{\mathbb{E}}[R^I]\Big)$$
$$- \underbrace{P^i_{IS}SI^2}_{\zeta^i}\Big(-R^S R^I - \bar{\mathbb{E}}[-R^S R^I]\Big) + \frac{1}{2}\underbrace{P^i_{II}I^2}_{\omega^i}\Big((R^I)^2 - \bar{\mathbb{E}}[(R^I)^2]\Big).$$

where $\bar{\mathbb{E}}[\cdot]$ denotes risk-neutral expectation

- Sources of risk driving excess returns are changes in:
  1. realized variance of index return
  2. IV
  3. realized covariation between returns and IV (downside risk)
  4. realized variance of IV (overall tail risk)

- Exposures: gamma (γ^i), vega (ν^i), vanna (ζ^i), volga (ω^i).

Factor-mimicking portfolios

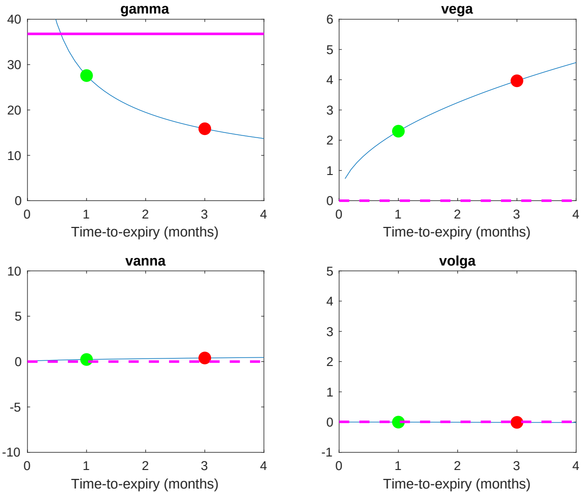
- Develop four-factor model for pricing options.
- Factor mimicking portfolios = Tradable option strategies such that (ideally!):
  - Factor 1: exposure only to realized return variance:

    gamma>0, vega=0, vanna=0, volga=0
  - Factor 2: exposure only to change in IV:

    gamma=0, vega>0, vanna=0, volga=0
  - ...
- Construct strategies using Black-Scholes greeks
  - Robust
  - Easily implementable
  - (Using fully specified SV model introduces model-dependency and complexity)

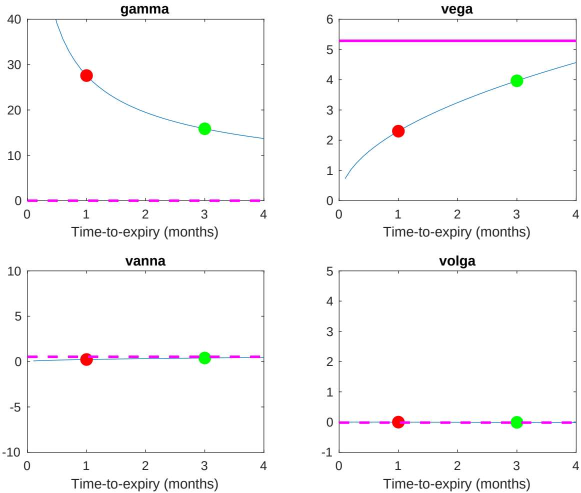
GMA factor

- Long short-maturity straddle
- Short long-maturity straddle (vega-weighted)



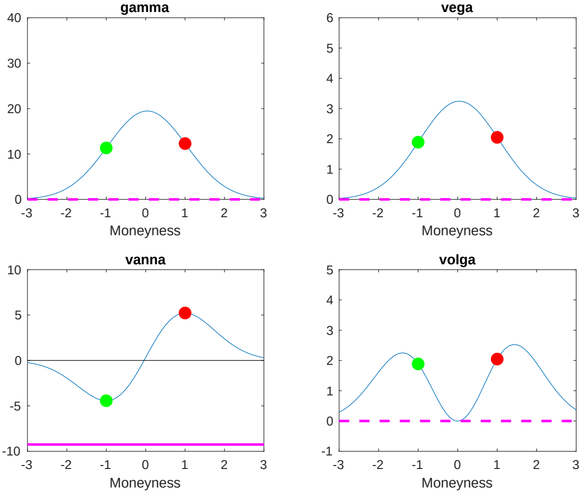
VGA factor

- Long long-maturity straddle
- Short short-maturity straddle (gamma-weighted)



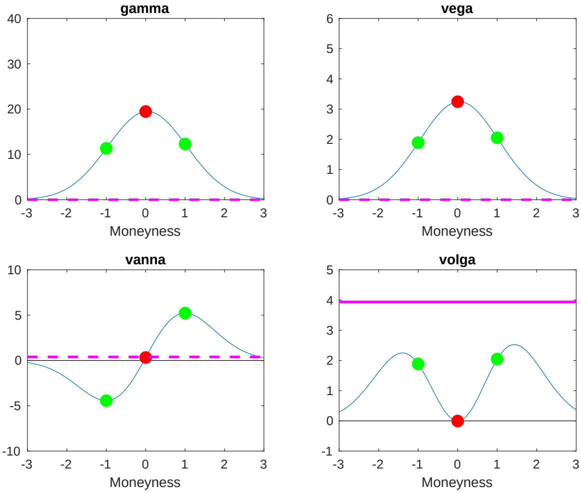
VNA factor

- Long OTM put option
- Short OTM call option (vega-weighted)



VLG factor

- Long OTM put option
- Long OTM call option
- Short ATM straddle (vega-weighted)



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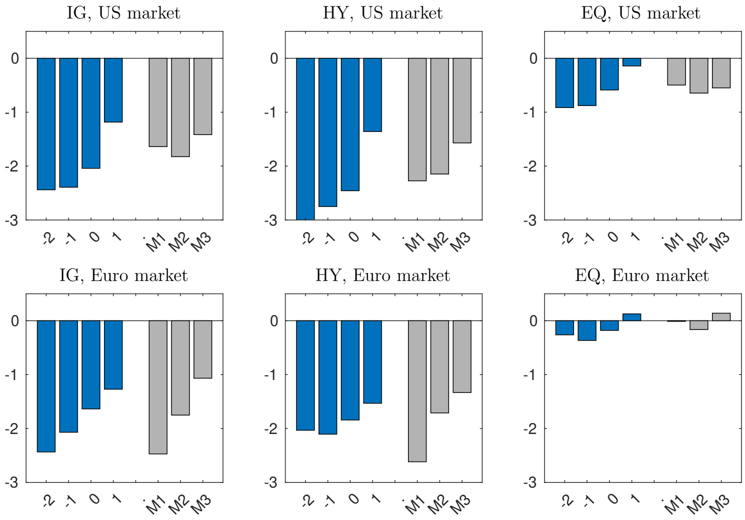
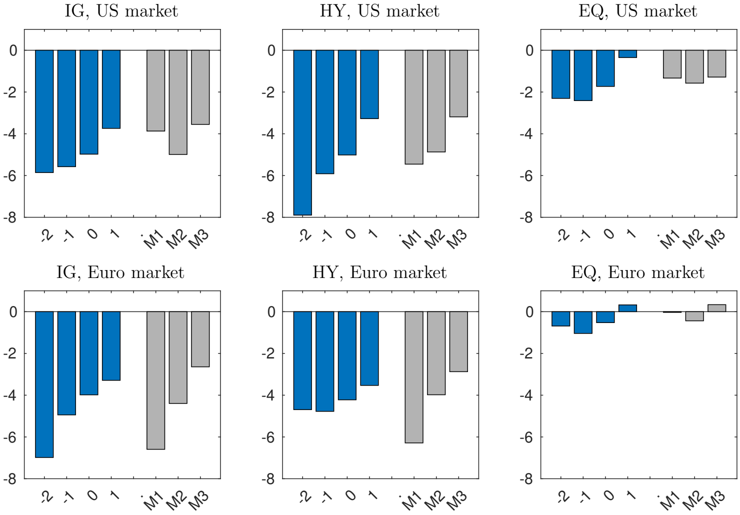
Data

- Credit indexes, credit index options, defaults, and recovery rates are from Markit.
- Equity index options are from CBOE and Eurex
- On each observation date select
  - First three monthly expirations among the options that have more than two weeks to expiration: M1, M2, M3
  - For each maturity, four levels of moneyness, $m = -2, -1, 0, 1$.
- Test assets are portfolios of **delta-hedged options**.
  - Use OTM options (puts for $m < 0$ and calls for $m > 0$). ATM options are puts.
  - For each index, 7 equally-weighted portfolios sorted on moneyness and maturity
- Daily returns
- Sample period is from January 1, 2013 to April 3, 2023

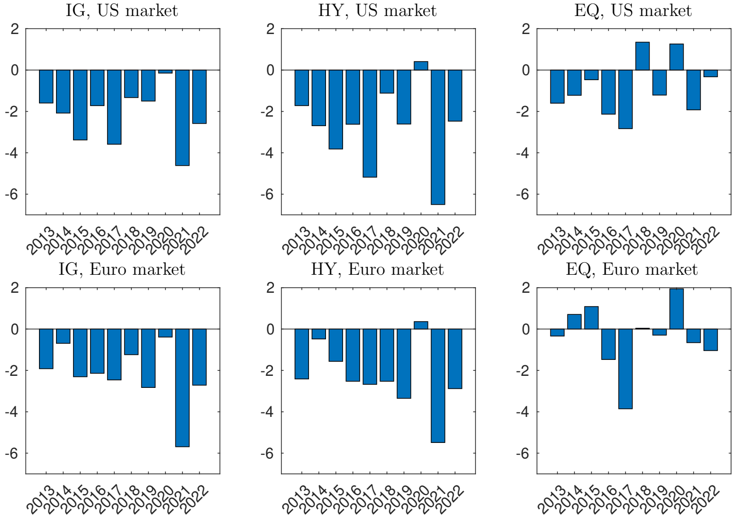
Performance of option portfolios, Sharpe ratios



Performance of option portfolios, *t*-statistics



Average Sharpe ratios on option portfolios per year



Performance of factor-mimicking portfolios

- Three possible option strategies for each factor:
  - Vega and gamma: 1M vs. 2M, 1M vs. 3M, and 2M vs. 3M
  - Vanna and volga: 1M, 2M and 3M
- Form equally-weighted portfolios

	GMA			VGA			VNA			VLG		
	IG	HY	EQ	IG	HY	EQ	IG	HY	EQ	IG	HY	EQ
Panel A: US market												
Mean	**-0.64**	**-0.71**	-0.19	**-0.21**	**-0.32**	-0.11	**-1.35**	**-1.88**	**-0.99**	-0.15	0.14	-0.25
	(-4.77)	(-4.84)	(-1.42)	(-2.93)	(-4.15)	(-1.59)	(-4.80)	(-7.98)	(-3.37)	(-1.00)	(0.98)	(-1.67)
Std.dev.	0.39	0.41	0.45	0.22	0.22	0.19	0.81	0.71	0.89	0.44	0.45	0.38
SR	-1.66	-1.71	-0.42	-0.93	-1.46	-0.58	-1.66	-2.65	-1.11	-0.33	0.31	-0.67
Skew	1.58	3.43	2.97	0.05	0.75	1.17	0.50	-0.39	0.96	-0.43	7.73	2.64
Kurt	10.6	35.1	25.8	11.8	12.9	13.3	10.3	13.3	55.8	9.3	216.5	32.9
N obs	2521	2504	2560	2521	2504	2560	2508	2483	2560	2501	2482	2560
Panel B: European market												
Mean	**-0.60**	**-0.71**	-0.02	-0.12	**-0.14**	-0.03	**-0.84**	**-1.05**	-0.55	-0.16	-0.04	-0.08
	(-4.65)	(-5.79)	(-0.14)	(-1.69)	(-2.10)	(-0.60)	(-2.51)	(-3.36)	(-1.85)	(-1.55)	(-0.41)	(-0.69)
Std.dev.	0.43	0.37	0.43	0.22	0.22	0.16	0.77	0.72	0.81	0.42	0.40	0.27
SR	-1.41	-1.95	-0.04	-0.53	-0.63	-0.21	-1.09	-1.46	-0.67	-0.38	-0.11	-0.28
Skew	5.77	2.11	2.86	-0.42	1.08	0.98	-0.89	0.04	1.59	3.80	0.82	0.61
Kurt	99.4	17.6	19.7	28.2	13.8	10.4	41.5	8.1	43.1	87.0	23.7	15.4
N obs	2521	2461	2547	2521	2461	2547	2510	2459	2547	2510	2445	2547

Performance of long-short factor-mimicking portfolios

- Three long equity - short credit strategies for each factor
- Risk-weighted
- Form equally-weighted portfolios

	GMA		VGA		VNA		VLG	
	EQ-IG	EQ-HY	EQ-IG	EQ-HY	EQ-IG	EQ-HY	EQ-IG	EQ-HY
Panel A: US market								
Mean	**0.64**	**0.74**	0.09	**0.22**	0.51	**1.18**	0.11	-0.29
	(3.89)	(4.85)	(1.45)	(2.96)	(1.57)	(3.41)	(0.46)	(-1.42)
Std.dev.	0.54	0.49	0.26	0.26	1.33	1.14	0.75	0.67
SR	1.19	1.51	0.35	0.84	0.38	1.03	0.14	-0.43
Skew	0.45	1.27	0.09	-0.32	0.08	1.43	1.70	-0.27
Kurt	16.5	18.2	11.8	13.0	29.8	25.0	22.4	76.1
N obs	2521	2504	2521	2504	2508	2483	2501	2482
Panel B: European market								
Mean	**0.75**	**0.81**	0.10	**0.12**	0.26	0.46	0.24	0.19
	(5.17)	(6.49)	(1.60)	(2.12)	(0.68)	(1.26)	(1.35)	(1.13)
Std.dev.	0.54	0.45	0.25	0.24	1.21	1.05	0.70	0.66
SR	1.38	1.79	0.41	0.48	0.22	0.44	0.34	0.28
Skew	-3.05	1.04	0.57	-0.25	1.29	-0.13	-8.24	0.74
Kurt	91.7	10.8	19.3	11.2	46.5	10.3	240.0	58.0
N obs	2494	2436	2494	2436	2486	2433	2486	2419

Agenda

In-sample performance of factor pricing model

- Test assets: 7 option portfolios
- 5 factors: *IDX*, *GMA*, *VGA*, *VNA*, and *VLG*

	IG	HY	EQ
		Panel A: US market	
Average μ	-2.71	-3.31	-1.12
Average t	-5.11	-5.57	-1.57
Max SR, options	2.94	3.28	1.72
Max SR, factors	2.77	3.64	1.37
Max SR, options+factors	3.05	3.95	1.87
GRS p-value	0.03	0.00	0.02
Average $\lvert\alpha\rvert$	0.14	0.27	0.27
Average $\lvert t\rvert$	0.53	0.90	0.95
Average R^2	0.82	0.87	0.93
N obs	2447	2382	2560

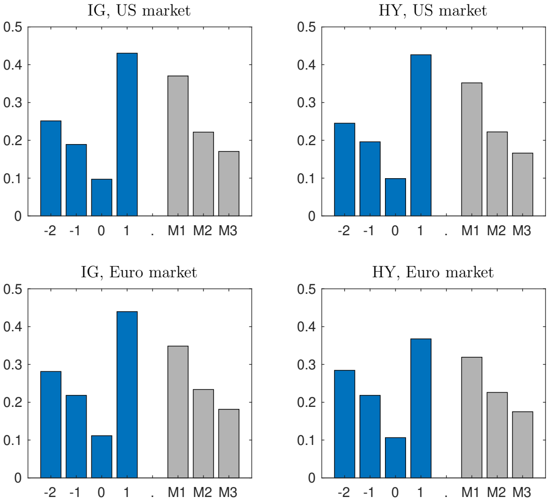
Equity-based model applied to credit

- Can equity-based factor model price the credit options?
- Three model specifications:
  1. Unconditional model (*mdl1*)
  2. Betas function of option greeks (*mdl2*)
  3. Betas function of option greeks and IVs (*mdl3*)
- Include both contemporaneous and one-day lagged factors

	Average $\lvert\alpha\rvert$		Average $\lvert t\rvert$		Average R^2	
	IG	HY	IG	HY	IG	HY
			Panel A: US market			
mdl1	1.66	2.30	3.57	4.41	0.26	0.27
mdl1+F_t^1	0.67	0.98	2.03	2.72	0.73	0.74
mdl1+F_t^1+F_t^2	0.62	0.86	2.11	2.54	0.79	0.82
mdl2	1.61	2.31	3.38	4.52	0.27	0.29
mdl2+F_t^1	0.71	0.92	2.00	2.80	0.73	0.74
mdl2+F_t^1+F_t^2	0.67	0.81	2.16	2.66	0.79	0.82
mdl3	1.56	2.23	3.45	4.58	0.28	0.31
mdl3+F_t^1	0.71	0.92	2.04	2.79	0.74	0.74
mdl3+F_t^1+F_t^2	0.71	0.81	2.30	2.74	0.79	0.83

PCA on residuals

- PCA on the residual excess returns of the 14 credit options
- 1st PC explains 70% of residual variation
- Construct credit option residual factor with SR of -1.72 in US

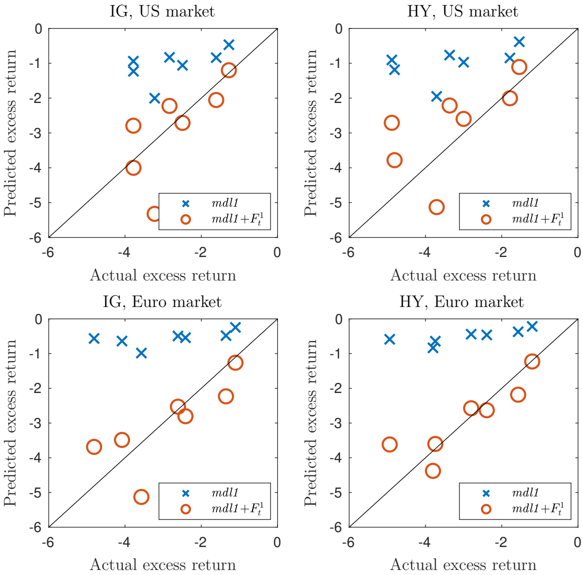


Augmented model applied to credit

- Add credit option residual factor, F_t^1, to factor model

	Average $\lvert\alpha\rvert$		Average $\lvert t\rvert$		Average R^2	
	IG	HY	IG	HY	IG	HY
			Panel A: US market			
mdl1	1.66	2.30	3.57	4.41	0.26	0.27
mdl1$+F_t^1$	0.67	0.98	2.03	2.72	0.73	0.74
mdl1$+F_t^1+F_t^2$	0.62	0.86	2.11	2.54	0.79	0.82
mdl2	1.61	2.31	3.38	4.52	0.27	0.29
mdl2$+F_t^1$	0.71	0.92	2.00	2.80	0.73	0.74
mdl2$+F_t^1+F_t^2$	0.67	0.81	2.16	2.66	0.79	0.82
mdl3	1.56	2.23	3.45	4.58	0.28	0.31
mdl3$+F_t^1$	0.71	0.92	2.04	2.79	0.74	0.74
mdl3$+F_t^1+F_t^2$	0.71	0.81	2.30	2.74	0.79	0.83

Fit of augmented model



Agenda

Structural demand for credit index options

CVA hedge losses prompt focus on swaptions and guarantees

Banks are turning to credit swaptions and guarantees to reduce the earnings volatility that arises when hedging the credit valuation adjustment capital charge in Basel III.

A demand-based option pricing model

- Spot price S_t and risk-free rate r exogenous (determined in larger market):

$$\frac{dS_t}{S_t} = \mu dt + \sigma(x_{t^-})dZ_t$$

$\rightarrow$ x_t volatility Markov Chain with intensity $\lambda_{x,i,j}$.

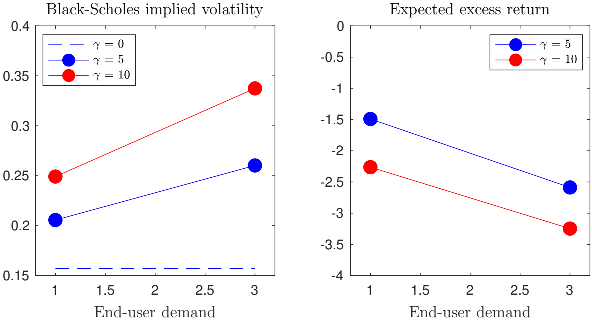
- Derivative price P_t determined so that risk-averse marginal liquidity providers (LPs) hold $\theta(y_t)$ (exogenous 'supply' by end-users).

$\rightarrow$ y_t order-flow Markov chain with intensity $\lambda_{y,i,j}(x_t)$.

- Given conjecture $P(t, S_t, x_t, y_t)$, solve for the optimal LP's holdings of stock $n^*(t, S_t, P_t, x_t, y_t)$ and derivative $q^*(t, S_t, P_t, x_t, y_t)$.
- Option market clearing $q^*(t, S_t, P_t, x_t, y_t) = \theta(y_t)$ implies that $P(t, S, x, y)$ satisfies an (easily solved) system of coupled BS-type PDEs (PDE)

Model predictions

- Solve for 2 volatility states and 2 demand states:



- Consistent with large negative SRs on credit options and residual factor
- Three testable predictions btw. residual factor and order flow
  1. Positive contemporaneous relation
  2. Relation is stronger when dealers are more risk averse
  3. Expected excess factor return more negative after high order flow.

Daily order-flow in credit index options

- Transaction data from swap data repositories
- Reported swap notionals capped at 100, 110, or 320 million

	IG			HY		
	N trades	N capped	Volume	N trades	N capped	Volume
			Panel A: US market			
Mean	18.67	11.93	1.55	14.89	4.91	0.96
5th pcntile	3	2	0.25	2	0	0.10
Median	15	10	1.35	13	4	0.82
95th pcntile	40	28	3.49	34	14	2.30
			Panel B: European market			
Mean	10.87	7.30	0.79	6.24	2.57	0.37
5th pcntile	1	1	0.09	0	0	0.00
Median	8	6	0.64	5	2	0.27
95th pcntile	24	18	2.05	17	9	1.10

Regression of credit option residual factor on order flow

- Estimate versions of

$$PC1_t = \beta^0 + (\beta^1 + \beta^2 HKM_{t-1}) flow_t + \beta^3 flow_{t-1} + \epsilon_t$$

- Flow in each market is averaged across IG and HY
- HKM: Intermediary capital ratio
- Model predictions: $\beta^1 > 0$, $\beta^2 < 0$, and $\beta^3 < 0$

	N trades			N capped			Volume		
				Panel A: US market					
Flow	**0.014**	**0.014**	0.012	**0.026**	**0.029**	**0.030**	**0.023**	**0.026**	**0.028**
	(2.059)	(1.961)	(1.695)	(3.306)	(3.828)	(4.291)	(3.181)	(3.620)	(4.298)
Flow × HKM	—	0.113	—	—	**-0.730**	—	—	-0.618	—
	—	(0.178)	—	—	(-1.984)	—	—	(-1.854)	—
Lagged flow	—	—	0.010	—	—	-0.008	—	—	-0.009
	—	—	(1.003)	—	—	(-0.861)	—	—	(-0.992)
R^2	0.003	0.003	0.004	0.016	0.019	0.016	0.014	0.018	0.016
N obs	2547	2547	2545	2547	2547	2545	2547	2547	2545
				Panel B: European market					
Flow	**0.018**	**0.018**	**0.019**	**0.038**	**0.038**	**0.041**	**0.031**	**0.030**	**0.039**
	(2.578)	(2.621)	(2.598)	(4.471)	(4.495)	(5.787)	(3.931)	(3.938)	(5.958)
Flow × HKM	—	-0.451	—	—	-0.585	—	—	-0.453	—
	—	(-0.621)	—	—	(-1.309)	—	—	(-0.905)	—
Lagged flow	—	—	-0.002	—	—	-0.009	—	—	**-0.018**
	—	—	(-0.373)	—	—	(-1.178)	—	—	(-2.810)
R^2	0.007	0.007	0.007	0.030	0.031	0.031	0.023	0.023	0.027
N obs	2509	2456	2508	2509	2456	2508	2509	2456	2508

Conclusion

- Consistently much more negative Sharpe ratios on **delta-hedged** credit index options than equity index options
- Difficult to obtain in CDJT2024 or DEFS2024 structural model
- Survives adding transaction costs.
- Equity-based factor model cannot price credit index options
- Order-flow risk help explain variation in the credit option residual factor

LP's problem

$$\mathcal{J}(t, S_t, x_t, y_t) = \max_{n_t, q_t} \mathbb{E}[\int_0^T dW_t - \frac{\gamma}{2}(dW_t)^2]$$

$$s.t \quad dW_t = rW_t dt + n_{t^-}(dS_t - rS_t dt) + q_{t^-}(dP_t - rP_t dt)$$

- Conjecture $P(t, S_t, x_t, y_t)$. Apply Itô's lemma to $P(t, \ldots)$, plug into wealth dynamics, and derive HJB equation.
- Solve for the optimal $n^*(t, S_t, x_t, y_t), q^*(t, S_t, x_t, y_t)$ to maximize $\mathcal{J}$ and impose $q^*(t, S_t, x_t, y_t) = \theta(y_t)$ to show that $P(t, S, x, y)$ must satisfy:

$$\begin{aligned}
0 =& \frac{\partial}{\partial t} P(t, S, x, y) + rS \frac{\partial}{\partial S} P(t, S, x, y) + \frac{1}{2}\sigma(x)^2 S^2 \frac{\partial^2}{\partial S^2} P(t, S, x, y) - rP(t, S, x, y) \\
&+ \sum_{i,j} \mathbf{1}_{\{x=i\}} \lambda_{x,i,j} \Delta_{x,i,j} P(t, S, x, y)(1 - \gamma\, \theta(y) \Delta_{x,i,j} P(t, S, x, y))) \\
&+ \sum_{i,j} \mathbf{1}_{\{y=i\}} \lambda_{y,i,j}(x) \Delta_{y,i,j} P(t, S, x, y)(1 - \gamma\, \theta(y) \Delta_{y,i,j} P(t, S, x, y))
\end{aligned}$$

(subject to boundary condition $P(T, S, x, y) = \max[K - S, 0]$) Back

Theorems

We prove that:

- If volatility is constant then order flow risk is not priced.
  Black-Scholes complete market economy...

- If volatility is stochastic, then order flow risk is priced iff the representative dealer is risk-averse:

$$\{P(t,S,x,y) = P(t,S,x)\ \forall x,y\} \iff \{\gamma\theta(y) = 0\ \forall y\}.$$

- The expected excess return on a delta-hedged option portfolio, μ_V, satisfies:

$$\mu_V \gtreqless 0 \quad \iff \quad \gamma\,\theta(y) \gtreqless 0\ \forall y.$$

Back

Sharpe Ratios in Simulated Structural Model Economy

	IG			EQ		
v_t	0.0040	0.0089	0.0140	0.0040	0.0089	0.0140
Panel A: Outright options						
Mean	-2.02	-3.19	-3.91	-1.31	-2.50	-3.20
Std.dev.	3.30	3.13	3.03	3.05	2.82	2.62
SR	-0.61	-1.02	-1.29	-0.43	-0.89	-1.22
Skew	0.84	0.71	0.59	2.00	1.36	1.01
Kurt	6.8	5.5	4.1	23.9	13.1	7.8
Panel B: Delta-hedged options						
Mean	-0.83	-1.40	-1.51	-1.52	-2.25	-2.41
Std.dev.	1.07	0.85	0.70	1.16	0.97	0.82
SR	-0.78	-1.65	-2.15	-1.31	-2.31	-2.92
Skew	0.62	0.66	0.68	1.13	0.69	0.35
Kurt	5.2	6.4	4.4	15.7	9.9	4.6

Table IA.20: Option returns in simulated data
Summary statistics of daily conditional excess returns on credit and equity index options simulated from a structural credit risk model. The options are two-month, at-the-money put options. The model is from Collin-Dufresne et al. (2024) with parameter estimates from Doshi et al. (2024). Conditional on three different values of the variance state variable, 100,000 observations are simulated. Means, standard deviations, and Sharpe ratios ("SR") are annualized.

Back

Sharpe Ratios: Impact of Transaction costs and rebalancing frequency

	IG			HY			EQ		
Holding period	Daily	Expiry	Expiry	Daily	Expiry	Expiry	Daily	Expiry	Expiry
T-cost	—	—	7.5%	—	—	7.5%	—	—	1.5%
Panel A: US market									
Mean	**1.93**	**1.81**	**0.91**	**2.16**	**2.16**	**1.26**	0.62	0.32	0.14
	(5.11)	(5.93)	(2.98)	(5.36)	(8.03)	(4.69)	(1.75)	(1.27)	(0.56)
Std.dev.	0.89	1.24	1.24	0.88	0.95	0.95	1.11	1.00	1.00
SR	2.18	1.46	0.73	2.46	2.28	1.33	0.56	0.32	0.14
Skew	-2.22	-2.00	-2.00	-3.12	-0.79	-0.79	-2.66	-1.53	-1.53
Kurt	17.2	9.1	9.1	24.8	4.2	4.2	18.6	8.6	8.6
N obs	2485	2501	2501	2465	2512	2512	2560	2537	2537
Panel B: European market									
Mean	**1.77**	**1.97**	**1.07**	**1.96**	**1.82**	**0.92**	0.17	0.22	0.04
	(5.26)	(9.15)	(4.97)	(5.42)	(8.31)	(4.20)	(0.50)	(0.80)	(0.16)
Std.dev.	0.85	0.83	0.83	0.82	0.92	0.92	1.06	1.02	1.02
SR	2.08	2.37	1.29	2.38	1.98	1.00	0.16	0.22	0.04
Skew	-3.21	-0.84	-0.84	-2.95	-1.81	-1.81	-2.71	-1.06	-1.06
Kurt	27.3	5.0	5.0	23.0	13.9	13.9	18.0	4.8	4.8
N obs	2480	2507	2507	2369	2425	2425	2545	2527	2527

Table IA.9: Performance of Straddles
Summary statistics of daily excess returns on straddles. Means, standard deviations, and Sharpe ratios ("SR") are annualized. t-statistics in parentheses are corrected for heteroscedasticity and serial correlation up to 63 lags (equal to three months) using the approach of Newey and West (1987). Mean estimates that are significant at the 5% level are highlighted in bold. Sample period is from January 1, 2013 to April 3, 2023.

Back