A Search for Sterile Neutrinos at the NOνA Far Detector

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Abstract

NOνA is a long baseline neutrino experiment designed to study neutrino oscillations. It consists of two functionally identical detectors each located 14 mrad off-axis from the NuMI neutrino beam generated at Fermilab, with one detector located about a kilometer from the beam source, and the other 810 km away in Ash River, Minnesota. With the longest distance between detectors and the ability of the NuMI beam to produce a beam of either neutrinos or anti-neutrinos, NOνA is the most sensitive experiment to CP violating effects in the neutrino sector in the world. While the primary physics goals of NOνA are to make measurements of the remaining unknown 3 flavor oscillation parameters, the experiment has the capability to perform more exotic analyses.

This thesis focuses on a search for sterile neutrinos in a $3+1$ model. The analysis presented searches for a deficit in the rate of neutral current events at the far detector using the near detector to constrain the predicted spectrum. The comparison between the observed and predicted spectra is translated into a measurement of the expanded PMNS mixing matrix elements, $|U_{\mu 4}|^2$ and $|U_{\tau 4}|^2$, assuming a value of $\Delta m^2_{41} \sim O(1\,\text{eV}^2)$. This analysis was performed using data taken between February 2014 and May 2015 corresponding to 3.52×10^{20} protons on target. The best fit values for the matrix elements were $|U_{\mu 4}|^2 = 0.xy \pm a.bc$ and $|U_{\tau 4}|^2 = 0.vw \pm d.ef$, consistent with the no sterile neutrino hypothesis. At the end of this thesis there is a short discussion of future sensitivity improvements using a larger dataset.

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This is the dedication.

Acknowledgments

These people were cool.

1

A Brief History of Neutrinos

1.1 Introduction

The neutrino was first postulated by Wolfgang Pauli as a possible explanation for the continuous spectrum of electrons emitted from nuclear β decay [1]. This decay was originally thought to be the emission of an electron from an atom, resulting in a different nucleus, via the process,

$$N \to N' + e \tag{1.1}$$

where N and N' are the parent and daughter nuclei, respectively. In a two body decay such as this, the momenta and energies of the outgoing particles are exactly constrained. Pauli's new particle explained the continuous spectrum of electron energy via a modified decay process:

$$N \to N' + e + \nu \tag{1.2}$$

where ν is the outgoing neutral particle. Pauli's original proposal called the new particle the neutron, but this name was later used to name the massive neutral nucleon discovered by Chadwick in 1932 [2]. Three years after Pauli's idea, Fermi proposed a model for nuclear β decay that included the new particle, which he coined the neutrino, or little neutral one [3].

1.2 First Detection of Neutrinos

Twenty years passed from Fermi's model proposal before neutrinos were discovered experimentally. Fred Reines and Clyde Cowan made the discovery by placing a detector near a nuclear reactor as a source of neutrinos and observing inverse β decay [4, 5]. The neutrinos observed were anti-electron neutrinos, thus the following was the observed process.

$$p + \bar{\nu}_e \rightarrow n + e^+ \tag{1.3}$$

Reines earned the Nobel Prize in Physics in 1995 for the detection of the neutrino.

In 1962, the muon neutrino was discovered at Brookhaven National Laboratory using the first neutrino beam [6] in a scheme still used in neutrino experiments today. The beam was generated by colliding protons with a target, producing pions that decayed into muons and muon neutrinos. The resultant beam then passed through thick steel, absorbing everything but the neutrinos. Leon Lederman, Melvin Schwartz, and Jack Steinberger won the Nobel Prize in Physics in 1988 for the discovery of the muon neutrino.

The last generation of neutrino, the tau neutrino, was discovered at Fermilab by the DONUT collaboration in 2000 [7].

1.3 Evidence of Neutrino Oscillations

Pontecorvo first postulated neutrino oscillations between neutrinos and anti-neutrinos, analogous to $K^0/\bar{K}^0$ oscillations, in 1957 [8]. Nothing came of the proposal immediately, but the idea was later revived and modified to solve the solar neutrino problem. The physics community initially viewed neutrino oscillations with skepticism and believed the experiments to be flawed, but over time oscillations have become an unmistakable and accepted phenomenon.

The solar neutrino problem was born from a large discrepancy between the theoretical and observed number of neutrinos produced by the sun. Neutrinos were used as a study for solar models because photons take a thousand years to escape the dense nuclear plasma to the surface of the sun, but neutrinos are unimpeded. The models, which have been confirmed today, describe a somewhat complicated chain of nuclear reactions, many of which produce neutrinos. Each individual process contributes a neutrinos in a different energy spectrum, but all of the neutrinos are created as electron neutrinos.

The experimental observations and theoretical predictions were both published in 1968. Ray Davis designed an experiment underground in the South Dakota Homestake mine consisting of a tank of an ultra pure chlorine cleaning solution capable of neutrino capture via the process

$$\nu_e + {}^{37}\mathrm{Cl} \rightarrow {}^{37}\mathrm{Ar} + e^- \tag{1.4}$$

The argon atoms could be collected and counted for a direct measurement of the neutrino flux [9]. Meanwhile, John Bahcall precisely calculated the expected neutrino flux [10], and the observed rate was found to be about one third of the predicted rate. Pontecorvo revived his theory with the modification of allowing ν_e to ν_μ oscillations [11], but the idea was still not taken seriously and it was another 20 years before the solar neutrino problem was confirmed.

Beginning in 1989, multiple experiments with different methodologies confirmed the solar neutrino problem. Kamiokande, a water Cherenkov detector, measured a rate deficit in 1989 [12]. Two experiments measured solar neutrinos via the reaction

$$\nu_e + {}^{71}\mathrm{Ga} \rightarrow {}^{71}\mathrm{Ge} + e^- \tag{1.5}$$

and measured a similar deficit, SAGE in 1991 [13] and GALLEX in 1992 [14]. With results from three different experimental methods all showing similar rate deficits, the solar neutrino problem could no longer be relegated to an experimental error.

Evidence soon emerged for oscillations with atmospheric neutrinos as well. These neutrinos are produced when cosmic rays collide with particles in the atmosphere and decay, predominantly via the following channels.

$$
\begin{aligned}
\pi^{+/-} &\rightarrow \mu^{+/-} + \nu_\mu/\bar{\nu}_\mu \\
\mu^{+/-} &\rightarrow e^{+/-} + \nu_e/\bar{\nu}_e + \bar{\nu}_\mu/\nu_\mu
\end{aligned} \tag{1.6}
$$

Thus, the expected ratio of muon family neutrinos to muon family neutrinos was expected to be 2. Kamiokande measured this ratio in 1992 and found the ratio to be much closer to 1 [15]. Furthermore, the ratio seemed to be dependent on zenith angle, with the measurement being nearly 2 for neutrinos coming from directly overhead, and dropping as the angle increased. Super-Kamiokande (or Super-K), the successor to Kamiokande, improved upon this measurement in 1998 [16], providing the most definitive evidence of neutrino oscillations to that point.

A resolution to the solar neutrino problem did not have to wait much longer with detector technologies capable of discerning different neutrino interaction types. SNO was designed as a heavy water (D_2O) Cherenkov detector experiment to be sensitive to both the flux of electron neutrinos and the flux of all neutrinos. In 2002, it released results for these measurements, finding what was then the expected deficit in electron neutrino flux, but a total flux consistent with the standard solar model, see Fig. 1.1 [17]. With this result, neutrino oscillations were confirmed, and subsequent experiments now measure oscillation parameters with precision.

1.4 Possible Evidence of Sterile Neutrinos

There exists some evidence of more than three neutrinos, but the number of active neutrinos is constrained by measurements of the width of the Z boson. LEP has measured the number of active neutrinos to be 2.984 ± 0.008 [18], so the discoveries of the ν_e, ν_μ, and ν_τ leave no room for new active neutrinos. (Strictly speaking, there could be other active neutrinos if they had mass greater than half the mass of the Z boson so the Z could not decay to them, but the evidence that does exist suggests a mass

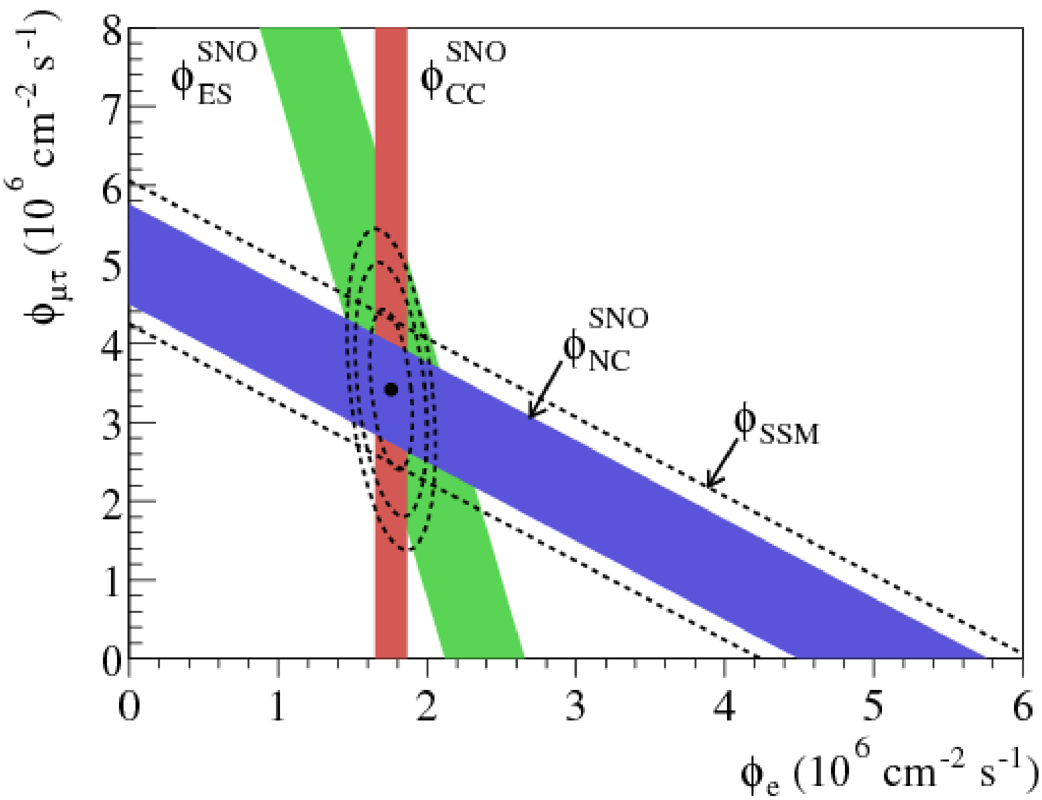

Figure 1.1: The measurement of different event rates at SNO [17]. The red band represents ν_e CC interactions with the deuterium neutron, an interaction only sensitive to electron neutrinos. The blue band represents neutral current scattering off of the deuterium nucleus, an interaction sensitive to the total neutrino flux. The green band represents elastic scattering of the neutrino off the deuterium electron, an interaction sensitive to all neutrino flavors, but not completely independent of neutrino flavor. The dashed straight lines represent the flux prediction by the standard solar model. The point represents the best fit for the flux of electron neutrinos and the flux for the combined muon and tau neutrinos.

splitting from the other neutrino states much smaller than this.)

The first evidence for an additional neutrino came from the Liquid Scintillator Neutrino Detector, or LSND, in 1995. This experiment searched for $\bar{\nu}_e$ appearance in a $\bar{\nu}_\mu$ beam. When it found an excess of events, it reported a measurement of a mass splitting between neutrino states Δm^2 of $O(1\ \text{eV}^2)$ [19]. The mass splitting affects the frequency of neutrino oscillations, and will be explained in greater depth in chapter 2. The measurement from LSND is incompatible with the mass splittings measured in both the atmospheric and solar oscillation experiments, suggesting the addition of at least one more neutrino. However, based on the result from LEP, this new neutrino can not couple to the Z boson, hence the suggestion for a sterile neutrino.

Many other experiments have tried to search for the existence of sterile neutrinos. The MiniBooNE experiment at Fermilab searched for both ν_e appearance in a ν_μ beam and $\bar{\nu}_e$ appearance in an $\bar{\nu}_\mu$ beam. While they first reported no event excess in 2007 [20], their more results show excesses in both modes [21] that could be consistent with some sterile neutrino models. Not all experiments have found evidence of sterile neutrinos, however. The most recent results from KARMEN in 2002 [22] and NOMAD in 2003 [23] showed no evidence of oscillations at the same mass scale as LSND. This list experimental evidence (or lack thereof) is by no means exhaustive, but it should be clear that there is not yet a scientific consensus on the existence of sterile neutrinos.

Today, most neutrino experiments have some form of analysis searching for a sterile neutrinos. The theory that would govern sterile oscillations (discussed in detail in section 2.4) is well understood, so individual experiments can try to measure or set limits on the various parameters introduced by adding sterile neutrinos to our current models. Recent measurements have come from an atmospheric neutrino measurement by Super-K in 2015 [24], a reactor experiment analysis by Daya Bay in 2014 [25], a short baseline detector analysis at the T2K near detector in 2015 [26], and a long baseline detector analysis by MINOS in 2011 [27].

NOνA is a long baseline neutrino experiment with a near detector, thus capable of performing both short baseline and long baseline sterile neutrino analyses; this thesis focuses on the long baseline. The analysis performed searches for a deficit in the number of neutral current events at the far detector, using the near detector data to constrain the predicted spectrum. Neutral currents are insensitive to the flavors of the standard 3 active neutrinos, so a rate deficit would point to the existence of a sterile neutrino.

2

Theory of Neutrino Oscillations

The idea of neutrino oscillations was first proposed by Pontecorvo in 1957 [8], but his proposal described oscillations between neutrinos and anti-neutrinos. In 1962, after the discovery of the muon neutrino, Maki, Nakagawa, and Sakata proposed the theory that described oscillations between neutrino flavors due to differing neutrino flavor and mass eigenstates [28]. This chapter describes the modern formalism in detail and uses natural units where $\hbar = c = 1$, except where otherwise noted.

2.1 The PMNS Matrix

In the Standard Model, neutrinos only interact via the W and Z bosons as shown by the Feynman diagrams in Fig. ??. From these diagrams, it is clear that neutrinos always interact in a definite flavor eigenstate, $|\nu_\alpha\rangle$. Furthermore, when a neutrino is produced from a W boson, the flavor is always determined by the associated charged lepton shown in eq. 2.1.

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}, \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}, \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} \tag{2.1}$$

On the other hand, neutrinos propagate through spacetime with a definite mass, $|\nu_i\rangle$ an eigenstate of the free Hamiltonian. The flavor states can be written as a superposition of the mass states via

$$|\nu_\alpha\rangle = \sum_{i=1}^{n} U^*_{\alpha i} |\nu_i\rangle , \tag{2.2}$$

where n is the number of neutrinos, and U is the unitary PMNS matrix, named after Pontecorvo, Maki, Nakagawa, and Sakata. The PMNS matrix is unitary, and would reduce to the identity matrix if neutrinos did not oscillate between flavor states. Yet since it does provide the mechanism for flavor transitions, it can be thought of as analogous to the quark sector CKM matrix.

2.2 Vacuum Oscillations

In this section, the basics of neutrino oscillations are developed by considering oscillations in a vacuum. The neutrinos are treated as plane waves, as in [29], with the assumption that the neutrino is actually localized in space put in by hand. A careful, rigorous analysis treating the neutrinos as plane waves in [30] reproduces the same results.

Consider a neutrino in a state of definite flavor α at time $t = 0$, $|\nu(0)\rangle = |\nu_\alpha\rangle$. This state is in a superposition of mass eigenstates. The time evolution of this neutrino is simply the time evolution of the individual mass states. In a vacuum, this adds a phase factor to each mass state.

$$|\nu_\alpha(t)\rangle = \sum_i U^*_{\alpha i} e^{-i(E_i t - \mathbf{p_i} \cdot \mathbf{x})} |\nu_i\rangle \tag{2.3}$$

With the neutrino at position $\mathbf{x} = L$ at time t, the dot product evaluates to $\mathbf{p_i} \cdot \mathbf{x} = p_i L$. Eq 2.3 can then simplified by making use of the fact that neutrinos are ultra-relativistic, allowing for several assumptions. First, the time, t, is replaced by the distance, L. Next, the energy of each mass state is approximated to be the same energy, $E_i = E$. Last, the momentum is expanded as $p_i = \sqrt{E^2 - m_i^2} \approx E - m_i^2/2E$. With these assumptions, eq. 2.3 simplifies as:

$$|\nu_\alpha(L)\rangle = \sum_i U_{\alpha i}^* e^{-im_i^2 L/2E} |\nu_i\rangle . \tag{2.4}$$

The mass eigenstate inside the sum is then re-expressed in terms of flavor eigenstates using the inverse of eq. 2.2 and unitarity of U.

$$|\nu_\alpha(L)\rangle = \sum_{\alpha'} \sum_i U_{\alpha i}^* U_{\alpha' i} e^{-im_i^2 L/2E} |\nu_\alpha'\rangle . \tag{2.5}$$

Eq. 2.5 can then be used to find the probability that the original neutrino in flavor state α has transitioned (or survived) as flavor state β. First, the matrix element $\langle \nu_\beta | \nu_\alpha(L) \rangle$ is computed.

$$\langle \nu_\beta | \nu_\alpha(L) \rangle = \sum_{\alpha'} \sum_i U_{\alpha i}^* U_{\alpha' i} e^{-im_i^2 L/2E} \langle \nu_\beta | \nu_\alpha' \rangle = \sum_i U_{\alpha i}^* U_{\beta i} e^{-im_i^2 L/2E} \tag{2.6}$$

The last equality in eq. 2.6 follows from the orthogonality of individual flavor eigenstates. The probability of the flavor transition is then the square of this matrix element.

$$P(\nu_\alpha \to \nu_\beta) = | \langle \nu_\beta | \nu_\alpha(L) \rangle |^2 = \sum_{i,j} U_{\alpha i}^* U_{\beta i} U_{\beta j}^* U_{\alpha j} e^{-i(m_i^2 - m_j^2)L/2E} \tag{2.7}$$

It is standard to rewrite the mass squared difference as $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$. Eq. 2.7 is then manipulated using the properties of unitary matrices.

$$\begin{aligned} P(\nu_\alpha \to \nu_\beta) &= \sum_{i,j} U_{\alpha i}^* U_{\beta i} U_{\beta j}^* U_{\alpha j} + \sum_{i,j} U_{\alpha i}^* U_{\beta i} U_{\beta j}^* U_{\alpha j} (e^{-i\Delta m_{ij}^2 L/2E} - 1) \\ &= \delta_{\alpha\beta} + \sum_{i,j} U_{\alpha i}^* U_{\beta i} U_{\beta j}^* U_{\alpha j} (e^{-i\Delta m_{ij}^2 L/2E} - 1) \end{aligned} \tag{2.8}$$

The remaining summed term is further simplified making use of two facts. When $i = j$, the complex phase is 0 as $\Delta m_{ii}^2 = 0$, and thus these terms vanish. Second, the terms with $i < j$ are complex conjugates of those with $i > j$, and $z + z^* = 2\Re(z)$ for any complex number z.

$$P(\nu_\alpha \to \nu_\beta) \quad = \quad \delta_{\alpha\beta} + 2\sum_{i>j} \Re\left[U^*_{\alpha i}U_{\beta i}U^*_{\beta j}U_{\alpha j}(e^{-i\Delta m^2_{ij}L/2E} - 1)\right] \tag{2.9}$$

Both pieces of this term are split into their real and imaginary parts, and simplified using the trigonometric identity $\cos 2\theta - 1 = -2\sin^2\theta$. Defining $\mathcal{U} \equiv U^*_{\alpha i}U_{\beta i}U^*_{\beta j}U_{\alpha j}(e^{-i\Delta m^2_{ij}L/2E} - 1)$ and $\phi \equiv \Delta m^2_{ij}L/2E$:

$$\begin{aligned}
\Re(\mathcal{U}) &= \Re\left[U^*_{\alpha i}U_{\beta i}U^*_{\beta j}U_{\alpha j}(e^{-i\Delta m^2_{ij}L/2E} - 1)\right] && (2.10)\\
&= \Re\left\{\left[\Re(U^*_{\alpha i}U_{\beta i}U^*_{\beta j}U_{\alpha j}) + i\Im(U^*_{\alpha i}U_{\beta i}U^*_{\beta j}U_{\alpha j})\right]\left[\cos\phi - i\sin\phi - 1\right]\right\} && (2.11)\\
&= \Re\left\{\left[\Re(U^*_{\alpha i}U_{\beta i}U^*_{\beta j}U_{\alpha j}) + i\Im(U^*_{\alpha i}U_{\beta i}U^*_{\beta j}U_{\alpha j})\right]\left[-2\sin^2(\phi/2) - i\sin\phi\right]\right\} && (2.12)\\
&= \Re\left\{-2\Re(U^*_{\alpha i}U_{\beta i}U^*_{\beta j}U_{\alpha j})\sin^2(\phi/2) + \Im(U^*_{\alpha i}U_{\beta i}U^*_{\beta j}U_{\alpha j})\sin\phi\right.\\
&\qquad \left.-i\left[\Re(U^*_{\alpha i}U_{\beta i}U^*_{\beta j}U_{\alpha j})\sin\phi + 2\Im(U^*_{\alpha i}U_{\beta i}U^*_{\beta j}U_{\alpha j})\sin^2(\phi/2)\right]\right\} && (2.13)\\
&= -2\Re(U^*_{\alpha i}U_{\beta i}U^*_{\beta j}U_{\alpha j})\sin^2(\phi/2) + \Im(U^*_{\alpha i}U_{\beta i}U^*_{\beta j}U_{\alpha j})\sin\phi && (2.14)
\end{aligned}$$

Inserting the expression from eq. 2.14 into eq. 2.9, we find:

$$\begin{aligned}
P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} \quad &- \quad 4\sum_{i>j}\Re(U^*_{\alpha i}U_{\beta i}U^*_{\beta j}U_{\alpha j})\sin^2\left(\frac{\Delta m^2_{ij}L}{4E}\right)\\
&+ \quad 2\sum_{i>j}\Im(U^*_{\alpha i}U_{\beta i}U^*_{\beta j}U_{\alpha j})\sin\left(\frac{\Delta m^2_{ij}L}{2E}\right), && (2.15)
\end{aligned}$$

which is the standard equation for neutrino oscillations. It can now be seen that the distance the neutrino travels, its energy, and the different mass splittings all affect the frequency of oscillation. Ideally, neutrino oscillations would be studied by having neutrinos with a fixed energy profile (preferably monoenergetic) and varying the baseline. However, neutrino detectors are incredibly large, so in practice the baseline is fixed and the oscillation probability is studied as a function of neutrino energy.

For the case of survival probability, $\alpha = \beta$ and eq. 2.15 simplifies further. The imaginary piece from eq. 2.15 drops out, as

$$\Im(U^*_{\alpha i}U_{\alpha i}U^*_{\alpha j}U_{\alpha j}) = \Im(|U_{\alpha i}|^2|U_{\alpha j}|^2) = 0. \tag{2.16}$$

The survival probability is then given by:

$$P(\nu_\alpha \to \nu_\alpha) = 1 - 4\sum_{i>j}|U_{\alpha i}|^2|U_{\alpha j}|^2 \sin^2\left(\frac{\Delta m^2_{ij}L}{4E}\right). \tag{2.17}$$

Add simplification for 2 neutrino model

Comment on mixing parameter as amplitude

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9.1 Conclusions

The results of this analysis are consistent with no sterile neutrinos.

9.2 Future Improvements

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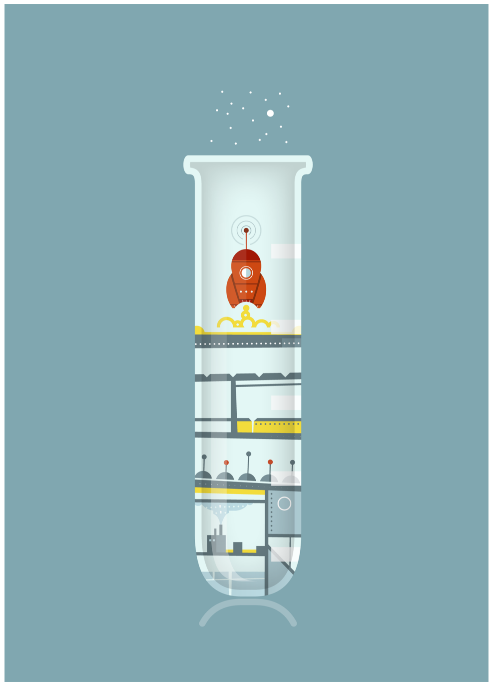

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